• Properties of fluids :-

Matter

→ solid
→ liquid
→ gases

→ About the molecules in the solids, liquids, gases.

→ solids can resist the external forces.

- → Tensile force.
- → compressive force.
- → shear force.

→ ~~liquids~~ Fluid has no tensile force. It can resist compressive force.

Fluid :-

It is defined as a substance which is capable to flowing.

- No shape, but conforms to the shape of the containing vessel.

Fluids

- Ideal fluids
- practical (or) Real fluids

Ideal fluids :-

→ which have "No viscosity", "No surface tension"

they are incompressible.

→ These are imaginary fluids.

Real fluids :-

→ which are available in nature.

→ which has viscosity, surface tension and compressible.

Quantity	unit	symbol	value in 'SI' units
Area	are	a	$100 m^2$
"	hectare	ha	$10,000 m^2$.
Time	minute	min	60 s.
"	hour	h	60 min = 3600 s
"	day	d	24 h = 86,400 s.
mass	tonne	t	1000 kg
Volume	litre	L	$10^{-3} m^3 = [1 dm^3]$
Dynamic viscosity	Poise	P	$10^{-1} N.s/m^2$.
"	centipoise	cP	$10^{-3} N.s/m^2$.
Kinematic viscosity	stoke	s	$10^{-4} m^2/s$
Pressure of fluid	bar	bar	$100 kN/m^2 = 10^5 pa$

specific Gravity :-

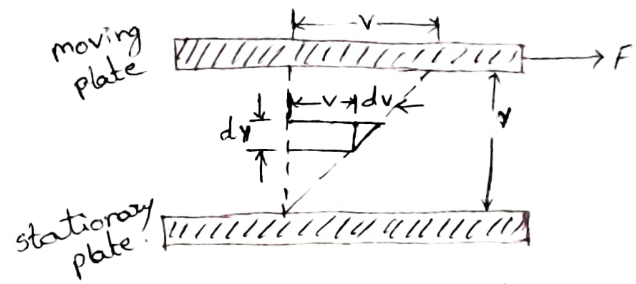
temperature at 4°c = $9810 N/m^3$.

s.g of mercury varies from - 13.5 to 13.6

P.T.O

viscosity :-

viscosity is that property of a fluid by virtue of which it offers resistance to the movement of one layer of fluid over an adjacent layer.



$$\tau = \frac{F}{A} = \mu \frac{V}{y} = \mu \frac{dv}{dy}$$

$$\mu = \frac{\tau}{dv/dy}$$ μ = proportionality const.

coefficient of viscosity.

(or)

Dynamic viscosity :- Defined as the shear stress required to produce unit rate of angular deformation

units :- $N.s/m^2$ (or) $kg/m.s$.

Mass density:- Specific Mass:-

It is the mass of the matter occupied in unit volume at a standard temperature and pressure.

- S.I unit – kg/m^3

$\rho = \frac{m}{V}$

- $P\uparrow$ M.D$\uparrow$ • $T\uparrow$ M.D$\uparrow$
- It is measured by "pycnometer" "hydrometer".

matter	ρ
• water	1000
• mercury	13600

specific weight (γ or weight):- weight of the matter per unit volume (or) unit wt.

- SI unit = $\frac{N}{m^3}$.

$\gamma = \frac{W}{V} = \frac{mg}{V} = \rho g$ $\quad \therefore \gamma = \rho g$

g: Acceleration due to gravity. * Dim:- $ML^{-2}T^{-2}$ (or) FL^{-3}

- specific weight of water, $\gamma = \rho g$

$\gamma = 1000\ kg/m^3 \times 9.81\ m/sec^2$

$= 9810\ N/m^3 = 9.81\ kN/m^3 \simeq 10\ kN/m^3$

mercury = $133.4\ kN/m^3$

specific volume:- Volume of fluid per unit wt.

$V_s = \frac{1}{\rho}$ • units = m^3/kg.

specific gravity:-(S) Ratio of the mass density of any matter to the mass density of a standard fluid.

$$S = \frac{\text{specific wt of a matter}}{\text{specific wt of a standard fluid}} = \frac{\rho}{\rho_{water}} = \frac{\gamma}{\gamma_{water}}$$

matter	sp. gravity
water	1.0
mercury	13.6

"no units (ratio)".

Assign

(P) calculate the sp. wt, sp. mass, sp volume and sp. gt of a liquid having a volume of $6m^3$ and weight of 44 kN. (sp. wt = $9.81\ kN/m^3$, $g = 9.81\ m/sec^2$)

sol:- $V = 6m^3$; $w = 44\ kN$

sp. wt $\gamma = \frac{w}{V} = \frac{44}{6} = 7.33\ kN/m^3$

sp. mass $\rho = \frac{\gamma}{g} = \frac{7.33 \times 1000}{9.81} = 747.5\ kg/m^3$

sp. vol $V_s = \frac{1}{\rho} = \frac{1}{747.5} = 0.00134\ m^3/kg$

sp. gr S $S = \frac{\gamma_{liquid}}{\gamma_{water}} = \frac{7.33}{9.81} = 0.747$

Kinematic viscosity:-

The ratio of the dynamic viscosity 'μ' and the mass density 'ρ' is known as "kinematic viscosity". Denoted by 'ν'

units:- m^2/s

('nu') $\nu = \frac{\mu}{\rho}$

Assign

* newtonian fluids
- non-newtonian fluids

Surface tension:- (σ)(sigma):-

surface tension is a measure of liquids tendency to take a spherical shape, caused by the mutual attraction of the liquids molecules.

cohesion:- Force of attraction between the molecules of the same liquid.

Adhesion:- Force of attraction between the molecules of different liquids (or) between the liquid molecules and solid boundary containing the liquid.

* surface tension is due to cohesion between particles at the surface of liquid.

units:- (N/m) Dimensions:- FL^{-1} (or) MT^{-2}.

* $t\uparrow$ $s.t\downarrow$

Pressure intensity inside a droplet:-

- consider a droplet radius 'r'.

internal pressure 'p'.(πr^2)

External pressure 'σ' surface tension acting around the circumference $(2\pi r)$

$$p(\pi r^2) = \sigma(2\pi r)$$

$$\boxed{p = \frac{2\sigma}{r}}$$

prob

$r = 0.5$ mm

assume $\sigma = 0.073$ N/m.

$$= \frac{2\sigma}{r} = \frac{2\times 0.073}{0.5} = 292\ N/m^2$$

Pressure intensity inside a soap bubble:-

- Soape bubble has two surfaces, one inside and other outside.
- These two surfaces contributes the same amount of tensile forces due to surface tension.
- radius 'r' and tensile force due to surface tension is equal to "$2\sigma(2\pi r)$"
- pressure is $p(\pi r^2)$.

$$p(\pi r^2) = 2\sigma(2\pi r)$$

$$\boxed{p = \frac{4\sigma}{r}}$$

Pressure intensity inside a liquid Jet:-

- consider a liquid jet of radius 'r', length 'l'.

Internal pressure 'p' in excess of the outside pressure intensity.

- If jet is cut into two halfs the pressure 'p' is for projected area $(2rl)$ and surface tension is '$(2l)$' acting along the length of the jet.

$p(2\pi l) = \sigma (2l)$

$$\boxed{p = \frac{\sigma}{r}}$$

Prob: A 20mm dia soap bubble has an internal pressure 27.576 N/m² greater than the outer atmosphere. then the S.T σ = _ N/m

$\Delta P = \frac{8\sigma}{d} \Rightarrow 27.576 = \frac{8\times\sigma}{20\times10^{-3}} = 0.0689$ N/m

Capillarity :-

The phenomenon of rise or fall of a liquid surface ~~ten~~ relative to the adjecent general level of liquid in small diameter tubes. The rise of liquid surface is ~~designated~~ called capillary rise and lowring is called capillary depression.

• units :- cm (or) mm of liquids.

$$h = \frac{4\sigma\cos\theta}{\gamma d}$$

Vapour pressure :

→ In a closed vessel at a constant temperature, the liquid molecules break away from the liquid surface and enter the air space in vapour state.

→ when the air above the liquid surface is saturated with liquid vapour molecules then the pressure exerted on liquid surface is called V.P

→ V.P ↑ with 'Temp'.

→ V.p for water → 2.3 kpa
mercury → 0.16 pa.

Pascal's law :-

Pressure at any point in a fluid at rest has the same magnitude in all directions.

→ when a certain pressure is applied at any point in a fluid at rest, the pressure is equally transmitted in all the directions and to every other point in the fluid.

→ This fact was established by B. pascal in 1653.

Atmospheric, Absolute, Gage and vacuum pressures:-

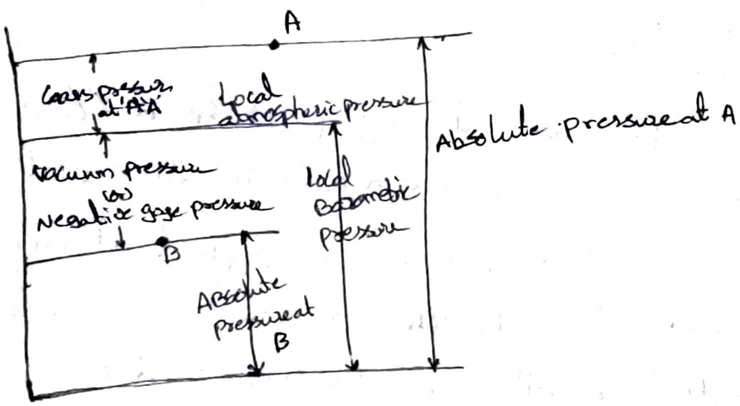
The atmospheric air exerts a normal pressure upon all surfaces with which it is in contact, and it is known as atmospheric pressure.

→ It is measured by means of a barometer. so it is called as barometric pressure.

→ At sea level under normal conditions the equivalent values of the A.T is $10.1043 \times 10^4 \, N/m^2$.

Absolute zero pressure :-

→ when the pressure is measured above absolute zero is called absolute zero pressure.



Absolute pressure = Atmospheric pressure + Gage pressure

Absolute pressure = Atmospheric pressure − Vacuum pressure.

Pressure measurement :-

Manometers:-

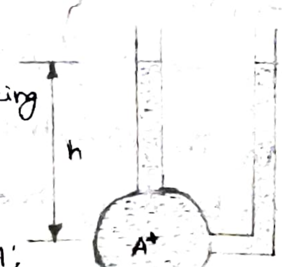
The pressure is measured by the instrument called "piezometer". The piezometer employed for pressure measurement are called manometer. The manometers may be classified as (i) simple manometers (ii) Differential manometers, and (iii) micro- manometers.

simple manometers:-

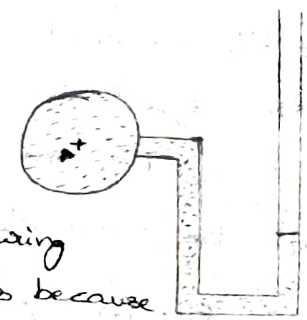
→ This can be used for measuring positive pressure at A.

→ The height of liquid in the piezometer h, above the point 'A', gives the pressure at that point, $P_A = \gamma h$.

γ = sp.gr. h = height



For positive pressure 'A'.



→ This type of manometer can only be used for measuring small and moderate pressures because as the pressure gets higher the length of piezometer becomes correspondingly larger.

For negative pressure at 'A'.

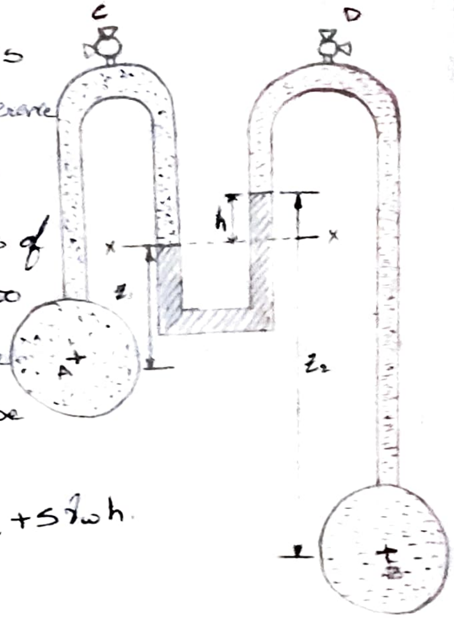
→ For −ve pressure the piezometer may be shaped as shown in fig (b) and pressure at 'A' will be

$$P_A = -\gamma h.$$

Differential manometer:-

→ A differential manometer is used to measure the difference in pressure at two points.

→ The manometer consists of a glass U-tube connected to the points between which the pressure difference is to be determined.



$$P_A - S_1 \gamma_w Z_1 = P_B - S_2 \gamma_w Z_2 + S \gamma_w h$$

γ_w = sp wt of water

$$P_A - P_B = S\gamma_w h + S_1 \gamma_w Z_1 - S_2 \gamma_w Z_2$$

and in terms of head of water.

$$\frac{P_A - P_B}{\gamma_w} = Sh + S_1 Z_1 - S_2 Z_2$$

If the manometer liquid is mercury = S = 13.6, and the liquid in both the containers is water, then $S_1 = S_2 = 1.0$, and further if both the containers are at the same elevation then $Z_2 - Z_1 = h$

$$\frac{P_A - P_B}{\gamma_w} = (S-1)h = 12.6h.$$

For the same liquid other than water, $S_1 = S_2$ and the pressure difference in terms of water head,

$$\frac{P_A - P_B}{\gamma_w} = (S - S_1)h$$

In terms of head of liquid of specific gravity S_1.

$$\frac{P_A - P_B}{\gamma} = \frac{(S - S_1)h}{S_1} = \left(\frac{S}{S_1} - 1\right)h$$

If we use a lighter liquid than water the U-tube will then have to be used in an inverted position.

Equating pressures along x-x

$$P_A - S_1 \gamma_w Z_1 = P_B - S_2 \gamma_w Z_2 - S\gamma_w h.$$

giving the pressure difference,

$$P_A - P_B = S_1 \gamma_w Z_1 - S_2 \gamma_w Z_2 - S\gamma_w h$$

and in terms of head of water,

$$\frac{P_A - P_B}{\gamma_w} = S_1 Z_1 - S_2 Z_2 - Sh.$$

If the both the containers are at the same level, then $Z_1 - Z_2 = h$

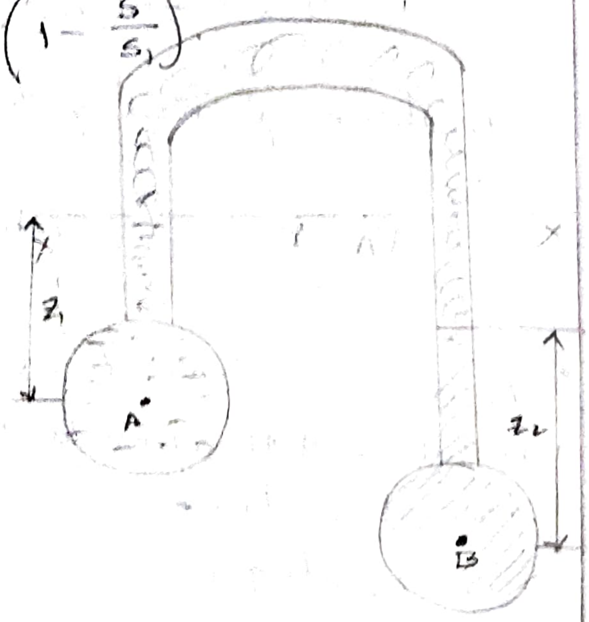
If both liquids are water $(S_1 = S_2 = 1.0)$ then,

$$\frac{P_A - P_B}{\gamma_w} = h - Sh = h(1-S).$$

$$\frac{P_A - P_B}{\gamma_w} = 0.20h$$

If the liquids are same $S_1 = S_2$

$$\frac{P_A - P_B}{\gamma} = h\left(1 - \frac{S}{S_1}\right)$$



P) The left leg of a U-tube mercury manometer is connected to a pipe line conveying water, the level of mercury in the leg being 0.6m below the center of pipe line, and the right leg is open to atmosphere. The level of mercury in the right leg is 0.45m above in the left leg and the space above mercury in the right leg contains Benzene (sp.gr = 0.88) to a height of 0.3m. Find the pressure in the pipe.

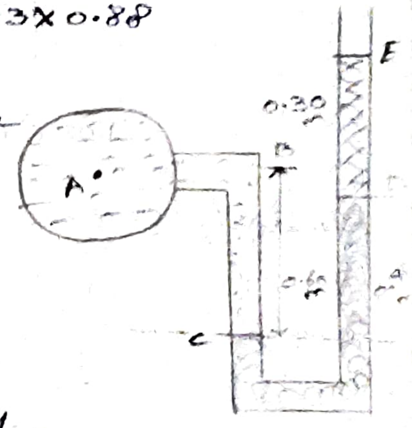
sol:- In the accompanying figure the pressure at c and c' are equal. Thus computing the pressure head c and c' from either side and equaling the same, we get.

$$\frac{P_A}{w} + 0.6 = 0.45 \times 13.6 + 0.3 \times 0.88$$

$$\frac{P_A}{w} = 5.784 \text{ m of water}$$

$P_A = (5.784 \times 9810)$

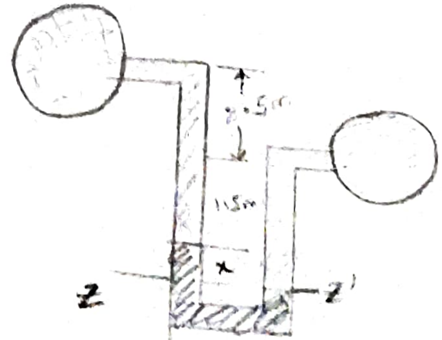
$= 5.67 \times 10^4 \, N/m^2$



$P_A = (5.784 \times 1000)$

$= 5.784 \times 10^3 \, Kg(f)/m^2$

P) As shown in fig. pipe 'M' contains carbon tetrachloride of sp.gr 1.594 under a pressure of 1.05 Kg(f)/cm² and pipe 'N' contains oil of sp.gr 0.8. If the pressure in the pipe N is 1.75 Kg(f)/cm² and manometric fluid is mercury, find the distance diff 'x' between the levels of mercury.



sol:- Equate the pressure heads at 'z' and z' as shown,

Pressure head at 'z' in terms of water column

$$= \left[\frac{1.05\times10^4}{1000} + (2.5+1.5)\times(1.594) + x(13.6)\right]$$

$= 16.876 + 13.6x$

similarly pressure head at z' in terms of water column

$$= \left[\frac{1.75\times10^4}{1000} + (1.5\times0.8) + x(0.8)\right]$$

$= 18.7 + 0.8x$

equating the two, we get

$16.876 + 13.6x = 18.7 + 0.8x$

$12.8x = 1.824$

$x = \frac{1.824}{12.8} = 0.1425\,m = 14.25\,cm$

Q) A differential manometer is connected at the two points A and B of two pipes as shown. The pipe 'A' contains a liquid of sp.gr = 1.5 while pipe 'B' contains a liquid of sp gr = 0.9. The pressures at 'A' and 'B' are 1 kgf/cm² and 1.80 kgf/cm². Find the difference in mercury level in the differential manometer.

sol:-

$S_1 = 1.5 \times 1000 \therefore \rho_1 = 1500$

$S_2 = 0.9 \times 1000 \therefore \rho_2 = 900$

Pressure at A, $P_A = 1\ kgf/cm^2 \Rightarrow 1\times10^4\ kgf/m^2$

$= 10^4 \times 9.81\ N/m^2 \ (\because 1\,kgf = 9.81\,N)$

Pressure at B, $P_B = 1.8\ kgf/cm^2$

$= 1.8\times10^4\ kgf/m^2$

$= 1.8\times10^4\times9.81\ N/m^2$

Density of mercury $= 13.6\times1000\ kg/m^3$

Taking x-x as datum line.

Pressure above x-x in the left limb

$= 13.6\times1000\times9.81\times h + 1500\times9.81\times(2+3) + P_A$

$= 13.6\times1000\times9.81\times h + 1500\times9.81\times(2+3) + 10^4\times9.81.$

Right limb:

$= 900 \times 9.81 \times (h+2) + P_B + \cancel{(3 \times 10^4 \times 9.81 \times (2 \times 3))}$ P_B

$= 900 \times 9.81 \times (h+2) + 1.8 \times 10^4 \times 9.81$

equating two pressures.

$13.6 \times 1000 \times 9.81 h + 7500 \times 9.81 + 9.81 \times 10^4$

$= 900 \times 9.81 \times (h+2) + 1.8 \times 10^4 \times 9.81$

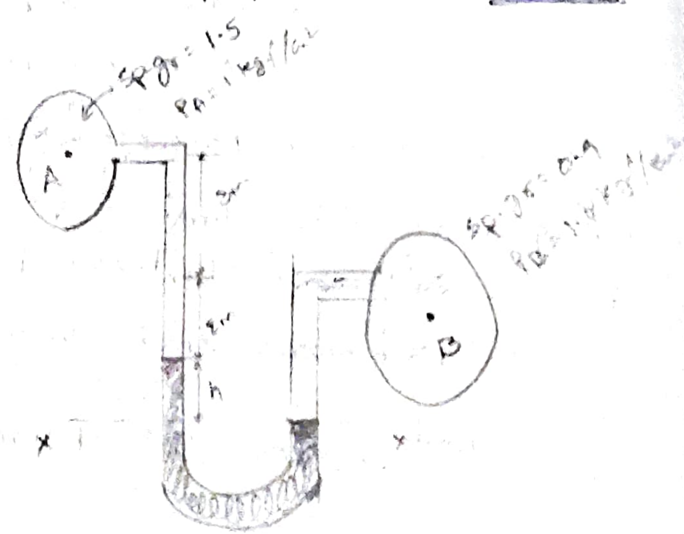
Dividing by 1000×9.81, we get

$13.6h + 7.5 + 10 = (h + 2.0) \times 0.9 + 18$

$13.6h + 17.5 = 0.9h + 1.8 + 18 = 0.9h + 19.8$

$(13.6 - 0.9)h = 19.8 - 17.5$

$h = \frac{2.3}{12.7} = 0.181\ m = \underline{18.1\ cm}$



Assign

A pipe contains an oil of sp.gr 0.9, A differential manometer connected at the two points A and B shows a difference in mercury level at 15cm. Find the difference of pressures at the two points.

Hydrostatic Forces:-

Total pressure and centre of pressure:-

Total pressure is defined as the force exerted by a static fluid on a surface either plane (or) curved, when the fluid comes in contact with the surface. This force always acts normal to the surface.

Centre of pressure is defined as the point of application of the total pressure on the surface.

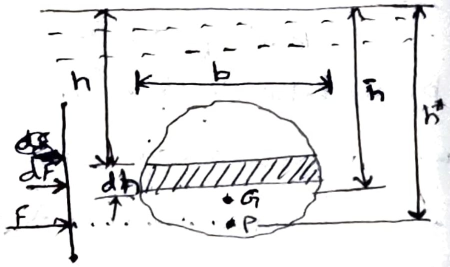
There are four cases of submerged surfaces on which the total pressure force and center of pressure is to be determined.

- → Vertical plane surface
- → Horizontal plane surface
- → Inclined plane surface
- → Curved surface.

Vertical plane surface submerged in liquid :-

Let
A = Total area of the surface
$\bar{h}$ = Distance of C.G of the area from free surface of liquid
G = Centre of gravity of plane surface
P = Centre of pressure.
h^* = Distance of centre of pressure from free surface of liquid

Total pressure :- ($\bar{h}$)



$p = \rho g h$.

Area of the strip $= dA = b \times dh$

Total pressure force on strip
$dF = p \times \text{Area}$.
$= \rho g h \times b \times dh$.

$\therefore$ Total pressure force on the total surface.

$F = \int dF = \int \rho g h \times b \times dh = \rho g \int h \times b \times dh$.

$\int b \times h \times dh = \int h \times dA$.

= moment of surface area about the free surface of liquid.

= Area of surface × Distance of C.G from the free surface

$= A \times \bar{h}$

$F = \rho g A \times \bar{h}$. —— ①

centre of pressure :- (h^*)

→ This is calculated by using the "principle of moments".

→ which states that the moment of the resultant force about an axis is equal to the sum of moments of the components about the same axis.

Hence moment of the force 'F' about free surface of the liquid $= F \times h^*$ ⟶ ②

moment of force dF, acting on a strip about free surface of liquid.

$= dF \times h$.

$= \rho g h \times b \times dh \times h$.

Sum of moments of all such forces about free surface of liquid.

$= \int \rho g h \times b \times dh \times h = \rho g \int b \times h \times h \, dh$.

$= \rho g \int b h^2 dh = \rho g \int h^2 dA$.

$\int h^2 dA = \int b h^2 dh$.

$= I_o$.

$\therefore$ Sum of moments about free surface

$= \rho g I_o$. ⟶ ③

by equating (2) (3)

$$F \times h^* = \rho g I_o$$

$$F = \rho g A \bar{h}.$$

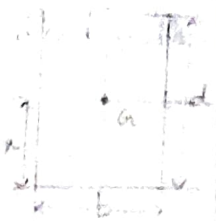
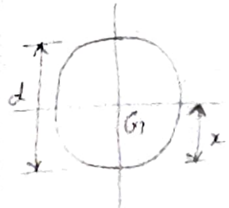
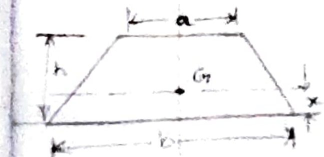
$$\rho g A \bar{h} \times h^* = \rho g I_o$$

$$h^* = \frac{\rho g I_o}{\rho g A \bar{h}} = \frac{I_o}{A \bar{h}}$$

By parallel axis, we have

$$I_o = I_G + A \times \bar{h}^2$$

$$h^* = \frac{I_G + A\bar{h}^2}{A\bar{h}} = \frac{I_G}{A\bar{h}} + \bar{h}.$$

Plane surface	C.G from the base	Area	M.I about an axis passing the C.G I_G	M.I about base (I_o)
1) Rectangle 	$x = \frac{d}{2}$	bd	$\frac{bd^3}{12}$	$\frac{bd^3}{3}$
2) Triangle 	$x = \frac{h}{3}$	$\frac{bh}{2}$	$\frac{bh^3}{36}$	$\frac{bh^3}{12}$
	$x = \frac{d}{2}$	$\frac{\pi d^2}{4}$	$\frac{\pi d^4}{64}$	-
	$x = \left(\frac{2a+b}{a+b}\right)\frac{h}{3}$	$\frac{(a+b)}{2} \times h$	$\left(\frac{a^2+4ab+b^2}{36(a+b)}\right) \times h^3$	

(P) A rectangular plane surface is 2m wide and 3m deep. It lies in vertical plane in water. Determine T.P and position of centre of pressure on the plane surface when its upper edge is horizontal

(a) coincides with water surface

(b) 2.5m below the free water surface.

Sol:-

(a)

$F = \rho g A \bar{h}$

$b = 2m.$

$d = 3m.$

$A = 3 \times 2 = 6m^2$

$\bar{h} = \frac{1}{2} \times 3 = 1.5m$

$F = 1000 \times 9.81 \times 6 \times 1.5$

$= 88290 N.$

$$h^* = \frac{I_G}{A\bar{h}} + \bar{h}$$

$$I_G = \frac{bd^3}{12} = 4.5m^4$$

$$h^* = 2.0m$$

b) $F = \rho g A \bar{h}$

$\bar{h} = 2.5 + \frac{3}{2} = 4.0\,m$

$F = 1000 \times 9.81 \times 6 \times 4.0$

$= 235440\,N$

$h^* = \frac{I_G}{A\bar{h}} + \bar{h}$

$= \frac{4.5}{6.0 \times 4.0} + 4.0$

$= 0.1875 + 4.0 = 4.187\,m$

(Q) A 3.6 m by 1.5 m wide rectangular gate "MN" is vertical and is hinged at point 0.15 m below the C.G of the gate. The total depth of water is 6m. what horizontal force must be applied at the bottom of the gate to keep the gate closed.

Sol:-

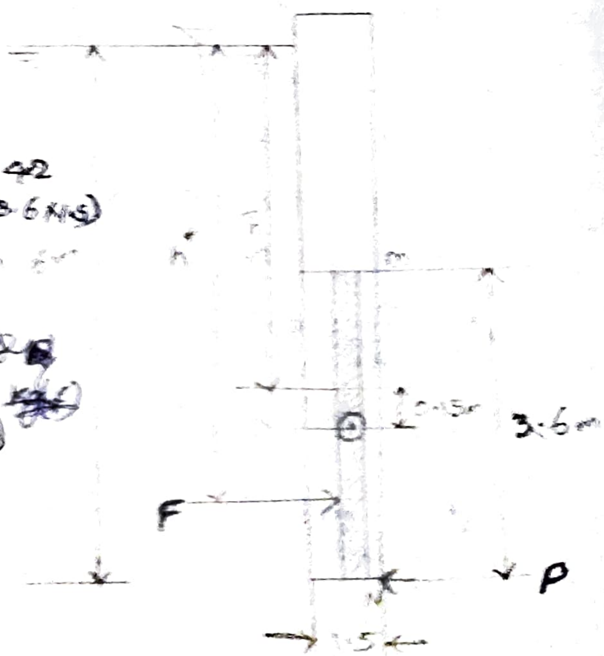
$F = \rho g \bar{h} A$

$= 1000 \times 9.81 \times 4.2 \times (3.6 \times 1.5)$

$\bar{h} = (6 - 1.8) = 4.2\,m$

$= \cancel{222490.8}$

$= 22680\,kg(f)$



$F = 1000 \times 9.81 \times 4.2 \times (3.6 \times 1.5)$

$\bar{h} = (6 - 1.8) = 4.2\,m$

$= \cancel{22680} = 222490.8\,N$

$22680\,kg\,(f)$

$h^* = \bar{h} + \frac{I_G}{A\bar{h}}$

$= 4.2 + \frac{\frac{1.5 \times (3.6)^3}{12}}{(3.6 \times 1.5)} = (4.2 + 0.257)$

$= 4.457\,m$

Let 'F' be the force required to be applied at the bottom of the gate to keep it closed.

By taking moments of all the forces about the hinge and equating to zero for equilibrium.

$F(1.8 - 0.15) - 22680\,(0.257 - 0.15) = 0$

$\therefore F = \frac{22680 \times 0.107}{1.65} = 1471\,kg\,(f).$

Horizontal plane surface submerged in liquid:-

consider a plane horizontal surface immersed in a static fluid. As every point of the surface is at the same depth from the free surface of the liquid, the pressure intensity will be equal on the entire surface and equal to,

$p = \rho g h$, where 'h' is depth of surface.

Let A = Total area of surface

Then total force, F, on the surface

$= p \times \text{Area} = \rho g \times h \times A$

$= \rho g A \bar{h}$

where,

$\bar{h}$ = Depth of C.G from the free surface of liquid $= h$.

h^* = Depth of centre of pressure from free surface $= h$.

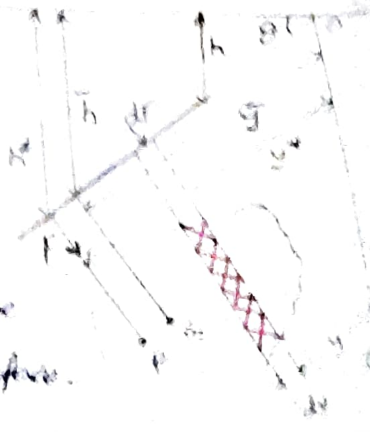
Inclined plane surface submerged in liquid:-

A = Total area of inclined surface

$\bar{h}$ = depth of C.G of inclined area from free surface

h^* = Distance of centre pressure from free surface of liquid

θ = Angle made by the plane of the surface with free liquid surface.



Let the plane of the surface, if produced meet the free liquid surface at 'O'. Then O-O is the axis ∥le to the plane of the surface

$\bar{y}$ = Distance of the C.G of the inclined surface from O-O

y^* = distance of the centre of pressure from O-O

$p = \rho g h$

$dF = p \times \text{Area of strip} = \rho g h \times dA$

$F = \int dF = \int \rho g h \times dA$

But from Fig. $\dfrac{h}{y} = \dfrac{\bar{h}}{\bar{y}} = \dfrac{h^*}{y^*} = \sin\theta$

$h = y \sin\theta$

$F = \int \rho g \times y \times \sin\theta \times dA = \rho g \sin\theta \int y\, dA$

$\int y\, dA = A\bar{y}$

$\therefore F = \rho g \sin\theta\, \bar{y} \times A$

$= \rho g A \bar{h}$.

Centre of Pressure :-

$$dF = \rho g h\, dA$$

moment of the force, dF, about axis o-o.

$$= dF \times y = \rho g y \sin\theta\, dA \times y$$

$$= \rho g \sin\theta\, y^2 dA$$

sum of moments

$$= \int \rho g \sin\theta\, y^2 dA$$

$$= \rho g \sin\theta \int y^2 dA$$

$\int y^2 dA$ = m.o.I of the surface about o-o $= I_o$ —(2)

sum of moments of all forces about o-o =

$$= \rho g \sin\theta\, I_o$$

moment of the total force, F,

$$= F \times y^* \quad \text{—(3)}$$

equate.

$$F \times y^* = \rho g \sin\theta\, I_o$$

$$y^* = \frac{\rho g \sin\theta\, I_o}{F}$$

$$y^* = \frac{h^*}{\sin\theta}, \ [F = \rho g A\bar{h}]$$

and I_o by the theorem of ||le axis $= I_G + A\bar{y}^2$

$$\frac{h^*}{\sin\theta} = \frac{\rho g \sin\theta}{\rho g A\bar{h}} [I_G + A\bar{y}^2]$$

$$h^* = \frac{\sin^2\theta}{A\bar{h}} [I_G + A\bar{y}^2]$$

$$\frac{\bar{h}}{\bar{y}} = \sin\theta \text{ (or) } \bar{y} = \frac{\bar{h}}{\sin\theta}$$

$$h^* = \frac{\sin^2\theta}{A\bar{h}} \left[I_G + A \times \frac{\bar{h}^2}{\sin^2\theta}\right]$$

$$\boxed{h^* = \frac{I_G \sin^2\theta}{A\bar{h}} + \bar{h}}$$

(P) A rectangular plane surface 2m wide and 3m deep lies in water in such a way that ~~the~~ its plane makes an angle of 30° with the free surface of water. Determine the t.p and position of centre of pressure when the upper edge is 1.5m below the free water surface.

sol:- width of plane surface = $b = 2m$

Depth - $d = 3m$

angle, $\theta = 30°$

depth ~~of~~ from water surface = 1.5m

$$F = \rho g A\bar{h}$$

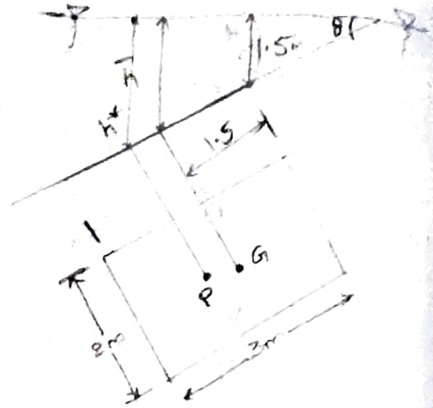
$\rho = 1000\ kg/m^3$

$A = b \times d = 3 \times 2 = 6m^2$.

$\bar{h}$ = Depth of C.G from water surface

$= 1.5 + 1.5 \sin 30^\circ$

$= 1.5 + 1.5 \times \frac{1}{2} = 2.25m$



$F = 1000 \times 9.81 \times 6 \times 2.25 = 132435\ N.$

b) Centre of pressure:-

$h^* = \frac{I_G \sin^2\theta}{A\bar{h}} + \bar{h}$, $I_G = \frac{bd^3}{12} = 4.5\ m^4$

$h^* = \frac{4.5 \times \sin^2 30^\circ}{6 \times 2.25} + 2.25 = 2.333m$.

P) An annular plate 3m external dia and 1.5m internal dia is immersed in water with its greatest and least depths below water surface as 3.6 and 1.2m respectively. Determine the t.p and position of the c.p on the face of the plate.

Sol:- External area $= \frac{\pi}{4}(3)^2 = (2.25\pi)m^2$

Internal area $= \frac{\pi}{4}(1.5)^2 = (0.56\pi)m^2$

net area of the plate $= (2.25\pi - 0.56\pi) = 5.31m^2$

$\bar{h} = \frac{3.6 + 1.2}{2} = 2.4m.$

The total pressure on one face of the plate is

$F = \rho g . A . \bar{h}$

$= 1000 \times 9.81 \times 5.31 \times 2.4$

$= 125018.64\ N.$

$h^* = \bar{h} + \frac{I_G \sin^2\theta}{A\bar{h}}$

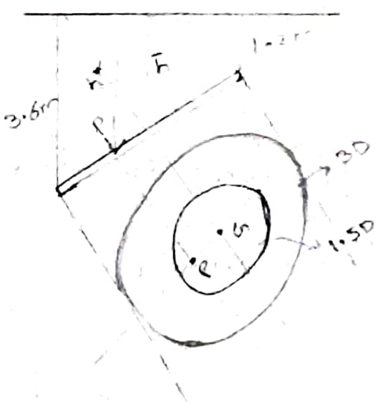
$= 2.4 +$

$I_G = \frac{\pi}{64}(D^4 - d^4) = \frac{\pi}{64}(3^4 - 1.5^4).$

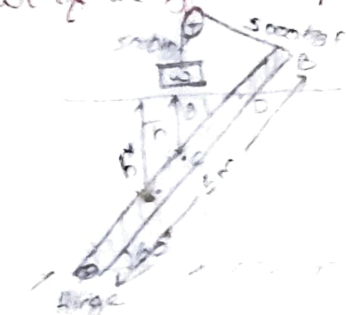
$\sin\theta = \frac{2.4}{3} = 0.8m.$

$h^* = 2.4 + \frac{\frac{\pi}{64}(3^4 - 1.5^4) \times (0.8)^2}{\frac{\pi}{4}[3^2 - 1.5^2] \times 2.4}$

$= 2.59m$



1) A rectangular gate 5 m × 2 m is hinged at its base and inclined at 60° to the horizontal as shown. To keep the gate in a stable position, a counter weight of 5000 kgf is attached at the upper end of the gate as shown. Find the depth of water at the gate begins to fall. Neglect the wt of the gate and friction at the hinge and pulley.



Curved surface submerged in liquid –

→ pressure intensity on the area dA is $= \rho g h$

$$dF = p \times \text{Area}$$
$$= \rho g h \times dA \quad \text{—(1)}$$

This force dF acts normal to the surface.

Total pressure force on the curved surface should be $F = \int \rho g h \, dA$ —(2)

$$F = \sqrt{F_x^2 + F_y^2} \quad \text{—(3)}$$

$$\tan\phi = \frac{F_y}{F_x} \quad \text{—(4)}$$

Resolving the forces dF given by equation (2), in "x and y"

$$dF_x = dF \sin\theta = \rho g h \, dA \sin\theta$$
$$dF_y = dF \cos\theta = \rho g h \, dA \cos\theta$$

Total forces in the x and y

$$F_x = \int dF_x = \int \rho g h \, dA \sin\theta = \rho g \int h \, dA \sin\theta \quad \text{—(5)}$$
$$F_y = \int dF_y = \int \rho g h \, dA \cos\theta = \rho g \int h \, dA \cos\theta \quad \text{—(6)}$$

Fig (b) shows the enlarged area dA. From this fig i.e., ΔEFG,

$$EF = dA$$
$$FG = dA \sin\theta$$
$$EG = dA \cos\theta$$

Thus in equation (5) $dA \sin\theta = FG =$ vertical projection of the area dA and hence the expression $\rho g \int h \, dA \sin\theta$

F_x = Total pressure force on the projected area of the curved surface on vertical plane

also, $dA \cos\theta = EG =$ horizontal projection of dA and hence $h \, dA \cos\theta$ is the volume of the liquid contained in the elementary area dA upto free surface of the liquid

$$F_y = \rho g \int h \, dA \cos\theta$$

weight of liquid supported by the curved surface upto free surface of liquid

2 Fluid Kinematics

streamline :-

A streamline is an imaginary line drawn through a flowing fluid in such a way that the tangent to it at any point gives the direction of the velocity of flow at that point.

stream tube :- A stream tube is a tube imagined to be formed by a group of streamlines passing through a small closed curve, which may (or) may not be circular. The concept of stream-tube is quite useful in analysing several fluid flow problems, since the entire flow field may be divided into a large number of stream tubes thus yielding a clear picture of the actual pattern of flow

path line :- A path line ~~made~~ may be defined as the line traced by a single fluid particle as it moves over a period of time

streak-line :

streak-line may be defined as a line that is traced by a fluid particle passing through a fixed point in a flow field.

— streakline

- - - path line

Types of Fluid flow :-

1) steady and unsteady flows
2) uniform and non-uniform flows
3) Laminar and turbulent flow
4) compressible and incompressible flow
5) Rotational and irrotational flow.
6) one - two and three dimensional flows

study and unstudy flow:-

study flow is defined as that type of flow in which the fluid characteristics like velocity, pressure, density etc at a point do not change with time ~~Thus~~

$$\left(\frac{\partial v}{\partial t}\right)=0,\left(\frac{\partial p}{\partial t}\right)=0 \text{ etc}$$

→ In unstudy flow velocity, pressure, etc are changes with the time.

$$\left(\frac{\partial v}{\partial t}\right)\neq 0,\left(\frac{\partial p}{\partial t}\right)\neq 0$$

uniform and non-uniform flow:-

Uniform flow is defined as that type of flow in which the velocity at any given time dose not changes with respect to space (ie length of direction of the flow). change of

$$\left(\frac{\partial v}{\partial s}\right)=0$$

∂v = velocity
~~∂s = change of time~~
∂s = length of flow in the direction s

non-uniform flow is that type of flow in which the velocity at any given time changes with respect to space

$$\left(\frac{\partial v}{\partial s}\right)\neq 0$$

Laminar and Turbulent flows:-

Laminar flow is defined as that type of flow in which the fluid particles moves along well-defined paths or stream line and all the stream-lines are straight and parallel.

Turbulent flow is that of flow in which the fluid particles move in a zig-zag way

For a pipe flow the type of flow is determined by a non-dimensional number $\frac{VD}{\nu}$

D = Diameter of pipe
V = mean velocity of flow in pipe
ν = kinematic viscosity of fluid

$Re < 2000$ that is laminar flow
$Re > 4000$ that is turbulent flow
$4000 > Re < 2000$ may be laminar (or) turbulent.

Compressible and In compressible flows:-

Compressible flow is that type of flow in which the density of the fluid changes from point to point

$$\rho \neq \text{Const}$$

Incompressible flow is that type of flow in which density is constant for the fluid flow.

ρ = const.

Rotational and irrotational flows:-

Rotational ~~is~~ flow is that type of flow in which the fluid particles while flowing along stream lines, also rotate about their own axis. And if the fluid particles while flowing along stream-line do not rotate about their own axis then that type of flow is called irrotational flow.

One, two- and three-Dimensional flows:-

One-dimensional flow is that type of flow in which the flow parameter such as velocity is a function of time and one space co-ordinate only say 'x'. For a steady one-dimensional flow, the velocity is a function of one-space-co-ordinates only. The variation of velocities in other two mutually perpendicular direction is assumed negligible.

$$u = f(x), v = 0 \text{ and } w = 0.$$

→ Two dimensional flow is that type of flow in which the velocity is a function of time and two rectangular space co-ordinates say 'x' and 'y'.

$$u = f_1(x,y), v = f_2(x,y) \text{ and } w = 0$$

→ Three dimensional flow is that type of flow in which the velocity is a function of time and three mutually perpendicular directions.

$$u = f_1(x,y,z), v = f_2(x,y,z) \text{ and } w = f_3(x,y,z).$$

Continuity Equation:-

The equation based on the principle of conservation of mass is called continuity equation. Thus for a fluid flowing through the pipe at all the cross-sections, the quantity of fluid per second is constant.

V_1 = Average velocity at cross-section 1-1

ρ_1 = Density at section 1-1

A_1 = Area of pipe at section 1-1.

and V_2, ρ_2, A_2 are corresponding values at section 2-2

Then rate of flow at section 1-1 = $\rho_1 V_1 A_1$

Rate of flow at section 2-2 = $\rho_2 A_2 V_2$

As law of conservation of mass.

$$\rho_1 A_1 V_1 = \rho_2 A_2 V_2$$

For in-compressible fluids $\rho_1 = \rho_2$.

$$\boxed{A_1 V_1 = A_2 V_2}$$

P) The diameters of a pipe at the sections 1 & 2 are 10 cm and 5 cm respectively. Find the discharge through the pipe if the velocity of water ~~by~~ flowing through the pipe at section 1 is 5 m/s. Determine also the velocity at section 2.

Sol^n

$D_1 = 10 cm = 0.1 m$

$A_1 = \frac{\pi}{4}(D_1^2) = \frac{\pi}{4}(0.1)^2 = 0.007854 m^2$.

$V_1 = 5 m/s$.

$D_2 = 15 cm = 0.15 m$

$A_2 = \frac{\pi}{4}(0.15)^2 = 0.01767 m^2$.

$Q = A_1 V_1$

$= 0.00785 \times 5 = 0.03927 m^3/s$.

$V_2 = \frac{A_1 V_1}{A_2} = 2.22 m/s$.

continuity Equation in three-D:-

Consider a fluid element of length dx, dy, dz in the direction of x, y, z. let u, v, w are the velocity components in x, y, z directions respecibly.

mass of fluid enter into the face ABCD.

= ρ x velocity in x-direction x Area of ABCD.

= $\rho \times u \times (dy \times dz)$

Then mass of fluid leaving the face EFGH per second = $\rho u dy dz + \frac{\partial}{\partial x}(\rho u dy dz) dx$

Gain of mass in x-direction

= mass through ABCD - mass through EFGH per second.

$= \rho u dy dz - \rho u dy dz - \frac{\partial}{\partial x}(\rho u \cdot dy \cdot dz) dx$

$= \frac{-\partial}{\partial x}(\rho u\, dy\, dz) \cdot dx$.

$= -\frac{\partial}{\partial x}(\rho u)\, dy\, dz\, dx$.

y-direction $= \frac{-\partial}{\partial y}(\rho v)\, dy\, dz\, dx$.

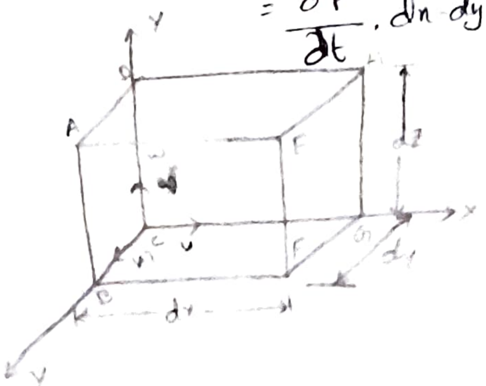
z-direction: $-\frac{\partial}{\partial z}(\rho w)\, dx\, dy\, dz$.

net gain of mass $= -\left[\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w)\right] dx\, dy\, dz$.

$$\Rightarrow \frac{\partial}{\partial t}(\rho \cdot dx \cdot dy \cdot dz) \text{ (or) } \frac{\partial \rho}{\partial t}\, dx\,dy\,dz$$

$$-\left[\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w)\right] dx\,dy\,dz$$

$$= \frac{\partial \rho}{\partial t} \cdot dx\,dy\,dz.$$



Velocity potential:- It is defined ~~as~~ ϕ (phi).

It is defined as a scalar function of space and time such that its negative derivative with respect to any direction gives the fluid velocity in that direction.

$$u = \frac{-\partial \phi}{\partial x}$$

$$v = \frac{-\partial \phi}{\partial y}$$

$$w = \frac{-\partial \phi}{\partial z}$$

The continuity equation for an incompressible stedy flow is $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$

Sub u, v, w values in the above equation

$$\frac{\partial}{\partial x}\left(\frac{-\partial \phi}{\partial x}\right) + \frac{\partial}{\partial y}\left(-\frac{\partial \phi}{\partial y}\right) + \frac{\partial}{\partial z}\left(-\frac{\partial \phi}{\partial z}\right) = 0$$

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} = 0$$

For 2-D $\Rightarrow \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0$

Stream Function:- It is defined as the scalar function of space and time, such that its partical derivative with respect to any direction gives the velocity component at right angles to that direction. It is denoted by ψ (Psi). It is defined ~~f~~only for two dimentinal flow

$$v = \frac{\partial \psi}{\partial x}$$

$$-u = \frac{\partial \psi}{\partial y}$$

The countinuity equation for two dimential flow is

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0.$$

sub u, v values

$$\frac{\partial}{\partial x}\left(-\frac{\partial\psi}{\partial y}\right)+\frac{\partial}{\partial y}\left(\frac{\partial\psi}{\partial x}\right)=0$$

$$-\frac{\partial^2\psi}{\partial y\partial x}+\frac{\partial^2\psi}{\partial x\partial y}=0.$$

P) The velocity potential function is given by $\phi=5(x^2-y^2)$ calculate the velocity component at the point (4,5)

sol:- $\phi=5(x^2-y^2)$

$$\frac{\partial\phi}{\partial x}=10x$$

$$\frac{\partial\phi}{\partial y}=-10y.$$

But velocty components

$$u=-\frac{\partial\phi}{\partial x}=-10x$$

$$v=-\frac{\partial\phi}{\partial y}=10y.$$

At point (4,5)

$u=-40$ units

$V=10\times5=50$ units.

1P) A stream function is given by $\psi=5x-6y$ calculate Resultant and direction at.

sol:- $\psi=5x-6y.$

$$\frac{\partial\psi}{\partial x}=5 \text{ and } \frac{\partial\psi}{\partial y}=-6$$

$$u=-\frac{\partial\psi}{\partial y}=-(-6)=6$$

$$v=\frac{\partial\psi}{\partial x}=5 \text{ units/sec}$$

Resultant $=\sqrt{u^2+v^2}=\sqrt{61}=7.81$ units/sec

Direction $=\tan\theta=\frac{v}{u}=\frac{5}{6}=0.833$

$\theta=\tan^{-1}0.833=39°48'.$

P) Two velocity components are given in the following case, Find the third component they satisfy the continuity equation

a) $u=x^3+y^2+2z^2$; $v=-x^2y-yz-xy$

b) $u=\log(y^2+z^2)$; $v=\log(x^2+z^2)$

sol:- For an incompressible flow the continuity eqution.

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0$$

a) In this case.

$u = x^3 + y^2 + 2z^2$

$\frac{\partial u}{\partial x} = 3x^2$

$v = -x^2y - yz - xy$

$\frac{\partial v}{\partial y} = -x^2 - z - x.$

$\therefore$ By substuting in the equ.

$3x^2 - x^2 - z - x + \frac{\partial w}{\partial z} = 0$

$\frac{\partial w}{\partial z} = x + z - 2x^2.$

$\partial w = (x + z - 2x^2)\partial z$

By integrating ~~[illegible]~~ $w = \left(xz + \frac{z^2}{2} - 2x^2z\right) + \text{Const.}$
$f(x, y)$

(b)

$u = \log(y^2 + z^2)$

$\frac{\partial u}{\partial x} = 0.$

$v = \log(x^2 + z^2)$

$\frac{\partial v}{\partial y} = 0.$

$w = f(x, y)$

$w = \log(x^2 + y^2).$

Flow net:- A grid obtained by drawing a series of equipotential lines and stream line ~~and the~~ is called a flow net.

Uses:- The flow net is an important tool in analysing two-dimensional irrotational flow problems.

Equipotential line:- A line along which the velocity potential ϕ is constant, is called equipotential line.

Dynamics of Fluid Flow

Equation of motion:-

$$F_x = m \cdot a_x$$

In the fluid flow, the following forces are present:-

(i) F_g, gravity force

(ii) F_p, the pressure force

(iii) F_v, Force due to viscosity.

(iv) F_t, force due to turbulence.

(v) F_c, Force due to compressibility.

the net force, $F_x = (F_g)_x + (F_p)_x + (F_v)_x + (F_c)_x$

→ If the force is due to compressibility, F_c is negligible,

net $F_x = (F_g)_x + (F_p)_x + (F_v)_x$

Equation of motions are called "Reynold's equation of motion".

→ For, Flow, where (F_t) is neglible, it is called "navier - stoker Equation".

iii) If the flow is assumed to be ideal, viscous force (F_v) is zero then equation of motion is known as "Euler's Equation of motion".

Euler's Equation of motion:-

This is equation of motion in which the forces due to gravity and pressure are taken into consideration.

consider a stream-line in which flow is taking place in s-direction as shown. consider a ~~circular~~ cylindrical element of cross-section 'dA' and length 'ds'.

→ pressure force pdA in the direction of flow.

→ pressure force $\left[p + \frac{\partial p}{\partial s} ds\right] dA$ opposite to the direction of flow.

→ weight of the element $\rho g dA ds$.

let 'θ' is the angle b/w the direction of flow and the line of action of the weight of element.

The resultant force on the fluid element in the direction of s must be equal to the mass of fluid element x acceleration in the direction 's'.

$$p\,dA - \left(p + \frac{\partial p}{\partial s} ds\right) dA - \rho g\, dA\, ds \cos\theta = \rho\, dA\, ds \times a_s \quad \text{—(1)}$$

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$$p\,dA - \left(p + \frac{\partial p}{\partial s} ds\right) dA - \rho g\, dA\, ds \cos\theta = \rho\, dA\, ds \times a_s \quad \text{—(1)}$$

where as a_s is the acceleration in the direction of 's' [a_s = Rate of change of velocity per time]

$a_s = \frac{dv}{dt}$, where v is a function of s and t.

$= \frac{\partial v}{\partial s}\frac{ds}{dt} + \frac{\partial v}{\partial t} = \frac{v\,\partial v}{\partial s} + \frac{\partial v}{\partial t} \left\{ \because \frac{ds}{dt} = v \right\}$

If the flow is steady, $\frac{\partial v}{\partial t} = 0$.

$\therefore \quad a_s = \frac{v \cdot \partial v}{\partial s}$

substituting the value of a_s in equation (1) and simplifying the equation: we get

$-\frac{\partial p}{\partial s} ds\, dA - \rho g\, dA\, ds \cos\theta = \rho\, dA\, ds \times \frac{v\,\partial v}{\partial s}$

Divide by $\rho\, ds\, dA$ $-\frac{\partial p}{\rho\, \partial s} - g\cos\theta = \frac{v\,\partial v}{\partial s}$

(or) $\frac{\partial p}{\rho\, \partial s} + g\cos\theta + v \cdot \frac{\partial v}{\partial s} = 0$

From fig,

$\cos\theta = \frac{dz}{ds}$

$\frac{1}{\rho}\frac{dp}{ds} + g\frac{dz}{ds} + \frac{v\,dv}{ds} = 0$

(or) $\frac{dp}{\rho} + g\,dz + v\,dv = 0$ $\longrightarrow$ Eulers eqution of motion

Bernoulli's Equation: from

$\longrightarrow \int \frac{dp}{\rho} + \int g\,dz + \int v\,dv = \text{constant}$

If flow is incompressible, ρ is constant

$\frac{p}{\rho} + gz + \frac{v^2}{2} = \text{constant}$

Devide with 'g'

$\frac{p}{\rho g} + z + \frac{v^2}{2g} = \text{Const}$

$\frac{p}{\rho g} + \frac{v^2}{2g} + z = \text{Const}$

z → potential energy per unit wt.

$\frac{p}{\rho g}$ → Pressure energy per unit wt

$\frac{v^2}{2g}$ → Kinetic energy per unit wt

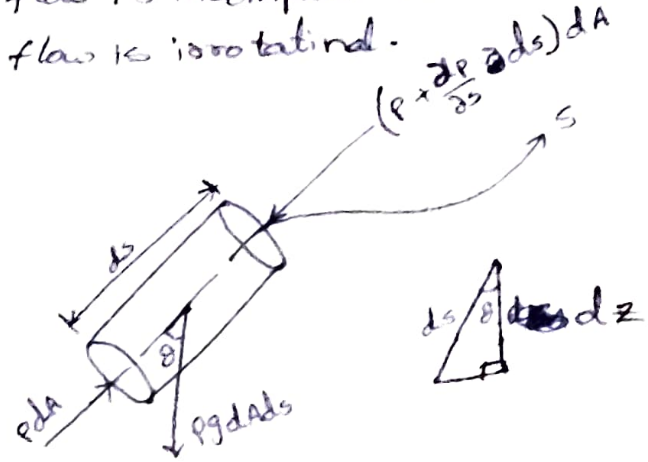
Assumptions:-

→ The fluid is ideal. i.e viscosity is zero

→ The flow is steady.

→ The flow is incompressible.

→ The flow is irrotational.



(Q) water is flowing through a pipe of 5cm dia under a pressure of 29.43N/cm² and with mean velocity of 2.0m/s. Find total head or total energy per unit weight of the water at a cross-section, which is 5m above the datum line

Sol:-

dia of pipe = 5cm = 0.5m.

$P = 29.43 N/cm^2 = 29.43 \times 10^4 N/m^2$.

$V = 2.0 m/s$

$Z = 5m$

Pressure head $= \frac{P}{\rho g} = \frac{29.43 \times 10^4}{1000 \times 9.81} = 30m$

Kinetic head $= \frac{V^2}{2g} = \frac{2 \times 2}{2 \times 9.81} = 0.204m$

Total head $= \frac{P}{\rho g} + \frac{V^2}{2g} + Z = 30 + 0.204 + 5 = 35.204 m.$

(Q) The water is flowing through a pipe having dia 20cm and 10cm at section 1 & 2. The rate of flow through pipe is 35lts/s. The section 1 is 6m above datum and sec 2 is 4m above datum. If the pressure at section 1 is 39.24 N/cm². Find the intensity of pressure at section 2.

Sol:-

At section 1, $D_1 = 20cm = 0.2m$.

$A_1 = \frac{\pi}{4}(0.2)^2 = 0.314 m^2$

$P_1 = 39.24 N/cm^2$

$= 39.24 N/m^2$

$Z_1 = 6.0m$

Section 2, $D_2 = 0.10m$

$A_2 = \frac{\pi}{4}(0.1)^2 = 0.00785 m^2$

$Z_2 = 4m$; $P_2 = ?$

Rate of flow $= Q = 35 lit/s = \frac{35}{1000} = 0.35 m^3/s$

$Q = A_1V_1 = A_2V_2$

$V_1 = \frac{Q}{A_1} = 1.114 m/s.$

$V_2 = \frac{Q}{A_2} = 4.456 m/s.$

apply bernoulli's eqution

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2$$

$$\frac{39.24 \times 10^4}{1000 \times 9.81} + \frac{(1.114)^2}{2 \times 9.81} + 6.0 = \frac{P_2}{1000 \times 9.81} + \frac{(4.456)^2}{2 \times 9.81} + 4.0$$

$P_2 = 40.27 N/cm^2$

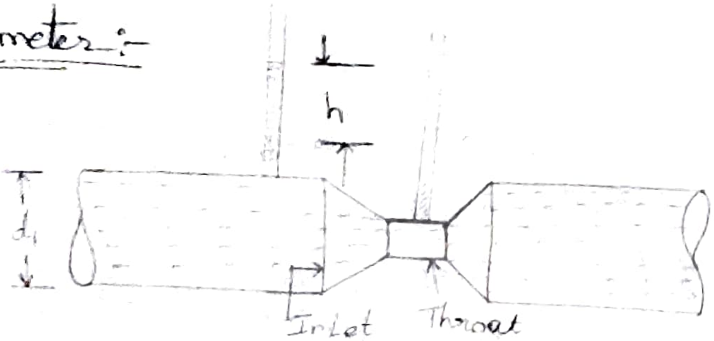
Bernoulli's eqution for Real fluid:-

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2 + h_L$$

Practical Applications of bernoulli's equations :-

1) venturimeter
2) orifice meter
3) pitot-tube.

venturimeter :-



→ A venturimeter is a device used to measuring the rate of a flow of a fluid through a pipe.
→ It is first demonstrated by B venturi in 1797.
→ A short converging part
→ Throat
→ Diverging part.

Rate of flow through venturimeter :-

d_1 = Dia at section 1
P_1 = pressure at sec 1
V_1 = velocity of fluid at sec 1
a = area of the sec = $\frac{\pi}{4} d_1^2$.

Applying bernoulli's equation at section 1 & 2

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2$$

As pipe is horizontal $Z_1 = Z_2$.

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} \quad (or)$$

$$\frac{P_1 - P_2}{\rho g} = \frac{V_2^2}{2g} - \frac{V_1^2}{2g}$$

$\frac{P_1 - P_2}{\rho g} = h$. Difference of pressure heads at section 1 & 2

$$h = \frac{V_2^2}{2g} - \frac{V_1^2}{2g} \longrightarrow (1)$$

Applying continuity equation.

$$a_1 V_1 = a_2 V_2 \text{ (or) } V_1 = \frac{a_2 V_2}{a_1}$$

sub V_1 is eqution.

$$h = \frac{V_2^2}{2g} - \frac{\left(\frac{a_2 V_2}{a_1}\right)^2}{2g}$$

$$= \frac{V_2^2}{2g}\left[1 - \frac{a_2^2}{a_1^2}\right] = \frac{V_2^2}{2g}\left[\frac{a_1^2 - a_2^2}{a_1^2}\right]$$

$$V_2^2 = 2gh \frac{a_1^2}{a_1^2 - a_2^2}$$

$$V_2 = \sqrt{2gh \frac{a_1^2}{a_1^2 - a_2^2}} = \frac{a_1}{\sqrt{a_1^2 - a_2^2}} \sqrt{2gh}$$

$Q = a_2 V_2$

$$\Rightarrow a_2 \frac{a_1}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh} = \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh}.$$

$$Q_{act} = C_d \times \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh}.$$

- "C_d = co-efficient of venturimeter and its value is less than '1'."

- Value of 'h' given by diff u-tube manometer:-

Case I :- Let the differential manometer contains a liquid which is heavier than the liquid flowing through the pipe.

S_h = sp. gr of the heavier liquid.

S_o = sp. gr of the liquid flowing through pipe

x = Difference of the heavier liquid column in u-tube.

$$h = x\left[\frac{S_h}{S_o} - 1\right].$$

Case II :- If the differential manometer contain a liquid which is lighter than the liquid flowing through the pipe, the value of h is given by

$$h = x\left[1 - \frac{S_L}{S_o}\right]$$

Inclined venturimeter with diff u-tube manometer

Case III :- Let the differential manometer contains heavier liquid then 'h' is given as

$$h = \left(\frac{P_1}{\rho g} + Z_1\right) - \left(\frac{P_2}{\rho g} + Z_2\right) = x\left[\frac{S_h}{S_o} - 1\right].$$

Case IV :- Lighter liquid than the liquid flowing through the pipe.

$$h = \left(\frac{P_1}{\rho g} + Z_1\right) - \left(\frac{P_2}{\rho g} + Z_2\right) = x\left[1 - \frac{S_L}{S_o}\right]$$

Q) An oil of sp. gr 0.8 is flowing through a venturi meter having inlet dia 20cm and throat dia 10cm. The oil - mercury differential manometer shows a reading of 25cm calculate the discharge of oil through the horizontal venturi meter Take $C_d = 0.98$

Sol:- sp. gr of oil: $S_o = 0.8$

$S_h = 13.6$

Reading of diff manometer $x = 25cm$

$$h = x\left[\frac{S_h}{S_o} - 1\right]$$

$$= 25\left[\frac{13.6}{0.8} - 1\right] \text{cm of oil} = 25[17-1]$$

$= 400$cm of oil.

$d_1 = 20cm$

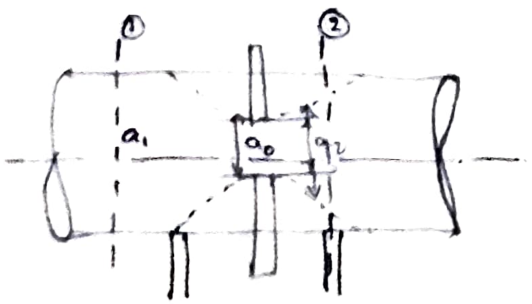
$a_1 = \frac{\pi}{4} \times 20^2 = 314.16cm^2$

$d_2 = 10cm$

$a_2 = 78.54cm^2$

$C_d = 0.98$

$Q = 70465 cm^3/s = 70.465 lit/s.$



Orifice meter (or) orifice plate:-

→ The orifice diameter is kept generally 0.5 times the diameter of the pipe through it may vary from 0.4 to 0.8 times the pipe dia.

→ A differential manometer is connected at section(1), which is at a distance of about 1.5 to 2.0 times the pipe diameter upstream from the orifice plate, at section(2), which at a distance of about half the dia of the orifice on the downstream side from the orifice plate.

Applying Bernoulli's equation at section(1)

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2$$

$$\left(\frac{P_1}{\rho g} + Z_1\right) - \left(\frac{P_2}{\rho g} + Z_2\right) = \frac{V_2^2}{2g} - \frac{V_1^2}{2g}$$

$$h = \frac{V_2^2}{2g} - \frac{V_1^2}{2g}$$

$$2gh = V_2^2 - V_1^2$$

$$V_2 = \sqrt{2gh + V_1^2} \quad \text{--- (1)}$$

Now section(2) is at the vena-contracta and 'a_2' represents the area at the vena-contract. If 'a_0' is the area of orifice then, we have

$$c_c = \frac{a_2}{a_0}$$

where, c_c = co-efficient of contraction

$$a_2 = a_0 \times c_c \quad \text{--- (2)}$$

$$a_1 V_1 = a_2 V_2 \text{ (or) } V_1 = \frac{a_2}{a_1} \times V_2 = \frac{a_0 c_c}{a_1} \cdot V_2 \quad \text{--- (3)}$$

sub V_1 in eqution (1)

$$V_2 = \sqrt{2gh + \frac{a_0^2 c_c^2 V_2^2}{a_1^2}}$$

$$V_2^2 = 2gh + \left(\frac{a_0}{a_1}\right)^2 c_c^2 \cdot V_2^2 \text{ (or) } V_2^2\left[1 - \left(\frac{a_0}{a_1}\right)^2 c_c^2\right] = 2gh.$$

$$V_2 = \frac{\sqrt{2gh}}{\sqrt{1 - \left(\frac{a_0}{a_1}\right)^2 c_c^2}}$$

$\therefore$ The discharge $Q = V_2 \times a_2 = V_2 \times a_0 c_c \cdot \left[\because a_2 = a_0 c_c \text{ from (ii)}\right]$

$$= \frac{a_0 c_c \sqrt{2gh}}{\sqrt{1 - \left(\frac{a_0}{a_1}\right)^2 c_c^2}} \quad \text{--- (4)}$$

$$c_d = c_c \frac{\sqrt{1 - \left(\frac{a_0}{a_1}\right)^2}}{\sqrt{1 - \left(\frac{a_0}{a_1}\right)^2 c_c^2}}$$

$$c_c = c_d \frac{\sqrt{1 - \left(\frac{a_0}{a_1}\right)^2 c_c^2}}{\sqrt{1 - \left(\frac{a_0}{a_1}\right)^2}}$$

sub in this value of c_c in eqution 4

$$Q = a_0 \times c_d \frac{\sqrt{1 - \left(\frac{a_0}{a_1}\right)^2 c_c^2}}{\sqrt{1 - \left(\frac{a_0}{a_1}\right)^2}} \times \frac{\sqrt{2gh}}{\sqrt{1 - \left(\frac{a_0}{a_1}\right)^2 c_c^2}}$$

$$\boxed{Q = \frac{c_d a_0 \sqrt{2gh}}{\sqrt{1 - \left(\frac{a_0}{a_1}\right)^2}} = \frac{c_d a_0 a_1 \sqrt{2gh}}{\sqrt{a_1^2 - a_0^2}}}$$

P) An orifice meter with orifice dia 15cm is inside in a pipe of 30cm dia. The pressure diff of manometer on two sides of the orifice meter reading of 50cm. Find the rate of flow. sp. of oil is 0.9; $C_d = 0.64$.

sol:- $d_0 = 15$cm. $\qquad Q = 137414.25 \text{ cm}^3/\text{s}$.

$a_0 = 176.7 \text{ cm}^2$

$d_1 = 30$cm

$a_1 = 706.85 \text{ cm}^2$

$S_0 = 0.9$

$x = 50$ cm of mercury

$$h = x\left[\frac{S_g}{S_0} - 1\right] = 50\left[\frac{13.6}{0.9} - 1\right] \text{ cm of oil.}$$

$$= 50 \times 14.11 = 705.5 \text{cm}$$

Pitot-tube:-

→ It is device used for measuring the velocity of flow at any point in a pipe.

→ It is based on the principal that if the velocity of flow at a point become zero, the pressure there is increased due to the converting of k.E into p.E

Consider two points '1' & '2' at the same level in such a way that point '2' is just as the inlet of the pitot-ptube and poin '1' is far away from the tube

P_1 = pressure at point '1'.

V_1 = velocity of flow at '1'

P_2 = pressure at point '2'

V_2 = velocity of flow at '2'.

H = depth of tube in liquid

h = rise of liquin in the tube above the free surface

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2$$

but $Z_1 = Z_2$ point '1' and '2' are on the same line $V_2 = 0$.

$\frac{P_1}{\rho g}$ = pressure head at (1) = H.

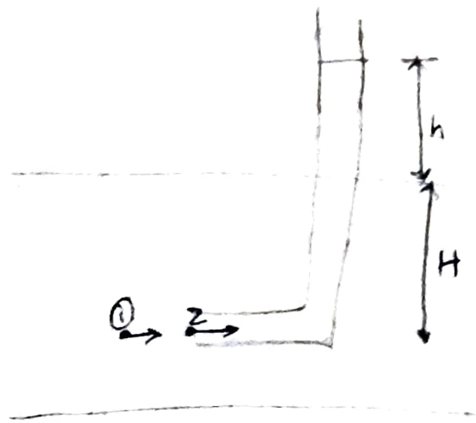
$\frac{P_2}{\rho g}$ = pressure head at (2) = [h + H]

$$H + \frac{V_1^2}{2g} = h + H$$

$$\therefore h = \frac{V_1^2}{2g} \quad (or) \quad V_1 = \sqrt{2gh}$$

$$(V_1)_{act} = C_v \sqrt{2gh}$$

C_v = Co-efficent of pitot-tube.



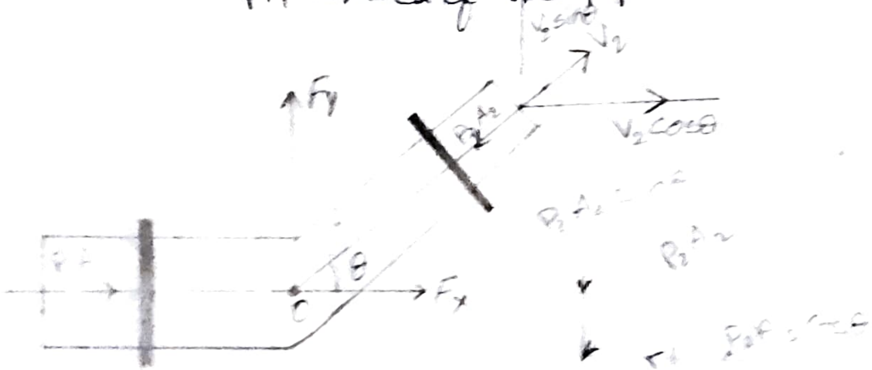
Force exerted by flowing fluid on a pipe bend:-

The impulse-momentum equation is used to determine the resultant force exerted by a flowing fluid on a pipe bend.

v_1 = velocity of flow at section (1)

P_1 = Pressure of flow

A_1 = Area of the pipe



Net force acting on fluid in the direction of x = Rate of change of momentum in x-direction

$P_1A_1 - P_2A_2\cos\theta - F_x$ = (mass per sec) (change of velocity)

$= \rho Q$ (Final velocity in the direction of x - Initial velocity in the direction of x).

$= \rho Q(v_2\cos\theta - v_1)$

$$F_x = \rho Q(v_1 - v_2\cos\theta) + P_1A_1 - P_2A_2\cos\theta$$

The momentum equation in y-direction gives

$$0 - P_2A_2\sin\theta - F_y = \rho Q(v_2\sin\theta - 0)$$

$$F_y = \rho Q(-v_2\sin\theta) - P_2A_2\sin\theta$$

$$R = \sqrt{F_x^2 + F_y^2}$$

$$\tan\theta = \frac{F_y}{F_x}$$

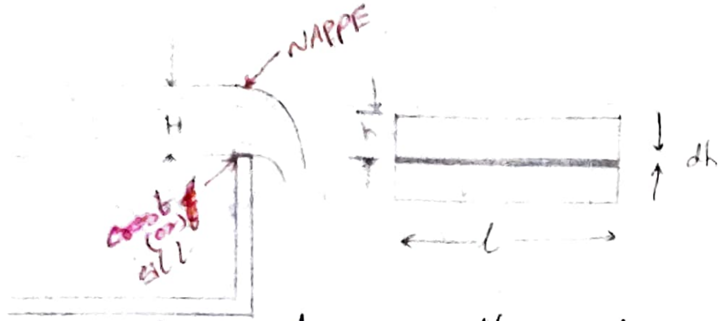
Notch:- It is a device used for measuring the rate of flow of a liquid through a small channel or a tank.

Weir:- It is a concrete (or) masonry structure placed in an open channel over which the flow occurs.

Nappe:- The sheet of water flowing through a notch (or) over a weir is called nappe.

Crest:- The bottom edge of notch (or) a top of a weir over which the water flows.

Discharge over a Rectangular notch :-



H = Head of water over the crest.

l = length of the notch or weir.

Area of strip = $L \times dh$

Theoretical velocity = $\sqrt{2gh}$

$$dQ = C_d \times \text{Area of strip} \times \text{Theoretical velocity}$$

$$= C_d \times L \times dh \times \sqrt{2gh}.$$

$$Q = \int_0^H C_d \times L \times \sqrt{2gh}\, dh = C_d \times L \times \sqrt{2g} \int_0^H h^{1/2}\, dh$$

$$= C_d \times L \times \sqrt{2g} \left[\frac{h^{1/2+1}}{\frac{1}{2}+1} \right]_0^H = C_d \times L \times \sqrt{2g} \left[\frac{h^{3/2}}{3/2} \right]$$

$$\boxed{Q = \frac{2}{3} C_d \times L \times \sqrt{2g}\, [H]^{3/2}}$$

Discharge over a triangular notch :-

H = head of water above the v-notch

θ = angle of notch.

Consider a horizontal strip of water of thickness 'dh' at a depth of 'h' from the free surface of water

$$\tan\frac{\theta}{2} = \frac{AC}{OC} = \frac{AC}{(H-h)}$$

$$AC = (H-h)\tan\frac{\theta}{2}$$

$$\text{width of strip} = AB = 2AC = 2(H-h)\tan\frac{\theta}{2}$$

$$\text{Area of strip} = 2(H-h)\tan\frac{\theta}{2} \times dh$$

Theoretical velocity of water = $\sqrt{2gh}$

$$dQ = C_d \times \text{Velocity} \times \text{Area}$$

$$= C_d \times 2(H-h)\tan\frac{\theta}{2} \times dh \times \sqrt{2gh}$$

$$= 2C_d (H-h)\tan\frac{\theta}{2} \times \sqrt{2gh} \times dh$$

Total discharge : $Q = \int_0^H 2C_d (H-h) \tan\frac{\theta}{2} \times \sqrt{2gh} \times dh$

$$= 2C_d \times \tan\frac{\theta}{2} \times \sqrt{2g} \int_0^H (H-h) h^{1/2}\, dh$$

$$= 2 \times C_d \times \tan\frac{\theta}{2} \times \sqrt{2g} \int_0^H (H h^{1/2} - h^{3/2})\, dh$$

$$= 2 \times C_d \times \tan\frac{\theta}{2} \times \sqrt{2g}\left[\frac{H h^{3/2}}{3/2} - \frac{h^{5/2}}{5/2}\right]_0^H$$

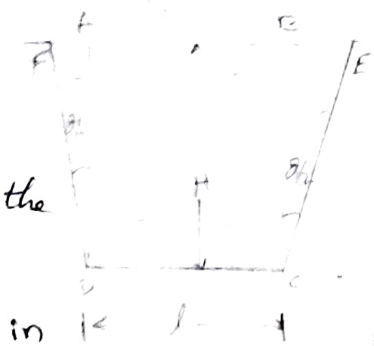
$$= 2 \times C_d \times \tan\frac{\theta}{2} \times \sqrt{2g}\left[\frac{2}{3} H \cdot H^{3/2} - \frac{2}{5} H^{5/2}\right]$$

$$= 2 \times C_d \times \tan\frac{\theta}{2} \times \sqrt{2g}\left[\frac{2}{3} H^{5/2} - \frac{2}{5} H^{5/2}\right]$$

$$= 2 \times C_d \times \tan\frac{\theta}{2} \times \sqrt{2g}\left[\frac{4}{15} H^{5/2}\right].$$

$$\boxed{Q = \frac{8}{15} C_d \times \tan\frac{\theta}{2} \times \sqrt{2g} \times H^{5/2}}$$

Discharge over a trapezoidal notch :-



H = Height of water over the notch

l = length of the Crest of the notch.

C_{d_1} = Co-eficent of discharge in rectangular notch

C_{d_2} = Coeficent of discharge in traingular notch.

$$Q_1 = \frac{2}{3} \times C_{d_1} \times l \times \sqrt{2g} \times H^{3/2}$$

$$Q_2 = \frac{8}{15} C_{d_2} \times \tan\frac{\theta}{2} \times \sqrt{2g} \times H^{5/2}.$$

trapezoil

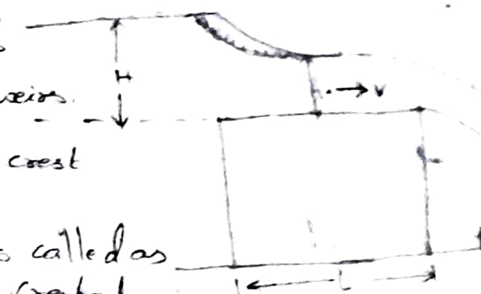
$$= \frac{2}{3} C_{d_1} L \sqrt{2g} \times H^{3/2} + \frac{8}{15} C_{d_2} \times \tan\frac{\theta}{2} \times \sqrt{2g} \times H^{5/2}$$

co-efficient of velocity (C_v) -

$$C_v = \frac{\text{Actual velocity of jet at vena-contracta}}{\text{theoretical velocity}}$$

$$= \frac{V}{\sqrt{2gh}}$$

Discharge over a broad-crested weir:-



→ Having a wide Crest is known as broad-crested weirs.

H = height of water above crest

L = length of the crest

If $2L > H$, then weir is called as broad-Crested weir

If $2L < H$, then weir is called as narrow-Crested weir

let h = Head of water at the middle of weir which is constant

V = velocity of flow over the weir

By applying ~~bernoll~~ bernoullis equation,

$$0 + 0 + H = 0 + \frac{V^2}{2g} + h$$

$$\frac{V^2}{2g} = H - h$$

$$V = \sqrt{2g(H-h)}$$

$$Q = C_d \times A \times V \Rightarrow C_d \times L \times h \times \sqrt{2g(H-h)}.$$

$$= C_d \times L \times \sqrt{2g\,(Hh^2 - h^3)} \quad \text{—(1)}$$

The discharge is max at $Hh^2 - h^3$.

$$\frac{d}{dh}(Hh^2 - h^3) = 0 \text{ (or) } 2h \times H - 3h^2 = 0$$

$$h = \frac{2}{3} H.$$

Sub in equ (1)

$$\boxed{Q_{max} = 1.705 \times C_d \times L \times H^{3/2}}$$

Boundary layer theory

Laminar Boundary layer:-

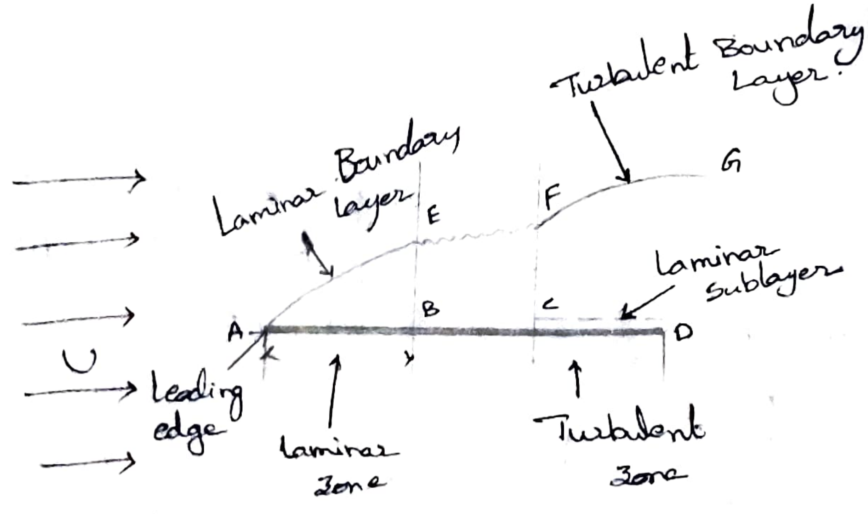
Near the leading edge of the surface of the plate, where the thickness is small, the flow in the boundary layer is laminar through the main flow is turbulent. This layer of the fluid is said to be laminar boundary layer. The length of the plate from the leading edge, upto which laminar boundary layer exists, is called laminar zone.

$$(Re)_x = \frac{U x x}{\nu}$$

x = distance from leading edge.

U - Free-stream velocity of fluid.

ν = Kinematic viscosity of fluid.



Turbulent Boundary layer:-

If the length of the plate is go on incresing the laminar boundary layer becomes unstable and motion of fluid with it, is disturbed and irregular which leads to a transition from a laminar to turbulent boundary layer. This short length over which the boundary layer flow changes from laminar to turbulent is called transition zone.

Laminar sub-layer:- This is the region in the turbulent boundary layer zone, adjecent to the solid surface of the plate. In this zone, the velocity variation is influenced only by viscous effects.

$$\tau_0 = \mu\left(\frac{\partial u}{\partial y}\right)_{y=0} = \mu \frac{u}{y}$$

Boundary layer Thickness - It is defined as the distance from the boundary of the solid body measured in the y-direction to the point, where the velocity of the fluid is approximatly equal to 0.99 times the free strem velocity (U) of the fluid.

δ_{Lam} = Thickness of laminar boundary layer

δ_{tur} = Thickness of turbulent boundary layer

δ' = Thickness of laminar sub-layer

Displacement thickness:- It is defined as the distance measured perpendicular to the boundary of the solid body, by which the boundary should be displaced to compensate for the reduction in flow rate on account of boundary layer formation. It is denoted by (δ^*).

$$\delta^* = \int_0^\delta \left(1 - \frac{u}{U}\right) dy.$$

momentum thickness (θ):- momentum thickness is defined as the distance measured perpendicular to the boundary of the solid body, by which the boundary should be displaced to compensate for the reduction in momentum of the flowing fluid on account of boundary layer formation. It is denoted by (θ).

$$\theta = \int_0^\delta \frac{u}{U}\left[1 - \frac{u}{U}\right] dy.$$

Energy thickness:- It is defined as the distance measured perpendicular to the boundary of the solid body by which the boundary should be displaced to compensate for the reduction in K.E of the flowing fluid on account of boundary layer formation. It is denoted by δ^{**}

$$\delta^{**} = \int_0^\delta \frac{u}{U}\left[1 - \frac{u^2}{U^2}\right] dy$$

Drag force on a flat plate due to boundary layer:-

$$\frac{\tau_0}{\rho U^2} = \frac{\partial}{\partial x}\left[\int_0^\delta \frac{u}{U}\left[1 - \frac{u}{U}\right] dy\right]$$

Von karman momentum integral equation

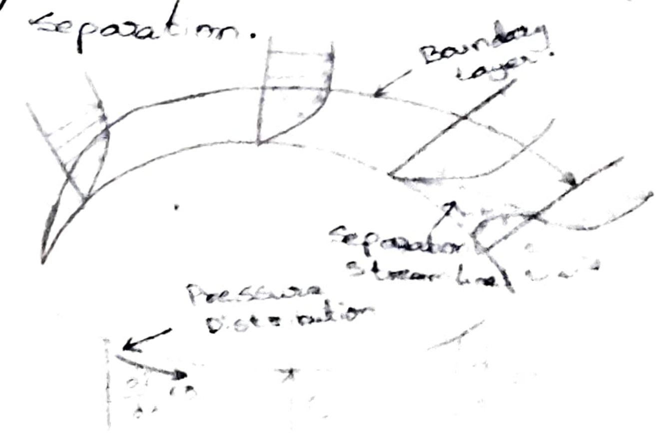
$$\frac{\tau_0}{\rho u^2} = \frac{\partial \theta}{\partial x}$$

This is applied to:-

1) Laminar boundary layer
2) Transition boundary layer
3) Turbulent boundary layer

separation of boundary layer:-

The point on the body at which the boundary layer is on the verge of separation from the surface is called Point of separation.



Thickness of laminar boundary layer and coefficient of drag from Blasius solution is given

$$\delta = \frac{4.91x}{\sqrt{Re_x}}$$

Re_x = Reynolds number

$$C_D = \frac{1.328}{\sqrt{Re_L}}$$

$Re = 5\times10^5$

Local coeficent of Drag (C_D^*):-

$$C_D^* = \frac{\tau_0}{\frac{1}{2}\rho U^2}$$

Average co-efficient of Drag force:-

$$C_D = \frac{F_D}{\frac{1}{2}\rho A U^2}.$$

$$Re_L = \frac{\rho U L}{\mu}$$

$$\delta = 5.48x\frac{x}{\sqrt{Re_x}}$$

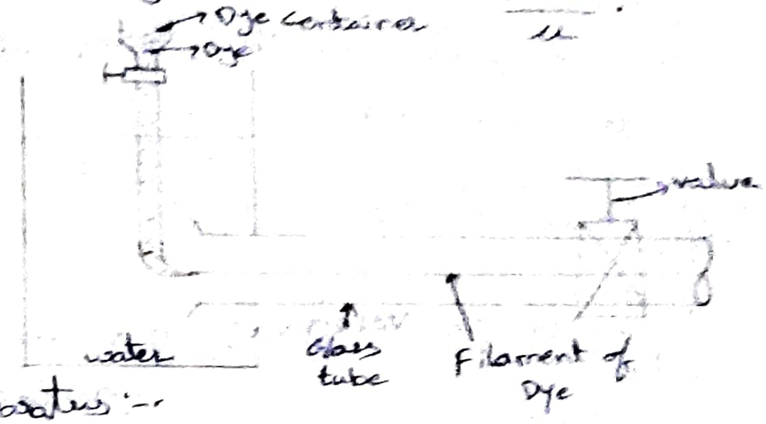
$$C_D = \frac{1.46}{\sqrt{Re_L}}$$

$$F_D = 0.73 b\mu U\sqrt{\frac{\rho U L}{\mu}}$$

Closed Conduit Flow

Reynold's Experiment -

The type of flow is determined from the reynolds number i.e. $\frac{\rho V d}{\mu}$.



Apparatus :-

(i) A tank containing water at constant head

(ii) A small tank containing dye.

(iii) A glass tube having a bell-mouthed entrance at one end a regulating valve at other ends.

~~head loss~~ Loss of head $h_f \propto V^n$

where n varies from 1.75 to 2.0

Darcy-weisbach formula

$$h_f = \frac{4 \cdot f \cdot L \cdot V^2}{d \times 2g}.$$

h_f = loss of head due to friction

f = co-efficent of friction which is a function of reynolds number

$= \frac{16}{Re}$ for $Re < 2000$ (viscous flow)

$= \frac{0.079}{Re^{1/4}}$ for Re ~~varing~~ varying from 4000 to 10^6.

L = length of pipe

v = mean velocity of flow.

d = dia of pipe.

chezy's formula for loss of head due to friction in pipe:-

$$\boxed{h_f = \frac{f'}{\rho g} \times \frac{P}{A} \times L \times V^2}$$

$V = C\sqrt{mi}$ — v = mean velocity.

$i = \frac{h_f}{L}$ — C = chezy's constant

$C = \sqrt{\frac{\rho g}{f'}}$ — i = loss of head per unit length

$m = d/4$.

Hydraulic Gradient and Total energy line

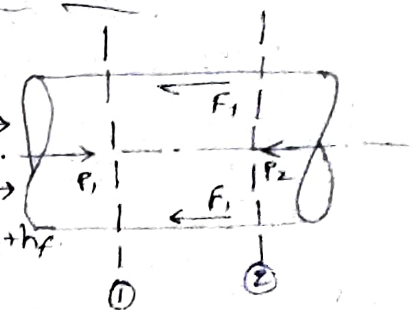
Hydraulic Gradient – It is defined as the line which gives the sum of pressure head $\left(\frac{P}{w}\right)$ and datum head (Z) of a flowing fluid in a pipe w.r.t some reference line.

Total Energy line:- It is defined as the line which gives the sum of pressure head, datum head and K.H of a flowing fluid in a pipe w.r.t reference line.

Equation for head loss in pipes due to friction -

Darcy - weisbach eqution:-

Applying bernoulli's



$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2 + h_f$$

$Z_1 = Z_2$

$V_1 = V_2$

$$\frac{P_1}{\rho g} = \frac{P_2}{\rho g} + h_f \text{ (or) } h_f = \frac{P_1}{\rho g} - \frac{P_2}{\rho g} \quad \text{——— (1)}$$

Frictinal resistance = Frictional resistance per unit wetted area per unit velocity × wetted area × velocity²

$F_1 = f' \times \pi d L \times V^2$

$= f' \times P \times L \times V^2$

Forces acting on the fluid b/w section 1-1 & 2-2

1) Pressure force at 1-1 = $P_1 \times A$.

2) Pressure force at 2-2 = $P_2 \times A$.

3) Frictional force F_1 as shown (all horizontal forces)

$$P_1 A - P_2 A - F_1 = 0 \quad \text{—— (2)}$$

$$(P_1 - P_2) A = F_1 \Rightarrow f' \times P \times L \times v^2$$

$$P_1 - P_2 = \frac{f' \times P \times L \times v^2}{A}$$

From eqn (1)

$$P_1 - P_2 = \rho g h_f$$

$$\rho g h_f = \frac{f' \times P \times L \times v^2}{A}$$

$$h_f = \frac{f'}{\rho g} \times \frac{P}{A} \times L \times v^2 \quad \text{—— (3)}$$

$$\frac{P}{A} = \frac{\text{wetted perimeter}}{\text{Area}} = \frac{\pi d}{\frac{\pi}{4} d^2} = \frac{4}{d}$$

$$h_f = \frac{f'}{\rho g} \times \frac{4}{d} \times L \times v^2 = \frac{f'}{\rho g} \times \frac{4 L v^2}{d}$$

—— (4)

$$\frac{f'}{\rho g} = \frac{f}{2} \Rightarrow$$

$$h_f = \frac{4f}{2g} \cdot \frac{L v^2}{d} \Rightarrow \boxed{\frac{4 f L v^2}{2g \times d}}$$

Fluid friction:-

It is the force of friction exerted by liquids on objects moving through them (or) along which they are flowing.

↳ It depends up on size and shape of the object.

Loss of energy in pipes:-

→ when a fluid is flowing through a pipe, the fluid experiences some resistance due to which some of the energy of fluid is lost. The loss of energy is classified as -

- Major losses → Friction loss in pipes.
- minor losses →

↳ Loss of head due to sudden enlargement:-

$$h_e = \frac{(V_1 - V_2)^2}{2g}$$

Sudden contraction:-

$$h_c = \frac{0.5 V_2^2}{2g}$$

$$h_c = \frac{k V_2^2}{2g}; \quad K = \left[\frac{1}{C_c} - 1\right]^2$$

$$C_c = \frac{A_c}{A_2}$$

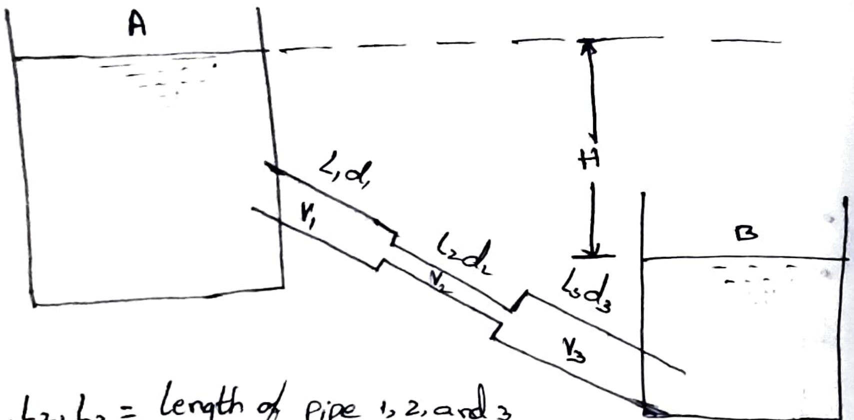
Loss at entrance of pipe:-

$$h_i = 0.5\frac{V^2}{2g}$$

Loss at exit of pipe:-

$$h_o = \frac{V^2}{2g}$$

Flow through pipes in series (or) Flow through compound pipes:-



L_1, L_2, L_3 = length of pipe 1, 2, and 3
d_1, d_2, d_3 = dia of pipe 1, 2, 3
V_1, V_2, V_3 = Velocity of flow through pipes 1, 2, 3
f_1, f_2, f_3 = co-efficient of friction for pipes 1, 2, 3.
H = Diff in water level. two tanks.

$$Q = A_1V_1 = A_2V_2 = A_3V_3$$

The diff in liquid surface levels is equal to the sum of the total head loss in the pipes

$$H = \frac{0.5V_1^2}{2g} + \frac{4f_1L_1V_1^2}{d_1\times 2g} + \frac{0.5V_2^2}{2g} + \frac{4f_2L_2V_2^2}{d_2\times 2g} + \frac{(V_2-V_3)^2}{2g} + \frac{4f_3L_3V_3^2}{d_3\times 2g} + \frac{V_3^2}{2g}$$

If minor loss is neglected,

$$H = \frac{4f_1L_1V_1^2}{d_1\times 2g} + \frac{4f_2L_2V_2^2}{d_2\times 2g} + \frac{4f_3L_3V_3^2}{d_3\times 2g}$$

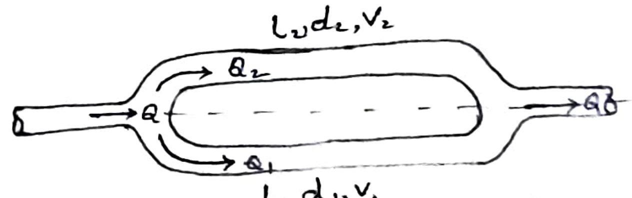
co-efficent of friction is same for all pipes

$$f_1 = f_2 = f_3 = f.$$

$$H = \frac{4fL_1V_1^2}{d_1\times 2g} + \frac{4fL_2V_2^2}{d_2\times 2g} + \frac{4fL_3V_3^2}{d_3\times 2g}$$

$$\boxed{H = \frac{4f}{2g}\left[\frac{L_1V_1^2}{d_1} + \frac{L_2V_2^2}{d_2} + \frac{L_3V_3^2}{d_3}\right]}$$

Flow through parallel pipes:-



$$Q = Q_1 + Q_2$$

$$\frac{4f_1L_1V_1^2}{d_1\times 2g} = \frac{4f_2L_2V_2^2}{d_2\times 2g}$$

$f_1 = f_2$, then; $\frac{L_1V_1^2}{d_1\times 2g} = \frac{L_2V_2^2}{d_2\times 2g}$

Pipe Networks:-

→ A pipe network is an interconnected system of Pipes forming several loops (or) circuits.

Conditions for network of Pipes:-

→ The flow into each junction must be equal to the flow out of the junction.

$h_f = r Q^n$; h_f = head loss
n = For turbulent flow is '2'.

$$h_f = \frac{4 \times f \times L \times V^2}{2gd}$$

$$= \frac{4fL \times \left(\frac{Q}{A}\right)^2}{2gd} = \frac{4fL \times Q^2}{D \times 2g \times \left(\frac{\pi}{4} D^2\right)^2}$$

$$= \frac{4fL \times Q^2}{2g \times \left(\frac{\pi}{4}\right)^2 \times D^5}$$

$$h_f = r Q^2$$

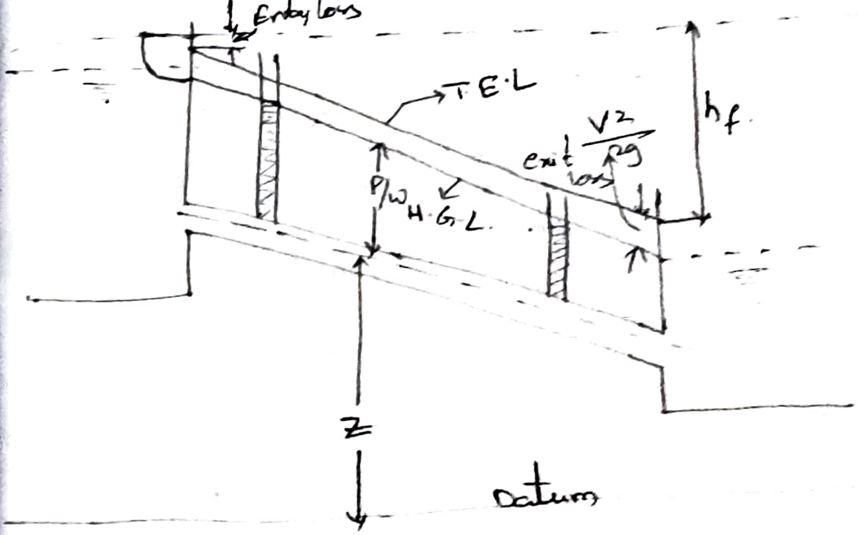
Hardy Cross method:-

$$\Delta Q = \frac{-\Sigma r Q_0^2}{\Sigma 2 r Q_0}$$

ΔQ = Correction factor

Q_0 = assumed discharge

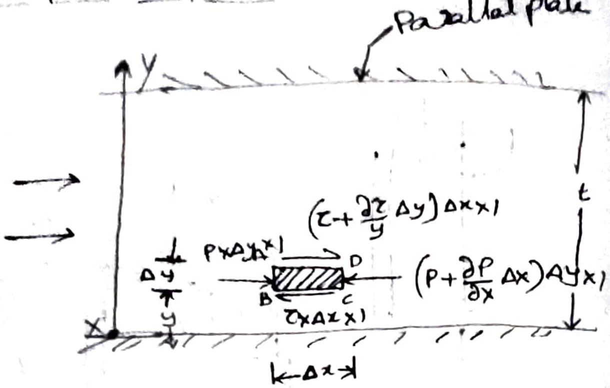
H.G.L & T.E.L:-



→ Laminar flow is possible only at low velocities and when the fluid is highly viscous.

→ $R_e < 2000$ → laminar
$R_e > 4000$ → Turbulent

Flow between two parallel plates:-



1) The pressure force, $p \times \Delta y \times 1$ on face AB.

2) The pressure force, $(p+\frac{\partial p}{\partial x}\Delta x)\Delta y \times 1$ on face CD.

3) The shear force, $\tau \times \Delta x \times 1$ on face BC

4) The shear force, $(\tau \times \frac{\partial \tau}{\partial y}\Delta y)\Delta x \times 1$ on face AD.

Resolving the forces.

$$p\Delta y \times 1 - \left(p+\frac{\partial p}{\partial x}\Delta x\right)\Delta y \times 1 - \tau\Delta x \times 1 + \left(\tau+\frac{\partial \tau}{\partial y}\Delta y\right)\Delta x \times 1 = 0.$$

(or) $$-\frac{\partial p}{\partial x}\Delta x\,\Delta y + \frac{\partial \tau}{\partial x}\Delta y\,\Delta x = 0.$$

Dividing by $\Delta x\,\Delta y$, we get $-\frac{\partial p}{\partial x} + \frac{\partial \tau}{\partial y} = 0.$

$$\frac{\partial p}{\partial x} = \frac{\partial \tau}{\partial y} \quad \text{—(1)}$$

i) velocity distribution:- To obtain velocity distribution across a section, the value of shear stress $\tau = \mu\frac{du}{dy}$ from newton's law of viscosity.

$$\frac{\partial p}{\partial x} = \frac{\partial}{\partial y}\left(\mu\frac{du}{dy}\right) = \mu\frac{\partial^2 u}{\partial y^2}$$

$$\frac{\partial^2 u}{\partial y^2} = \frac{1}{\mu}\frac{\partial p}{\partial x}.$$

Integrating the above eqution w.r.t y, we get

$$\frac{\partial u}{\partial y} = \frac{1}{\mu}\frac{\partial p}{\partial x}y + c_1$$

Integrating again

$$u = \frac{1}{\mu}\frac{\partial p}{\partial x}\frac{y^2}{2} + c_1 y + c_2 \quad \text{—(2)}$$

c_1, c_2 are constants of integration.

i) $y=0;\ u=0$ (ii) $y=t,\ u=0$

by sub in equ '2' $y=0,\ u=0$;

$$0 = 0 + c_1 \times 0 + c_2$$

$$c_2 = 0.$$

by sub in eqution '2' $y=t,\ u=0$.

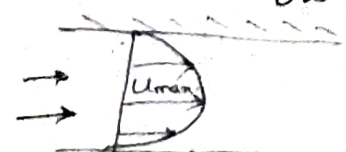
$$0 = \frac{1}{\mu}\frac{\partial p}{\partial x}\frac{t^2}{2} + c_1 \times t + 0.$$

$$c_1 = -\frac{1}{\mu}\frac{\partial p}{\partial x}\cdot\frac{t^2}{2\times t} = -\frac{1}{2\mu}\frac{\partial p}{\partial x}t.$$

Sub c_1 & c_2 in eqution '2'.

$$u = \frac{1}{2\mu}\frac{\partial p}{\partial x}\cdot y^2 + y\left(-\frac{1}{2\mu}\frac{\partial p}{\partial x}t\right)$$

$$u = -\frac{1}{2\mu}\frac{\partial p}{\partial x}[ty - y^2]. \quad \text{—(3)}$$



Ratio of maximum velocity to avg velocity:-

velocity is maximum, when $y = t/2$ sub in equation '3'

$$U_{max} = -\frac{1}{2\mu}\frac{\partial p}{\partial x}\left[t \times \frac{t}{2} - \left(\frac{t}{2}\right)^2\right]$$

$$= -\frac{1}{2\mu}\frac{\partial p}{\partial x}\left[\frac{t^2}{2} - \frac{t^2}{4}\right] = -\frac{1}{2\mu}\frac{\partial p}{\partial x}\frac{t^2}{4} = -\frac{1}{8\mu}\frac{\partial p}{\partial x}t^2 \quad \text{—(4)}$$

Avg velocity is obtained by dividing the discharge (Q) across the section by the area of the section ($t \times 1$)

considering strip

dQ = velocity at a distance y × Area of strip

$$= -\frac{1}{2\mu}\frac{\partial p}{\partial x}[ty - y^2] \times dy \times 1$$

$$Q = \int_0^t -\frac{1}{2\mu}\frac{\partial p}{\partial x}(ty - y^2)\,dy$$

$$= -\frac{1}{2\mu}\frac{\partial p}{\partial x}\left[\frac{ty^2}{2} - \frac{y^3}{3}\right]_0^t = -\frac{1}{2\mu}\frac{\partial p}{\partial x}\left[\frac{t^3}{2} - \frac{t^3}{3}\right]$$

$$= -\frac{1}{2\mu}\frac{\partial p}{\partial x}\frac{t^3}{6} = -\frac{1}{12\mu}\frac{\partial p}{\partial x}t^3$$

$$\bar{u} = \frac{Q}{Area} = -\frac{\frac{1}{12\mu}\frac{\partial p}{\partial x}\cdot t^3}{t \times 1}$$

$$= -\frac{1}{12\mu}\frac{\partial p}{\partial x}t^2 \quad \text{—(5)}$$

Dividing 4 & 5 equations

$$\frac{U_{max}}{\bar{u}} = \frac{-\frac{1}{8\mu}\frac{\partial p}{\partial x}t^2}{-\frac{1}{12\mu}\frac{\partial p}{\partial x}t^2} = \frac{12}{8} = \frac{3}{2} \quad \text{—(6)}$$

(iii) Drop of pressure head for a given length:-

From equation '5'

$$\bar{u} = -\frac{1}{12\mu}\frac{\partial p}{\partial x}t^2$$

$$\frac{\partial p}{\partial x} = -\frac{12\mu\bar{u}}{t^2}$$

Integrate w.r.t 'x' we get.

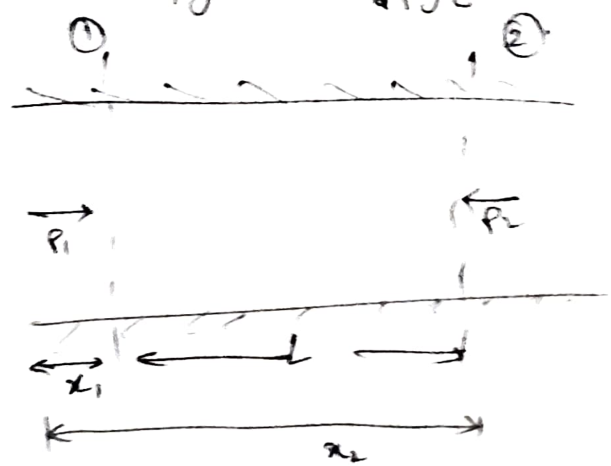
$$\int_2^1 dp = \int_2^1 -\frac{12\mu\bar{u}}{t^2}\,dx$$

$$P_1 - P_2 = -\frac{12\mu\bar{u}}{t^2}(x_1 - x_2) = \frac{12\mu\bar{u}}{t^2}(x_2 - x_1)$$

$$P_1 - P_2 = \frac{12\mu\bar{u}L}{t^2}\left[\because x_1 - x_2 = L\right]$$

If h_f is drop of pressure head

$$h_f = \frac{P_1 - P_2}{\rho g} = \frac{12\mu\bar{u}L}{\rho g t^2} \quad \text{—(7)}$$



shear stress distribution:-

$$\tau = \mu \frac{du}{dy}$$

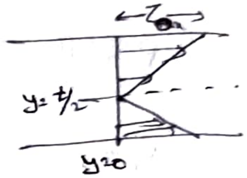
$$\tau = \mu \frac{du}{dy} = \mu \frac{\partial}{\partial y}\left[-\frac{1}{2\mu}\frac{\partial p}{\partial x}(ty - y^2)\right]$$

$$= \mu\left[-\frac{1}{2\mu}\frac{\partial p}{\partial x}(t-2y)\right].$$

$$\tau = -\frac{1}{2}\frac{\partial p}{\partial x}[t-2y].$$

max shear stress(τ_0).

$$\tau_0 = \frac{-1}{2}\frac{\partial p}{\partial x}t.$$



→ unit-5

→ **Laws of fluid friction for laminar flow:-**

The frictional resistance in the laminar flow is.

→ proportinal to the velocity of flow

→ Independent of the pressure.

→ proportinal to the area of surface in contact

→ Independent of the nature of the surface in contact

→ greatly affected by the variation of the temperature of the flowing fluid.

laws of Fluid friction for turbulent flow:-

→ Proportinal to (velocity)^n, where the index n varies from 1.72 to 2.0.

→ Independent of the pressure.

→ proportinal to the density of the flowing fluid.

→ slightly affected by the variation of the temp of the flowing fluid.

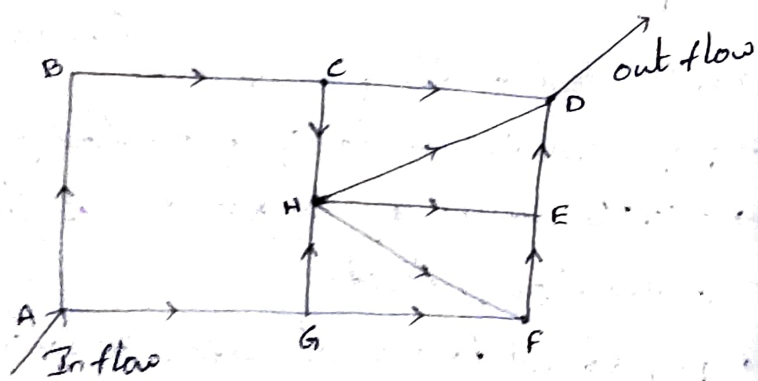
→ proportional to area of surface in contact.

Pipe networks –

→ A group of interconnected pipes forming several loops or circuits is called a network of pipes.

• The conditions to be satisfied in any network of pipes

→ According to the principle of continuity the flow into each junction must be equal to the flow out of the junction.



For example at junction A, the inflow must be equal to the flow through AB and AG.

→ In each loop, the loss of head due to flow in clockwise direction must be equal to the loss of head due to flow in anticlockwise direction.

→ The Darcy-weisbach equation must be ~~equ~~ satisfied for flow in each pipe.

→ According to darcy-weisbach equation the loss of head h_f through any pipe discharged at the rate Q

$$h_f = r Q^n.$$

where 'r' is a proportionality factor which can be determined for each pipe knowing the friction factors f, the length 'L' and the dia 'D' of the pipe.

$$\left[r = \frac{fL}{2g(\pi/4)^2 D^5} = \frac{fL}{12.10 D^5} \right]$$

n is an exponent having a numerical value ranging from 1.7 to 2.00

Hardy cross method :-

→ Assume a most suitable ~~dis~~ distribution of flow that satisfies continuity at each junction.

→ with the assumed value of Q, compute the head losses of each pipe using

$$h_f = r Q^n$$

→ consider different loops and compute the net head loss around each circuit considering the head loss is clockwise flow is +ve anti-clockwise is '–ve'.

Q_0 = assumed discharge

Q = correct discharge.

$$Q = Q_0 + \Delta Q$$

head loss for the pipe is

$$h_f = rQ^n = r(Q_0 + \Delta Q)^n.$$

For the complete circuit

$$\Sigma h_f = \Sigma rQ^n = \Sigma r(Q_0 + \Delta Q)^n$$

By expanding

$$\Sigma rQ^n = \Sigma r[(Q_0^n + nQ_0^{n-1}\Delta Q + \dots].$$

ΔQ is small compared with Q_0.

$$\Sigma rQ^n = \Sigma rQ_0^n + \Sigma rnQ_0^{n-1}\Delta Q.$$

For the correct distribution the circuit is balanced and $\Sigma rQ^n = 0$

$$\Sigma rQ_0^n + \Delta Q\,\Sigma rnQ_0^{n-1} = 0.$$

$$\boxed{\Delta Q = -\frac{\Sigma rQ_0^n}{\Sigma rnQ_0^{n-1}}}$$