WAVES AND OSCILLATIONS

A Motion which repeats itself after equal intervals of time is called a period motion (or) Harmonic motion. The Examples of the periodic motion are the spin of the earth, the motion of a Satellite around a planet, Vibrations of atoms in molecules etc. A body or a particle is said to possess Oscillatory or Vibratory motion, if it moves backs and forth repeatedly about the mean position. The pendulum of clock swings back and forth is said to perform an Oscillatory. Similarly the motion of prongs of tuning fork, motion of Simple pendulum, the Vertical Oscillations of a loaded Spring, to and fro motion of the piston of an engine are the Examples of Oscillatory motion.

SIMPLE HARMONIC MOTION:

This is a Special type of periodic motion in which the body moves again and again over the Same path about a fixed point (equilibrium position). It is defined as the motion of an Oscillatory particle which is acted upon by a restoring force; which is directly

proportional to the displacement but opposite to it in direction.

following are the characteristics of SHM:

(a) The Motion is periodic

(b) The Motion is along a straight line about the mean or equilibrium position

(c) The acceleration is proportional to displacement

(d) Acceleration is directed towards the mean (or) equilibrium position.

THE SIMPLE HARMONIC OSCILLATOR:-

When a particle or body moves such that it's acceleration is always directed towards a fixed point and varies directly as it's distance from that point. The particle or body is called to execute SHM. The particle or body executing S.H.M is called Simple harmonic Oscillator.

EQUATION OF MOTION OF SIMPLE HARMONIC OSCILLATOR

Consider a particle p of mass m; executing SHM, about equilibrium position O along x-axis as shown in fig (1)
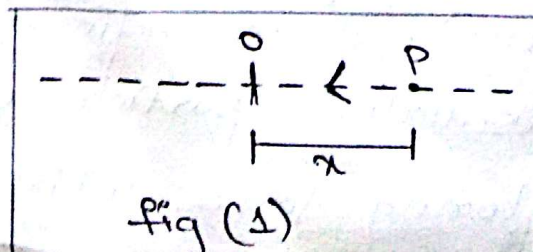

fig (1)

By definition, the force under which the particle is oscillating is proportional to its displacement directed towards mean position. Let x be displacement of P from 0 at any instant. The instantaneous force, F acting upon P is given by

$$F \propto -x \quad (or)$$

$$\boxed{F = -kx \longrightarrow (1)}$$

where k is proportionality factor which represents the force per unit displacement. The Negative Sign is used to show that the force F is opposite to the displacement.

According to Newton's Second Law of Motion, the resting force on mass, m produces an acceleration d^2x/dt^2 in the mass, So that

$$Force = Mass \times Acceleration$$

i.e

$$\boxed{F = m \times \frac{d^2x}{dt^2} \quad - (2)}$$

from ① and ②

$$m \frac{d^2x}{dt^2} = -kx \quad (or) \quad \frac{d^2x}{dt^2} = -\frac{k}{m} x.$$

Let us put; $\frac{k}{m} = \omega^2$. Thus;

$$\boxed{\frac{d^2x}{dt^2} + \omega^2 x = 0 \quad - ③}$$

This is known as differential equation of Simple Harmonic Oscillator.

Now;

For the Simple Harmonic Oscillator equation;

$$\frac{d^2x}{dt^2} + \omega^2 x = 0$$

Let us assume trial solution of the form

$$x = C e^{\alpha t}$$

where C and α are orbitary constants. Differentiating it on both sides we get;

$$\frac{dx}{dt} = C\alpha e^{\alpha t} \text{ and}$$

$$\frac{d^2x}{dt^2} = C\alpha^2 e^{\alpha t}$$

Substituting these Values in Simple harmonic Oscillator, we have.

1. $C\alpha^2 e^{\alpha t}, \omega^2 C e^{\alpha t} = 0$

$\Rightarrow C e^{\alpha t} (\alpha^2, \omega^2) = 0$ (or)

$\alpha^2, \omega^2 = 0$ $(\because C \neq 0 \text{ and } e^{\alpha t} \neq 0)$

where, $j = \sqrt{(-1)}$

$\therefore \alpha = \pm \sqrt{(-\omega^2)} = \pm j\omega$

Now

$$\therefore x = Ce^{+j\omega t} \quad \text{and} \quad x = Ce^{-j\omega t}$$

So, the general Solution can be written as;

$$x = C_1 e^{+j\omega t} + C_2 e^{-j\omega t}.$$

where C_1 and C_2 are Orbitary Constants

further,

$$x = C_1 (\cos \omega t + j \sin \omega t) + C_2 (\cos \omega t - j \sin \omega t)$$

$$\Rightarrow x = (C_1 + C_2) \cos \omega t + j (C_1 - C_2) \sin \omega t$$

Now

Let us put;

$$C_1 \text{ and } C_2 = a \sin \emptyset \quad \text{and} \quad j (C_1 - C_2) = a \cos \emptyset$$

where a and $\emptyset$ are new Constants

$$\therefore x = a \sin \emptyset \cos \omega t + a \cos \emptyset \sin \omega t$$
$$x = a \sin (\omega t + \emptyset)$$

This is the Solution of the equation of Simple Harmonic Oscillator.

CHARACTERISTICS OF SIMPLE HARMONIC MOTION:

1. DISPLACEMENT :

The displacement of any particle at any executing S.H.M is given by

$$x = a \sin (\omega t + \emptyset)$$

The Maximum displacement from the mean position amplitude. Here the amplitude is a.

2. VELOCITY :

The Velocity V of the oscillating particle can be obtained by differentiating equation.

$$x = a \sin(\omega t + \phi)$$

$$\therefore V = \frac{dx}{dt} = \omega a \cos(\omega t + \phi) = \omega \sqrt{a^2 - x^2}$$

At the mean position i.e, at $x=0$, the Velocity is; $V_{max} = \omega a$. The Velocity is zero at the Extreme positions.

3. PERIODIC TIME :

Time taken for one complete oscillation is defined as periodic time and is denoted by T.

Let t is increased by $2\pi/\omega$ in eq;

$$x = a \sin(\omega t + \phi) \text{ then};$$

$$x = a \sin\left(\omega\left(t + \frac{2\pi}{\omega}\right) + \phi\right)$$

$$x = a \sin(\omega t + 2\pi + \phi)$$

$$x = a \sin(\omega t + \phi)$$

This Shows that the displacement repeats itself after a time $\left(\frac{2\pi}{\omega}\right)$. Therefore, $\frac{2\pi}{\omega}$ is known as periodic time

$$\therefore T = \frac{2\pi}{\omega}$$

Now

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{\dfrac{d^2x/dt^2}{x}}} \qquad \left(\because \omega = \left. \frac{d^2x}{dt^2} \right/ x \right)^{1/2}$$

$$= 2\pi \sqrt{\frac{x}{(d^2x/dt^2)}} = 2\pi \sqrt{\frac{displacement}{acceleration}}$$

4. FREQUENCY :

The Number of Oscillations made in One Second is called as frequency and is denoted by n or ν

Hence

$$n \,(or)\, \nu = \frac{1}{T} = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

RELATION BETWEEN DISPLACEMENT, VELOCITY & ACCELERATION

By putting $\omega = \frac{2\pi}{T}$ in the expressions of displacement, velocity and acceleration and taking $\phi = 0$ for simplicity we get;

$$x = a \sin(\omega t + \phi) = a \sin(2\pi t/T)$$

$$\frac{dx}{dt} = a\omega \cos(\omega t + \phi) = a\omega \cos(2\pi t/T)$$

$$\frac{d^2x}{dt^2} = -a\omega^2 \sin(\omega t + \phi) = -a\omega^2 \sin\left(\frac{2\pi t}{T}\right)$$

ENERGY OF SIMPLE HARMONIC OSCILLATOR !

A Simple Harmonic Oscillator possess k.E as well as P.E. The k.E is due to its velocity while the P.E is an account of its displacement from the equillibrium position. During Oscillation both the energies Vary but their Sum is Conserved if no dissipative forces are present. Here we shall calculate both these energies

KINETIC ENERGY :

The general equation of Simple harmonic Oscillator is given by

$$x = a \sin(\omega t + \phi)$$

where x = displacement of Oscillator and time t.

The Velocity of the oscillator at any point t is given by

$$\frac{dx}{dt} = a \omega \cos(\omega t + \phi)$$

k.E of the oscillator $= \frac{1}{2} m \left(\frac{dx}{dt}\right)^2$

m = mass of oscillator

$$k = \frac{1}{2} m \left[a \omega \cos(\omega t + \phi)\right]^2$$

$$= \frac{1}{2} m a^2 \omega^2 \cos^2(\omega t + \phi) \longrightarrow \text{①}$$

POTENTIAL ENERGY!

Incase of Simple harmonic Oscillator, the force is proportional and opposite to the displacement x i.e

$$F = -kx$$

where the Constant k gives the force per Unit displacement.

We know that force can also be Expressed in terms of PE (v) by the following relation.

$$F = -\frac{dv}{dx}$$

Hence,

$$\frac{dv}{dx} = kx$$

On integrating; we get;

$$v = \frac{1}{2} kx^2 + c$$

But the P.E $v = 0$ when displacement $x = 0$.

Thus $c = 0$

$$v = \frac{1}{2} kx^2 = -\frac{1}{2} k \left[a \sin (\omega t + \phi) \right]^2$$

$$= \frac{1}{2} ka^2 \sin^2 (\omega t + \phi)$$

$$= \frac{1}{2} m \omega^2 a^2 \sin^2 (\omega t + \phi) \quad (\because \omega^2 = k/m) \rightarrow ②$$

TOTAL ENERGY!

The total energy is sum of K.E and P.E
Hence

$$E = K.E + P.E = K + U$$

$$\Rightarrow E = \frac{1}{2}ma^2\omega^2 \cos^2(\omega t + \phi) + \frac{1}{2}m\omega^2 a^2 \sin^2(\omega t + \phi)$$

$$\Rightarrow E = \frac{1}{2}ma^2\omega^2 \longrightarrow ③$$

$$\Rightarrow E = \frac{1}{2}ma^2(2\pi n)^2 = 2\pi^2 n^2 a^2 m \longrightarrow ④$$

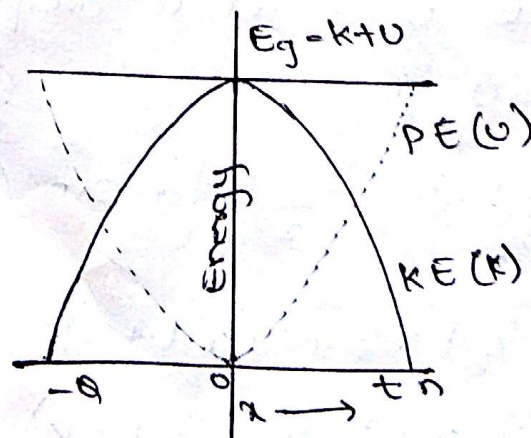where, $\omega = 2\pi n$ and n = frequency of oscillator

It is clear that from Eq.(4) that the total Energy of the oscillator is

(i) directly proportional to square of amplitude (a^2)

(ii) directly proportional to the square of frequency (n^2) or inversely proportional to time period (T^2)

The distribution of energy versus displacement is shown in figure.

It can be seen that maximum K.E and P.E is also called equal to the T.E of the Oscillator. The reason is that the energy of the Oscillator changes from maximum K.E at the mean position to the maximum P.E at the extreme position and back to maximum K.E

COMBINATION OF TWO MUTUALLY PERPENDICULAR SIMPLE HARMONIC VIBRATIONS: [LISSAJOUS FIGURES]

1. EQUAL FREQUENCIES:

Let us consider the case when two SHM's have the same frequency (or time period) one acting along the y-axis. Let the two variations are represented by

$$x = a \sin(\omega t + \phi) \longrightarrow ①$$
$$y = b \sin(\omega t) \longrightarrow ②$$

where a and b are the amplitudes of x and y. Vibrations respectively. The x motion is ahead of y motion by angle ϕ. i.e, the phase difference b/w two vibrations is ϕ

The equation of resultant vibration can be obtained by eliminating t b/w eq ① and ②

Vibrations respectively. The x motion is ahead of y motion by angle ϕ i.e, the phase difference b/w two Vibrations is ϕ

The equation of resultant vibration can be obtained by eliminating t between Eq ① and ②

from Eq ②; we have;

$$\sin(\omega t) = y/b$$

$$\therefore \cos(\omega t) = \sqrt{(1-\sin^2\omega t)} = \sqrt{(y^2/b^2)-1}$$

Expanding
Eq ① and Substituting the Values of $\sin(\omega t)$ & $\cos(\omega t)$

we get

$$\frac{x}{a} = \sin\omega t \cos\phi + \cos\omega t \sin\phi$$

$$\Rightarrow \frac{x}{a} = \frac{y}{b}\cos\phi + \sqrt{(1-y^2/b^2)}\sin\phi$$

$$\Rightarrow \frac{x}{a} - \frac{y}{b}\cos\phi = \sqrt{\left(1-\frac{y^2}{b^2}\right)}\sin\phi$$

Squaring on b.s; we get

$$\left(\frac{x}{a}-\frac{y}{b}\right)\cos\phi^2 = \left(1-\frac{y^2}{b^2}\right)\sin^2\phi$$

$$\Rightarrow \frac{x^2}{a^2} + \frac{y^2}{b^2}\cos^2\phi - \frac{2xy}{ab}\cos\phi = \sin^2\phi - \frac{y^2}{b^2}\sin^2\phi$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2}(\cos^2\phi + \sin^2\phi) - \frac{2xy}{ab}\cos\phi = \sin^2\phi$$

$$\Rightarrow \quad \frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{2xy}{ab}\cos\phi = \sin^2\phi \quad \longrightarrow \quad \text{(3)}$$

The equation represents an oblique ellipse, which is the resultant path of the particle. Here we consider the following important cases:

(i) When $\phi = 0$:- (two vibrations are in phase):

In this case ; $\sin\phi = 0$ and $\cos\phi = 1$

The Eq (3) becomes ;

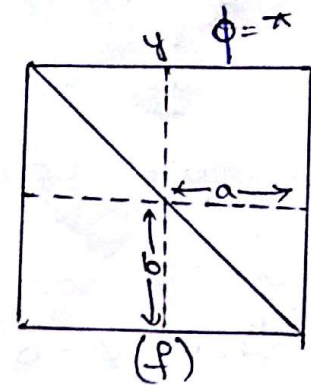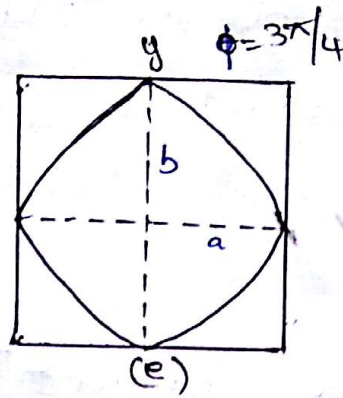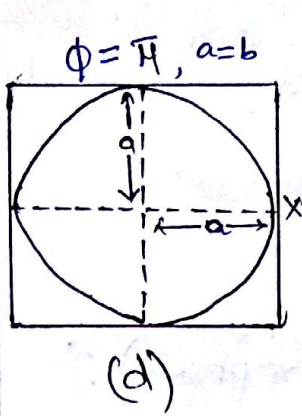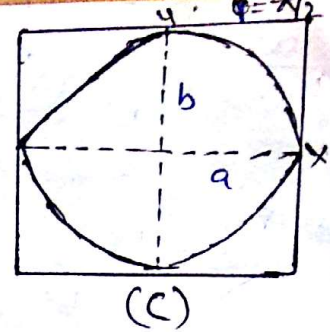$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{2xy}{ab} = 0$$
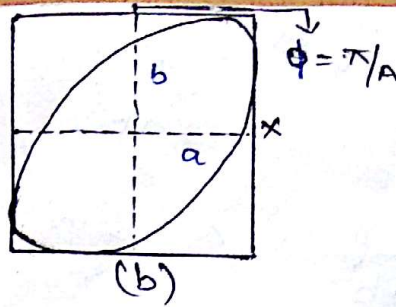
(or) $\left(\frac{x}{a} - \frac{y}{b}\right)^2 = 0$ (or) $\pm\left(\frac{x}{a} - \frac{y}{b}\right) = 0$

$$\therefore \quad y = \pm\frac{b}{a}x \quad \longrightarrow \quad \text{(4)}$$

This represents two coincident straight lines passing through the origin and include to x-axis at the angle θ, given by

$$\theta = \tan^{-1}\left(\frac{b}{a}\right)$$

This is the resultant path of particles as shown in fig.

(a) (b) (c)

$\phi = \pi$, $a=b$ (d) $\phi = 3\pi/4$ (e) $\phi = \pi$ (f)
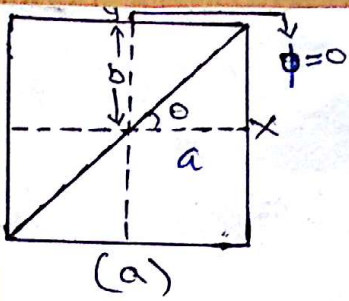
(ii) when $\phi = \dfrac{\pi}{4}$, we have

$$\sin\phi = \frac{1}{\sqrt{2}} \text{ and } \cos\phi = \frac{1}{\sqrt{2}}$$

Now Eq (3) becomes;

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{2xy}{ab} \times \frac{1}{\sqrt{2}} = \frac{1}{2} \longrightarrow \text{⑤}$$

This represents an oblique Ellipse as shown in fig (b)

(iii) when $\phi = \dfrac{\pi}{2}$, we have

$$\sin\phi = 1 \text{ and } \cos\phi = 0$$

The Eq. (3) reduces to

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \longrightarrow \text{⑥}$$

The resultant path in an ellipse whose major axes coincide with the co-ordinate axis as shown in fig (c). If $a = b$ then $x^2 + y^2 = a^2$. So the resultant path of the particle is a circle of radius 'a' as shown in fig (d)

(iv) when $\phi = \dfrac{3\pi}{4}$, we have

$$Sin\phi = \dfrac{1}{\sqrt{2}} \quad \text{and} \quad Cos\phi = -\dfrac{1}{\sqrt{2}}$$

Then Eq ③ becomes

$$\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} - \dfrac{2xy}{ab}\left(-\dfrac{1}{\sqrt{2}}\right) = \dfrac{1}{0} \longrightarrow ④$$

This represents an oblique ellipse as shown in fig (c)

(v) when $\phi = \pi$, we have

$$Sin\phi = 0 \quad \text{and} \quad Cos\phi = -1$$

Now eq. (3) reduces to

$$\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} + \dfrac{2xy}{ab} = 0$$

$$\left(\dfrac{x}{a} + \dfrac{y}{b}\right)^2 = 0$$

$$\pm\left(\dfrac{x}{a} + \dfrac{y}{b}\right) = 0 \quad (or) \quad y = \pm\dfrac{b}{a}x \longrightarrow ⑧$$

This again represents a pair of coincident straight line passing through the origin and inclined to x-axis at an angle θ given by

$$\theta = \tan^{-1}\left(\frac{-b}{a}\right)$$

This is shown in fig (f)

The resultant path traced out by a particle when it is acted open; Simultaneously by two S.H.M's ;. at right angles to each other is known as lissajous figure.

Damped Oscillations :-

In case of an ideal Harmonic oscillator, the amplitude of vibration remains constant for an infinite time. Such vibrations are called free vibrations and the frequency of vibration is called as natural frequency. In practise, the vibrations of a freely falling body gradually diminish in amplitude and ultimately die away. The reason being that the oscillatory system is always subjected to frictional forces araising from air resistance such vibrations are known as Damped vibrations.

Damped Harmonic Oscillator :-

For an ideal Harmonic oscillator, the amplitude of vibrations remains constant for an infinite time. When a body vibrate in air or in any other medium which offers resistance to its motion, the amplitude of vibration decreases gradually & ultimately the body comes to rest. This is due to the fact that the body is subjected to frictional forces araising from air resistance. The motion of the body is known as damped simple harmonic motion. As an example, if we displace a pendulum from its equilibrium position it will oscillate with a decreasing amplitude and finally come to rest in equilibrium position.

EQUATION OF DAMPED HARMONIC OSCILLATOR :-

The damped system is subjected to :

(i) The restoring force which is proportional to displacement but, oppositely directed. This is written as $-\mu x$ where μ is a constant of proportionality or force constant.

ii) A frictional force proportional to velocity but oppositely directed. This may be written as $-\gamma (dx/dt)$, where γ is the frictional force per unit velocity.

since force = mass × acceleration = $md x^2/dt^2$.

Therefore the equation of motion of the particle is given by

$$m \frac{d^2x}{dt^2} = -\mu x - \gamma \frac{dx}{dt}$$

$$\frac{d^2x}{dt^2} + \frac{\gamma}{m} \frac{dx}{dt} + \frac{\mu}{m} x = 0$$

$$\boxed{\frac{d^2x}{dt^2} + 2b \frac{dx}{dt} + w^2 x = 0} \longrightarrow (1)$$

where $\frac{\gamma}{m} = 2b$ and $\frac{\mu}{m} = w^2$

This is D.E of Damped Harmonic motion.

Solution of the Equation :-

Eq(1) is a differential equation of second degree.
Let its solution be

$$x = A \cdot e^{\alpha t} \longrightarrow (2)$$

where A and α are arbitary constants.

Diff (2) w.r.t. to 't' on b.s;

$\cdot \dfrac{dx}{dt} = A\alpha e^{\alpha t}$ and $\dfrac{d^2 x}{dt^2} = A\alpha^2 e^{\alpha t}$.

sub. these values in (1) we have;

$A\alpha^2 e^{\alpha t} + 2bA\alpha e^{\alpha t} + w^2 A e^{\alpha t} = 0$

$A e^{\alpha t}(\alpha^2 + 2b\alpha + w^2) = 0$

as, $A e^{\alpha t} \neq 0$. $\quad \therefore \alpha^2 + 2b\alpha + w^2 = 0$

This gives $\alpha = -b \pm \sqrt{b^2 - w^2}$

The General solution of eq (1) is given by;

$$x = A_1 e^{(-b + \sqrt{b^2 - w^2})t} + A_2 e^{(-b - \sqrt{b^2 - w^2})t} \longrightarrow (3)$$

where A_1 and A_2 are arbitary constants.

$$x_1 = A_1 e^{(-b + \sqrt{b^2 - w^2})t} + A_2 e^{(-b - \sqrt{b^2 - w^2})t}$$

Let, $\sqrt{b^2 - w^2} = R$

At $t = 0$; $x = A$ & $\dfrac{dx}{dt} = 0$.

$$\therefore A = A_1 + A_2.$$

$$\dfrac{dx}{dt} = A_1 (-b + R) e^{(-b + R) \cdot t} + A_2 (-b - R) e^{(-b - R)t}.$$

$$0 = A_1 (-b + R) + A_2 (-b - R)$$

From; $A = A_1 + A_2$

$A(-b - R) = A_1 (-b + R) + A_2 (-b + R)$

$\quad 0 = A_1 (-b + R) + A_2 (-b - R)$

$\quad (-) \qquad\qquad t^? \qquad\qquad (-)$

$A(-b + R) = A_2 [-b + R + b + R]$

$A(-b + R) = 2 A_2 R \Rightarrow A_2 = \dfrac{A}{2R}(-b + R)$

Therefore, $A_1 = \frac{A}{2P}(b+P)$

$$x = \frac{A}{2P}(b+P)e^{(-b+P)t} + \frac{A}{2P}(-b+P)e^{(-b-P)t}$$

$$x = \frac{A}{2}e^{-bt}\left[\left(1+\frac{b}{P}\right)e^{Pt} + \left(1-\frac{b}{P}\right)e^{Pt}\right]$$

$$x = \frac{Ae^{-bt}}{2}\left[1+\frac{b}{\sqrt{b^2-\omega^2}}e^{(\sqrt{b^2-\omega^2})t} + \left(1-\frac{b}{\sqrt{b^2-\omega^2}}\right)e^{-(\sqrt{b^2-\omega^2})t}\right]$$

DIFFERENT DAMPING CONDITION :-

Case 1 :- (overdamped motion) :

When $b^2 > \omega^2$. In this case $\sqrt{b^2-\omega^2}$ is real and less than b. Now the powers

$$(-b+\sqrt{b^2-\omega^2}) \text{ and } (-b-\sqrt{b^2-\omega^2})$$

in eq(3) are both negative. Thus the displacement x consists of two terms, both dying off exponentially to zero without performing any oscillations. The rate of decrease of displacement is governed by $(-b+\sqrt{b^2-\omega^2})t$ as the other term reduces to zero quickly relative to it. In this case, the body once displaced returns to its equilibrium position quite slowly without performing any oscillation. This type of motion is called overdamped or dead beat.

This type of motion is shown by pendulum moving in a thick oil or by a dead beat moving coil galvanometer.
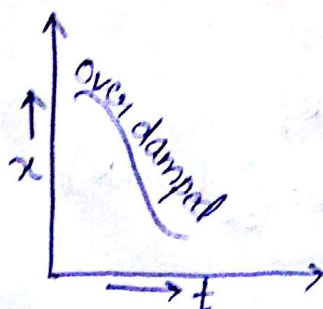
Case - II (Culical Damping) :-

When $b^2 - w^2$. If we put $b^2 = w^2$ in eq(3), then this solution does not satisfy the D.E (1). Let us consider that $\sqrt{b^2 - w^2}$ is not zero but this is equal to very small quantity h ;

i.e ; $\sqrt{b^2 - w^2} = h \longrightarrow 0$.

Now eq(3) reduces to ;

$$x = A_1 e^{(-b+h)t} + A_2 e^{(-b-h)t}$$

$$= e^{-bt} \left[A_1 e^{ht} + A_2 e^{-ht} \right]$$

$$= e^{-bt} \left[A_1 (1 + ht + \cdots) + A_2 (1 - ht + \cdots) \right]$$

$$= e^{-bt} \left[(A_1 + A_2) + ht (A_1 - A_2) + \cdots \right]$$

$$= e^{-bt} \left[p + qt \right]$$

where $p = A_1 + A_2$ and $q = h(A_1 - A_2)$

Eq (4) represents a possible form of solution. It is clear from eq(4) that as t increases, the factor $(p+qt)$ increases but the factor e^{-bt} decreases. In this way the displacement x first increases due to factor $(p+qt)$ but at the same time reversal occurs due to exponential term e^{-bt} and the displacement approaches zero as t increases. It is also clear that in this case the exponent is $-bt$ while in the first case it was more than $-bt$, hence in this case the particle tends to acquire its position of equilibrium much rapidly than in I case. Such a motion is called critical Damped motion.

This type of motion is exhibited by many pointer instruments such as voltmeter, ammeter etc, in which point moves to correct position and comes to rest without any oscillation in minimum time.

Case III : (Under Damped motion) :-

When $b^2 < w^2$. In this case $\sqrt{b^2 - w^2}$ is imaginary.

Let us write,

$$\sqrt{b^2 - w^2} = i\sqrt{w^2 - b^2} = i\beta.$$

where, $\beta = \sqrt{w^2 - b^2}$ and $i = \sqrt{-1}$

where ; eq(3) becomes;

$$x = A_1 e^{(-b + i\beta)t} + A_2 e^{(-b - i\beta)t}$$

$$x = e^{-bt}(A_1 e^{i\beta t} + A_2 e^{-i\beta t})$$

$$x = e^{-bt}\left(A_1(\cos\beta t + i\sin\beta t) + A_2(\cos\beta t - i\sin\beta t)\right)$$

$$x = e^{-bt}\left((A_1 + A_2)\cos\beta t + i(A_1 - A_2)\sin\beta t\right)$$

$$x = e^{-bt}\left(a\sin\emptyset \cos\beta t + a\cos\emptyset \sin\beta t\right)$$

where, $a\sin\emptyset = A_1 + A_2$. and $a\cos\emptyset = i(A_1 - A_2)$

$$\therefore x = e^{-bt}\left(a\sin(\beta t + \emptyset)\right)$$

$$\Rightarrow x = a e^{-bt}\left(\sin\left(\sqrt{w^2 - b^2}\right)t + \emptyset\right) \longrightarrow ⑤$$

This equation represents the simple harmonic motion with amplitude . $a e^{-bl}$ and time period ;

$$T = \frac{D\pi}{\beta} = \frac{D\pi}{\sqrt{w^2 - b^2}}.$$

The amplitude of the motion is continuously decreasing to the factor e^{-bt} which is called the Damping factor. Because of the value of; $\sin((\sqrt{w^2-b^2})t+\phi)$ varies between +1 and -1 therefore the amplitude also varies between ae^{-bt} and $-ae^{-bt}$. The decay of amplitude depends upon the damping coefficients b. It is called "Under Damped motion". In this case the period is slightly increased or frequency is decreased because the period is now $2\pi/\sqrt{w^2-b^2}$ while in the absence of damping it was $2\pi/w$
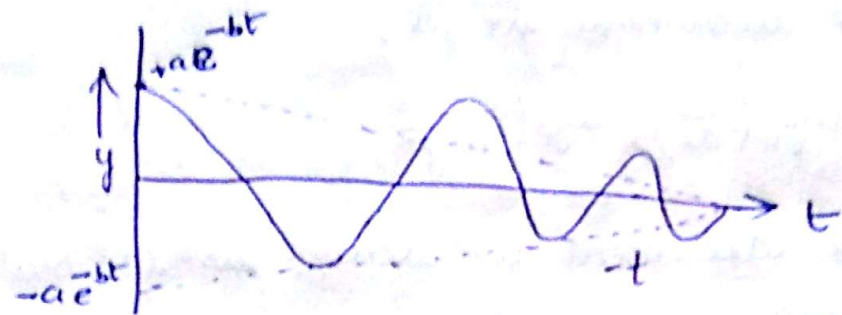
The example of this type of motion is the motion of pendulum in air, the motion of the coil of ballistic Galvanometer or the electric oscillations of L·C·R circuit.

LOGARITHMIC DECREMENT:-

Logarithmic Decrement measures the rate at which the amplitude dies away. The amplitude of damped Harmonic oscillator is given by;

amplitude $= a \cdot e^{-bt}$ at $t=0$,
amplitude $a_0 = 0$.

Let $a_1, a_2, a_3 \ldots$ be the amplitudes at time;
$t = T, 2T, 3T\ldots$ respectively where;
$T=$ period of oscillation then;

$$a_1 = ae^{-bt}$$

$$a_2 = ae^{-b(2T)}$$

$$a_3 = ae^{-b(3T)}$$

$$\cdots\cdots$$

From these equations we get;

$$\frac{a_0}{a_1} = \frac{a_1}{a_2} = \frac{a_2}{a_3} = \cdots = e^{bT} = e^\lambda \quad (\lambda = bT)$$

λ is known as Logarithmic decrement.

Taking the natural logarithmic we get;

$$\lambda = \log_e\left(\frac{a_0}{a_1}\right) = \log_e\left(\frac{a_1}{a_2}\right) = \log_e\left(\frac{a_2}{a_3}\right) = \cdots$$

"Thus logarithmic decrement is defined as the Natural logarithmic of the ratio between two successive maximum amplitudes which are seperated by one period."

FORCED VIBRATIONS :-

Forced vibrations can be defined as the vibrations in which the body vibrates with a frequency other than its natural frequency under the action of an external periodic force.

Theory of forced vibration :-

Equation of forced vibrations :-

The forces acted upon the particle are :

1) A restoring force proportional to the displacement but oppositely directed, given by

$$-\mu x .$$ where, μ = force constant

2. A fructional force proportional to the velocity but oppositely directed, given by

$$-r\frac{dx}{dt}$$ where, $r=$ fructional force per unit velocity

3. The external periodic force, represented by,

$$F\sin(pt)$$

where, $F=$ the maximum value of the force

$$P/2\pi = \text{frequeny}$$

so the total force acting on the particle is given by,

$$-\mu x - r\frac{dx}{dt} + F\sin(pt)$$

The impressed periodic table force is called driver and the body executing forced vibrations is called driven oscillator.

By Newton's second law of motion this must be equal to the product of mass m of the particle and its instaneous acceleration i.e, $m\cdot\frac{d^2x}{dt^2}$

Hence,

$$m\frac{d^2x}{dt^2} = -\mu x - r\frac{dx}{dt} + F\sin pt.$$

$$m\frac{d^2x}{dt^2} + r\frac{dx}{dt} + \mu x = F\sin pt.$$

$$\frac{d^2x}{dt^2} + \frac{r}{m}\frac{dx}{dt} + \frac{\mu}{m}x = \frac{F}{m}\sin pt$$

$$\boxed{\frac{d^2x}{dt^2} + 2b\frac{dx}{dt} + w^2 x = f\sin pt}$$

where, $\frac{r}{m}=2b$; $\mu/m = w^2$. & $\frac{F}{m}=f$

eq(1) is the differential eqn of the motion of the particle

Condition of Amplitude Resonance :-

In case of forced vibrations,

$$A = \frac{f}{\sqrt{(\omega^2 - p^2)^2 + 4b^2 p^2}}$$

The above expression shows that the amplitude varies with the frequency of the force p. For a particular value of P, the amplitude becomes maximum. The phenomenon is known as Amplitude Resonance. The amplitude is maximum when;

$$\sqrt{(\omega^2 - p^2)^2 + 4b^2 p^2} \text{ is minimum.}$$

$$\frac{d}{dp}\left((\omega^2 - p^2)^2 + 4b^2 p^2\right) = 0.$$

$$2(\omega^2 - p^2)(-2p) + 4b^2(2p) = 0.$$

$$\omega^2 - p^2 = 2b^2.$$

$$P = \sqrt{(\omega^2 - 2b^2)}$$

Thus the amplitude is maximum when the frequency $P/2\pi$ of the impressed force becomes $\sqrt{(\omega^2 - 2b^2)}/2\pi$. This is the resonant frequency. This gives frequency of the system both in presence of damping i.e, $\sqrt{\omega^2 - 2b^2}/2\pi$ and in absence of damping i.e $\omega/2\pi$.

If the damping is small then it can be neglected and the condition of maximum amplitude reduced to,

$$P - \omega$$

By putting condition ③ in ①, we get;

$$\therefore A_{max} = \frac{f}{\sqrt{(w^2 - w^2 + 2b^2)^2 + 4b^2(w^2 - 2b^2)}}$$

$$= \frac{f}{\sqrt{4b^2 w^2 - 4b^4}}$$

$$= \frac{f}{2b\sqrt{w^2 - b^2}}$$

$$= \frac{f}{2b\sqrt{p^2 + b^2}} \qquad (\because p^2 = w^2 - 2b^2)$$

Forcing Frequency $\longrightarrow$
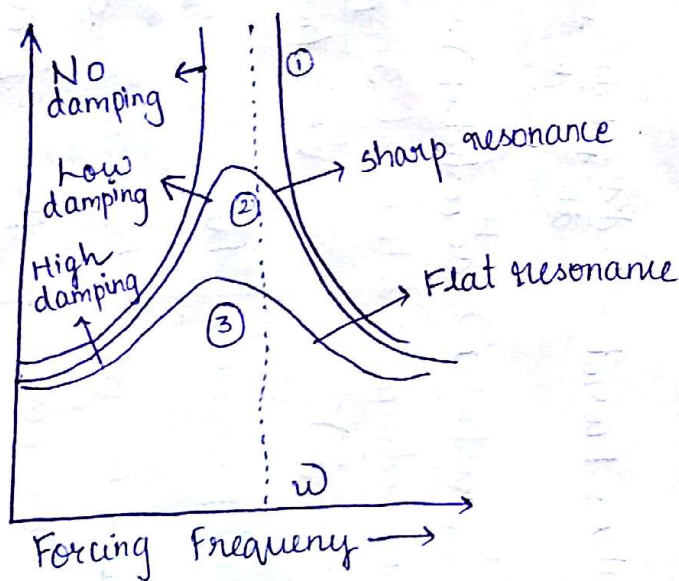
It was observed that the pack of the curves moves towards the left. It is also observed that the value of A, which is different for different values of b (damping), diminishes as the value of b increases

For smaller values of b, the fall in the curve about $w = p$ is steeper than for large values. This shows that the value of damping greater is the departure of amplitude of forced vibration from the maximum value and vice-versa.

SHARPNESS OF RESONANCE :-
If the frequency changes from this value, the amplitude falls. When the fall in amplitude for a small departure from the resonance condition is very large the resonance is said to be sharp. On the other hand if the fall in

amplitude is small, the resonance is terms as flat. Thus the sharpness of resonance means the rate of fall in amplitude, with the change of forcing frequency on each side of resonance frequency.

Hence smaller is damping, sharper is resonance or larger is the damping, flater is the resonance.

ANALOGY b/w MECHANICAL AND ELECTRICAL OSCILLATOR.

	Mechanical Oscillator	electrical oscillator.
Independent term	time (t)	time (t)
Dependent term	displacement (x)	charge (q)
Inertial term	mass (m)	Inductance (L)
Resistance	mechanical resistance (r)	electrical resistance (R)
Stiffness constant	spring constant (k)	Reciprocal of capacitance
Differential equation	$m\frac{d^2x}{dt^2}+r\frac{dx}{dt}+kx=F\cos\omega t$	$L\frac{d^2q}{dt^2}+R\frac{dq}{dt}+\frac{q}{C}=E_0\cos(\omega t)$
Solution	$ae^{-\frac{bt}{2}}\cos(\omega t+E)+\frac{F}{\omega z_m}\cos(\omega t-E)$	$ae^{-\frac{bt}{2}}\cos(\omega t+E)+\frac{E_0}{\omega-z}\cos(\omega t-E)$
Impedance	mechanical impedance ; $Z_m=\sqrt{r^2+(m\omega-\frac{k}{\omega})^2}$	electrical impedance ; $Z=\sqrt{R^2+(\omega L-\frac{1}{\omega C})^2}$
Resistance	frequency independent	frequency, independent.
Reactance	frequency dependent	frequency dependent
Impedance	Collective opposition offered to flow of current by two or more elements in the circuit.	

Acoustics

Introduction :

The branch of Physics that deals with the process of production, propagation and reception of Sound in a room or auditorium is called Acoustics. W. C. Sabine, Professor of Physics in Harvard University made a systematic study of Sound in the year 1911 and hence can be called the pioneer in the field of Acoustics.

Echo Vs Reverberation :

The reflection of Sound off the distant surface is called Echo while Reverberation is defined as the prolongation of Sound even after the source is switched off.

Reverberation Time (T) :

It is the Time required for the intensity of sound to drop by one millionth of (10^{-6}th) of its initial value. It is also defined as the time required for the intensity to drop by 60 decibels.

Absorption Coefficient (a) :

The coefficient of absorption of a material is defined as the ratio of the Sound energy absorbed by the surface to that of the total sound energy incident on the surface.

$$\text{Absorption coefficient}, \ a = \frac{\text{Sound energy absorbed by the surface}}{\text{Total sound energy incident on the surface}}$$

Any given material partially absorbs sound energy and partially reflects sound energy. An open window on the other hand completely transmits sound energy (which can be assumed as absorption) such that sound energy is not at all reflected. Hence it can be considered as Ideal absorbent. Therefore Absorption coefficient is measured in the units of OWU that is Open window units. Obviously an open window has got an absorption coefficient of 1.0 OWU.

Sabine's Formula for Reverberation time:

W. C. Sabine, Professor of Physics in Harward University proposed the following formula for Reverberation time (T).

$$T = \frac{0.165 \, V}{\Sigma \, a \, S}$$

where (V) is the Volume of the hall.

(a) is the absorption coefficient of the material

(S) is the surface area of the material.

Basic requirements of Acoustically good hall:

1. Volume of the auditorium is decided by the purpose that it serves. The following are the requirements

Lecture halls — 2.8 to 3.7 m³ per seat

Movie theatres — 3.7 to 4.2 m³ per seat

Music halls — 4.2 to 5.6 m³ per seat

It is very much clear that a music hall requires larger volume while a lecture hall requires smaller volume. Also height plays an important role in deciding the volume. The ratio between the ceiling height and breadth should be 2:3.

2. The shape of the wall and ceiling should be so as to provide uniform distribution of sound throughout the hall.

3. Reverberation time should neither be small nor be large.

$$T_{speech} = 0.5 \text{ to } 1 \text{ second}$$

$$T_{Music} = 1 \text{ to } 2 \text{ second}.$$

4. The sound heard must be sufficiently loud in every part of the hall.

5. Successive syllables spoken must be clear and distinct.

MREC

6. Sound should not get concentrated in one part of the hall.

7. Extraneous noise should be excluded with sufficiently sound proof boundaries.

Factors affecting the architectural acoustics and their remedies:

REVERBERATION:

Large reverberation in a hall leads to overlapping of successive sounds that results in loss of clarity in hearing while small reverberation results in inadequate loudness. Therefore optimum reverberation time is required, as given by Sabine's formula, that is satisfactory to both the speaker and the audience.

Remedy:

1. Provision of windows and ventillators that can be opened and closed to make the reverberation time optimum.

2. Usage of heavy curtains with folds.

3. Covering the floors with carpets.

LOUDNESS:

Erecting good absorbent materials all along the hall will definitely make the time of reverberation smaller. Though it minimises the chances of confusion between the different syllables, the intensity of sound might go below the level of intelligibility of hearing. Therefore sufficient loudness at every point is very much essential.

Remedy:

1. Using large sounding boards behind the speaker, facing the audience.

2. Low ceilings reflect sound energy towards the audience.

3. Positioning loud speakers at appropriate locations to achieve uniform distribution of sound intensity.

MREC

FOCUSSING:

Focussing surfaces such as spherical or concave surfaces on the walls or ceiling of the hall produce concentration of sound in particular regions while sound does not reach few parts.

Remedy:

Curved surfaces should be covered with absorbent material.

ECHO:

Reflection of sound off the distant surface is called Echo and it causes lot of confusion.

Remedy:

Echoes can be ~~covered by~~ avoided by covering the long distant walls with absorbent material.

AIR BORNE NOISE:

The extraneous noise that reaches a hall through open windows, doors and ventillators is known as Air-borne noise.

Remedy:

(i) using heavy glass in doors, windows and ventillators
(ii) Using double doors and windows with insulating material between them.
(iii) by avoiding openings for pipes and ventillators.

STRUCTURE BORNE NOISE:
The noises which are conveyed through the structure of the building are known as Structural noises.
Remedy: Mechanical equipments such as refrigerator, lifts etc produce vibration in the structure. These can be checked by insulating the equipments properly.

INSIDE NOISE:
The noises which are produced inside the hall or rooms in big offices are called Inside noises.
Remedy: (i) Machinery like type-writers should be placed on absorbent pads.
(ii) Any engine inside the hall should be fitted on the floor with a wood between the machine and the floor.
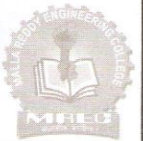
Ultrasonics

Introduction: Sound waves of frequencies ranging from 20 Hz to 20 kHz are called sonic waves or audible waves, as they are perceived by human ear. Sound waves with the frequencies lesser than 20 Hz are called Infrasonics, while those whose frequency is greater than 20 kHz (i.e., beyond the audible limit) are called Ultrasonic waves or Supersonics. Though humanbeings are not capable of hearing Ultrasonic waves, certain animals like dogs, bats, and marine animals like whales, sharks, dolphins have the ability to hear the high frequency sounds.

Bats and dolphins are known to generate ultrasonic waves and use the reflections of the waves to find their way. The waves reflected from the surrounding objects are perceived by the bats and from the time elapsed between the generation and reflection of the pulses, the direction and distance of the objects are determined. The large ears of bats are specialised to detect these sounds.
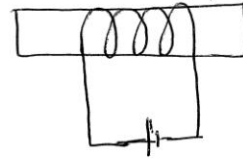
Light is strongly absorbed by sea-water and hence the radius of visibility is limited. Relatively, ultrasonic waves are less absorbed by sea-water. Thus marine animals use ultrasonic pulses to locate fish, to avoid obstacles.

In this chapter we will be learning production, detection of ultrasonic waves. Also we will be learning the properties of ultrasonics and their applications.

Magnetostriction effect :-

When a ferromagnetic rod such as iron, or nickel is kept in a magnetic field parallel to its length, the rod suffers a change in its length. This phenomenon is known as magnetostriction effect. This change in length depends on the magnitude of the field and the nature of the material. This effect was discovered by Joule in 1847.

Explanation : The adjacent figure shows a wire wound around the ferromagnetic rod. When the D.C field is turned on current flows through the wire. Let the magnetic field associated with this d.c. current increases the length of the rod. When the polarity of the d.c. field is reversed then length of the rod decreases. When an A.C. field is applied ~~the length of~~ the rod elongates and contracts for each half cycle of the A.C. signal i.e., the rod vibrates with a frequency twice that of the A.C. signal. The amplitude of vibration is usually small, but if the frequency of the A.C. signal coincides with the natural frequency of the rod, the amplitude of vibration increases due to resonance.
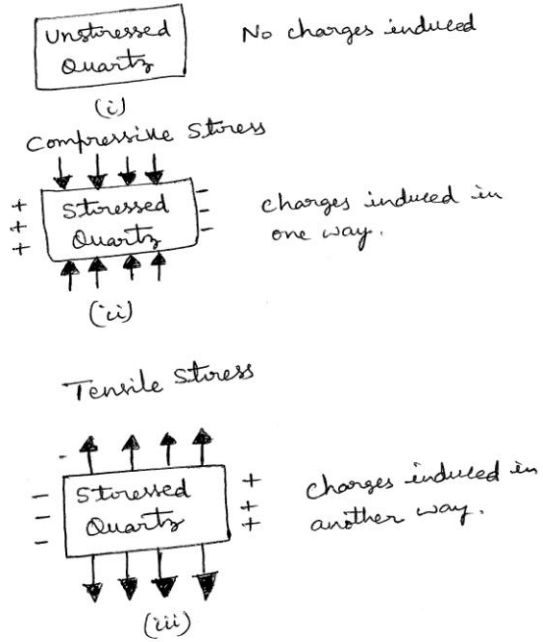
~~Piezoelectric effect~~ :-

Piezo electric effect:

When crystals like quartz or tourmaline are stressed along any pair of opposite faces, electric charges of opposite polarity are induced in the opposite faces perpendicular to the stress. This is known as Piezo electric effect.

Unstressed Quartz — No charges induced

(i)

Compressive Stress

Stressed Quartz — charges induced in one way.

(ii)

Tensile Stress

Stressed Quartz — charges induced in another way.

(iii)

Explanation:

Fig (i) shows an unstressed quartz crystal. As there is no mechanical stress, electrical charges are not induced.

When a compressive stress is applied on the quartz crystal, electrical charges will be induced on two opposite faces, perpendicular to the direction of the mechanical stress. Let the charges be induced as shown in fig (ii).

When the direction of the mechanical stress is reversed, ie; if a tensile stress is applied instead of compressive stress then the electrical charges will be induced in a way opposite to that of the previous case, as shown in fig (iii).

Piezo electric Crystal (Additional information).

Quartz crystal is the most popular Piezo electric crystal. It has the shape of a hexagonal prism with a pyramid attached to each end; as shown in the adjacent figure. The axis along the longest dimension of the natural crystal is called Optic axis or Z-axis. The three lines, which pass through the opposite corners of the crystal, constitute its three x-axes or electrical axes. Similarly, the three lines which are perpendicular to the sides of the hexagon ~~form~~ form the three Y-axes which are known as Mechanical axes.
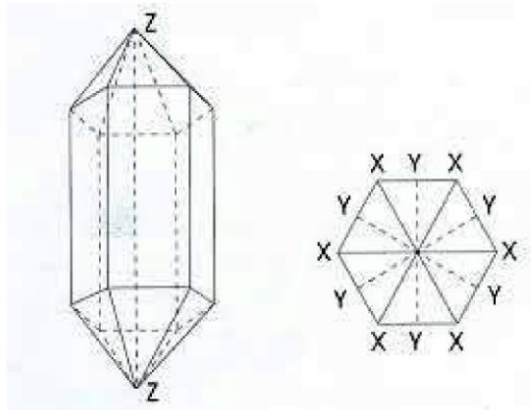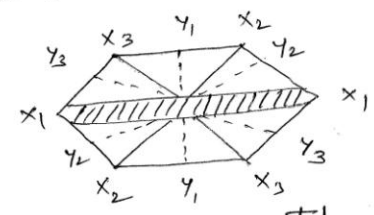

Thin plates of the quartz crystal cut perpendicular to one of its x-axis are known as x-cut plates. Similarly thin plates cut perpendicular to one of its y-axis are known as Y-cut plates.

x-cut plates generate longitudinal mode of Ultrasonic vibrations while Y-cut plates generate transverse mode of vibrations.
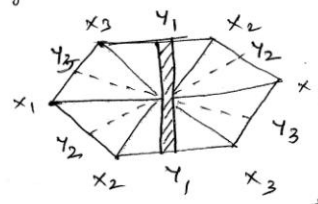
X-cut crystal

Y-cut crystal

Inverse Piezo electric effect:

When crystals like Quartz or Tourmaline are subjected to electric field on the opposite faces then they undergo either contraction or expansion in a perpendicular direction. This is known as ~~piezo~~ Inverse piezo electric effect.

Explanation :-

Fig (i) shows a non-electrified quartz crystal. As there is no electrical field, neither contraction nor expansion is observed.

When an electric field is applied as shown in fig(ii), a compressive force acts on the crystal such that the crystal is subjected to contraction.

When the electric field is reversed as shown in fig (iii), a tensile force acts on the crystal such that the crystal is subjected to expansion.

When an A.c. field is applied to the opposite faces of the crystal, it undergoes contraction and expansion alternatively in the perpendicular direction.

| Non electrified Quartz |
| --- |

(i)

Contraction

+	Electrified Quartz	−
+		−

(ii)

Expansion

−	Electrified Quartz	+
−		+
−		+

(iii)

Production of ultrasonic waves :—

1. Magnetostriction Method :—

The general principle involved in generating ultrasonic waves is to cause some dense material to vibrate very rapidly. The vibrations produced by this material then cause the air surrounding the material, to begin vibrating with the same frequency. These vibrations then spread out in the form of ultrasonic waves.
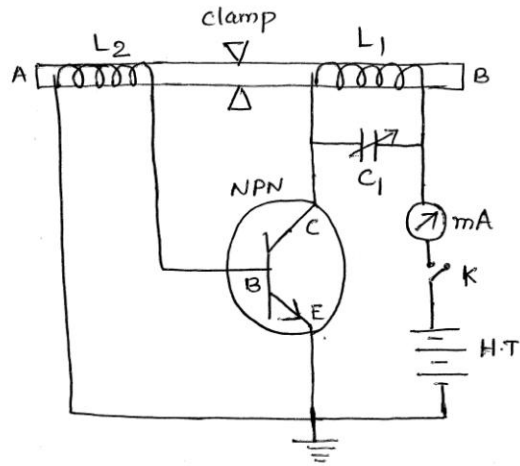
construction :

1. Figure shows the ferro magnetic rod AB wound would by the coils L_1 & L_2.

2. A variable capacitor C_1 is connected parallel to the coil L_1. One end of the capacitor (C_1) is connected to the collector (C) of the n-p-n transistor. The other end of the capacitor (C_1) is connected to battery via the milli Ammeter and the key (K). The battery helps in providing necessary biasing.

2. Clamp helps in properly holding the rod AB. The coil L_2 is connected to the base (B) and the emitter (E) of the n-p-n NPN transistor, as shown in the figure.

Working:

When the key (K) is on, a D.C. current is passed through the L_1 coil which produces a stationary magnetic field around it and it induces an emf in L_2 coil. The induced emf is fed to the base of the transistor and amplified and again gives to the

L_1 coil. Due to the additional emf in L_1 coil, the magnetic field in L_1 coil is changed. In this way a varying magnetic field is produced around the specimen which produces mechanical vibrations and generates Ultrasonic waves. If the frequency of the oscillator circuit (which can be varied by changing c_1) is equal to the natural frequency of the specimen, then it vibrates with maximum amplitude and ~~they~~ their frequencies can be represented as

$$f = \frac{1}{2\pi\sqrt{LC}}$$

$$\text{and} \quad f = \frac{1}{2l}\sqrt{\frac{Y}{\rho}}$$

where l - length of ferromagnetic rod

Y - Young's modulus of the material of the specimen.

ρ - density of the rod

Advantages :-

1, By using this method we can produce ultrasonic waves of frequency upto 300 kHz .

2, Magnetostrictive materials are easily available and inexpensive.

Disadvantages :-

1, Frequencies beyond 300 kHz can not be generated.

2, As the rod dimensions are influenced by the temperature, production of constant single frequency is not possible.

crystal

2. Piezo electric method :

Principle :

    Piezo electric specimen is subjected to an A.C. electric field, which induces mechanical vibrations in the specimen. In short, Inverse Piezo electric effect is the principle behind this method of generating Ultrasonics.

Construction

1. Figure shows a Piezo electric specimen placed between the plates of A and B of the capacitor which are in turn connected to the coil (L_3).

2. The variable capacitor (C_1) is connected parallel to the coil (L_1). One end of the capacitor (C_1) is connected to the collector (C) of the NPN transistor while the other end is connected to the battery, via milli ammeter and key (K). The battery helps in providing necessary biasing.

3. Coil (L_2) is connected to the base (B) and emitter (E) of the NPN transistor, as shown in the figure.

4.

Working :

    When the key (K) is turned ON, current starts flowing and the capacitor (C_1) gets charged. This capacitor (C_1) then discharges through the coil (L_1). The discharge through the coil (L_1) produces a magnetic field, which induces an emf in the coil (L_2). The induced emf in the coil (L_2) is fed to the base (B) of the NPN transistor. This acts as feedback to the transistor. Thus the oscillations in the coil (L_1) and (L_2) are sustained. These electrical oscillations in (L_1) and (L_2)
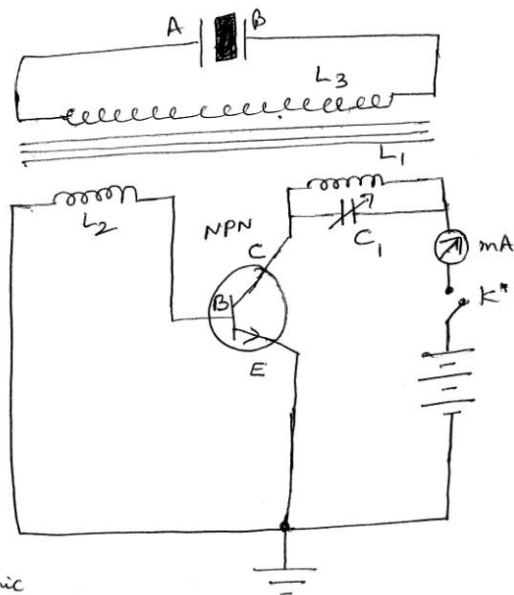
induce emf in the coil (L_3) by means of mutual induction. The induced emf is applied to the Piezo electric specimen via the plates A and B of the capacitor. The alternating emf thus applied on the piezo electric specimen induces mechanical vibrations in the specimen, thereby producing Ultrasonic waves.

~~Merits~~ If (l) is the length of the Piezo electric specimen, (Y) and (P) are the Young's modulus and density of the specimen respectively, then the frequency of the Ultrasonics produced is given by

$$f = \frac{1}{2l} \sqrt{\frac{Y}{P}}$$

Merits:

1. High frequency waves upto 500 KHz frequency can be produced.

Demerits:

1. It is relatively expensive.

Detection of Ultrasonic waves:

1. Piezo electric detector:
   In this method, Ultrasonic waves are applied to one pair of faces of a quartz crystal. As a result, opposite charges are induced on the other pair of opposite faces. These charges, being small, are amplified and detected.

2. Kundt's tube:
   A Kundt's tube can be used to detect Ultrasonic waves of relatively longer wavelengths. Stationary Ultrasonic waves are produced in air contained in a long tube. Lycopodium powder sprinkled along the inner surface of the tube collects into small heaps at the nodes and is blown off at the antinodes. The distance between two successive heaps is equal to $(\lambda/2)$. Thus by knowing

the frequency of the waves velocity of the Ultrasonic waves can be calculated.

Sensitive flame method:

When a narrow sensitive flame is moved ~~through~~ in a the medium where of Ultrasonic waves, are present, it is observed that the flame remains steady at the positions of antinodes and flickers at the nodes. By measuring the distance between the adjacent nodes, the value of $(\lambda/2)$ can be determined. Thus velocity can be calculated by knowing the frequency of the Ultrasonic wave.

Thermal detector method:

A fine platinum wire probe is used in this method. When stationary Ultrasonic waves are produced in a medium, there occurs a change in temperature at nodes. As the Platinum probe moves through the medium, its resistance changes at nodes. The change in the resistance of Platinum wire is detected by using a sensitive Wheatstone bridge.

Properties of Ultrasonic waves:

1. Ultrasonic waves have high frequencies and therefore they are highly energetic

2. The speed of Ultrasonic waves depends on their frequency. It increases with increase in frequency.

3. Owing to their small wavelengths, Ultrasonic waves can be transmitted over long distances without any appreciable loss of energy.

4. When Ultrasonic waves are passed through a liquid, stationary Ultrasonic waves are set up in the liquid due to the incident and reflected waves. This standing wave forms compressions and rarefactions in the liquid medium which act as parallel rulings of a diffraction grating.

Applications of ultrasonic waves :—

Ultrasonic waves have a variety of applications in different branches of science and technology. some important applications are:

(1) communication :

a) SONAR:
The word SONAR stands for sound navigation and ranging. The highly directional ultrasonic waves can be used for locating objects submerged under sea water. In SONAR, an Ultrasonic beam is directed in different direction into Sea. In the absence of an obstacle, the Ultrasonic pulses do not return to the ship. In the presence of an obstacle, pulses are reflected from the obstacle and are ~~pick~~ picked up by the receiver. If the speed of the Ultrasonics in sea water is known, distance of the object, '(l) is determined using the equation

$$\boxed{l = \frac{(v)(t)}{2}}$$

where (v) is the velocity of Ultrasonic waves
(t) is the time elapsed between the transmitted and reflected pulses.

b) Fish finder :
Ultrasonics can be used to locate shoals of fish, utilising the fact that the swimming bladder of fish is filled with air, that scatters ultrasonic waves. ~~Some of the sea~~ Some of the sea animals such as whales and dolphins use Ultrasonics to locate their prey and even to converse with each other In the depths of the sea, visibility is highly restricted because of the strong absorption of light by water. It may be therefore that these animals use Ultrasonics that ~~is~~ are relatively less absorbed.

(2) Industrial application :

a) Ultrasonic Drilling :
The magnetostriction vibrator is made of thin isolated ferromagnetic plates of high magnetostriction, such as Nickel. A coil is wound on the needle through
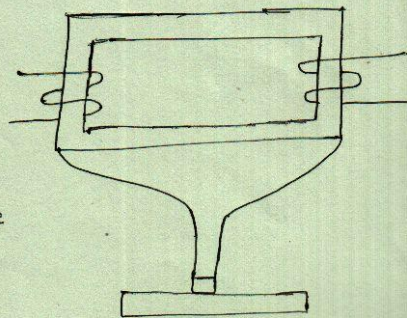
Ultrasonic drill

which alternating current is passed. The resulting magnetic field magnetises the core and changes the length of the vibrator. This action rapidly chips away the work-piece in a pattern controlled by the tool shape.

(b) Ultrasonic welding :

The surfaces of the work pieces are cleaned and held together. They are subjected to Ultrasonic oscillations ~~where the~~ at the spot where they are welded. The Ultrasonic energy converts to heat at the contact area as a result of friction arising between the surfaces. As the temperature of surface layers exceeds the recrystalisation point, the layers melt and bond together to form a strong joint.

(3) Biological and medical applications :

(i) High intensity Ultrasonic waves are used for killing bacteria, germs and insects. Therefore they are used to Sterilise milk.

(ii) These waves are used to destroy diseased tissues. This use of Ultrasonic waves is Known as Knifeless surgery.

(iii) Ultrasound is widely available, easy-to-use and lesser expensive than other imaging methods. It gives a clear picture of soft tissues that do not show up well on X-ray images. Ultrasound scanning is the preferred imaging method for the diagnosis and monitoring of pregnant women.

MODULE-II: LASER AND FIBER OPTICS

A. LASERS

Introduction:

LASER is a device that emits light through a process of optical amplification based on the Quantum effect called stimulated emission of electromagnetic radiation. LASER is an acronym for Light Amplification by Stimulated Emission of Radiation.

Distinguish between conventional light and Laser light:

| Conventional light	Laser light
1. It emits photons in all directions with wide range of wavelengths.	1. It emits photons in a narrow, well-defined directional beam.
2. These are incoherent (No fixed phase among the photons emitted by the source).	2. These are highly coherent (constant phase relationship among the photons)
3. May be multi-wavelength	3. Single wavelength or color.
4. Intensity is low	4. Intensity is very high.
5. Ex.:- Electric Bulb, candle, etc.,	5. Ex.:- Ruby Laser, He-Ne Laser, etc.,

Characteristics of Laser light

Like ordinary light laser light is electromagnetic in nature. However, there are few characteristics not processed by the normal light. Some of the main characteristics of laser beam are mentioned below:

1. **High Monochromaticity,**
2. **Extremely Directional,**
3. **Extremely Coherence,**
4. **High Intensity and brightness**.

1. **Monochromaticity**: The light emitted from a laser is highly *monochromatic*, that is, it is of one unique wavelength (color). The light from a laser typically comes from one atomic transition with a **single precise wavelength**. So the laser light has small broadening width ($\Delta\lambda$) and is almost the purest monochromatic light.


For Ordinary white light, $\Delta\lambda$= 300 nm, For monochromatic light, $\Delta\lambda$= 5-10 nm
For Laser light, $\Delta\lambda$= 0.2 nm
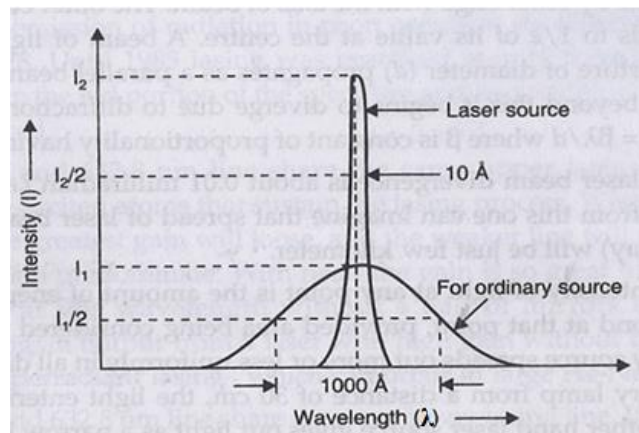
2. **Directionality:** We know that when light travels then it tends to spread out, this spreading of light is called divergence and the angle at which the light spreads is called angle divergence (ϕ). Generally divergence is more in ordinary light and lesser in laser light.

   Usually a laser generates less than one mille radian (0.001) divergent beam. This means that a beam from the laser will spread to less than 1cm diameter at a distance of 10 m from the laser. The laser beam divergence is illustrated in the adjacent figure.

divergence, $\phi = \frac{r_2 - r_1}{d_2 - d_1}$
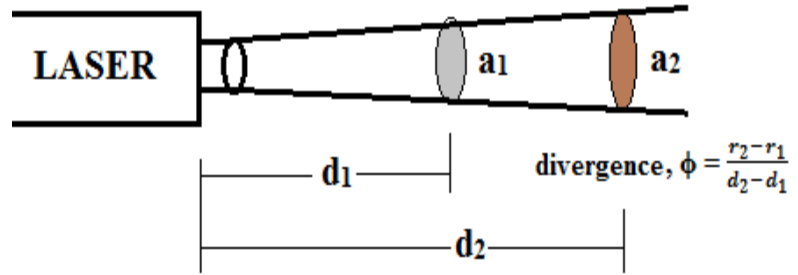
3. **Coherence:** The light coming from the Laser is said to be coherent, which means the group of photons (Laser beam) are in phase in space (Spatial) and time (Temporal). Laser light is much more coherent than ordinary light.
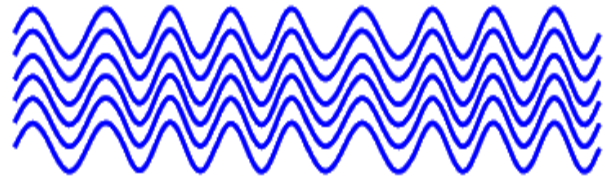

Figure: Coherent Light Waves

4. **Intensity and Brightness:** Intensity of a wave is defines as energy per unit time flowing through a unit normal area. In Laser light, energy is concentrated in small region of space with small wavelengths with greater intensity. A laser beam has brightness many times in magnitude greater than that of conventional sources due to high directional property of laser beam.

> **Therefore, Laser light is an extra ordinary light emitted under stimulated and amplified conditions, so that the beam is characterized by high intensity, specific directionality, high monochromaticity and high degree of coherence.**

## ABSORPTION, SPONTANEOUS AND STIMULATED EMISSION:

To describe the phenomenon of Absorption, spontaneous or stimulated emission, let us consider two energy levels, 1 and 2, of some atom or molecule of a given material, their energies being E_1 and E_2 ($E_1 < E_2$).

i. **Absorption:** Let us now assume that the atom is initially lying in level 1 (Adj. Fig.). If this is the ground level, the atom will remain in this level unless some external stimulus (photons) is applied to it. We shall assume that, a photon of frequency ν is incident on the material. In this case there is a finite probability that the atom will be raised to level 2. The energy difference $E_2 - E_1$ required by the atom to
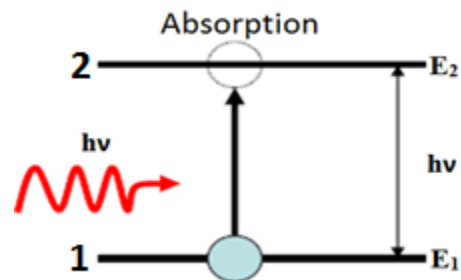
undergo the transition is obtained from the energy of the incident photon. This is the *absorption* process.

ii. **Spontaneous emission:** Let us now assume that the atom is initially in level 2. Since $E_2 > E_1$, the atom will tend to decay to level 1. The corresponding energy difference, $E_2–E_1$, must therefore be released by the atom. When this energy is delivered in the form of an electromagnetic wave called photon, the process will be called *spontaneous* (or *radiative*) *emission*.


Spontaneous emission
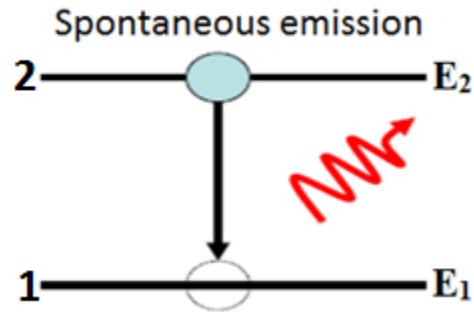
iii. **Stimulated emission:** Let us now suppose that the atom is found initially in level 2 and that a photon of frequency v_o is incident on the material (Adjacent Fig.). Since this photon has the same frequency as the atomic frequency, there is a finite probability that this photon will force the atom to undergo the transition 2→1. In this case the energy difference $E_2–E_1$ is delivered in the form of a photon that adds to the incident one. This is the phenomenon of *stimulated emission*.

Stimulated emission
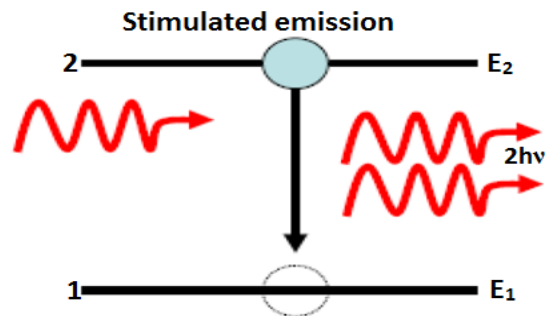
There is a fundamental difference between the spontaneous and stimulated emission processes. In the case of spontaneous emission, the atoms emits a photon that has no definite phase relation with that emitted by another atom. Furthermore, the photon can be emitted in any direction. In the case of stimulated emission, since the process is forced by the incident photon, the emission of any atom adds in phase to that of the incoming photon and along the same direction.

EINSTEIN'S COEFFICIENTS:

In 1916, **Albert Einstein** proposed that there are three processes occurring in the formation of an atomic spectral line. The three processes are referred to as **spontaneous emission, stimulated emission, and absorption**. With each is associated an Einstein coefficient which is a measure of the probability of that particular process occurring. Einstein considered the case of isotropic radiation of frequency v, and **energy density $\rho(v)$**.
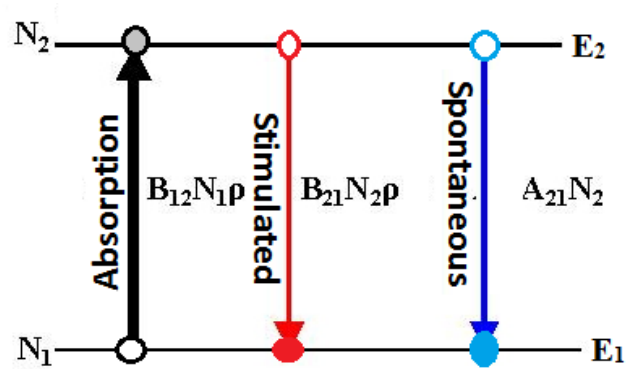
Let us consider N_1 and N_2 be the populations in the energy levels of energies E_1 and E_2 respectively in a system of atoms at a thermal equilibrium of temperature T.

Upward transition:

Absorption is the process by which a photon is absorbed by the atom, causing an electron to jump from a lower energy level E_1 to a higher one E_2. The process is described by the Einstein coefficient B_{12},

The absorption rate is directly proportional to N_1 and $\rho(v)$

$$\text{Therefore, } \textit{Rate of absorption} = B_{12}N_1\rho(v) \qquad \text{------------- (1)}$$

Here B_{12} is **Einstein's coefficient of absorption**.

Downward transition:

Spontaneous emission is the process by which an electron "spontaneously" (i.e. without any outside influence) decays from E_2 to E_1. The process is described by the Einstein coefficient A_{21}.

Spontaneous emission rate is directly proportional to N_2 only.

$$\text{Therefore, } \textit{Rate of spontaneous emission} = A_{21}N_2 \text{ ------------- (2)}$$

Here, A_{21} is the **Einstein's coefficient of spontaneous emission**.

Stimulated emission is the process by which an atomic electron in the excited E_2 is interacting with a photon of certain frequency may drop to a lower energy level E_1, transferring its energy to that photon. A new photon created in this manner has the same phase, frequency and direction of travel as same as the incident photon. The process is described by the Einstein coefficient B_{21}.

Stimulated emission rate is directly proportional to N_2 and $\rho(v)$.

$$\text{Therefore, } \textit{Rate of stimulated emission} = B_{21}N_2\,\rho(v) \text{ ---------- (3)}$$

Here, B_{21} is the **Einstein's coefficient of stimulated emission**.

Consider an ideal material with only two non-degenerate energy levels, at thermal equilibrium,

$$\textbf{Absorption = spontaneous emission + Stimulated emission} \qquad \text{------ (4)}$$

$$\textbf{i.e. } B_{12}N_1\rho(v) = A_{21}N_2 + B_{12}N_2\rho(v) \qquad \text{------- (5)}$$

$$B_{12}N_1\rho(v) - B_{12}N_2\rho(v) = A_{21}N_2$$

$$[B_{12}N_1 - B_{21}N_2]\,\rho(v) = A_{21}N_2$$

$$\rho(v) = \frac{A_{21}N_2}{[B_{12}N_1 - B_{21}N_2]} \quad \text{or} \quad \rho(v) = \frac{A_{21}N_2}{B_{21}N_2\left\{\left[\frac{N_1}{N_2}\left(\frac{B_{12}}{B_{21}}\right)\right]-1\right\}}$$

$$\text{or} \quad \rho(v) = \frac{A_{21}}{B_{21}}\frac{1}{\left\{\left[\frac{N_1}{N_2}\left(\frac{B_{12}}{B_{21}}\right)\right]-1\right\}} \qquad \text{-------------- (6)}$$

But, in thermal equilibrium, the Boltzmann's distribution law applies, so,

$$N_1 = N_0 exp\left(\frac{-E_1}{k_BT}\right) \; and \; N_2 = N_0 exp\left(\frac{-E_2}{k_BT}\right)$$

where, N_o is population in ground state and k_B is the Boltzmann's constant.

Therefore, $\frac{N_1}{N_2} = exp\left[\frac{E_2-E_1}{k_BT}\right] = exp\left[\frac{hv}{k_BT}\right]$ \qquad (Since, E_2-E_1= hv) \qquad -------- (7)

Substituting eq. (7) in eq. (6), we get

$$\rho(v) = \frac{A_{21}}{B_{21}}\frac{1}{\left\{\left[exp\left(\frac{hv}{k_BT}\right)\left(\frac{B_{12}}{B_{21}}\right)\right]-1\right\}} \qquad \text{------------- (8)}$$

According to Plank's law of black body radiation at temperature T we have the energy density $\rho(v)$ at frequency v is

$$\rho(v) = \frac{8\pi hv^3}{c^3}\frac{1}{\left\{exp\left(\frac{hv}{k_BT}\right)-1\right\}} \qquad \text{-------------- (9)}$$

Comparing eq. (8) and eq. (9), we get

$$\boxed{\frac{A_{21}}{B_{21}} = \frac{8\pi hv^3}{c^3}} \quad \text{----- (10)} \qquad\qquad \boxed{\frac{B_{12}}{B_{21}} = 1} \text{ -------- (11)}$$

Equations (10) and (11) show the relations between Einstein's coefficients B_{12}, B_{21} and A_{21}.

From the above relation eq. (10) shows that the ration of rate of spontaneous (A_{21}) to rate of stimulated emission (B_{21}) is **proportional to cube of the frequency** of the incident radiation. The eq. (11) shows that the rate of absorption is equal to the rate of stimulated emission, when the system in equilibrium.

POPULATION INVERSION:
    At thermal equilibrium, the no of atoms in level 2 is less than the no of atoms in level 1 (N_2<N_1). **The process of making of higher population level 2 than the population in level 1 is**

known as population inversion ($N_2 > N_1$) This is highly non equilibrium situation. When the population inversion exists, rate of stimulated emission is greater than rate absorption occurs and the light passing through the material is amplified.
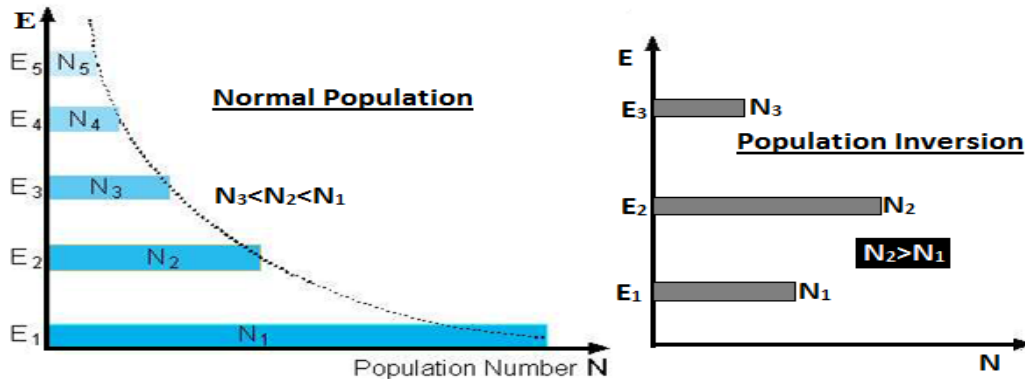
LASER PUMPING:

The process of achieving the population inversion by sending the atoms from lower level to higher levels through supply of energy to the lower level atoms is called **laser pumping**. The supply of energy is usually provided in the form of **light (Optical) or electric current (Electrical)**. But, more exotic sources such as chemical or nuclear reactions can also be used.

i. **Optical Pumping:** Optical pumping is a process in which light is used to raise or pump electrons from lower energy level in an atom or molecule to higher one. It is commonly used in solid state lasers (Ex.: Ruby Laser, etc.).
ii. **Electrical Pumping:** Electric discharge is common in gas lasers. Gas lasers have very narrow absorption band, pumping then in any flash lamp is not possible. In most of the cases population inversion is created by means of electric discharge. (Ex.: He-Ne Laser etc.)

META-STABLE STATE:

Metastable state is an excited state of an atom or other system with a longer lifetime than the other excited states. However, it has a shorter lifetime than the stable ground state. Atoms in the metastable state remain excited for a considerable time in the order of 10^{-6} to 10^{-3} s. During metastable state, all the parameters associated with state hold stationary values. A large number of excited atoms are accumulated in the metastable state.

The population of metastable state can exceed the population at a lower level thereby establishing population inversion in a lasing medium. Population inversion could not be created without a metastable state.

THREE LEVEL SCHEME:

We have seen that the key to laser action is to obtain a population inversion between two levels of energy E_1 and E_2 with $E_2 > E_1$, so that more atoms are in the level 2 than in the level 1. In the three-level lasers figure, we look for three levels in an atom such that $E_3 > E_2 > E_1$, with a fast decay between levels 3 and 2 and a slow decay between 2 and 1. Incident radiation of angular

frequency $v_{31} = (E_3 - E_1)/\hbar$ is used to raise as many atoms as possible from the level 1 to level 3. Note that a population inversion cannot be obtained between levels 3 and 1, because when the number of atoms N_3 in level 3 equals the number N_1 in level 1, absorption will be balanced by stimulated emission and the material will become transparent at the frequency v_{31}. If level 3 decays rapidly to level 2, a population inversion can be obtained between levels 2 and 1. An example is the ruby laser.
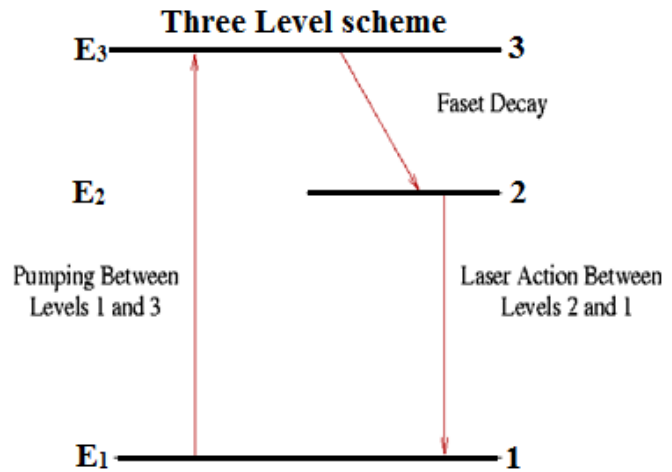
Three Level scheme

FOUR LEVEL SCHEME:

Except in special cases, such as the ruby, it is difficult to produce a population inversion between a ground state and an excited state, because initially all the atoms are likely to be in the ground state, and we have to get more than half the atoms into level 2 before a population inversion can be achieved. An easier approach is to use a four-level system in the adjacent figure and attempt to create a population

Four Level Scheme

inversion between two excited levels. We start with all the atoms in the ground state 1, and none in the excited states 2, 3 and 4 ($E_2 < E_3 < E_4$). Level 4 is chosen so that it has a fast decay to level 3, and pumping between levels 1 and 4 immediately produces a population inversion between levels 3 and 2. As level 2 begins to fill up by stimulated emission at the frequency ($E_3 - E_2$)/h, the population inversion will decrease. To minimize this, level 2 is chosen so that it has a fast decay to the ground state.
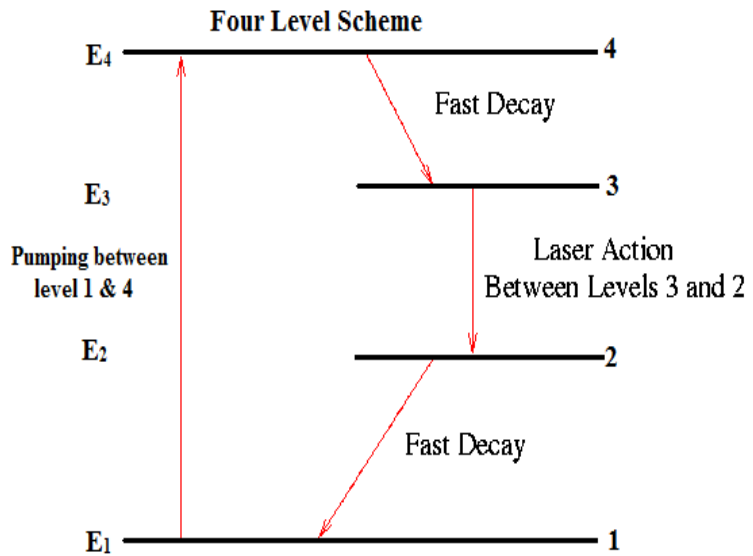
REQUIREMENTS FOR THE LASER SYSTEM:

The basic requirements of the Laser system are shown in below figure. It consists of three basic parts as given below:

    i. Pumping System
    ii. Active medium
    iii. Optical Resonator

i. **Pumping System:** Pumping source is the basic energy source for a laser. It gives energy to various atoms of laser medium & excites them, so that population inversion can take place and it is maintained with time. The excitation of atom occurs directly or through atom or atom collision.

There are various types of pumping systems depending on the nature of the active medium.


ii. **Active Medium:** Active medium is the heart of the laser system and is responcible for producing population inversion (gain) and subsequent generation of laser. It can be a solid, liquid, semiconductor or gas medium and can be pumped to higher energy state.
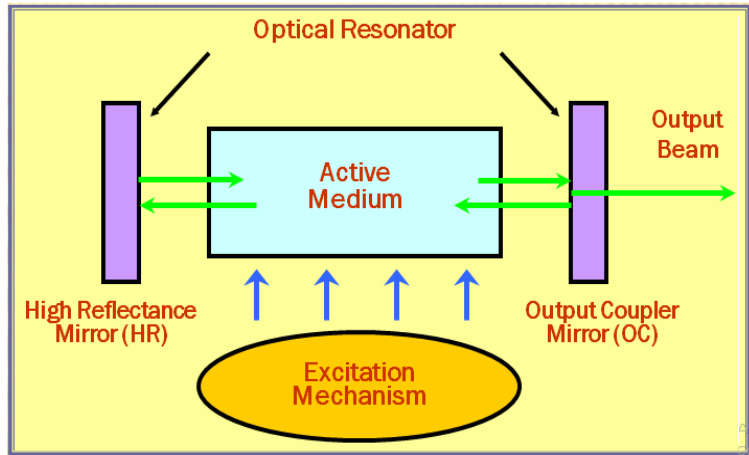
iii. **Optical resonator:** It plays a very important role in the production of laser beam from the laser system. It is a setup used to obtain amplification of slimulated photons, by oscillating them back and forth between two extreme limits. It consists of two plane or concave mirrors placed co-axially. One mirror is fully reflecting and other is partially reflecting.
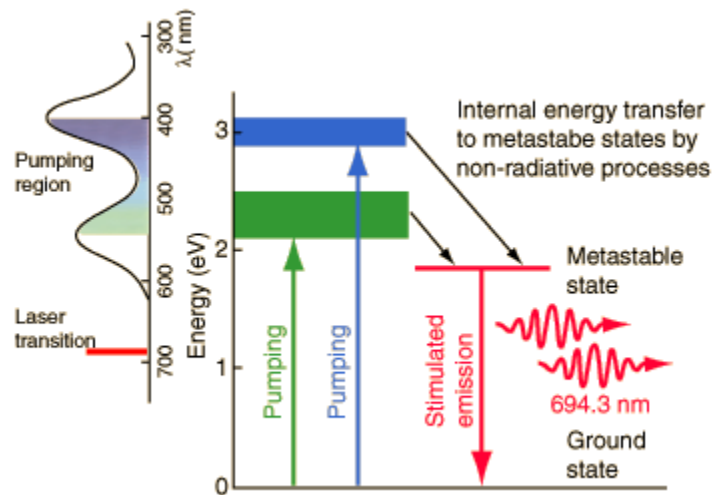
RUBY LASER:

The first working laser was built in 1960 by T.H. Maiman using a Ruby crystal and so called the Ruby Laser. This is also called **solid state laser or three level laser**. Ruby belongs to the family of gems consisting of Al_2O_3 with various types of impurities. For example in Ruby laser used the pink ruby contains 0.05% Cr atoms ($Al_2O_3+0.05\%Cr_2O_3$). The schematic diagram of the Ruby laser is as follows:
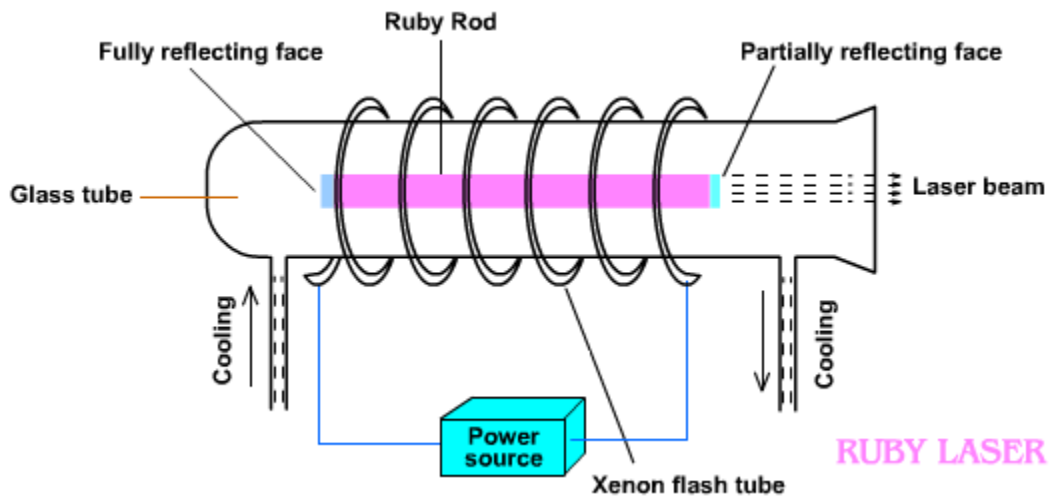
Construction: The Ruby laser consists of a ruby rod whose length is few centimeter and diameter is 0.5 cm. This is made up of with chromium (Cr^{3+}) doped Al_2O_3 material. Both the ends of the ruby rod are silvered such that one end is fully reflecting and the other end is partially reflecting. The ruby rod is surrounded by helical xenon flash lamp tube which provides the optical pumping to raise Cr^{+3} ions to upper energy level. The light from the xenon flash tube was focused by the cylindrical cavity onto the ruby rod, thereby exciting the chromium atoms which were responsible for the laser action.

Working: The ruby laser is a three-Level system (Adjacent Fig.) since only three energy levels are involved in the process of stimulated emission. The depopulation of the ground state for population inversion is achieved by exciting the atoms of the ruby crystal with intense light from a xenon flash lamp. Thus the atoms are excited from the ground state (level 1) to an upper state (level 3) by means of absorption. From the energy level 3, the atoms are transferred to energy level 2 without emitting radiation (non-radiative transfer). The energy level 2 is called met stable level since the atoms stay at this level for a longer interval of time. Finally, the atoms return to the ground state from the meta stable level through the process of stimulated emission giving rise to an intense laser light at **6943Aº**. The laser beam comes out in the form of a pulse of very short duration (about a millisecond).

Applications: They are still used in a number of applications where short pulses of red light are required.

  i. Holography's around the world produce holographic portraits with ruby lasers, in sizes up to a meter squared.
  ii. Many non-destructive testing labs use ruby lasers to create holograms of large objects such as aircraft tires to look for weaknesses in the lining.
 iii. Ruby lasers were used extensively in **tattoo and hair removal**.

Drawbacks:

  i. The laser requires high pumping power because the laser transition terminates at the ground state and more than half of ground state atoms must be pumped to higher state to achieve population inversion.
  ii. The efficiency of ruby laser is very low because only green component of the pumping light is used while the rest of components are left unused

iii. The laser output is not continues but occurs in the form of pulses of microseconds duration.

HELIUM-NEON (He-Ne) LASER:

The He-Ne laser active medium consists of two gases which do not interact form a molecule. Therefore He-ne laser is one type of **atomic gas laser** and also called as **Four level laser**.

Construction:
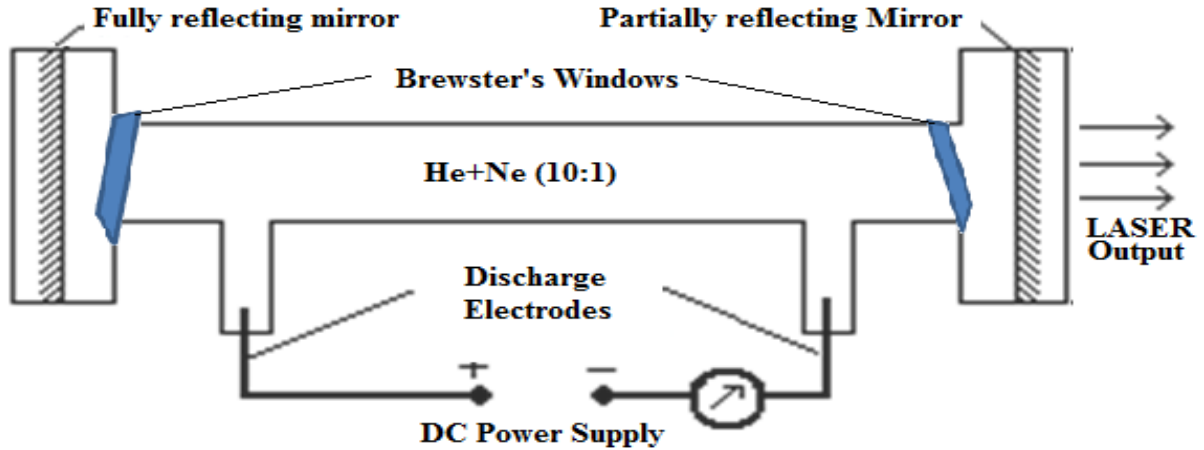
He - Ne gas laser consists of a gas discharge tube of length 30cm and diameter of 1.5cm. The tube is made up of quartz and is filled with a mixture of Neon under a pressure of 0.1mm of Hg. The Helium under the pressure of 1mm of Hg, the ratio of He-Ne mixture of about 10:1, hence the no. of helium atoms are greater than neon atoms. The mixtures is enclosed between a set of parallel mirrors forming a resonating cavity, one of the mirrors is completely reflecting and the other partially reflecting in order to amplify the output laser beam.

Working:

In the He-Ne laser the light produced by atomic transitions within the Neon atom. The Helium does not directly produce laser light but it acts as a buffer gas, this purpose of which is to assist/help the atoms of the other gas to produce lasing action.

The active energy levels of He and Neon atoms are show in adjacent figure. In helium there are three active energy levels named as F_1, F_2 and F_3 where as in Neon, there are six active energy levels named as E_1, E_2, E_3, E_4, E_5, and E_6. In Helium, the metastable states are F_2 and F_3, where as in Neon, E_4 and E_6. When a discharge is passed through the gaseous mixture electrons are accelerated in the tube these
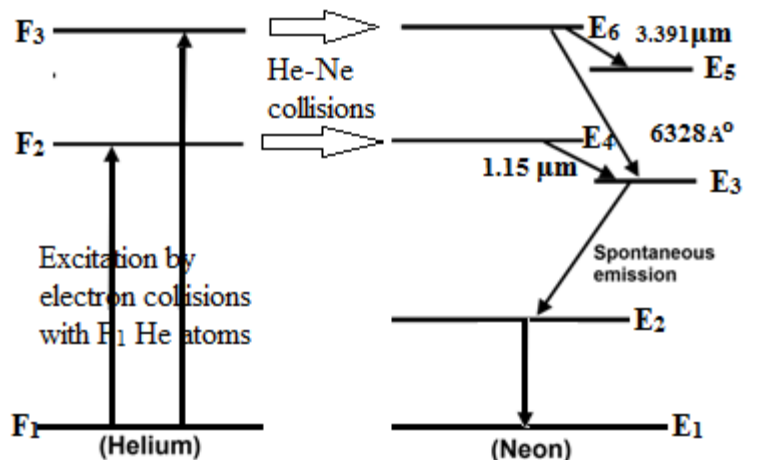
accelerated electrons collide with the helium atoms and excite them to higher energy levels (F_2 and F_3) since the levels are metastable energy levels, He atoms spend sufficiently long time. Inelastic collision of the excited helium atoms (F_2 and F_3) with the ground state (E_1) Neon atoms results in transfer of energy to the neon atoms, exciting them into E_4 and E_6 states. This is due to the coincidence of energy levels between the helium and neon atoms. Since E_4 and E_6 are metastable states, hence the population inversion takes place between the E_6 and E_4 with respect to E_5 and E_3. The stimulated emission takes place between $E_6 \rightarrow E_3$ gives a laser light of wave length **6328Aº** and the stimulated emission between $E_6 \rightarrow E_5$ gives a laser light wavelength of **3.39μm**. Another stimulated emission between $E_4 \rightarrow E_3$ gives a laser light wavelength of **1.15μm**. The neon atoms undergo spontaneous emission from $E_3 \rightarrow E_2$ and $E_5 \rightarrow E_2$. Finally the neon atoms are returned to the ground state E_1 from E_2 by non-radiative diffusion and collision process.

After arriving the ground state, once again the neon atoms are raised to $E6$ & $E4$ by excited helium atoms thus we can get **continuous wave** output from He-Ne laser. But some optical elements placed insides the laser system are used to absorb the infrared laser wavelengths 3.39μm and1.15μm. Hence the output of He-Ne laser contains only a single wavelength of 6328Aº and the output power is about few milliwatts.

Applications:

i.   The Narrow red beam of He-Ne laser is used in supermarkets to read bar codes (Bar-code scanners).
ii. The He-Ne Laser is used in Holography in producing the 3D images of objects.
iii. He-Ne lasers have many industrial and scientific uses, and are often used in laboratory demonstrations of optics.

SEMICONDUCTOR (Diode) LASER (GaAs Laser):

Semiconductor lasers also known as *quantum well lasers* are smallest, cheapest, can be produced in mass, and are easily scalable. They are basically p-n junction diode, which produces light of certain wavelength by recombination of charge carrier when forward biased, very similar to the light-emitting diodes (LEDs). LEDs possess spontaneous emission, while laser diodes emit radiation by stimulated emission.

Principle: In the case direct band gap semiconductors there is a large possibility for direct recombination of hole and electron emitting a photon. GaAs is a direct band gap (1.44 eV) semiconductor and hence it is used to make lasers and light emitting diodes. The wave lengths of the emitted light depend on the band hap of the material.

Construction:

The P^+ and N^+ regions of the diode are obtained by heavily doped p- and n-regions of GaAs. The thickness of the p-n junction layer is very narrow at the junction, the side walls are well polished and parallel to each other. Since the refractive index of GaAs is high, the reflectance at the material air interface is sufficiently large so that the external mirrors are not necessary to produce multiple reflections. The p-n junction is forward biased by connecting positive terminal to p-type and negative terminal to n-type.
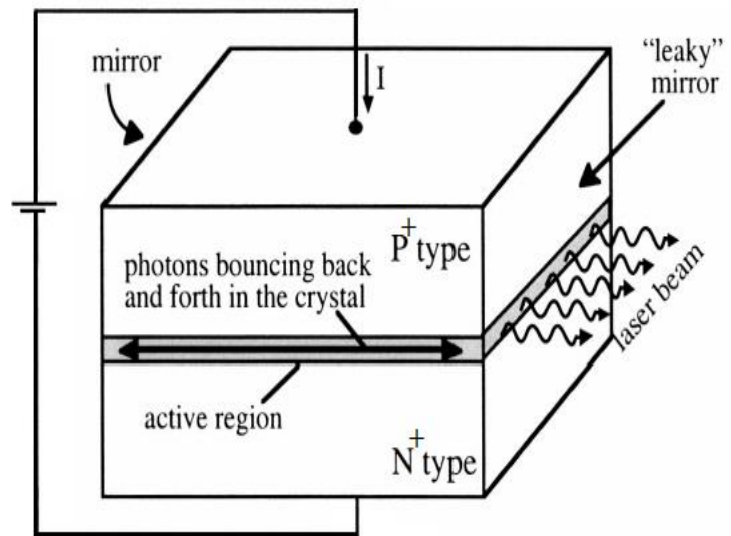
Working:

The population inversion can be obtained by injecting electrons and holes in to the junction from the n-region and p-region by means of forward bias voltage. When the forward bias is not connected, no electrons and holes present in the depletion region. When small forward bias voltage is given to the p-n junction then small number of electrons and holes will injected into the depletion region from respective regions. When
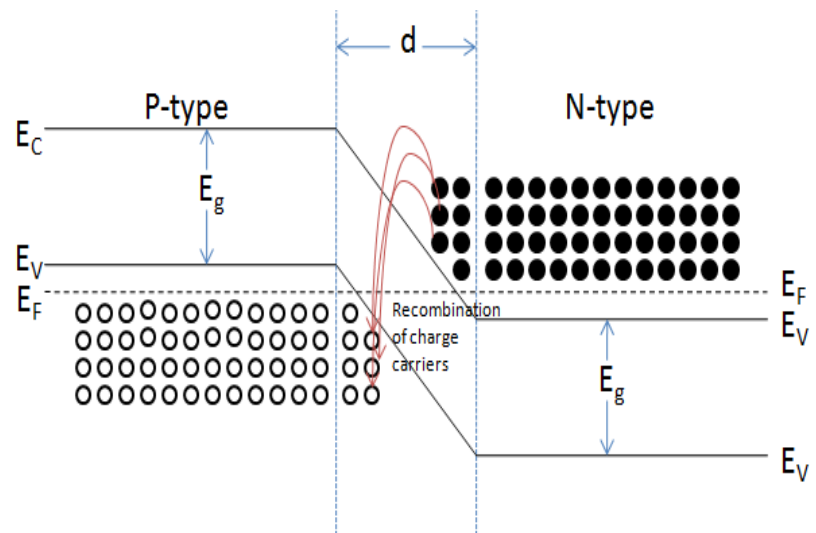
relatively a large current of the order of 10^4 A/cm^2 is passed through the junction then large number of electrons and holes will be injected into the depletion region as shown in above figure. Then the direct recombination processes take place between holes and electrons in the depletion region and release the photons. Further the emitted photons increase the rate of recombination. Thus more number of photons produced having same phase and frequency of the induced photons.

The wavelength of the emitted radiation depends on the energy band gap of the semiconductor material. The energy gap of the GaAs Semiconductor is 1.44 eV then it emits laser light of wavelength ~8600 A°.

$$\lambda = \frac{hc}{E_g} = \frac{6.625 \times 10^{-34} \times 3 \times 10^8}{1.44} = 8626 \text{ A}^o$$

The efficiency of the laser emission increases when a cooling arrangement provided to the laser diodes.

APPLICATIONS OF LASER: There are many scientific, military, medical, industrial and commercial laser applications which have been developed since the invention of the laser.

i. **Laser Drilling:** Laser drilling is one of the few techniques for producing holes in hard materials with desired diameter. The diameter of these holes as small as 5 microns.
Laser-drilling is used in many applications, including the oil gallery of some engine blocks, aerospace turbine-engine cooling holes, laser fusion components and printed circuit board micro-vias.

ii. **Laser Welding and Cutting:** The highly collimated beam of a Laser can be further focused to a microscopic dot of extremely high energy density for welding and cutting. The automobile industry makes extensive use of CO_2 laser with powers up to several KW for computer controlled welding on auto assembly lines.

iii. **Lasers in Data Storage:** The reading and writing (Store) of the data on a compact disc (CD or DVD) is done with semiconductor laser.

iv. **Lasers in scientific research:**
   a) Lasers are used to clean delicate pieces of art, develop hidden finger prints
   b) Laser are used in the fields of 3D photography called holography
   c) Using lasers the internal structure of micro organisms and cells are studied very accurately
   d) Lasers are used to produce certain chemical reactions.

v. **Laser in Medicine:**
   a) The heating action of a laser bean used to remove diseased body tissue
   b) Lasers are used for elimination of moles and tumours, which are developing in the skin tissue.
   c) Laser beam is used to correct the retinal detachment by eye specialist.

vi. **It is used in biomedical science.**

vii. **It is used in 3D photography.**

viii. **It can be used for measuring velocity of light, to study spectrum of matters, to study Raman effect.**

ix. **It is used in holography.**

x. **It is also used in military, like LIDAR.**

xi. **It is used to accelerate some chemical reactions.**

xii. **It is also used in nuclear fusion reactions.**

B. FIBER OPTICS

STRUCTURE OF AN OPTICAL FIBER:

Optical Fiber: A very thin, long, flexible, transparent, cylindrical dielectric medium which guide the light signal propagated through it.

It consists of three parts:

1. The core
2. The cladding,
3. The outer jacket.

Core   Cladding   Sheath

- The core is the inner part of the fiber, which guides the light signal.
- The cladding surrounds the core completely.
- The refractive index of the core (n_1)is greater than the cladding(n_2) to satisfy the total internal reflection ($n_1 > n_2$)
- The outer jacket provides the mechanical protection to the fiber.
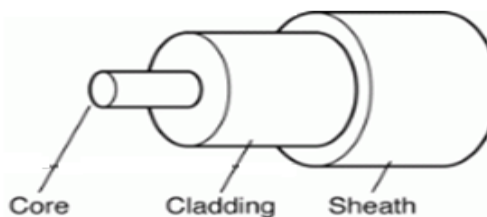
The core and cladding diameters are about few microns. Optical fibers are arranged in bundles called optical cables and used to transmit light signals over long distances.

PRINCIPLE AND WORKING OF OPTICAL FIBER:

Principle: The transmission of light in optical fiber is based on the principle of **Total Internal reflection**.

Let n_1 and n_2 be the refractive indices of core and cladding respectively such that $n_1 > n_2$. Let a light ray travelling from the medium of refractive index n_1 to the refractive index n_2 be incident with an angle of incidence θ and the angle of refraction r. By Snell's law,

$$n_1 \sin \theta = n_2 \sin r \qquad \text{---------- (1)}$$

The refracted ray bends towards the normal as the ray travels from rarer medium to denser medium. On the other hand, the refracted ray bends away from the normal as it travel from denser medium to rarer medium. In Optical fiber, the light ray travelled from core (denser) to cladding (rarer) medium, there is possibility of total internal reflection, if the angle of incidence is greater than the critical angle (θ_c).

Critical Angle: When a light ray moves from high refractive index (core) medium to low refractive index (cladding) medium and for a particular angle of incidence the refraction angle (r) is 90° then the angle of incidence is known as critical angle (θ_c).

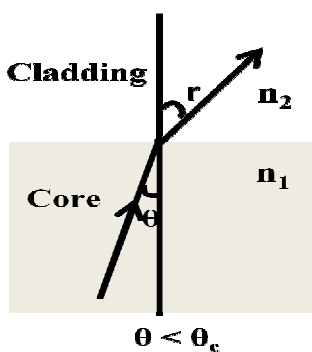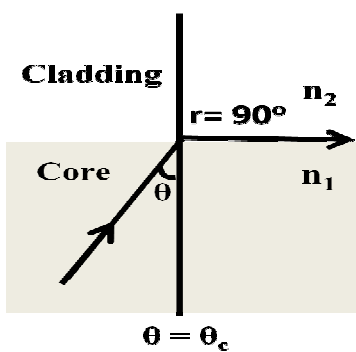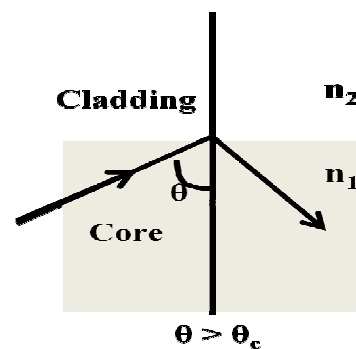|		
$\theta < \theta_c$	$\theta = \theta_c$	$\theta > \theta_c$
Fig.1	Fig.2	Fig.3

i. When $\theta < \theta_c$, then the ray refracted into the second medium as shown in above fig.1.
ii. When $\theta = \theta_c$, then the ray travels along the interface of two media as shown fig. 2.
iii. When $\theta > \theta_c$, then the ray totally reflects into the same medium (core) as shown in fig.3.

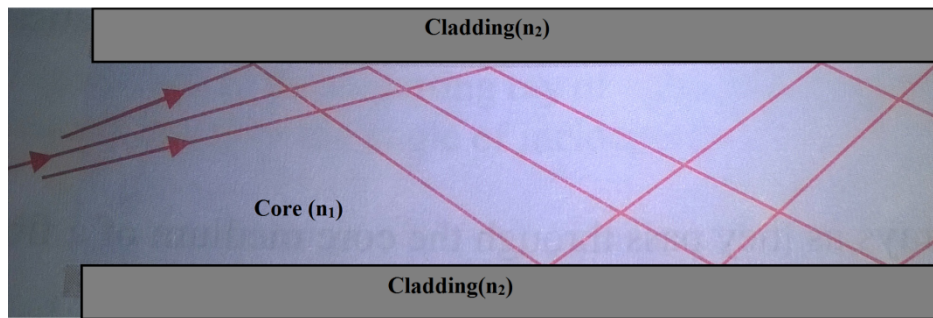The critical angle can calculated from Snell's law, suppose if $\theta = \theta_c$ then $r = 90^\circ$, hence

$$n_1 \sin \theta_c = n_2 \sin 90^\circ$$
$$\sin\theta_c = \frac{n_2}{n_1}$$
or
$$\boldsymbol{\theta_c = sin^{-1}\left(\frac{n_2}{n_1}\right)}$$

This is known as critical angle of mediums of refractive indices n_1 and n_2 ($n_1 > n_2$).

The light signal once entered the fiber and takes total internal reflection with in the core as shown in fig.4, it will continue to propagate till the other end of the fiber.

ACCEPTANCE ANGLE:

The maximum angle of incidence to launch the beam at its one end to enable the entire light to pass through the core is called **acceptance angle**.

When we launch the light beam in to the fiber at its one end the entire light may not pass through the core and propagate. Only the rays which make the angle of incidence greater than critical angle undergo total internal reflection and propagate through the core and all other rays are lost. Let us consider a ray enters the core of refractive index $\boldsymbol{n_1}$ from air medium of refractive index $\boldsymbol{n_o}$ with an angle of incidence $\boldsymbol{\alpha_i}$ at the interface of air and core and incident at the interface of core and cladding with an angle of incidence $\boldsymbol{\theta}$ as shown in below figure.
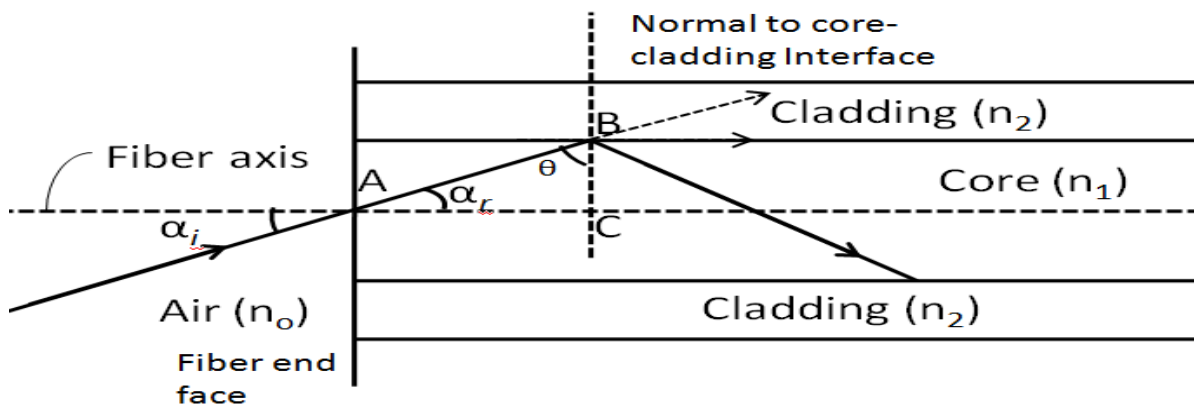
If the angle of incidence at the interface of air and core is maximum ($\alpha_i = \alpha_{max}$), the angle of incidence at the interface of core and cladding is equal to critical angle ($\theta = \theta_c$). If the angle α_i is less than the α_{max}, the angle of incidence at the interface of core and cladding would be greater than the critical angle ($\theta > \theta_c$), and further total internal reflections cascades through the fiber.

$$From\ fig., \Delta ABC,$$
$$\alpha_r = 90 - \theta$$
$$At\ air - core\ interface, the\ Snell's\ law\ is,\ \quad \frac{\sin \alpha_i}{\sin \alpha_r} = \frac{n_1}{n_o}$$

$$\sin \alpha_i = \frac{n_1}{n_o} \sin \alpha_r$$
$$\sin \alpha_i = \frac{n_1}{n_o} \sin(90 - \theta)$$
$$\sin \alpha_i = \frac{n_1}{n_o} \cos \theta$$
$$\boldsymbol{\sin \alpha_{max}} = \frac{n_1}{n_o} \boldsymbol{\cos \theta_c} \text{ (When } \alpha_i = \alpha_{max}, \theta = \theta_c) \quad \text{----- (1)}$$

Snell's law at core and cladding interface gives,

At critical angle ($\theta = \theta_c$), the angle of refraction is 90°

$$n_1 \sin \theta_c = n_2 \sin 90$$
$$\sin \theta_c = \frac{n_2}{n_1}$$
$$\cos \theta_c = \sqrt{1 - sin^2\theta_c} = \sqrt{1 - \frac{n_2^2}{n_1^2}} \quad \text{------ (2)}$$

From Eq. (1), we get, $\boldsymbol{\sin \alpha_{max}} = \frac{n_1}{n_o} \sqrt{1 - \frac{n_2^2}{n_1^2}} = \frac{\sqrt{n_1^2 - n_2^2}}{n_o}$

Thus,

$$\boldsymbol{\sin \alpha_{max}} = \frac{\sqrt{n_1^2 - n_2^2}}{n_o}$$

$$\boldsymbol{\alpha_{max} = sin^{-1}\left(\frac{\sqrt{n_1^2 - n_2^2}}{n_o}\right)} \quad \text{----- (3)}$$

This $\boldsymbol{\alpha_{max}}$ is known as **Acceptance angle.**

ACCEPTANCE CONE: The acceptance cone is derived by rotating the Acceptance Angle about the fiber axis. Light launched at the fiber end within this acceptance cone alone will be accepted and propagated to the other end of the fiber by total internal reflection. Larger acceptance angles make launching easier.
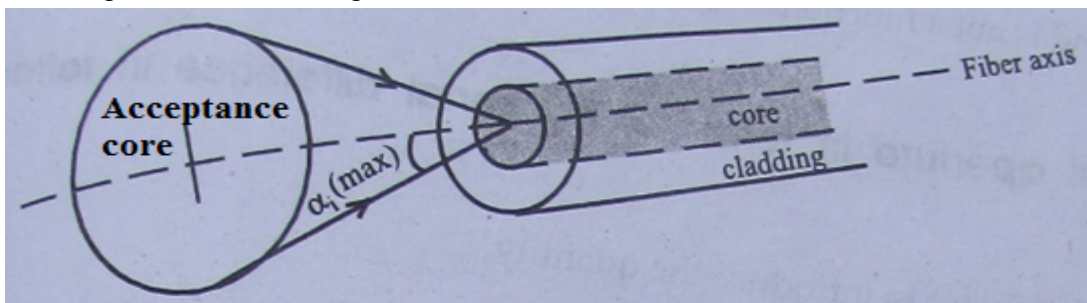
Numerical Aperture: The numerical aperture (*NA*) of an optical fiber is defined as sin of acceptance angle and is dimensionless number that characterizes the range (ability) of angles over which the system can accept light.

Therefore, $NA = \sin(\alpha_{max}) = \dfrac{\sqrt{n_1^2 - n_2^2}}{n_o}$ -------- (4)

If the refractive index of air is unity ($\mu_{air}=1$), thus the Eq. (4) reduces as,

$NA = \sqrt{n_1^2 - n_2^2} = n_1\sqrt{2\Delta}$ where $\Delta = \dfrac{n_1 - n_2}{n_1}$ called Fractional Index difference
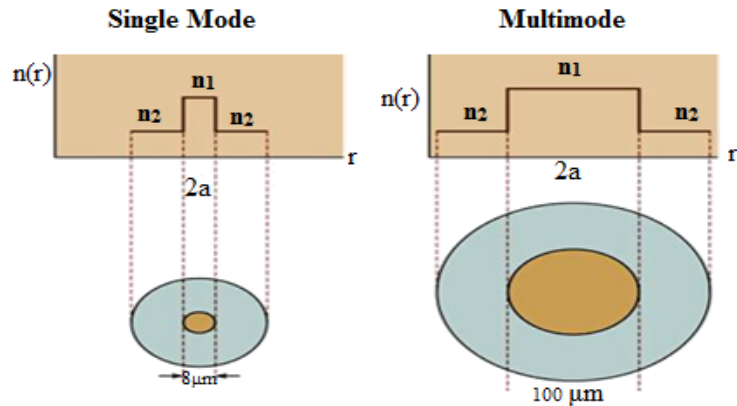
FIBER TYPES AND REFRACTIVE INDEX PROFILES:

Depending upon the refractive index profile of the core, optical fibers are classified into two types. They are:

      i. Step Index (SI) Fiber   ii. Graded Index (GI) fiber

i. Step Index (SI) Fiber: In step index fibers, the refractive index of the core is uniform throughout the medium and undergoes an abrupt (Step) change at the interface of core and cladding.

The light in the fiber propagates by bouncing back and forth from core-cladding interface. The step index fibers propagate both single and multimode signals within the fiber core. The light rays propagating through it are in the form of meridinal rays which will cross the fiber core axis during every reflection at the core – cladding boundary and are propagating in a zig – zag manner.

The variation of refractive index profile with radial distance *r* for the SI fiber is shown in the above figure.

$$n(r) = n_1 (\text{core})$$
$$= n_2 (\text{cladding})$$

The number of possible propagation modes in the core depends on the radius of the core and NA of the fiber and is given by V-number as

$$V = \frac{2\pi}{\lambda}a(NA)$$ Where '*a*' is the radius of the core and NA-Numerical Aperture.

The number of modes (paths) through the SI fiber is $= \dfrac{V^2}{2}$

The propagation modes in the SI Fiber in both the modes are shown below:

Single mode SI fiber   Multimode SI Fiber
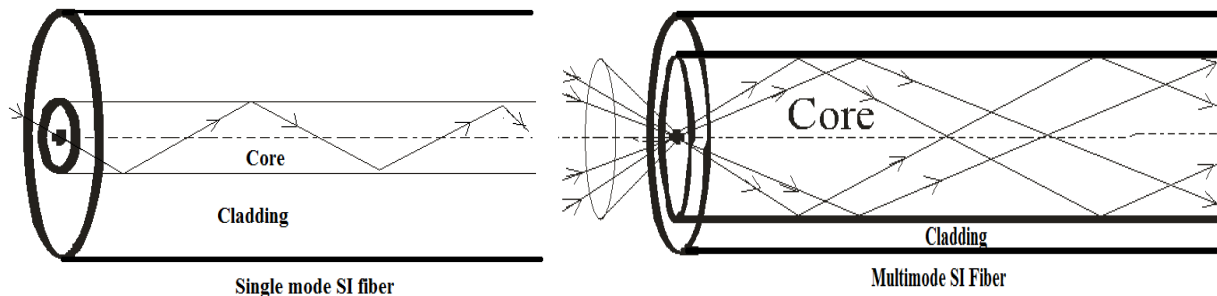
ii. **Graded Index (GI) Fiber:** In graded index fibers, the refractive index of the core varies gradually as a function of radial distance from the fiber center.

The refractive index of the core decreases as we move away from the centre. The refractive index of the core is made to vary in the form of parabolic manner such that the maximum refractive index is present at the centre of the core.

The variation of refractive index profile with radial distance r for the SI fiber is shown in the adjacent figure.

Graded Index Fiber

$$n(r) = n_1\sqrt{1 - 2\Delta\left(\frac{r}{a}\right)^{\alpha}} \ (0 \leq r \leq a) \text{ (core)}$$

$$= n_2 \text{ (cladding)}$$

Here α is the grading parameter which decides the variation of RI in core.

$\alpha = 1$ for linear grading,

$\alpha = 2$ for **parabolic** grading

$\alpha = \infty$ for Step Index grading.

The transmitted light signals travel through the core medium in the form helical (sine waves) rays, which will not cross the fiber axis at any time.

The number of modes propagated through the GI Fiber depends on the radius of the core and NA of the fiber. Therefore,

The possible number of modes propagated through the GI Fiber is $= \dfrac{V^2}{4}$

The propagation modes in the GI Fiber are shown below:

Graded Index Fiber
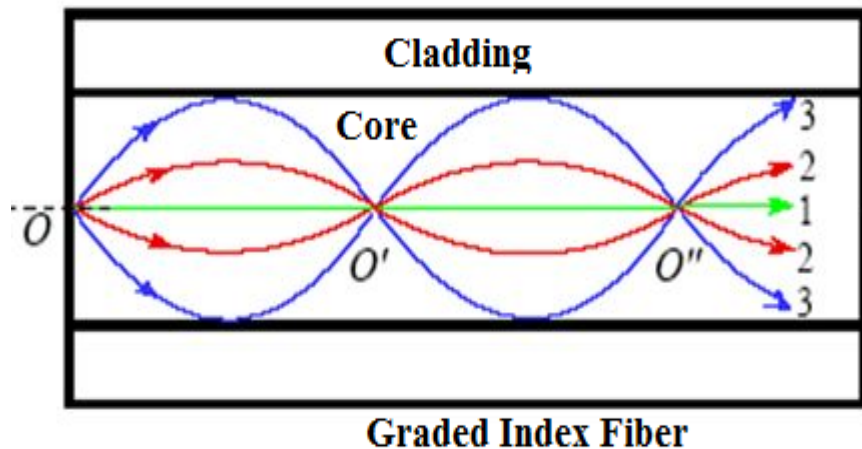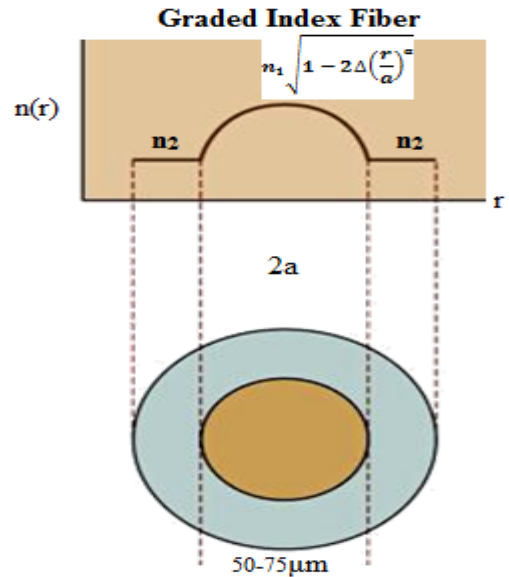
ATTENUATION:

Attenuation means loss of light energy as the light pulse travels from one end of the fiber cable to the other. It is also called as signal loss or fiber loss. It is directly proportional to the length of the cable. It limits the optical power which can reach the receiver at the destination end of the fiber.

Attenuation is mainly caused as a result of
  i. Absorption loss
  ii. Scattering loss and
  iii. Bending losses.
Attenuation is defined as the ration of input optical power (P_i) to the output optical power (P_o)
The following equation defines the signal attenuation as a unit of length:

$$\alpha(dB/km) = -\frac{10}{L}\log_{10}\left(\frac{P_O}{P_i}\right)$$

i. **Absorption loss:** Every material has a characteristic of absorbing a fraction of the incident light. Optical fibers are also no exception. This property is called intrinsic absorption. Besides the intrinsic absorption, the impurities whatsoever present in optical fiber also absorb light which is called impurity (Extrinsic) absorption. Such types of absorptions result in the reduction of the strength of the light signal propagating through the optical fiber cable.

ii. **Scattering loss:** Light signal scattering can be thought of as the deflection of a ray from a straight path, for example by irregularities in the propagation medium, particles or in the interface between the two media. Irregularities and defects (which are produced when optical fibers are manufactured) are main causes for the scattering of light in unexpected directions.
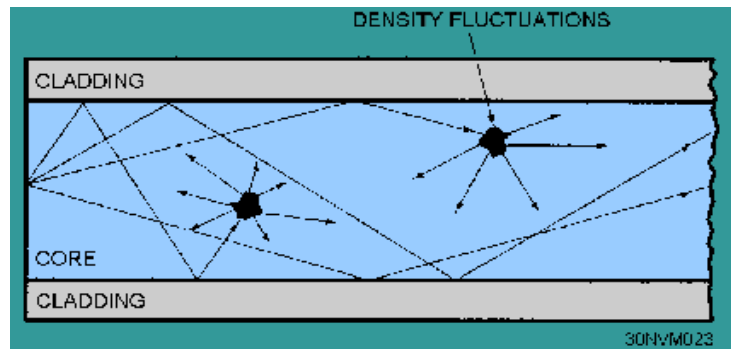
iii. **Bending losses:** This loss induced by physical stress on the fiber. Bending loss is classified according to the bend radius of curvature:

  a) Macrobend Loss   b) Microbend Loss

a) Macroscopic Bending: Macro-bend Losses are observed when a fiber bend's radius of curvature is large compared to the fiber diameter. These bends are a great source of loss when the radius of curvature is less than several centimeters.
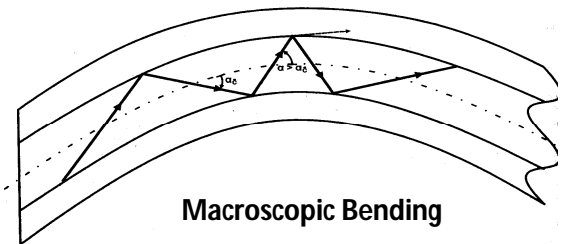
b) Microscopic Bending: Micro-bend Loss are caused by small discontinuities or imperfections in the fiber. Uneven coating applications and improper cabling procedure increases micro bend loss. External forces are also a source of micro bends.

Macroscopic Bending

Microscopic Bending
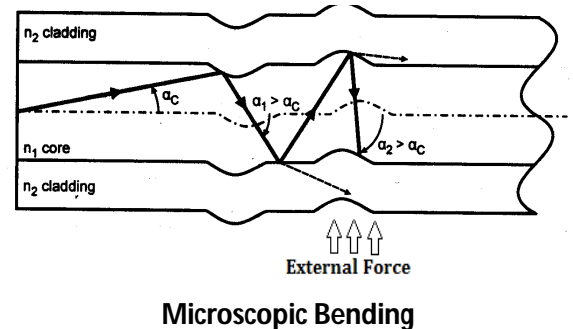
APPLICATIONS OF OPTICAL FIBERS:

Due to its variety of advantages optical fiber has a wide range of application in different fields namely:

    i.   Communication:

    ii. Medicine and

    iii. Sensors etc.,

i.   **COMMUNICATION:** Optical fibers are used as wave guides in the communication system. A typical block diagram of optical fiber communication system (OFCS) is shown in the following figure. It mainly consists of the following parts:

    a) Encoder, b) Transmitter, c) Waveguide, d) Receiver and d) Decoder.
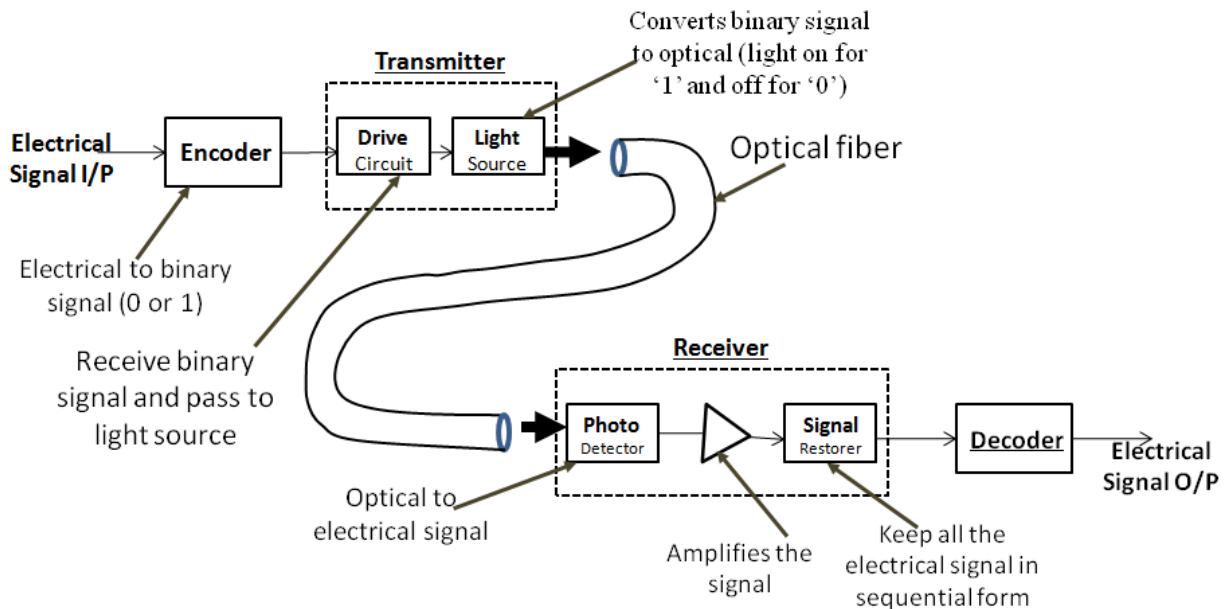

Fig.: Block Diagram of Optical Fiber Communication System

    a) Encoder: The audio signal (i.e., the words spoken by us) is converted into electrical signal which is an analog signal. Encoder is an electronic circuit that converts this analog signal into binary or digital signals.

    b) Transmitter: The digital signal from the encoder is fed to the transmitter which consists of two parts- Drive circuit and Light source. Drive circuit receives the digital signal from encoder and feeds it to the light source. Light source is usually LED or a Diode LASER. If digital '0' is received then light source will be turned OFF. If digital '1' is received then the light source will be turned ON. Thus light source converts electrical signals into optical signals.

    c) Waveguide: Now the Optical signals generated by the transmitter are fed to an optical fiber which acts as waveguide. The signal traverses over longer distances through these waveguides.

    d) Receiver: On the other side of the waveguide, he optical signal is received by the receiver which consists of Photo detector, amplifier and a signal restorer. The Photo detector

receives the optical signal and generates the equivalent electrical signals. These electrical signals are amplified by the amplifier. The signal restorer keeps all the electrical signals in a sequential form and supplies to decoder.

e) Decoder: It is an electronic system that converts the digital signal to analog signal.

ii. MEDICAL:

Optical fibers are generally used in Endoscopy. They are also used in LASER Angioplasty (Laparoscopic Surgery) which is usually used for operations in the stomach area such as appendectomies. A LASER Angioplasty usually makes use of three channels (bundles) of optical fibers. Channel 1(One optical fiber cable) is used to observe where exactly the cholesterol deposits are present. LASER of suitable power is sent through channel 2 to destroy the cholesterol deposits. Channel 3 is used to suck out the debris.

iii. SENSORS:

Another important application of optical fibers is in sensors. If a fiber is stretched or squeezed, heated or cooled or subjected to some other change of environment, there is usually a small but measurable change in light transmission.

Level Sensors:

A chamgered Optical fiber, containing of core alone is used in a level sensor. The condition here is that the refractive indices of air, core and liquid should such that $n_{air} < n_{core} < n_{liquid}$.

Fig (i)

A light signal from the source is fed to the fiber core as shown in fig. (i). The light signal reaches the detector after getting subjected to total internal reflections in the core. It is observed here that the liquid level did not touch the tip of the optical fiber yet and hence the light signal smoothly reaches the detector.

When the liquid raises to sufficient level as shown in figure (ii), at the point of core-liquid interface, total internal reflection cannot take place. The reason is $n_{core} < n_{liquid}$. Therefore the light signal gets leaked into liquid without reaching the detector.

Fig (ii)

Thus the light signal reaching the detector indicates lower liquid level while the detector does not receive the light signal, indicates sufficient level of the liquid.
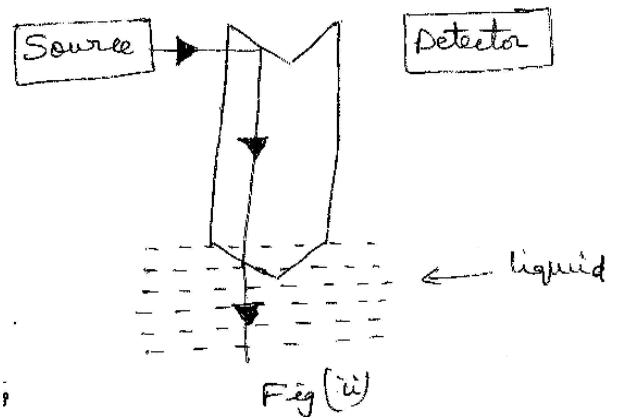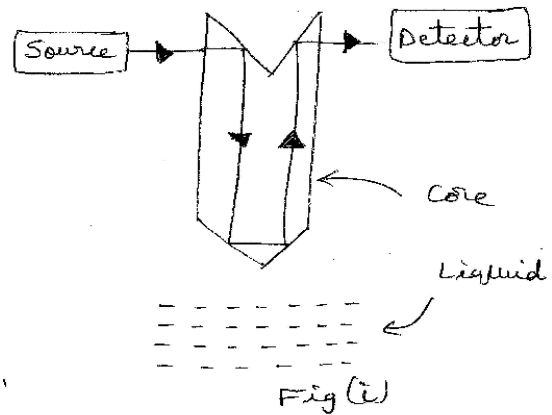
UNIT-V

NON DESTRUCTIVE TESTING

Non-Destructive testing is the use of noninvasive techniques to determine the integrity of a material, component or structure or quantitatively measure some characteristics of an object. It is the testing of materials, for surface or internal flaws or metallurgical condition, without interfering in any way with the integrity of the material or its suitability for service.

Does not destroys the test specimen after testing, allow the part to be used for its intended purpose

Destructive: The material is destroyed & connot be used after testing.ex-tensil,impact etc.

TYPES OF DEFECTS:

- CRACKING
- SPALLING
- STAINING
- CONSTRUCTION&DESIGN DEFECTS
- HONEYCOMBING
- DUSTING
- BLISTERING
- RAIN DAMAGE

CRACKING

Minor –shrinkage………repaired by cosmetic treatment

Major-over load of structure….need of investigation for causes

Major- (i)-Dormant-do not increase size once formed.

   (Ii)-Active-changes under load

   (iii)fine cracks-less than 1mm

        medium cracks-1mm-2mm

         wide cracks-greater than 2mm

SPALLING

De-lamination of the surface of concrete is know as spalling

--internal stress or external stress are responsible for spalling

CAUSES-1-external load producing highly stressed narrow zone

  2-corrosion of steel embedded in concret

  3-freeze-thaw effect of entrapped water

  4-chemical reactions,effloresecence,repeated wetting&drying

STAINING DEFECT

It caused by absorption of water.which contains minerals/salt.

Staining caused by efflorescence.

CONSTRUCTION & DESIGN DEFECT

1-choice of wall thickness , out of plumb of walls

2-Dfective of joint and bonds,lack of movement of joints , misalignment of joints

3-failure to connect inserting walls&columns

4-impropare drainage path causing staining.

5-poor layout that causes excessive torsion

6-cracking below beams due to inadequate gap for deflection.

HONEYCOMBING

It occurs when too much coarse aggregate appears on the surface with some cavities underneath.

DUSTING

Dusting is a surface defect which appears as fine powder on the concrete surface and come off when brushed.

BLISTERING

Blistering occur when the fresh concrete surface is sealed by trowelling trapping air or bleed waterf undert the surface.It avoided by delaying trowelling as long as possible and covering to prevent evaporation.

RAIN DAMAGE

Heavy rain may cause pitted surface or eroded surface on a concrete structure. such damage can be avoided by convexing newly placed concrete with plastic sheeting when rain.

Importance of NDT:

1. NDT increases the safety and reliability of the product during operation.
2. It decreases the cost of the product by reducing scrap and conserving materials, labor and energy.
3. It enhances the reputation of the manufacturer as a producer of quality goods. All of the above factors boost the sales of the product which bring more economical benefits for the manufacturer.
4. NDT is also used widely for routine or periodic determination of quality of the plants and structures during service.
5. This not only increases the safety of operation but also eliminates any forced shut down of the plants.

Six Most Common NDT Methods

▶ Visual Testing (VT)

▶ Dye Penetrant Testing (DPT)

▶ Magnetic Particle Testing (MPT)

▶ Ultrasonic Testing (UT)

▶ Eddy Current Testing (ECT)

▶ X-RAY Radiography Testing (RT)

1.Visual Testing

Visual testing is the most basic and common inspection method involves in using of human eyes to look for defects. But now it is done by the use special tools such

as video scopes, magnifying glasses, mirrors, or borescopes to gain access and more closely inspect the subject area.

Visual Testing Equipments

•Mirrors (especially small, angled mirrors),
•Magnifying glasses,
•Microscopes (optical and electron),
•Borescopes and fiber optic borescopes,
•Closed circuit television (CCTV) systems,
•Videoscope.

Robotic crawlers permit observation in hazardous or tight areassuch as air ducts, reactors, pipelines

2.Dye Penetrant Testing

This method is commonly used for detect the surface cracks or defects. Dye penetrant Testing (DPT) is one of the most widely used nondestructive Testing (NDT) methods. DPT can be used to inspect almost any material provided that its surface is not extremely rough.

Dye Penetrant Testing Process

Three liquids are used in this method.
1. Cleaner
2. Penetrant
3. Developer

At first the surface of the material that is to be tested is cleaned by a liquid. The liquid is called cleane. Then a liquid with high surface wetting characteristics is applied to the surface of the part and allowed time to seep into surface breaking defects. This liquid is called penetrant. After five or ten minutes the excess penetrant is removed from the surface. Then another liquid is applied to pull the trapped penetrant out the defect and spread it on the surface where it can be

seen. This liquid is called deveoper. After Dye Penetrant Testing there are surface cracks are Detected.

Advantages of Dye Penetrant Testing:

1.This method has high sensitivity to small surface discontinuities.
2. Large areas and large volumes of parts/materials can be inspected rapidly and at low cost.
3. Indications are produced directly on the surface of the part and constitute a visual representation of the flaw.

4. Aerosol spray can make penetrant materials very portable.

5. Penetrant materials and associated equipments are relatively inexpensive.

Limitations of Dye Penetrant Testing:

1.Only surface breaking defects can be detected.

2.Precleaning is critical since contaminants can mask defects.

3. The inspector must have direct access to the surface being inspected.
4. Surface finish and roughness can affect inspection sensitivity.
5. Post cleaning of acceptable parts or materials is required.
6. Chemical handling and proper disposal is required.

3.Magnetic Particle Testing:

This method is suitable for the detection of surface and near surface discontinuities in magnetic material, mainly ferrite steel and iron. Magnetic particle Testing (MPT) is a nondestructive testing method used for defect detection. MPT is fast and relatively easy to apply, and material surface

preparation is not as critical as it is for some other NDT methods.

Basic Principle of MPT:

In the first figure the magnetized metal has no crack and there only two poles that is north pole and south pole. And in second figure the magnetized metal has a crack and at the crack point there creates another north and south pole for the
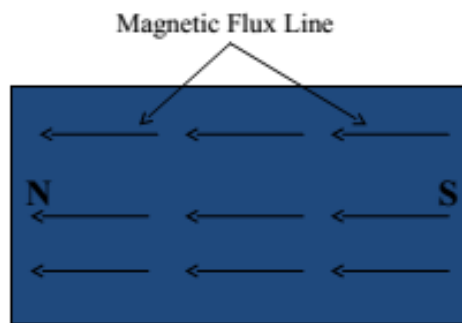
magnetic flux leakage.
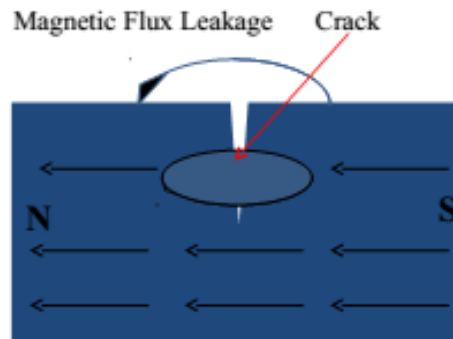

Fig.1: Magnetized Metal with no crack   Fig.2: Magnetized Metal with crack

Magnetic Particle Testing Process:

The first step in a magnetic particle testing is to magnetize the test component by a MPT equipment. If there any defects on the surface or near to the surface are present, the defects will create a leakage field. Then finely milled iron particles coated with a dye pigment are applied to the specimen. These particles are attracted to magnetic flux leakage fields and will cluster to form an indication directly over the defects. This indication can be visually detected under proper lighting conditions.

First the welding joint is magnetized by MPT equipment. Then finely milled iron particles are applied to the magnetized weld joint. Iron particles make a cluster at the welding joint for magnetic flux leakage because of welding defects.

Advantages Magnetic Particle Testing:

1.It does not need very stringent pre-cleaning operation.
2. It is the best method for the detection of surface and near to the surface cracks in ferromagnetic materials.
3. Fast and relatively simple NDT method.
4. Generally inexpensive.
5. Will work through thin coating.
6. Highly portable NDT method.
7. It is quicker.

Limitations of Magnetic Particle Testing

1.Material must be ferromagnetic.

2. Orientation and strength of magnetic field is critical.

3. Detects surface and near-to-surface discontinuities only.

4. Large currents sometimes require.

4.Ultrasonic Testing:

This technique is used for the detection of internal surface (particularly distant surface) defects in sound conducting materials. In this method high frequency sound waves are introduced into a material and they are reflected back from surface and flaws. Reflected sound energy is displayed versus time, and inspector can visualize a cross section of the specimen showing the depth of features.

Basic Principle of Ultrasonic Testing:

A typical UT system consists of several functional units, such as the pulses /receiver, piezoelectric transducer, and display devices. A pulses /receiver is an electronic device that can produce high voltage electrical pulses. Driven by the pulses, the transducer generates high frequency ultrasonic energy. The sound energy is introduced and propagates through the materials in the form of waves. When there is a discontinuity (such as a crack) in the wave path, part of the energy will be reflected back from the flaw surface. The reflected wave signal is transformed into an electrical signal by the piezoelectric transducer and is displayed on a screen. In the figure below, the reflected signal strength is displayed versus the time from signal generation, when a echo was received. Signal travel time can be directly related to the distance. From the signal, information about the reflector location, size, orientation and other features can sometimes be gained.

Oscilloscope, or flaw detector screen

29
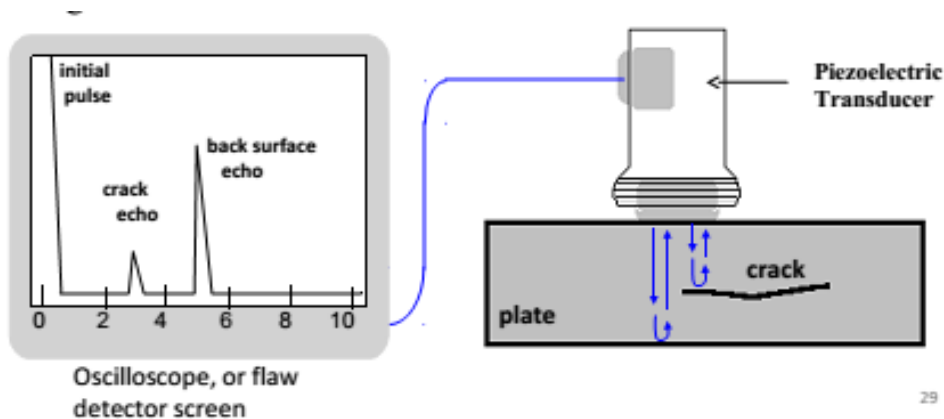
Advantages of Ultrasonic Testing:

1.Thickness and lengths up to 30 ft can be tested.
2. Position, size and type of defect can be determined.
3. Instant test results.
4. Portable.
5. Capable of being fully automated.
6. Access to only one side necessary.

Limitations of Ultrasonic Testing:

1. The operator can decide whether the test piece is defective or not while the test is in progress.
2.Considerable degree of skill necessary to obtain the fullest information from the test.
3.Very thin sections can prove difficult.

5.Eddy Current Testing:

This method is widely used to detect surface flaws, to measure thin walls from one surface only, to measure thin coatings and in some applications to measure depth. This method is applicable to electrically conductive materials only. In this method eddy currents are produced in the product by bringing it close to an alternating current carrying coil. The main applications of the eddy current technique are for the detection of surface or subsurface flaws, conductivity measurement and coating thickness measurement.

Electromagnetic Induction

•Eddy currents are created through a process called electromagnetic induction.
•When alternating current is applied to the conductor, such as copper wire, a magnetic field develops in and around the conductor.
•This magnetic field expands as the alternating current rises to maximum and collapses as the current is reduced to zero.

 If another electrical conductor is brought into the proximity of this changing magnetic field, the reverse effect will occur. Magnetic field cutting through the second conductor will cause an "induced" current to flow in this second conductor. Eddy currents are a form of induced currents!
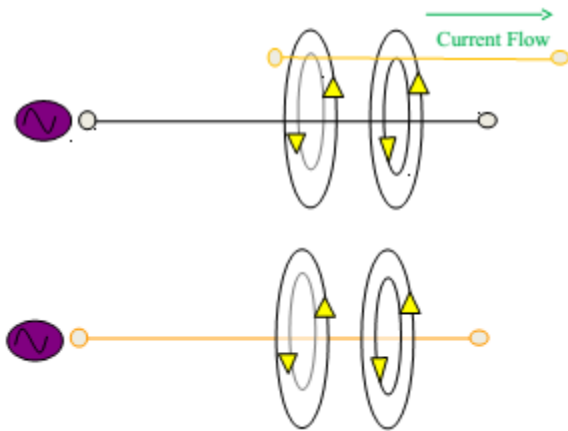
Crack Detection:
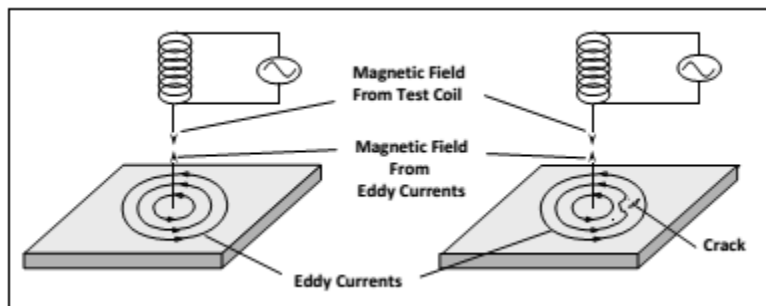
Crack detection is one of the primary uses of eddy current inspection. Cracks cause a disruption in the circular flow patterns of the eddy currents and weaken their strength. This change in strength at the crack location can be detected.

Advantages of Eddy Current Testing:

1.Sensitive to small cracks and other defects
2.Detect surface and near surface defects
3.Inspection gives immediate results
4.Equipment is very portable
5.Method can be used for much more than flaw detection
6.Inspects complex shapes and sizes of conductive materials

Limitations of Eddy Current Testing:

1.Only conductive materials can be inspected.
2.Surface must be accessible to the probe.
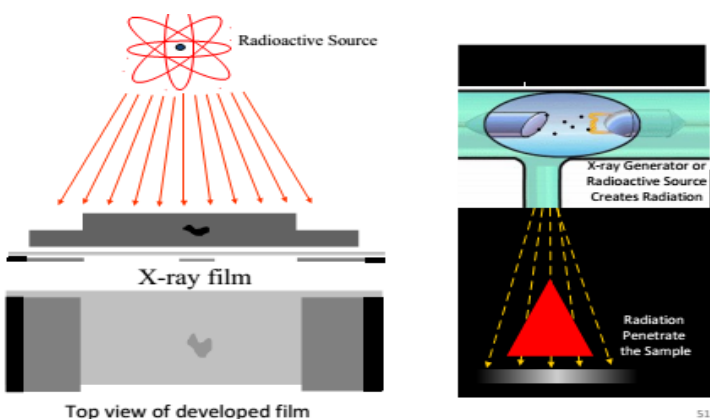3.Skill and training required is more extensive than other techniques.

4.Surface finish and roughness may interfere.

5.Depth of penetration is limited.

Radiography Testing:

Radiography Testing (RT), or industrial radiography is a nondestructive testing (NDT) method of inspecting materials for hidden flawsby using the ability of short wavelength electromagnetic radiation (high energy photons) to penetrate various materials. Radiographic Testing Method is nothing but to take the shadow picture of an object onto a film by the passage of X-ray or Gamma ray through it. It is the same as the medical radiography (X ray). Only difference in their wave length.

Radiography Testing Process:

The principles are the same for both X and Gamma radiography. In X-radiography the penetrating power is determined by the number of volts applied to the X-Ray tube - in steel approximately 1000 volts per inch thickness is necessary. To produce an X or Gamma radiograph, the film package is placed close to the surface of the subject. The source of radiation is positioned on the other side of the subject some distance away, so that the radiation passes through the subject and on to the film. After the exposure period the film is removed, processed, dried, and then viewed by transmitted light on a special viewer. Various radiographic and photographic accessories are necessary, including such items as radiation monitors, film markers, image quality indicators, darkroom equipment, etc. Where the last is concerned there are many degrees of sophistication, including fully automatic processing units. These accessories are the same for both X and Gamma radiography systems. Also required are such consumable items as radiographic film and processing chemicals.

Radioactive Source

X-ray film

Top view of developed film

X-ray Generator or Radioactive Source Creates Radiation

Radiation Penetrate the Sample

51

Essential Elements for Radiography Testing

1. A source of penetrating radiation, such as an X-ray machine.
2. The object to be radiographed, such as a weldment.
3. A recording or viewing device, usually photographic (X-ray) film enclosed in a light tight holder.
4. A qualified radiographer trained to produce a satisfactory exposure.
5. A person skilled in the interpretation of radiographs.

Radiation Safety

Ionizing radiation is an extremely important NDT tool but it can pose a hazard to human health. For this reason, special precautions must be observed when using and working around ionizing radiation. Complicating matters further is the fact that Gamma and X-ray radiation are not detectable by the human body. However,

the risks can be minimized when the radiation is handled and managed properly.

Advantages of Radiography Testing:

1. Information is presented pictorially.
2. A permanent record is provided which may be viewed at a time and place distant from the test.
3. Useful for thin sections.

4. Sensitivity declared on each film suitable for any material.

Limitations of Radiography Testing:

1. Possible health hazard.
2. Need to direct the beam accurately for two-dimensional defects.
3. Film processing and viewing facilities are necessary
4. Not suitable for automation.
5. Not suitable for surface defects.

6. X-RAY FLUORESCENCE:

An X-ray fluorescence (XRF) spectrometer is an x-ray instrument used for routine, relatively non-destructive chemical analyses of rocks, minerals, sediments and fluids.It is typically used for bulk analyses of larger fractions of

geological materials. One of the most widely used methods due to its relative ease, low cost of sample preparation and the stability and use of Xray spectrometers. One of the best analytical techniques to perform elemental analysis in all kinds of samples, no matter if liquids, solids or loose powders.

HISTORY OF XRF

• The history of XRF dates back all the way to 1895 when German physicist Wilhelm Conrad Roentgen accidentally discovered X-rays while studying cathode rays in high-voltage, gaseous discharge tube.

• However, the possible use of X-rays for analysis went unnoticed until 1913 when Henry Moseley established the specific relationship between the wavelength of a characteristic X-ray photon and the atomic number of the excited element.

• In 1925 X-rays were used for the first time to excite a sample, but the technique was only made practical in 1940 and the first commercial XRF spectrometers were produced in 1950.

FUNDAMENTAL PRICIPLES OF XRF

• XRF works on methods involving interactions between electron beams and x-rays with samples.

• Made possible by the behavior of atoms when they interact with radiation.

• When materials are excited with high-energy, short wavelength radiation (e.g., X-rays), they can become ionized.

• If the energy of the radiation is sufficient to dislodge a tightly held inner electron, the atom becomes unstable and an outer electron replaces the missing inner electron.

• When this happens, energy is released due to the decreased binding energy of the inner electron orbital compared with an outer one.

• The emitted radiation is of lower energy than the primary incident X-rays and is termed fluorescent radiation.

• Because the energy of the emitted photon is characteristic of a transition between specific electron orbitals in a particular element, the resulting fluorescent X-rays can be used to detect the abundances of elements that are present in the sample.

XRF - WORKING

• An XRF spectrometer works because if a sample is illuminated by an intense X-ray beam, known as the incident beam, some of the energy is scattered, but some is also absorbed within the sample in a manner that depends on its chemistry.

• The incident X-ray beam is typically produced from a Rh target, although W, Mo, Cr and others can also be used, on the application.

• When x-ray hits sample, the sample emits x-rays along a spectrum of wavelengths characteristic of the type of atoms present.

• If a sample has many elements present, the use of a Wavelength Dispersive Spectrometer allows the separation of a complex emitted X-ray spectrum into characteristic wavelengths for each element present.

• Various types of detectors used to measure intensity of emitted radiation.

• Examples of detectors used include the flow counter and the scintillation detector.

• Flow counters measure long wavelength(>0.15nm) x-rays typical of elements lighter than zinc.

• The scintillation detector is commonly used to analyze shorter wavelengths in the X-ray spectrum(K spectra of element from Nb to I; L spectra of Th and U).

• The intensity of the energy measured by these detectors is proportional to the abundance of the element in the sample.

• The exact value for each element is derived from standards from prior analyses from other techniques.
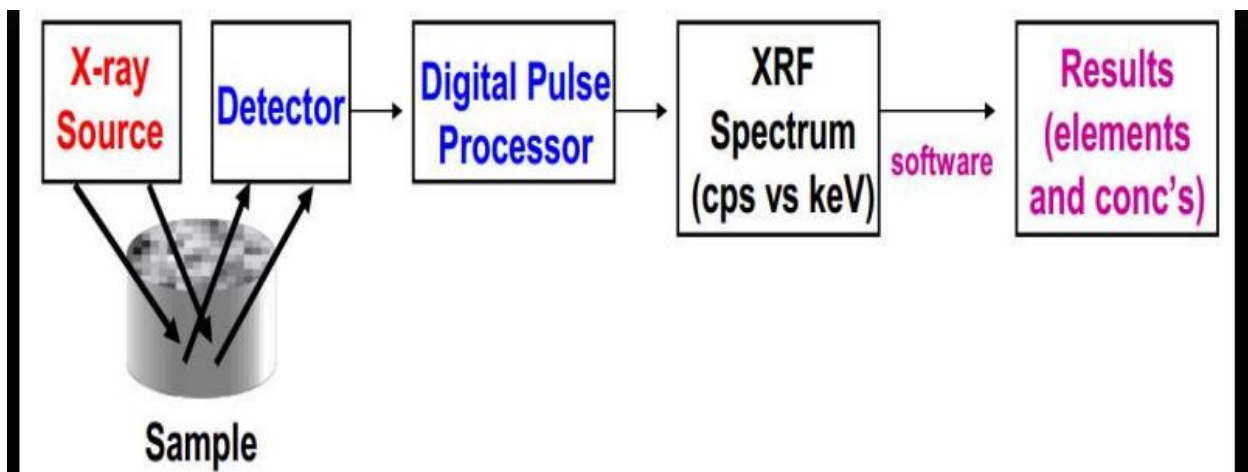
APPLICATIONS XRF

X-Ray fluorescence is used in a wide range of applications,
including
• research in igneous, sedimentary, and metamorphic petrology
• soil surveys
• mining (e.g., measuring the grade of ore)
• cement production
• ceramic and glass manufacturing
• metallurgy (e.g., quality control)
• environmental studies (e.g., analyses of particulate matter on air filters)
• petroleum industry (e.g., sulfur content of crude oils and petroleum products)
• field analysis in geological and environmental studies (using portable, hand-held
XRF spectrometers) X-ray fluorescence is limited to analysis of
• relatively large samples, typically > 1 gram
• materials that can be prepared in powder form and effectively homogenized
• materials for which compositionally similar, well-characterized standards are
available
• materials containing high abundances of elements for which absorption and
fluorescence effects are reasonably well understood

STRENGTHS & LIMITATIONS OF XRF

Strengths
X-Ray fluorescence is particularly well-suited for investigations that
involve:
• bulk chemical analyses of major elements (Si, Ti, Al, Fe, Mn, Mg, Ca, Na, K, P) in
rock and sediment
• bulk chemical analyses of trace elements (>1 ppm; Ba, Ce, Co, Cr,Cu, Ga, La, Nb,
Ni, Rb, Sc, Sr, Rh, U, V, Y, Zr, Zn) in rock and sediment
LIMITATIONS
In theory the XRF has the ability to detect X-ray emission from virtually all
elements, depending on the wavelength and intensity of incident x-rays.
However...
• In practice, most commercially available instruments are very limited in their
ability to precisely and accurately measure the abundances of elements with Z<11

in most natural earth materials.

• XRF analyses cannot distinguish variations among isotopes of an element, so these analyses are routinely done with other instruments.

• XRF analyses cannot distinguish ions of the same element in different valence states, so these analyses of rocks and minerals are done with techniques such as wet chemical analysis or Mossbauer spectroscopy.

Physics of Nanomaterials

Definition of Nanotechnology: "Nano Materials are the materials which have structure components with size less than 100nm at least in one dimension"

"Nanotechnology is the understanding and control of matter at dimensions of roughly 1 to 100 nanometers, where unique phenomena enable novel applications. Encompassing nanoscale science, engineering and technology, nanotechnology involves imaging, measuring, modeling, and manipulating matter at this length scale." At the nanoscale, the physical, chemical, and biological properties of materials differ in fundamental and valuable ways from the properties of individual atoms and molecules or bulk matter. Nanotechnology R&D is directed toward understanding and creating improved materials, devices, and systems that exploit these new properties.

Important:

1-Dimension : Thin films or surface coatings

2-Dimension : Nano Wires, Nano Tubes

3-Dimension : Nano Crystalline materials, Quantum Dots (Tiny Semiconducting Device)

Why the properties of nano particle are different from macroscopic particles ?

1. <u>Surface to Volume Ratio</u> : In the case of a spherical particle

Surface area = $4\prod r^2$

 Volume $=$ $(4/3) \prod r^3$

 Surface area to Volume Ratio = 3/r

 "Lesser the radius , Greater the ratio"

Similarly in the case of cube we have
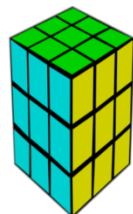
sides = 3
surface = 3^2 x 6 = 54
volume = 3^3 = 27

surface/volume = 2

sides = 2
surface = 2^2 x 6 = 24
volume = 2^3 = 3

surface/volume = 3

sides = 1
surface = 1^2 x 6 = 6
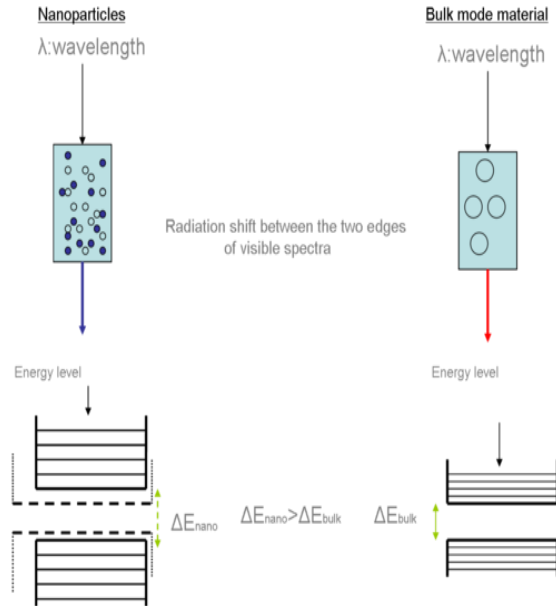volume = 1^3 = 1

surface/volume = 6

2. **<u>Quantum confinement in nano particles</u>**

Nano technology an emerging technology which has gained fame in every field of life from an excellent sunscreen to an electronic chip. This emerging technology has given excellent properties to even those elements which at one time were thought of being useless. For example Carbon is a non metal but when considered at the nano scale the carbon nano tubes are the best conductors .But what is the enigma beyond size if this size can make a non conductor an insulator what is the basic physics beyond it .Well the answer is simple and that is Quantum confinement.

When atoms are isolated energy levels are discrete. When atoms are closely packed, the energy levels splits and bands will be formed . Nano materials represents intermediate stage. When the materials sufficiently small in size (<10 nm), Organization of energy levels into which electrons can change. This phenomenon results from electrons and holes being squeezed into a dimension, called exciton Bohr radius.

The quantum confinement effect is observed when the size of the particle is too small to be comparable to the wavelength of the electron. To understand this effect we break the words like quantum and confinement, the word confinement means to confine the motion of randomly moving electron to restrict its motion in specific energy levels (discreteness) and quantum reflects the atomic realm of particles. So as the size of a particle decrease till we a reach a nano scale the decrease in confining dimension makes the energy levels discrete and this increases or widens up the band gap and

 ultimately the band gap energy also increases. Since the band gap and wavelength are inversely related to each other the wavelength decrease with decrease in size and the proof is the emission of blue radiation .Comparison of a bulk material and nano particle reveals that too from the diagram the blue wavelength and the red wavelength

Nanoparticles
λ:wavelength

Bulk mode material
λ:wavelength

Radiation shift between the two edges of visible spectra

Energy level

Energy level

ΔE_{nano} $\Delta E_{nano} > \Delta E_{bulk}$ ΔE_{bulk}

Electrical, optical and magnetic properties of nano particles:

Various properties of the materials like electrical, optical and magnetic are sensitively depend on the size of the matter. Thus the above properties greatly vary for a material in the bulk size to the same material in nano size.

Electrical properties:

If the material has at least one of the dimensions of the order of nano metre then it is called Quantum well. We know the energy of the particle inside the potential box is

$$E = \frac{n^2 h^2}{8mL^2}$$

Considering L = 1 cm, for electron the separation between the consecutive energy levels will be of the order of 10^{-14} eV which is quasi – continuous. In the case of L = 100 nm, the separation between consecutive energy levels is around 10^{-4}eV. Thus in nano scale range the energy levels are discrete.

However, the change in electrical properties cannot be generalized. In nano ceramics and magnetic nano composites, the electrical conductivity increases with the decrease in particle size whereas in metals electrical conductivity decreases with the reduction in particle size.

Optical properties:

In some of the materials, energy is related to wavelength (colour). Therefore the optical properties of the particle can be finely tuned depending on its size. Thus particles can be made to emit or absorb specific wave lengths of light by merely controlling their size. Gold nano spheres of 100 nm appear in Orange while 50 nm nano spheres appear in Green.

Magnetic properties:

The strength of a magnet is measured in terms of coercivity and saturation of magnetization. These values increase, with the decrease in the grain size and an increase in the specific surface area of the grain. Thus nano particles possess good magnetic properties.

<u>Synthesis of Nanomaterials:</u>

1). Physical Method
2). Chemical method

<u>Physical Method:</u>

<u>Synthesis of Nanomaterials by Laser Ablation:</u>

Since the discovery of laser decades ago, laser has been intensively used and studied for various applications including laser ablation. Even though the first experimental paper about laser ablation was reported as early as 1963, laser ablation was not employed for synthesizing nanomaterials with the purpose for gas sensing until mid 1990s.

Laser ablation means the removal of material from a surface by means of laser irradiation. The term "laser ablation" is used to emphasize the nonequilibrium vapor/plasma conditions created at the surface by intense laser pulse, to distinguish from "laser evaporation," which is heating and evaporation of material in condition of thermodynamic equilibrium. A typical schematic diagram of laser ablation is shown in the following figure. Briefly, there are two essential parts in the laser ablation device, a pulsed laser (CO_2 laser, Nd-YAG laser etc) and an ablation chamber. The high power of the laser beam induces large light absorption on the surface of target, which makes temperature of the absorbing material increase rapidly. As a result, the material on the surface of target vaporizes into laser plume. In some cases, the vaporized materials condensate into cluster and particle without any chemical reaction. In some other cases, the vaporized material reacts with introduced reactants to form new materials. The condensed particle will be either deposited on a substrate or collected through a filter system consisting of a glass fiber mesh. Then, the collected nanoparticle can be coated on a substrate through drop-coating or screen-printing process.
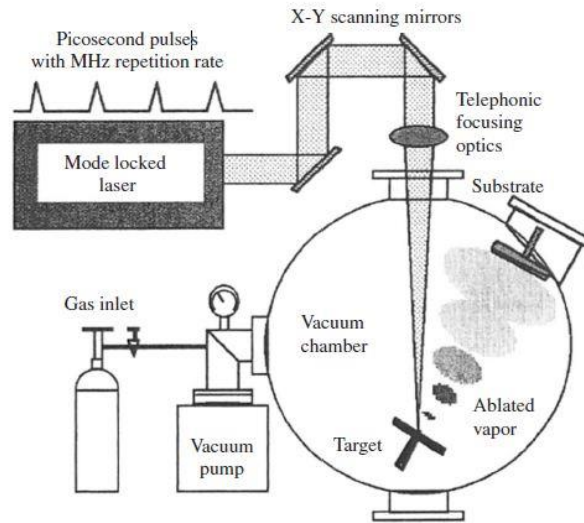
BALL MILLING: Ball milling is a method of production of nano materials by the process of a mechanical crushing. The mills are equipped with grinding media composed of wolfram carbide or steel. Small balls inside a drum-like cavity are rotated at high speeds and by gravity actions, they settle on a solid layer where they crushed into nanocrystals.

The following are the various types of ball mills:

1)   Attrition ball mill
2)   Planetary ball mill
3)   Vibrating ball mill
4)   Low and high energy ball mills

The significant advantage of this method is that it can be readily implemented commercially. Ball milling can be used to make carbon nanotubes and boron nitride nanotubes. It is a preferred method for preparing metal oxide nano crystals like Cerium (CeO2) and Zinc Oxide (ZnO).

Chemical method:

Chemical Vapour Deposition Method:

1.The vapour of the heated precursor is carried into the reaction chamber by the carrier gas (inert gases like Argon Neon).

2. The atoms in the vapour are relatively hotter when they enter into chamber.

3. They agglomerate around the relatively cooler atoms present in the reaction chamber forming nano clusters.

4. Once the required size dusters are formed they are sent on to the scraper and collected in particle collector.

5. In a different kind of arrangement, a substrate will be present in the reaction chamber and the hot atoms in the vapour get accumulated on the substrate.They involve in a chemical reaction either with the substrate or with a second kind of atoms taking the substrate as the platform.Thus atoms will be deposited layer by layer and the substrate is taken out once the thin film of required thickness is ready.
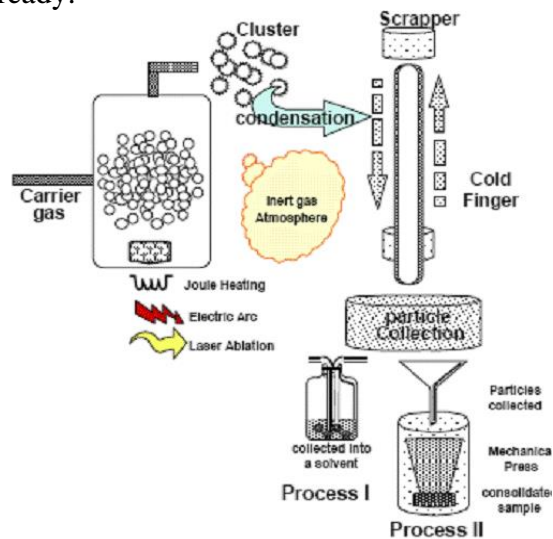
Sol-Gel Method:

 Colloid suspended in a liquid is called Sol. A suspension that keeps it shape is called Gel.

Steps Involved in Sol-Gel Method:

Step-1: Formation of different stable solutions of alkoxide or solvated metal precursors.(a **precursor** is a compound that participates in the chemical reaction that produces another compound)

Step 2: By Dehydration reaction ,we can form a Gel which results dramatic increase in viscosity of the solution.

step 3: Drying the gel, when water and other volatile liquids are removed from gel network.

The resulting monolith is termed a 'Xerogel'.

step 4:   In Dehydration,the surface bound M-OH groups are removed by calcining at a temparature upto 800^0 C.

Step-5: Densification and decomposition of Gels at high temparatures (T>800^0 C).The pores of Gel network will collapse.
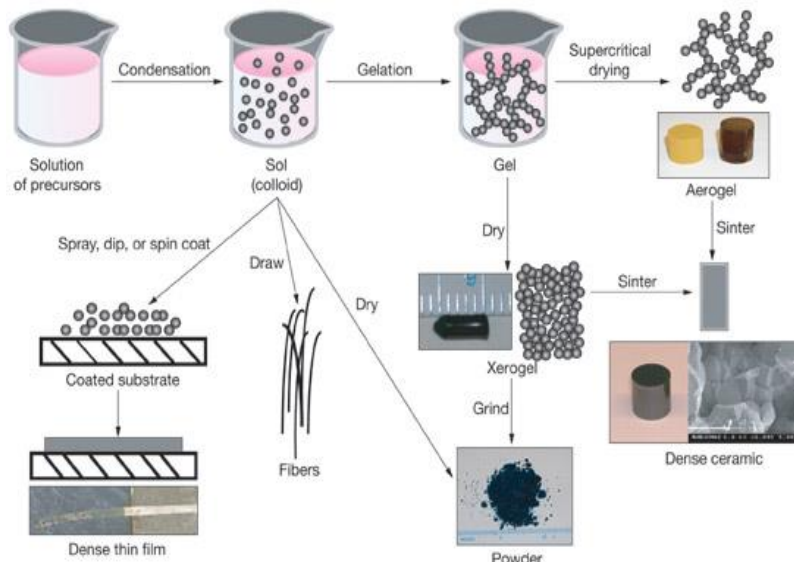
Carbon nanotubes (CNTs):

Carbon nanotubes (CNTs) are allotropes of carbon with a cylindrical nanostructure. Nanotubes have been constructed with length-to-diameter ratio of up to 132,000,000:1, significantly larger than for any other material. These cylindrical carbon molecules have unusual properties, which are valuable for nanotechnology, electronics, optics and other fields of materials science and technology. In particular, owing to their extraordinary thermal conductivity and mechanical and electrical properties, carbon nanotubes find applications as additives to various structural materials.

 Nanotubes are members of the fullerene structural family. Their name is derived from their long, hollow structure with the walls formed by one-atom-thick sheets of carbon, called graphene. These sheets are rolled at specific and discrete ("chiral") angles, and the combination of the rolling angle and radius decides the nanotube properties; for example, whether the individual nanotube shell is a metal or semiconductor. Nanotubes are categorized assingle-walled nanotubes (SWNTs) and multi-walled nanotubes (MWNTs).

Most single-walled nanotubes (SWNTs) have a diameter of close to 1 nanometer, and can be many millions of times longer. The structure of a SWNT can be conceptualized by wrapping a one-atom-thick layer of graphite called graphene into a seamless cylinder. SWNTs are an important variety of carbon nanotube because most of their properties change significantly. In particular, their band gap can vary from zero to about 2 eV and their electrical conductivity can show metallic or semiconducting behavior. Single-walled nanotubes are likely candidates for miniaturizing electronics. The most basic building block of these systems is the electric wire, and SWNTs with diameters of an order of a nanometer can be excellent conductors.

Multi-walled nanotubes (MWNTs) consist of multiple rolled layers (concentric tubes) of graphene. There are two models that can be used to describe the structures of multi-walled nanotubes. The interlayer distance in multi-walled nanotubes is close to the distance between graphene layers in graphite, approximately 3.4 Å.

Carbon Nanotubes Properties and Applications

There are numerous carbon nanotubes applications which take full advantage of CNTs unique properties of aspect ratio, mechanical strength, electrical and thermal conductivity. We've compiled the list below for you.

Properties:

- CNTs have High Electrical Conductivity
- CNTs have Very High Tensile Strength
- CNTs are Highly Flexible- can be bent considerably without damage
- CNTs are Very Elastic ~18% elongation to failure
- CNTs have High Thermal Conductivity
- CNTs have a Low Thermal Expansion Coefficient
- CNTs are Good Electron Field Emitters
- CNTs Aspect Ratio

Applications:
- CNTs Thermal Conductivity
- CNTs Field Emission
- CNTs Conductive Properties
- CNTs Energy Storage
- CNTs Conductive Adhesive
- Molecular Electronics based on CNTs
- CNTs Thermal Materials
- CNTs Structural Applications
- CNTs Fibers & Fabrics
- CNTs Catalyst Supports
- CNTs Biomedical Applications
- CNTs Air & Water Filtration

- Other CNT Applications

DIELELCTRIC PROPERTIES

* Dielectrics are basically insulating materials. All electrons are bound to their parent atoms/molecules and there are no free charges.

* Even with normal voltage, Electrons are not released. These are non metallic materials, they have high resistivity (ρ).

Ex:- glass, mica, plastic, air, etc.

Def:- A insulating substance which get electric properties by the polarization.

Electric polarization (P):-

When an electric field is applied to a dielectric material, the positive charges (of all atoms) are displaced along the field direction, while negative charges (of all atoms) are displaced in the opposite direction. The displacement of charges produce dipoles. "This process of producing dipoles by the influence of electric field is called electric polarization (P)".
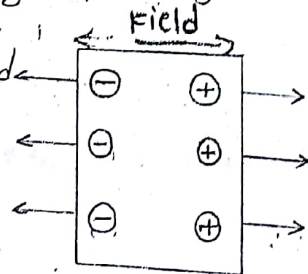
Polarizability (α):-

Induced dipolemoment is proportional to the intensity of electric field (E),

$$\mu \propto E$$

$$\mu = \alpha E$$

Where α - polarizability

DEF:- "Induced dipolemoment per unit electric field"

DIELECTRIC CONSTANT (K) (or) RELATIVE PERMITIVITY $(\mathcal{E}_r)$:-

"Relative permitivity is the ratio between permitivity of the medium $(\mathcal{E})$ and permitivity of free space $(\mathcal{E}_o)$". This is also called dielectric constant (K)

i.e $\boxed{\mathcal{E}_r = \dfrac{\mathcal{E}}{\mathcal{E}_o}}$

units:- Dimensionless

NOTE:- * For pure dielectric, $\underline{\mathcal{E}_r > 1}$
   * For air , $\mathcal{E}_r = 1$

ELECTRIC FLUX DENSITY (D):-

"The number of lines of force (electric force) passing through unit area perpendicularly is called electric flux density (D)". It is proportional to applied field.

$$D \propto E$$

$$D = \mathcal{E} E$$

$$D = \mathcal{E}_r \mathcal{E}_o E \longrightarrow ① \quad \left[\because from \ \mathcal{E}_r = \dfrac{\mathcal{E}}{\mathcal{E}_o} \ \longrightarrow \mathcal{E} = \mathcal{E}_r \mathcal{E}_o \right]$$

Where
   $\mathcal{E}$ - permitivity of medium

If 'P' is Polarization. Then

we know that, $D = \mathcal{E}_o E + P \longrightarrow ②$

from ① and ②

$$\varepsilon_r \varepsilon_0 E = \varepsilon_0 E + P$$

$$P = \varepsilon_r \varepsilon_0 E - \varepsilon_0 E$$

$$\boxed{P = \varepsilon_0 E (\varepsilon_r - 1)} \longrightarrow ③$$

ELECTRIC SUSCEPTIBILITY :-

When an electric field is applied to a dielectric material, the polarization takes place.

$$P \propto E$$

$$P = \varepsilon_0 \chi_e E$$

Where

χ - Electric susceptibility

$$\Rightarrow \chi_e = \frac{P}{\varepsilon_0 E} \longrightarrow ④$$

$$\chi_e = \frac{\varepsilon_0 E (\varepsilon_r - 1)}{\varepsilon_0 E} \quad (\because \text{from } ③)$$

$$\chi_e = \varepsilon_r - 1 \longrightarrow ⑤$$

DIFFERENT PROCESS OF POLARISATION :-

When a d.c (current) electric field is applied to a dielectric material, mainly the following processes are takes place.

1. Electronic polarization
2. Ionic polarization.
3. Orientational polarization.

1. ELECTRONIC POLARISATION :-

" On application of an electric field there is a small displacement between positively charged nucleus and the negative charged electrons (electron cloud) of an atom in opposite directions ". This phenomena is called
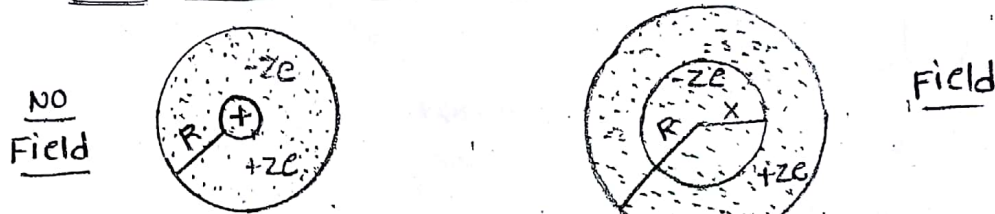
"electronic polarization":

Induced dipole moment $\mu_e \propto E$

$$\mu_e = \alpha_e E$$

Where, α_e - electronic polarizability.

It is independent on temperature

CALCULATION OF ELECTRONIC POLARIZABILITY (α_e) :-

NO Field

Field

A simplified classical model of an atom is shown in fig. Here the nucleus of charge 'ze' is surrounded by an electron cloud of charge '-ze', distributed in a sphere of radius R. The charge density 'ρ' is given by

$$\rho = \frac{-ze}{\frac{4}{3}\pi R^3} \longrightarrow ① \quad \left(\because \text{charge density} = \frac{\text{charge}}{\text{volume}} \right)$$

When an electric field of intensity 'E' is applied, the nucleus and electrons experience Lorentz force,

$$\text{Lorentz force} = -zeE$$

Due to electric field nucleus and electron cloud are seperated. When they are seperated a coulomb force develops between them,

$$\text{Coulomb force} = ze \left[\frac{\text{charge enclosed in sphere of radius } x}{4\pi\varepsilon_0 x^2} \right]$$

Where x- equilibrium distance.

$$= ze \ \frac{-ze \ x^3/R^3}{4\pi\varepsilon_o \ x^2}$$

$\therefore$ coulomb force $= \frac{-z^2e^2x}{4\pi\varepsilon_o R^3}$

$\left[\begin{array}{l} \therefore \text{ charge enclosed in the} \\ \text{sphere} = \frac{4}{3}\pi x^3 \rho \\ = \frac{4}{3}\pi x^3 \left[\frac{-ze}{\frac{4}{3}\pi R^3} \right] \ (\therefore \text{from } \textcircled{1}) \\ = -ze \ x^3/R^3 \end{array} \right.$

At equilibrium condition,

Lorentz force = coulomb force

$$-zeE = - \frac{z^2e^2x}{4\pi\varepsilon_o R^3}$$

$$E = \frac{zex}{4\pi\varepsilon_o R^3}$$

$$x = \frac{4\pi\varepsilon_o R^3 E}{ze} \longrightarrow \textcircled{2}$$

induced electric dipole moment

$\mu_e = zex$ $[\therefore \text{ dipole moment} = \text{charge} \times \text{distance}]$

$$\mu_e = ze \ \frac{4\pi\varepsilon_o R^3 E}{ze}$$

$$\mu_e = 4\pi\varepsilon_o R^3 E$$

$$\mu_e = \alpha_e E \longrightarrow \textcircled{3}$$

Where $\boxed{\alpha_e = 4\pi\varepsilon_o R^3}$ is called electronic polarizability,

It is independent of temperature.

$\longrightarrow$ electronic polarization, $P_e = N\mu_e$

Where N – Number of molecules per unit volume

$P_e = N\alpha_e E \longrightarrow \textcircled{4}$ $(\therefore \text{ from } \textcircled{3})$

We know that $P_e = \varepsilon_o E \ (\varepsilon_r - 1) \longrightarrow \textcircled{5}$

Compare equations $\textcircled{4}$ and $\textcircled{5}$. $N\alpha_e = \varepsilon_o (\varepsilon_r - 1)$

$$\boxed{\alpha_e = \frac{\varepsilon_o (\varepsilon_r - 1)}{N}} \longrightarrow \textcircled{6}$$

2. IONIC POLARISATION :-

" In ionic solids, on application of the electric field, there is a small displacement betwe cations and anions in opposite directions. This phenomena is called Ionic Polarisation ".

CALCULATION OF IONIC POLARIZABILITY (α_i) :-

Suppose an electric field is applied in the +ve x direction. The cations move to right by x_1 and the anions move to left by x_2

NO FIELD
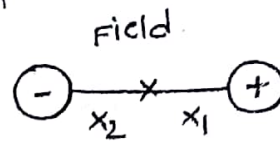
FIELD

Let each unit cell has one cation and one anion. The resultant dipolemoment per unit cell due to ionic displacement is

$$\mu_i = e(x_1 + x_2) \quad\text{—} \quad ⑦$$

If k_1 and k_2 are restoring-force constants of cation and anion -force due to applied field.

$$F = k_1 x_1 = k_2 x_2$$

$F = k_1 x_1$

$x_1 = F/k_1$

$$x_1 = \frac{eE}{m\omega_0^2}$$

Where m - mass of cation

ω_0 - angular frequency

$F = k_2 x_2$

$x_2 = F/k_2$

$$x_2 = \frac{eE}{M\omega_0^2}$$

Where M - Mass of anion

ω_0 - angular frequency

Substitute x_1 and x_2 values in eq⑦

$$\mu_i = e\left[\frac{eE}{m\omega_0^2} + \frac{eE}{M\omega_0^2}\right]$$

$$\mu_i = \frac{e^2 E}{\omega_o^2}\left[\frac{1}{m} + \frac{1}{M}\right] - \text{⑧}$$

$\therefore$ Ionic polarizability $\alpha_i = \frac{\mu_i}{E}$ $(\because \mu = \alpha E)$

$$\alpha_i = \frac{\dfrac{e^2 E}{\omega_o^2}\left[\dfrac{1}{m} + \dfrac{1}{M}\right]}{E}$$

$$\boxed{\alpha_i = \frac{e^2}{\omega_o^2}\left[\frac{1}{m} + \frac{1}{M}\right]} - \text{⑨}$$

3. ORIENTATIONAL POLARIZATION :-

"In some materials which have molecules (CH_3Cl) with permanent dipolemoment ($\because$ positive and negative charges do not coincide), applied electric field results in orienting these molecular dipoles along the field direction. This phenomenon is called orientational polarization".

NO FIELD   FIELD

Orientational polarization can be written as

$$P_o = N\vec{\mu} - \text{⑩}$$

$$P_o = N\frac{\mu^2}{3KT}E \qquad \left[\because \vec{\mu} = \frac{\mu^2}{3KT}E\right]$$

$$P_o = N\alpha_o E - \text{⑪}$$

Where $\alpha_o = \frac{\mu^2}{3KT}$ = orientational polarizability

$$\boxed{\alpha_o = \frac{P_o}{NE} = \frac{\mu^2}{3KT}} - \text{⑫}$$

From above equation, α_o is inversely proportional to absolute temperature of the material. So, α_o is depends on temperature.

∴ Total polarizability can be written as

$$\alpha = \alpha_e + \alpha_i + \alpha_o$$

∴ Total polarisation $P = N\alpha E$

$$P = \left[4\pi \varepsilon_0 R^3 + \frac{e^2}{\omega_0^2}\left(\frac{1}{m} + \frac{1}{M}\right) + \frac{\mu^2}{3KT}\right]NE \rightarrow ⑬$$

This equation is called 'Langevin - Debye' equation.

INTERNAL FIELD (or) LOCAL FIELD :-

"Total electric field acting on atoms or molecules of dielectric material is called "Internal-field" or "Local field".

To find an expression for internal field on a atom or a molecule, we consider a dielectric material in the electric field of intensity E, between the capacitor plates. as shown in fig ⓐ
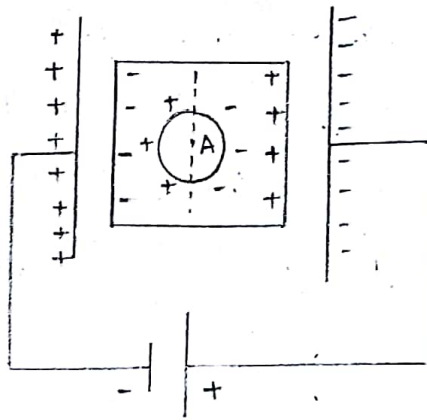
fig(a).

Let us imagine a small spherical region of the dielectric with an atom 'A' at the centre of the sphere. Let 'r' is the radius of the sphere.

The internal field at the atom 'A' can considered to be made up of the following four components namely E_1, E_2, E_3 and E_4.

FIELD E_1 :-

'E_1' is the field intensity at 'A' due to charge

on the plates of capacitor (i.e without dielectric)

$$E_1 = \frac{D}{\varepsilon_0} \qquad (\because D = \varepsilon E).$$

$$\implies E_1 = \frac{\varepsilon_0 E + P}{\varepsilon_0} \qquad (\because D = \varepsilon_0 E + P)$$

$$\implies \boxed{E_1 = E + \frac{P}{\varepsilon_0}}$$

FIELD E_2 :-

E_2' is the field intensity of 'A' due to polarized charges (induced charges) on the two sides of the dielectric,
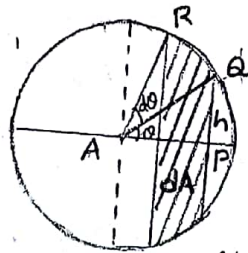
$$\boxed{E_2 = -P/\varepsilon_0}$$

FIELD E_3 :-

E_3' is the field intensity at 'A' due to all other dipoles (all other charges) of the atoms inside spherical region. If the imaginary sphere is highly symmetric

$$\boxed{E_3 = 0}$$

FIELD E_4 :-

E_4' is the field intensity at 'A' due to polarization charges on the surface of the sphere. If 'dA' is the surface area of the small element of the sphere of radius 'r' lying between 'θ' and θ+dθ, then

$$dA = 2\pi \, (PQ)(QR).$$

$$= 2\pi \,(r\sin\theta)(r\,d\theta)$$

$$dA = 2\pi r^2 \sin\theta \, d\theta \quad —①$$

fig (b).

$$\left[\begin{array}{l} \because \text{from } \triangle PAQ \\ \quad \sin\theta = \frac{PQ}{r} \\ \rightarrow PQ = r\sin\theta \\ \text{from sector } AQR, \\ \quad QR = r\,d\theta. \end{array} \right.$$

The charge dq on the surface dA is equal to the normal component of the polarization multiplied by surface area,

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$$dq = P\cos\theta \times dA$$

$$dq = P\cos\theta \times (2\pi\gamma^2 \sin\theta \, d\theta) - ② \quad (\because from①)$$

The field due to this charge at 'A', denoted by dE_4,

$$dE_4 = \frac{dq}{4\pi\varepsilon_0\gamma^2} \cos\theta$$

$$dE_4 = \frac{P\cos\theta(2\pi\gamma^2\sin\theta \, d\theta)\cos\theta}{4\pi\varepsilon_0\gamma^2} \quad (\because from②)$$

$$dE_4 = \frac{P}{2\varepsilon_0}\cos^2\theta \sin\theta \, d\theta$$

$\therefore$ The total field E_4 due to the charge on the surface of the entire sphere is obtained by integrating,

$$E_4 = \int_0^\pi dE_4 = \int_0^\pi \frac{P}{2\varepsilon_0}\cos^2\theta \sin\theta \, d\theta$$

$$= \frac{P}{2\varepsilon_0} \int_0^\pi \cos^2\theta \sin\theta \, d\theta$$

Let $x = \cos\theta$
$dx = -\sin\theta \, d\theta$
$-dx = \sin\theta \, d\theta$

if $\theta = 0, x = \cos 0 = 1$
if $\theta = \pi, x = \cos\pi = -1$

$$= \frac{P}{2\varepsilon_0} \int_1^{-1} x^2(-dx)$$

$$= \frac{-P}{2\varepsilon_0} \int_1^{-1} x^2 \, dx$$

$$= \frac{-P}{2\varepsilon_0} \left[\frac{x^3}{3}\right]_1^{-1}$$

$$= \frac{-P}{6\varepsilon_0} \left[(-1)^3 - (1)^3\right] = \frac{-P}{6\varepsilon_0}(-2) = \frac{P}{3\varepsilon_0}$$

$$\boxed{E_4 = \frac{P}{3\varepsilon_0}}$$

$\therefore$ Internal field $= E_{int} = E_1 + E_2 + E_3 + E_4$

$$= E + \frac{P}{\varepsilon_0} - \frac{P}{\varepsilon_0} + 0 + \frac{P}{3\varepsilon_0}$$

$$\boxed{E_{int} = E + \frac{P}{3\varepsilon_0}}$$

CLAUSIUS - MOSSOTTI EQUATION:-

Let us consider the dielectric having cubic struc-ture. Since there are no ions and permanent dipoles

in these materials, the ionic polarizability α_i; and orientational polarizability α_0 are zero.

$$\text{i.e}\quad \alpha_i = \alpha_0 = 0$$

Hence polarization $P = N\alpha_e E_{int}$ —①

$$\Rightarrow P = N\alpha_e \left(E + \frac{P}{3\varepsilon_0}\right) \quad \left[\because E_{int} = E + \frac{P}{3\varepsilon_0}\right]$$

$$\Rightarrow P = N\alpha_e E + N\alpha_e \frac{P}{3\varepsilon_0}$$

$$\Rightarrow P - \frac{N\alpha_e P}{3\varepsilon_0} = N\alpha_e E$$

$$\Rightarrow P\left[1 - \frac{N\alpha_e}{3\varepsilon_0}\right] = N\alpha_e E$$

$$\Rightarrow P = \frac{N\alpha_e E}{1 - \frac{N\alpha_e}{3\varepsilon_0}} \quad - \text{②}$$

we know that $P = \varepsilon_0 E (\varepsilon_r - 1)$ — ③

-from ② and ③

$$\frac{N\alpha_e E}{1 - \left(\frac{N\alpha_e}{3\varepsilon_0}\right)} = \varepsilon_0 E (\varepsilon_r - 1)$$

$$\Rightarrow \frac{N\alpha_e}{\varepsilon_0 (\varepsilon_r - 1)} = 1 - \left(\frac{N\alpha_e}{3\varepsilon_0}\right)$$

$$\Rightarrow \frac{N\alpha_e}{\varepsilon_0 (\varepsilon_r - 1)} + \frac{N\alpha_e}{3\varepsilon_0} = 1$$

$$\Rightarrow \frac{N\alpha_e}{\varepsilon_0}\left[\frac{1}{\varepsilon_r - 1} + \frac{1}{3}\right] = 1$$

$$\Rightarrow \frac{N\alpha_e}{\varepsilon_0}\left[\frac{3 + \varepsilon_r - 1}{3(\varepsilon_r - 1)}\right] = 1$$

$$\Rightarrow \frac{N\alpha_e}{3\varepsilon_0}\left[\frac{\varepsilon_r + 2}{\varepsilon_r - 1}\right] = 1$$

$$\boxed{\frac{N\alpha_e}{3\varepsilon_0} = \frac{\varepsilon_r - 1}{\varepsilon_r + 2}}$$

Where N - Number of atoms/molecules per unit volume

This relation is known as 'Clausius - Mossoti equation'.

$$\left[\begin{array}{l} \because P = N(\alpha_e + \alpha_i + \alpha_0)E_{int} \\ P = N(\alpha_e + 0 + 0)E_{int} \\ P = N\alpha_e E_{int} \end{array}\right.$$

from the various polarisation processes and from the concept of relaxation time for each process when the frequency of the applied field is much greater than the inverse of the relaxation time for a particular polarisation process, that particular polarisation process falls and so it does not contribute to polarisability. Thus, the decrease of total polarisability with increase in frequency is due to the disappearance of α_o, α_1 and α_e successively, i.e. with increase in frequency, gradually the corresponding polarisabilities are not able to follow the frequency variations and disappear gradually.
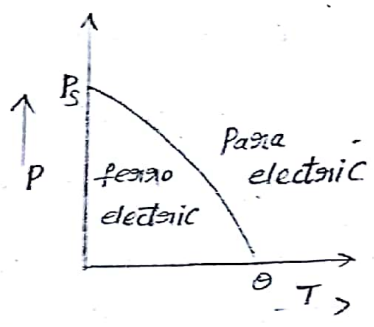
Ferro electricity:

Certain dielectrics exhibit polarization & dipole moment even in the absence of electric field, i.e. they exhibit 'spontaneous polarisation (P_S)'. This phenomenon is known as 'ferro electricity' and the dielectrics are known as 'ferro electrics'.

ex:-
1. Rochelle Salt
2. KDP (KH_2PO_4) - Potassium dihydrogen phosphate
3. Barium titanate $(BaTiO_3)$

Properties:-

1. As the temperature increases the spontaneous polarisation decreases and at a particular temperature

the spontaneous polarization Vanishes. This temperature is called curie temperature (θ) as show in fig
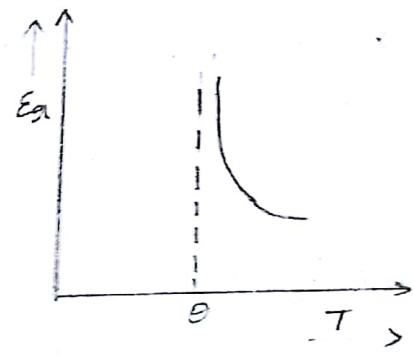

2. curie temperature can also be defined as the temperature at which ferro electric material converts into para electric material.

3. All ferro electric materials possess spontaneous polarisation below curie temperature $(T < θ)$

4. The dielectric constant changes with temperature according to relation,

$$\varepsilon_r = \frac{C}{T-θ}$$

where
c → curie constant
θ → curie temperature

* The dielectric constant at a ferro-electric material increases enormously as the temperature of the material reduces to its curie temperature.

5. Hysteresis

when the electric field is applied on a ferro-electric material then polarization takes place. This polarization always lags behind the applied field. This phenomenon is known as Hysteresis of ferro electric materials. This is similar to Hysteresis of ferro magnetic materials

A /m
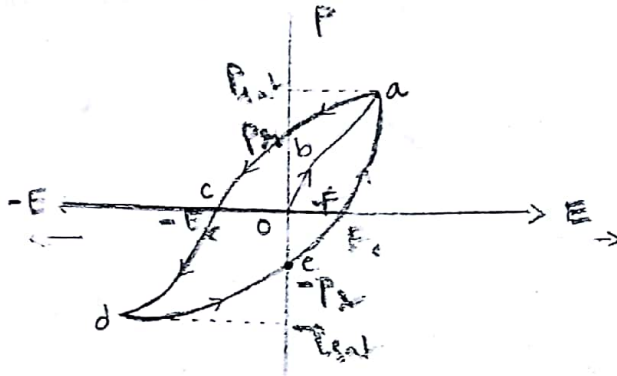
6. All the ferro electric crystals exhibit the piezo-electricity but all piezo electric crystals need not exhibit ferro electricity.

Applications:-

1. Due to high dielectric constant, they are used in the manufacture of small sized and large capacitance capacitors.

2. Due to Hysterisis property, they are used in the construction of memory devices and are used in computers

3. The Ferro electric materials show piezo-electric property, so they used to produce and detect the sound waves

Barium Titanate (BaTiO$_3$):-

The properties of ferro electric materials can be illustrated with the help of Barium titanate. The significance of ferro electrics is the anomalous dependence of its dielectric constant (ε_r) with temperature. The variation of ε_r with T for BaTiO$_3$ is shown in fig (a)

fig (a)