New Applications on Game Theory

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Abstract:

Game theory has been of importance on many fields of the social sciences since its rise to prominence more than fifty years ago (Lim, 1999). The subject first outlined zero-sum games, such that one person's gains are exactly equal net losses of the other participant. Turocy & von Stengel (2001) define game theory as a formal study of decision-making where several players must make choices that potentially affect the interests of other players. Turocy van Stengel (2001) highlighted that the game theory was first presented on the study of duopoly by Antoine Cournt in 1838. However Camerer (2003) discovered that the first known discussion of game theory occurred in a letter written by James Waldegrave in 1713. Game theory was further put on the spotlight by John von Nuemann in 1928 in a study of "theory of polar games". Lim (1999) also argued that game theory was firmly entrenched by von Neumann and Morgenstern in the realm of economics by providing a whole new way of looking at the competitive process, through the eyes of strategic interactions between economic players. More emphases on game theory was observed in 1949 when John Forbes Nash published his thesis titled Non Cooperative games, that's where the concept of equilibrium point (also known as Nash Equilibrium) was introduced (Hyksova, n.d.). Kerk (n.d.) pointed out that Nash equilibrium is based on the principle that the combination of strategies that players are likely to choose is one in which no player could do better by choosing a different strategy given the strategy the other chooses. Camerer (2003) stated that game theory is generally used in economics, political science, and psychology, as well as logic and biology. Game theory applies in many studies of competitive scenarios, therefore the problems are called games and the participants are called players. A player is defined by Osborne (2002) as an individual or group of individuals making a decision. Camerer *et al*., (2001) went on to outline the assumptions of the game theory as that, all players form beliefs based on analysis of what others might do, choose a best response given those beliefs, and adjust best responses and beliefs until they are equal. Camerer *et al*., (2001) emphasized that these assumptions are sometimes violated, meaning that not every player behaves rationally in difficult situations. Osborne & Rubinstein (1994) also highlighted that the basic assumption that motivates the game theory is that decision-makers are rational and they reason strategically. Osborne &Rubinstein (1994) further stated that decision-makers are aware of their alternatives and chooses their action deliberately after some process of optimization.

Keywords: Pure Strategy, Mixed Strategy and Games.

Introduction

Game theory applies whenever the actions of several agents are interdependent. Therefore the main aim of this chapter is to look at game theory with more emphasis on the dominance, Nash equilibrium, maxmin Strategies, mixed strategies, extensive games with perfect information, extensive games with imperfect information, zero-sum games and computation, and lastly on the bidding in auctions. According to Turocy & von Stengel (2001) the purpose of study in game theory is game. There players involved in a game are arranged in their preferences, their information, the strategic actions available to them, and how these influence the outcome. A high level description of a game specifies only what payoffs each individual or group can obtain by assistance of its members. Game theory is generally divided into two branches, which are non-cooperative and cooperative game theory (Osborne & Rubinstein, 1994). Lim (1999) further clarified that whether a game is cooperative and non-cooperative would depend on whether the players can communicate with one another. Non-cooperative game theory focuses on strategic choices resulting from interaction among competing players, each player chooses its strategy independently for improving its own utility (Lim, 1999). Tracy & von Stengel (2001) suggested that non-cooperative game theory specifically means, this branch of game theory explicitly represent the process in which players make choices out of their own interest. Turocy & von Stengel (2001) further suggested that in the model of non-cooperative game theory the details of the ordering and timing of players' choices are crucial in determining the outcome of a game. Several concepts such as the Nash equilibrium exist for solving non-cooperative games. Lim (1999) suggested that the Nash equilibrium concept may be applied to games in both normal and strategic form, and provides a solution where each player maximizes his payoff given the other players' strategies. While, non-cooperative game theory focuses on competitive scenarios, cooperative game theory provides analytical tools to study the behavior of rational players when they cooperate. The main focus of cooperative games describes the formation of cooperating groups of players that can strengthen the players' positions in a game. Lim (1999) views cooperative game theory concepts as sets of payoff combinations that satisfy both individual

		First Prisoner's Decision	
		Confess	*Hold Out*
Other Prisoner's Decision	*Confess*	10 years	25 years
	Hold Out	1 year	3 years

Figure 1 Prisoner's Dilemma; Source: Avinash & Nalebuff (1991)

The game table illustrates that the first prisoner will either get 10 years if he confesses or 25 if he does not. So if the other prisoner confesses, the first would also prefer to confess. If the other prisoner holds out, the first prisoner will get 1 year if he confesses or 3 if he does not, so again he would prefer to confess. Both players are said to have dominant strategies (Avinash & Nalebuff, 1991). A dominant strategy has payoffs such that, irrespective of the choices of other players, no other strategy would result in a higher payoff. An ultimate observation here is that if both prisoners use their confess, they do not reach an optimal outcome.

1.Nash equilibrium

The Nash equilibrium is a game theoretic solution concept that is normally applied in economics. As previously outlined, Nash equilibrium was introduced by John Nash in 1950 and has emerged as one of the fundamental concepts of game theory (Kerk, n.d.). Nash equilibrium is a solution concept of a game involving two or more players, in which each player is assumed to know the equilibrium strategies of the other players, and no player has anything to gain by changing only his own strategy (Osborne, 2002). However, Myerson (1999) viewed the concept of equilibrium as one of the most important and elegant ideas in game theory. Myerson (1999) also pointed out that a game can have many Nash equilibriums, and some of these equilibriums may be unreliable compared to what should be the outcome of a game. Some studies reflect that Nash equilibrium is concern about the actions that will be chosen by players in a strategic game (Osborne, 2002). Players have to know precisely what their opponents will choose (de Bruin, 2009). To do so, players should not base on the assumption that all players are rational. They rather focus on the basis of statistical information about previous game playing situations, if such information is available and reliable. Osborne (2002) also outlined an example where there is interaction between buyers and sellers. A buyer usually transacts only once with any given seller, or interacts repeatedly but anonymously. Each player chooses her action given her belief about the other players' actions. Osborne (2002) provided an example of Nash equilibrium on Prisoner's Dilemma.

	Quiet	Fink
Quiet	2, 2	0, 3
Fink	3, 0	1, 1

$$\frac{1}{2} \times (-2) + \frac{1}{2} \times 0 = -1$$

Figure 2 Prisoner's Dilemma; Source: Osborne (2002).

On the presented Prisoner's Dilemma, Osborne (2002) argued that (Fink, Fink) is an exclusive Nash equilibrium. This action pair is said to be the equilibrium because given that player one chooses Fink, player two is also better off by choosing Fink than Quiet. Presented on the right column of the table, it is observed that Fink yields off player one a payoff of 1, while Quiet yields a payoff of 0. Also given that player one chooses Fink, player two is better off choosing Fink than Quiet, presented on the bottom row of the table it is observed that Fink yields player two a payoff of 1 whereas Quiet yields one a payoff of 0.

2. Mixed strategies

Turocy and von Stengel (2001) outlined that a game in strategic form does not always have a Nash equilibrium in which each player definitely chooses one of the strategies. But players base their random selection of strategies on certain probabilities. Mixed strategies are defined as a probability distribution over the set of actions. However Rubinstein (1991) alternatively viewed mixed strategy as a belief held by all other players regarding a player's actions. Presented below is an illustration of mixed strategies equilibrium by an example of drunk driving; the police choose to set up checkpoints with probability 1/3. Assume if a player drinks Cola, he will get 0. If a player drinks Wine, he will get -2 with probability 1/3 and 1 with probability 2/3. Kockesen (n.d) also assumed that the value is the expected payoff;

$$\frac{1}{3} \times (-2) + \frac{2}{3} \times 1 = 0$$

The player is indifferent whether to drink Wine or Cola with any probability. If a player drinks Wine with probability of 1/2 and gets to the police check points, he gets an expected payoff of -1 and if he does not;

It is also outlined that the police are also indifferent about setting up checkpoints and any mixed strategy. This results on mixed strategy equilibrium. Osborne (2002) argued that the concept of mixed strategy equilibrium in a strategic game does not motivate the player to introduce randomness in their behaviour. Players normally randomize deliberately to influence the other player's behaviour. Pindyck & Rubinfeld (2009) emphasized that there is no Nash equilibrium on game theory under mixed strategies. Pindyck& Rubinfeld (2009) further explained mixed strategies by use of matching pennies. In the game each player chooses either heads or tails and both players reveal their coin at the same time. If both are heads or tails, player one wins and if coins do match, player two wins. Nevertheless Rubinstein (1991) is of the view that mixed strategy equilibrium is then as common knowledge opportunities, this is because all the actions to which a strictly positive probability is assigned are ideal, given the beliefs.

Extensive games with perfect information

Pindyck & Rubinfeld (2009) define extensive games as a representation of possible moves in a game in the form of decision tree. In strategic form games, players simultaneous choose their strategies without being aware of choices of other players. However with extensive games, players can over time be informed about the actions of other players (Turocy & von Stengel, 2001). This is also viewed as under perfect information since every player at some point becomes aware of the previous choices of other players. It is further highlighted that to avoid simultaneous movement on extensive game, only one player moves at a time. Osborne (2002) highlighted that this model allows the observation of the game in which each player can consider his plan of action not only at the beginning of the game but also at any point of time. However, strategic game restricts the observation of the game where each player chooses his plan of action once and for all. Extensive games can only consider unlimited possibilities, but the strategic game does not allow a player to reconsider his plan of action after some events in the game have unfolded. Extensive games with perfect information can be presented on a tree diagram, thus it’s also called a game tree with perfect information. Osborne (2002) outlined an example of extensive games with perfect information as follows:

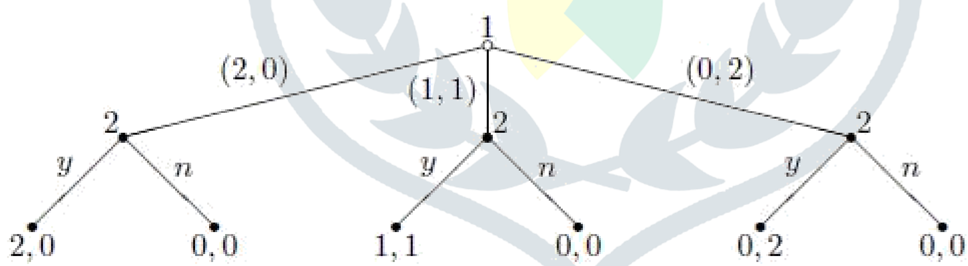

Figure 3 Prisoner's Dilemma; Source: Osborne (2002).

The above outlined diagram represents extensive games with perfect information on a tree diagram. The small circle at the top of the diagram represents the starting point of the game. The 1 above the small circle indicates that player 1 has to make the first move. The three branching points from the circle represents the possible actions of player 1 at the starting point of the game. The labels beside these branching points are the names of the actions to be taken. Each branching points leads to a small dot beside which is the label 2, indicating that player 2 takes an action after any history of length one. The labels beside the branching points that originate from these small dots and are the names of player 2's actions, y meaning *accept* and meaning *reject*. The numbers at the end of the branches are payoffs for player's preferences, the first number in each pair is payoffs for player 1 and the second number is payoff for player 2.

Extensive games with imperfect information

In environments with more than one player, each player's payoff is generally affected by the actions of the other players (Gipin & Sandholm, 2007). Thus, the ideal strategy of each player can depend on other players. Extensive games with imperfect information are one of the ways to deal with such strategies. Extensive games with imperfect information are defined as the games that are not fully observable. Osborne (2002) argued that when the player's information is imperfect in extensive games, a player need not to know what actions his rivals have taken before him. This means that when it is a player's turn to move, he does not have access to all of the information about the other player's decisions. Gilpin & Sandholm (2007) argued that such games, the decision of what to do at a point in time cannot generally be optimally made without considering decisions at all other points in time. This is because those other decisions affect the probabilities of being at different states at the current point in time.

Bellow is an illustration of an example of the extensive games with imperfect information, in the example this games emphasizes that moves by players are imperfectly observed. Assume numbers of players are lined up, each player has two options either buy a new iPad (B) or do not buy (N). The quality of a new iPad is high (H) with probability of (p) 0,1 or low (L) with probability of 1-p. The quality is common to all players. Player 1 observes a private signal of new iPad (H,L), which is correct with probability of (p) 0,1 and a choice of preceding player (A1,……..,An). The net payoff from purchasing an iPad is 1 if the quality is good and -1 if the quality is bad. Extensive games with imperfect information were further illustrated on a tree diagram by Osborne & Rubinstein (1994) on the following figure.

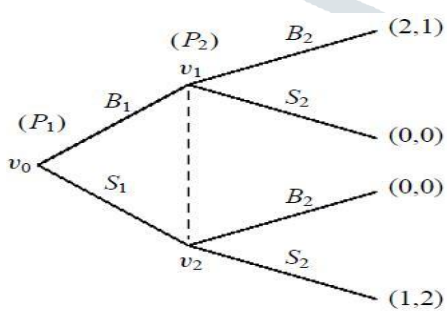

Figure 4 Prisoner's Dilemma; Source: Osborne & Rubinstein (1994)

In the above presented figure, the game starts at v0, and P2 must make a choice at branches v1 and v2 without knowing the choice of player P1. They are then connected with a dotted line and label the edges coming out with common labels, B2 and S2. Player P2 must make the choice of the edges with the same labels at both of these branches. These pair of branches (v1 and v2) is called the information sets.

CONCLUSION

This paper has viewed that game theory is not simply a matter of mathematics but concerns the real world, in the sense that involves decision-making by several players that also affect the interest of other players. But it does not mean that the purpose of game theory is to predict behaviour in the same sense as in sciences but it is capable of such things. Players are arranged in their preferences, their information, the strategic actions available to them, and how these influence their payoffs (returns). In situations where there are more than two players, a decision by player 1 does also affect the interest of other players (player 2). Game theory covers many aspects such as economics, political science, and psychology, as well as logic and biology. Game theory is also viewed as a broad subject but basically divided into two branches, non-cooperative games and cooperative games, which are sets of payoff combinations that satisfy both individual and group rationality. While non-cooperative games looks at a situation where each player maximises his payoff given the other players' strategies, which means players basically make choices out of their own interest. Game theory also views Nash equilibrium as a basic concept of the subject, but in situations of strategic games players base their random selection of strategies using certain probabilities. The subject looks at different scenarios that involve decision making and Nash equilibrium is said to be the most effective concept that deals with that with the famous application of Prisoner's Dilemma is also said to be the most common example to illustrate game theory.

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