# ASSOCIATOR IDEAL IN (-1, 1) RINGS

**By**

**P. SARADA DEVI[#1], MANTHA SRIKANTH[#2]**

[#] *Assistant professor, Department of mathematics*

*Malla Reddy Engineering College (Autonomous)*

*Hyderabad-500100, Telangana, India.*

[1]*Sarada.chakireddy@gmail.com*

[2]manthasrikanth9@gmail.com

**ABSTRACT:** Kleinfeld and Smith [1], have proved that Semiprime flexible weakly novikov rings are associative. In this paper we show that a semiprime 2-and 3- divisible (-1,1) ring satisfying weak novikov identity is associative.

**KEY WORDS:** Semiprime (-1,1) ring, (-1,1) ring, Associator ideal, Weakly novikov rings.

**1. Introduction:** A (-1,1) ring is a nonassociative ring in which

(a, b, b) = 0, i.e., (a, b, c) + (a, c, b) = 0 ---------------(1)

and (a, b, c) + (b, c, a) + (c, a, b) = 0 ------------------------(2) for all a, b, c in R.

In any nonassociative ring R, the associator (a, b, c) is defined by (a, b, c) = (ab)c – a(bc) and the commutator is defined by (a, b) = ab – ba. [3]. A non-associative ring R is said to be weakly novikov if it satisfies the identity

(w, a, bc) = b (w, a, c) ---------------- (3) for all w, a, b, c in R.

An alternative ring R is a ring in which (aa)b = a(ab) and b(aa) = (ba)a for all a, b in R. These equations are known as the left and right alternative laws respectively. The ring R is called semi prime, if for any ideal A of R, $A^2 = 0$ implies A = 0. [2]

The commutative center U of R is the set of all elements u ∈ R such that (u, R) = 0. Let A be the associator ideal of R. The associator ideal A consists of all finite sums of associators and left multiples of associators. R is called k - divisible if kx = 0 implies x = 0, x ∈ R and k is a natural number. Throughout this paper R will denote a 2 – and 3 – divisible (-1, 1) ring satisfying the weak novikov identity.

**2. Preliminaries:** In any ring the following identity

(wa,b,c) - (w,ab,c) + (w,a,bc) = w(a,b,c) + (w,a,b)c --------------(4) holds.

In any (-1, 1) ring R the following identities hold [2] & [ 4]:

(a, b, u) + (b, a, u) = 0 ---------------- (5)

(u, a, b) – 2 (b, a, u) = 0 ---------------- (6)

and $(c, c, (a, b)) = 0$ ---------------- (7) for all $u \in U$ and $(a, b, c) \in R$.

By linearizing the right alternative identity $(b, c, c) = 0$, we obtain

$$(wa, b,c) = w(a,b,c) \quad \text{----------------- (8)}$$

Now Comparing (4) and (8) implies $-(w,ab,c) + (w,a,bc) = (w,a,b)c$.

By substituting $w = c$ in this equation, we get that

$$(c,a,b)\, c = 0. \quad \text{------------------ (9)}$$

A linearization of (9) is $(w,a,b)\, c = -(c,a,b)w$. ----------------- (10)

For arbitrary elements a, b, x, y, z in R, we observe that

$$(w^{\pi}, b, (x^{\pi},y,c^{\pi})) = (\text{sgn}\pi)\, (w,b,(x,y,c)), \quad \text{----------------- (11)}$$

where $\pi$ is any permutation on the set $\{w,x,c\}$.

Also we get $(w,b^{\sigma},(x,y^{\sigma},c)) = (\text{sgn}\sigma)\, (w,b,(x,y,c))$, ----------------- (12)

Where $\sigma$ is any permutation on the set $\{b, y\}$.

Also, we get $(a^{\alpha},b,c^{\alpha})\, (x^{\alpha},y,z^{\alpha}) = (\text{sgn}\alpha)\, (a,b,c)\, (x,y,z)$, -----------------(13)

Where $\alpha$ stands for any permutation on the set $\{ a, c, x, z\}$.

By taking $q = (a,y,c)\, (x,y,z)$ and using (12) we get $2q=0$.

Using 2-divisible, we know that $q = 0$, so that $(a,y,c)\, (x,y,z) = 0$.

By linearizing this, we get

$$(a,b,c)\, (x,y,z) = -(a,y,c)\, (x,b,z). \quad \text{----------------- (14)}$$

**3. Main Results:**

**Lemma 1:** A is anticommutative

Proof: From (14) and (13), we have

$$(a,b,c)\, (x,y,z) = -(x,y,z)\, (a,b,c) \quad \text{----------------- (15)}$$

In view of (8) it suffices to take all finite sums of associators. This and (15) implies that A is Anticommutative

**Lemma 2**: - A is alternative.

**Proof**: - Let q be an arbitrary element in A and w, a, b, c are arbitrary elements in R.

Using (3), (8) and (10) we get (c,a,b). qw = - (c,a,b)q.w

By taking (c,a,b) =p, we have p.qw = – pq.w ----------------- (16)

for all p, q in A and all w in R.

By assuming that r is an element of A. we get

(p,q,r) + (q,p,r) = 0.

At this point A is both left and right alternative. Hence A is alternative.

**Lemma 3**: - If M is a 2-divisible anticommutative alternative ring, then $(M^2)$ $(M^2)$ = 0.

**Proof**: - Let w, a, b, c be arbitrary elements in M then we have (ab) (ca) = a(bc)a = – $a^2$ (bc) = 0, from alternative identities and anticommutativity. Linearizing this identity, we get (wb) (ca) = – (ab) (cw).

Applying this in conjunction with anticommutativity we observe that wb also anticommutes with ca so that 2 (wb) (ca) = 0.

Since M is 2-divisible (wb) (ca) = 0, so that $(M^2)$ $(M^2)$ = 0.

Using the above lemmas, we prove the following main result.

**Theorem 1**: - A semiprime 2-divisible (-1,1) weakly novikov ring R is associative.

**Proof:** - Let p, q be arbitrary elements of A, A is the associator ideal of R, and c is an arbitrary element of R. From (16), we have pq.c = – p.qc. Hence $A^2$ is a right ideal of R.

Also (c,p,q) = – (p,q,c) – (q,c,p) = – (p,q,c) + (q,p,c)

= – (pq).c+ p.qc + qp.c – q.pc is

an element of $A^2$. Since cp.q is in $A^2$, c.pq = – (c,p,q) + cp.q also is in $A^2$. Thus, $A^2$ is an ideal of R.

Then lemmas (1), (2) and (3) imply that the ideal $A^2$ of R squares to 0. Applying semiprime, A = 0 follows and so R must be associative.

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