https://doi.org/10.48047/AFJBS.6.Si4.2024.4286-4294





# Viscous dissipation effect on steady free convective hydromagnetic heat transfer flow of a reactive viscous fluid in a bounded domain

D. Chenna Kesavaiah<sup>1</sup>, Ch. Shashi Kumar<sup>2\*</sup>, M. Chitra<sup>3</sup>, Vuppala Lakshmi Narayana<sup>4</sup>

<sup>1</sup>Department of BS & H, Vignan's Institute of Management and Technology for Women, Kondapur (V), Ghatkesar (M), Medchal-Malkajgiri (Dist), Telangana-501301, India, Email: chennakesavaiah@gmail.com

<sup>2</sup>Department of Mathematics, VNR Vignana Jyothi Institute of Engineering & Technology, Hyderabad - 500090, Telangana, India, Email: skch17@gmail.com

<sup>3</sup>Department of Mathematics, Malla Reddy Engineering College (Autonomous), Maisammaguda, Kompally (Mandal), Medchal Malkajgiri (Dist)-500100, Telangana, India, Email: muddasanichitra@gmail.com

<sup>4</sup>Department of Mathematics, Vardhaman College of Engineering, Hyderabad, Telangana, India Email: vnarayana2006@gmail.com

\*Corresponding author: Ch. Shashi Kumar,

Department of Mathematics, VNR Vignana Jyothi Institute of Engineering & Technology, Hyderabad - 500090, Telangana, India Email: skch17@gmail.com

#### Article info

Volume 6, Issue Si4, 2024 Received: 2 April 2024 Accepted: 25 May 2024 Published: 29 July 2024 doi: 10.48047/AFJBS.6.Si4.2024.4286-4294 A study of steady free convective hydromagnetic flow of a reactive viscous fluid in a bounded domain has been analyzed in the present paper. The existence and uniqueness of the solution of momentum and energy equations are proved. By employing the perturbation technique, the momentum and energy equation are solved analytically. Results are presented for velocity and temperature profiles. It is shown that velocity profiles increases as Grashof number, heat source parameter and Prandtl number increases whereas the temperature profile increases as heat source parameter increases.

Keywords: Hydro magnetic flow, Free convection, Steady, Numerical solution

#### Abstract

## 1. Introduction

Many researchers carried on MHD (magnetohydrodynamics) flow of the boundary layer problem or a radiating gas inside a vertical pathway. The impact of this type of viscous dissipation term on an unsteady condition was often ignored. The influence of this heat dissipation function cannot be neglected from a practical point of view because of its momentous in several flow issues. It is the bearing lubricant that provides the source of temperature rise and geodynamic heating. For much lower velocity methods the impact of viscous dissipation in the temperature profile is comparatively small. The influence of viscous dissipation cannot be neglected in the manner concerning a dynamic temperature which is analogous to the attributed difference in heat transfer temperature. The boundary layer theory is utilized to analyze the viscous dissipation effect for both incompressible and compressible flows. Magneto hydrodynamics of an electrically conducting fluid is encountered in many problems in geophysics, astrophysics, engineering applications, and other industrial areas. Hydromagnetic free convection flows have a great significance for the applications in the field of stellar and planetary magnetospheres and aeronautics. Engineers employed magneto hydrodynamics principles in the design of heat exchangers pumps, in space vehicle propulsion, thermal protection, control and re-entry, and in creating novel power generating systems. Hydro magnetic flow and heat transfer problems have also become more important, industrially. In many metallurgical processes involving the cooling of many continuous strips of filaments by drawing them through an electrically conducting fluid subject to a magnetic field, the rate of cooling can be controlled and final product of desired characteristics can be achieved. Another important application of Hydromagnetic to metallurgy lies in the purification of molten metals from non-metallic inclusions by the application of a magnetic field [1-16].

In recent decades, more importance have been made to the topic of viscous dissipation effect on unsteady mixed convection, because of its utilitarian aspects in diverse fields as medical chemistry medicine engineering, industries etc. The effects of viscous dissipation on convective heat transfer is significant especially for higher velocity flows, highly viscous flows even at moderate velocities, for fluids with a moderate Prandtl number and moderate velocities with small wall-to-fluid temperature difference or with low wall heat fluxes. Free convection flow of heat and mass transfer problems has diverse applications in industrial processes. The Influence of a magnetic field on free convection plays a vital role in the fields of petroleum, engineering and agriculture. The final result of the magnetic field and thermal radiation on the induced magnetic field has attracted many researchers. It has been adopted in various fields such as aerodynamics, science and technology, astrophysics and geophysics. In polymer processing industry radiation effects play a major role in getting the final product, by adjusting the heat and mass transfer. The impact of heat and mass transfer plays a major role in distribution of moisture and temperature in agriculture, modeling the chemical equipment, emergence of fog, harm of a crop due to freeze, pollution in the ecosystem, etc. [17-33].

The natural convection flow between heated vertical plates is a classical problem that occurs in many physical phenomena and engineering applications and has received attention in recent years because of its various applications in the design of nuclear reactors, aircraft cabin insulation, thermal storage systems and cooling of electronic equipments. The study of heat transfer and flow of viscous fluids through and across a porous medium has wide ranging applications in various fields of science and engineering. As a result of its technological import to geothermal and reservoir engineering and cooling of nuclear reactors, etc. several researchers have studied such problems in channels, composed of porous materials [34-47].

From the existing literature most of the previous studies considered only the case of free convection with heat transfer, but little has been done in the direction of reactive flow. In this work we shall consider a steady free convective hydro magnetic flow of a reactive viscous fluid in a bounded domain. In section two we provide the mathematical formulation of the problem and method of solution. In section three gives the properties of solution while section four gives the description of the numerical method employed and section five deals with discussion of results.

## 2. Mathematical formulation and Solution of the problem

Consider a steady one-dimensional free convection flow of an incompressible electrically conducting viscous fluid on a finite plate and the temperatures of the flow assume Arrhineus dependence. The -axis is along the plate in the upward direction and -axis normal towards it. A transverse constant magnetic field is assumed to be negligible. By assuming a very small magnetic Reynolds number the induced magnetic field is neglected. The appropriate governing equation is given as:

$$\frac{dv'}{dy'} = 0 \tag{1}$$

$$v'\frac{dv'}{dy'} = v\frac{d^{2}u'}{dy'^{2}} + g\beta' (T' - T'_{\infty}) - \frac{\sigma B_{0}^{2}}{\rho}u' - \frac{v}{\rho}u'$$
(2)

$$v'\frac{dT'}{dy'} = \frac{k}{\rho C_p} \frac{d^2 T'}{dy'^2} - \mu \left(\frac{du'}{dy'}\right)^2 - \frac{Q_0}{\rho C_p} \left(T' - T'_{\infty}\right)$$
(3)

The appropriate boundary conditions are:

$$u(y) = 0, T(y) = T_{\infty}$$
 at  $y = \pm 1$  (4)

Introducing the following non-dimensional quantities and assume  $v_0 = 0$ 

$$u = \frac{u'}{u_0}, t = \frac{t'u_0^2}{\nu}, v = \frac{v'}{v_0}, \eta = \frac{u_0 y'}{\nu}, \theta = \frac{T' - T'_{\infty}}{T'_{w} - T'_{\infty}}$$
(5)

Equations (2) and (3) one gets the following non-dimensional equations governing the flow:

$$\frac{d^2u}{d\eta^2} - \left(M^2 + K\right)u = -Gr\theta \tag{6}$$

$$\frac{d^2\theta}{d\eta^2} + \Pr E\left(\frac{du}{d\eta}\right)^2 - Q\Pr\theta = 0$$
<sup>(7)</sup>

where

$$E = \frac{\rho u_0^2}{(T_w - T_\infty)}, \quad Gr = \frac{\rho \beta'(T_w - T_\infty)\nu}{u_0^3}, Q = \frac{Q_0}{\rho C_p u_0^2} \Pr = \frac{\mu C_p}{k}, M^2 = \frac{B_0^2 \nu \sigma}{\rho u_0^2}$$

The corresponding boundary condition in dimensionless form are reduced to

$$u = 0, \theta = 0 \qquad at \quad \eta = \pm 1 \tag{8}$$

The physical variables  $u, \theta$  and C can be expanded in the power of Eckert number (E). This can be possible physically as E for the flow of an incompressible fluid is always less than unity. It can be interpreted physically as the flow due to the Joules dissipation is super imposed on the main flow. Hence we can assume

$$u(\eta) = u_0(\eta) + Eu_1(\eta) + 0(E^2)$$
  

$$\theta(\eta) = \theta_0(\eta) + E\theta_1(\eta) + 0(E^2)$$
(9)

Using equation (9) in equations (6)–(7) and equating the coefficient of like powers of E, we have

$$u_0'' - M^2 u_0 = Gr\theta_0 \tag{10}$$

$$u_1'' - M^2 u_1 = Gr\theta_1 \tag{11}$$

$$\theta_0'' - Q \Pr \theta_0 = 0 \tag{12}$$

$$\theta_1'' - Q \operatorname{Pr} \theta_1 = -\operatorname{Pr} u_0'^2 \tag{13}$$

The corresponding boundary condition in dimensionless form are reduced to

$$u_0 = 0, \theta_0 = 0, u_1 = 0, \theta_1 = 0$$
 at  $\eta = \pm 1$  (14)

Using equations (10) - (12) under the boundary conditions (14), we obtain the solution of the velocity and temperature.

#### 3. Results and Discussion

In this section, numerical computations are presented in the form of non-dimensional velocity and temperature profiles. Also, numerical computation has been carried out for different values of the parameters entering into the problem. The values of Grashof number are taken to be large from the physical point of view. The large Grashof number corresponds to the cooling problem. Figures (1) and (2) reveal the effect of Prandtl number on velocity and temperature profiles. High thermal conductivity and are therefore a good choice for heat conducting liquids. As can be seen, liquid metals are very good heat transfer liquids. Interestingly, air is a decent heat transfer liquid as, well, whereas typical organic solvents are not. It is observed that the velocity increases while temperature decreases as the Prandtl number increases. Figures (3) and (4) showed that the velocity increases and the temperature do not change as Grashof number increases. Figures (5) and (6) showed the effect of Hartmann number on the velocity as well as temperature profiles. It is observed that the velocity decrease as the Hartmann number increases; while Hartmann number does not have noticeable effect on the temperature profile. Figures (7) and (8) reveal that Grashof number and activation energy parameter does not have noticeable effect on both velocity and temperature profiles respectively. It is shown that both velocity and temperature profiles increases as Frank-Kamenetskii parameter increases. Finally, it is observed in all that the flow is symmetric about the centre (i.e.,  $\eta = 0$ ).



Conclusion

In this paper we derived the energy and momentum equations governing free convective hydro magnetic flow of a reactive viscous fluid. We have shown the existence and uniqueness of solutions, and then presented numerical results for various non–dimensionless parameters graphically. From the present study we can make the following conclusions:

- The velocity profiles increases whereas temperature profile does not have noticeable effect with an increase of the free convection current.
- Using magnetic field we can control the flow characteristic and heat transfer.
- The velocity and temperature profiles increase as the Frank-Kamenetskii parameter increases.
- The velocity profile increases whereas the temperature profile decreases as the Prandtl number increases.

Nomenclature:	
Gr	Grashof number
Pr	Prandtl number
$M^2$	Magnetic parameter
K	Porous permeability
E	Eckert number
Q	Heat source parameter

# References

- 1. W. R. Derric and G. I. Stanley (1976): Elementary differential equations with applications, Addison-Wesley: Los Angeles, CA.
- D. Chenna Kesavaiah, G. Rami Reddy, Y. V. Seshagiri Rao (2022): Impact of thermal diffusion and radiation effects on MHD flow of Walter's Liquid model-B fluid with heat generation in the presence of chemical reaction, International Journal of Food and Nutritional Sciences, Vol. 11,(12), pp. 339- 359
- K. Venugopal Reddy, B. Venkateswarlu, D. Chenna Kesavaiah, N. Nagendra (2023): Electro-Osmotic flow of MHD Jeffrey fluid in a rotating microchannel by peristalsis: Thermal Analysis, Science, Engineering and Technology, Vol. 3, No. 1, pp. 50-66
- 4. D. Chenna Kesavaiah, G. Rami Reddy, G. Maruthi Prasada Rao (2022): Effect of viscous dissipation term in energy equation on MHD free convection flow past an exponentially accelerated vertical plate with variable temperature and heat source, International Journal of Food and Nutritional Sciences, Vol. 11,(12), pp. 165- 183
- Dr. Pamita, D. Chenna Kesavaiah, Dr. S. Ramakrishna (2022): Chemical reaction and radiation effects on magnetohydrodynamic convective flow in porous medium with heat generation, International Journal of Food and Nutritional Sciences, Vol. 11,(S Iss. 3), pp. 4715- 4733
- Anita Tuljappa, D. Chenna Kesavaiah, M. Karuna Prasad, Dr. V. Bharath Kumar (2023): Radiation absorption and chemical reaction effects on MHD free convection flow heat and mass transfer past an accelerated vertical plate, Eur. Chem. Bull. Vol. 12(1), pp. 618-632

- D. Chenna Kesavaiah, K. Ramakrishna Reddy, Ch. Shashi Kumar, M. Karuna Prasad (2022): Influence of joule heating and mass transfer effects on MHD mixed convection flow of chemically reacting fluid on a vertical surface, NeuroQuantology, Vol. 20 (20), pp. 786-803
- M. Bhavana and D. Chenna Kesavaiah (2018): Perturbation solution for thermal diffusion and chemical reaction effects on MHD flow in vertical surface with heat generation, International Journal of Future Revolution in Computer Science & Communication Engineering, Vol. 4 (1), pp. 215-220
- D. Chenna Kesavaiah, P. Govinda Chowdary, Ashfar Ahmed, B. Devika (2022): Radiation and mass transfer effects on MHD mixed convective flow from a vertical surface with heat source and chemical reaction, NEUROQUANTOLOGY, Vol.20 (11), pp. 821-835
- Ch. Shashi Kumar, K. Ramesh Babu, M. Naresh, D. Chenna Kesavaiah, Dr. Nookala Venu (2023): Chemical reaction and Hall effects on unsteady flow past an isothermal vertical plate in a rotating fluid with variable mass diffusion, Eur. Chem. Bull. Vol. 12 (8), pp. 4991-5010
- Chenna Kesavaiah DAMALA, Venkateswarlu BHUMARAPU, Oluwole Daniel MAKINDE (2021): Radiative MHD Walter's Liquid-B flow past a semi-infinite vertical plate in the presence of viscous dissipation with a Heat Source, Engineering Transactions, Vol. 69(4), pp. 373–401
- G. Rami Reddy, D. Chenna Kesavaiah, Venkata Ramana Musala and G. Bkaskara Reddy (2021): Hall effect on MHD flow of a viscoelastic fluid through porous medium over an infinite vertical porous plate with heat source, Indian Journal of Natural Sciences, Vol. 12 (68), pp. 34975-34987
- D. Chenna Kesavaiah and B. Venkateswarlu (2020): Chemical reaction and radiation absorption effects on convective flows past a porous vertical wavy channel with travelling thermal waves, International Journal of Fluid Mechanics Research, Vol. 47 (2), pp. 153-169
- B. Mallikarjuna Reddy, D. Chenna Kesavaiah and G. V. Ramana Reddy (2019): Radiation and Diffusion Thermo effects of viscoelastic fluid past a porous surface in the presence of magnetic field and chemical reaction with heat source, Asian Journal of Applied Sciences, Vol. 7 (5), pp. 597-607
- D. Chenna Kesavaiah, K Ramakrishna Reddy and G Priyanka Reddy (2019): MHD rotating fluid past a moving vertical plate in the presence of chemical reaction, International Journal of Information and Computing Science, Vol. 6 (2), pp. 142-154
- 16. D. Chenna Kesavaiah, Ikramuddin Sohail Md, R. S. Jahagirdar (2019): Radiation effect on slip flow regime with heat generation, Cikitusi Journal For Multidisciplinary Research, Vol. 6 (1), pp. 7-18
- 17. A. Barletta, E. Magyari, I. Pop and L. Storesletten (2007): Mixed convection with viscous dissipation in a vertical channel filled with a porous medium, Acta Mech., Vol. 194, pp. 123-140.
- D Chenna Kesavaiah, D Chandraprakash and Md Ejaz Ahamed (2019): Radiation effect on transient MHD free convective flow over a vertical porous plate with heat source, Journal of Information and Computational Science, Vol. 9 (12), pp. 535-550
- B Mallikarjuna Reddy, D Chenna Kesavaiah and G V Ramana Reddy (2018): Effects of radiation and thermal diffusion on MHD heat transfer flow of a dusty viscoelastic fluid between two moving parallel plates, ARPN Journal of Engineering and Applied Sciences, Vol. 13 (22), pp. 8863-8872
- Srinathuni Lavanya, D. Chenna Kesavaiah (2014): Radiation and Soret effects to MHD flow in vertical surface with chemical reaction and heat generation through a porous medium, International Journal of Computational Engineering Research, Vol. 04 (7), pp. 62-73

- 21. D. Chenna Kesavaiah, Ikramuddin Sohail Md, R. S. Jahagirdar (2018): MHD free convection heat and mass transfer flow past an accelerated vertical plate through a porous medium with effects of hall current, rotation and Dufour effects, Suraj Punj Journal For Multidisciplinary Research, Vol. 8 (11), pp. 46-62
- 22. D. Chenna Kesavaiah and R. S. Jahagirdar (2018): MHD free convective flow through porous medium under the effects of radiation and chemical reaction, Journal of Applied Science and Computations, Vol. 5 (10), pp. 1125-1140
- Srinathuni Lavanya and D. Chenna Kesavaiah (2017): Radiation effects on MHD natural convection heat transfer flow from spirally enhanced wavy channel through a porous medium, International Journal on Future Revolution in Computer Science & Communication Engineering ,Vol. 3(10), pp. 130-140
- Srinathuni Lavanya and D. Chenna Kesavaiah (2017): Heat transfer to MHD free convection flow of a viscoelastic dusty gas through a porous medium with chemical reaction, International Journal of Pure and Applied Researches, Vol. 3 (1), pp. 43 - 56
- D. Chenna Kesavaiah and A. Sudhakaraiah (2014): Effects of heat and mass flux to MHD flow in vertical surface with radiation absorption, Scholars Journal of Engineering and Technology, 2(2): pp. 219-225
- 26. Srinathuni Lavanya, D. Chenna Kesavaiah (2014): Magnetic field and radiation effects on MHD free convection heat and mass transfer flow through a porous medium with chemical reaction, Int. Journal of Applied Sciences and Engineering Research, Vol. 3, (4), pp. 850-868
- D. Chenna Kesavaiah and R. S. Jahagirdar (2018): Radiation absorption and chemical reaction effects on MHD flow through porous medium past an exponentially accelerated inclined plate, International Journal for Research in Applied Science & Engineering Technology, Vol. 6 (6), pp. 1370-1381
- S. S. Das, S. K. Sahoo and G. C. Dash (2006): Numerical solution of mass transfer effects on unsteady flow past an accelerated vertical porous plate with suction, Bull. Malays. Math. Sci. Soc., Vol. 29 (1), p. 33-42
- 29. Damala Ch Kesavaiah, P. V. Satyanarayana (2014): Radiation absorption and Dufour effects to MHD flow in vertical surface, Global Journal of Engineering, Design & Technology, Vol. 3 (2), pp. 51-57
- 30. Srinathuni Lavanya, D. Chenna Kesavaiah and A. Sudhakaraiah (2014): Radiation, heat and mass transfer effects on magnetohydrodynamic unsteady free convective Walter's memory flow past a vertical plate with chemical reaction through a porous medium, International Journal of Physics and Mathematical Sciences, Vol. 4 (3), pp. 57-70
- D. Chenna Kesavaiah, P. V. Satyanarayana, A. Sudhakaraiah, S Venkataramana (2013): Natural convection heat transfer oscillatory flow of an elastico-viscous fluid from vertical plate, International Journal of Research in Engineering and Technology, Vol. 2 (6), pp. 959-966
- 32. M. S. Alam, M. M Rahman and M. A. Amad (2006): Numerical study of the combined free-forced convection and mass transfer flow past a vertical porous plate in a porous medium with heat generation and thermal diffusion, Nonlinear Analysis: Modelling and Control. 11(4), pp. 331-343
- D. Chenna Kesavaiah, P. V. Satyanarayana (2013): MHD and Diffusion Thermo effects on flow accelerated vertical plate with chemical reaction, Indian Journal of Applied Research, Vol. 3 (7), pp. 310-314
- 34. S. Karunakar Reddy, D. Chenna Kesavaiah and M. N. Raja Shekar (2013): Convective heat and mass transfer flow from a vertical surface with radiation, chemical reaction and heat source/absorption, International Journal of Scientific Engineering and Technology, Vol. 2 (5), pp : 351-361
- A. K. Al-Hadhrami, L. Elliott and D. B. Ingham (2002): Combined free and forced convection in vertical channels of porous media, Transport Porous Media, Vol. 49, pp. 265-289

- 36. D. Chenna Kesavaiah, P. V. Satyanarayana and S. Venkataramana (2013): Radiation and Thermo Diffusion effects on mixed convective heat and mass transfer flow of a viscous dissipated fluid over a vertical surface in the presence of chemical reaction with heat source, International Journal of Scientific Engineering and Technology, Vol. 2 (2), pp: 56-72
- 37. S. Karunakar Reddy, D. Chenna Kesavaiah and M. N. Raja Shekar (2013): MHD heat and mass transfer flow of a viscoelastic fluid past an impulsively started infinite vertical plate with chemical reaction, International Journal of Innovative Research in Science, Engineering and Technology, Vol. 2 (4), pp.973-981
- D. Chenna Kesavaiah and A. Sudhakaraiah (2013): A note on heat transfer to magnetic field oscillatory flow of a viscoelastic fluid, International Journal of Science, Engineering and Technology Research, Vol. 2 (5), pp. 1007-1012
- 39. A. J. Chamkha, C. Issa, and K. Khanafer (2001): Natural convection due to solar radiation from a vertical plate embedded in a porous medium with variable porosity, J. Porous Media. 4: 69-77
- 40. D. Chenna Kesavaiah, P. V. Satyanarayana and S. Venkataramana (2013): Radiation effect on unsteady flow past an accelerated isothermal infinite vertical plate with chemical reaction and heat source, International Journal of Science, Engineering and Technology Research, Vol. 2 (3), pp. 514-521
- 41. L. L. Animasaun (2015): Effects of thermophoresis, variable viscosity and thermal conductivity on free convective heat and mass transfer of non-darcian MHD dissipative Casson fluid flow with suction and nth order of chemical reaction, Journal of the Nigerian Mathematical Society, Vol. 34 (1), pp. 11-31
- 42. Damala Ch Kesavaiah, A. Sudhakaraiah, P. V. Satyanarayana and S. Venkataramana (2013): Radiation and mass transfer effects on MHD mixed convection flow from a vertical surface with Ohmic heating in the presence of chemical reaction, International Journal of Science, Engineering and Technology Research, Vol. 2 (2), pp. 246 – 255
- D. S. Chauhan and V. Kumar (2009): Effects of Slip Conditions on Forced Convection and Entropy Generation in a Circular Channel Occupied by a Highly Porous Medium: Darcy Extended Brinkman-Forchheimer Model," Turkish. J. Eng. Env. Sci., Vol. 33, pp. 91-104
- 44. Damala Ch Kesavaiah, P. V. Satyanarayana and S. Venkataramana (2012): Radiation absorption, chemical reaction and magnetic field effects on the free convection and mass transfer flow through porous medium with constant suction and constant heat flux, International Journal of Scientific Engineering and Technology, Vol.1 (6), pp. 274-284
- 45. S. O. Adeansya and R. O. Ayeni (2008): Steady Flow of Reacting Temperature Dependent fluid past an impulsively started porous vertical surface with Newtonian heating, Journal of Nigerian Association of Mathematical Physics, Vol. 13, pp. 221-226.
- 46. G. Bhaskar Reddy, K. Malleswari, K. Venugopal Reddy, Ch. Shashi Kumar, Y. V. Seshagiri Rao, Dr. Nookala Venu (2023): Impact of magnetic field on an oscillatory flow of a non-Newtonian fluid with radiation and heat generation, Eur. Chem. Bull. Vol. 12 (11), pp. 600-613
- 47. Ch. Shashi Kumar, P. Govinda Chowdary, P. Sarada Devi, V. Nagaraju (2022): Radiation and chemical reaction effects on unsteady flow past an accelerated infinite vertical plate with variable temperature and uniform mass diffusion through a porous plate, Journal of Positive School of Psychology, Vol. 6 (4), pp. 10983-10991