Peristaltic Transport of a Conducting Fluid

in an Inclined Asymmetric Channel

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Abstract

In this chapter MHD peristaltic flow of an incompressible viscous fluid in an inclined asymmetric channel is investigated under long wavelength and low Reynolds number assumptions. The expressions for velocity distribution and the pressure rise are obtained. The effects of phase shift and Hartmann number on the pumping characteristics are discussed.

Keywords: peristaltic flow, MHD, asymmetric channel

Introduction: The word peristalsis stems from the Greek word Peristaltikos, which means clasping and compressing. Peristaltic pumping is a form of fluid transport generated in the fluid contained in a distensible tube when a progressive wave travels along the wall of the tube. It is an inherent property of many syncytial smooth muscle tubes, stimulation at any point can cause a contractile ring to appear

in the circular muscle of the gut, and this ring then spreads along the tube. In such a way, peristalsis occurs in the gastrointestinal tract, the bile ducts, other glandular ducts throughout the body, the ureters, and many other smooth muscle tubes of the body, Guyton and Hall (7).

The first attempt to study the fluid dynamics aspects of peristaltic transport with an experimental investigation was done by Latham (11). The earliest models of peristaltic pumping are based on the assumption of trains of periodic sinusoidal waves in infinitely long two-dimensional channels or axisymmetric tubes (Shapiro(18); Fung & Yih (5)). These models which were applied primarily to characterize the basic fluid mechanics of pumping process, fall into two classes: (1) the model developed by Fung & Yih which is restricted to small peristaltic wave amplitudes but has no restrictions on Reynolds number; and (2) the lubrication-theory model introduced by Shapiro et al., (19) in which effects of fluid inertial and wall curvature are neglected but no restrictions are placed on wave amplitude. A complete review of peristaltic transport is given by Jaffrin and Shapiro (9). The lubrication-theory model is applicable globally in the limit of totally including peristaltic waves and is found to be a reasonably accurate approximation of global pumping characteristics at a small Reynolds number and wall curvature, Jaffrin (10); Takabatake & Ayukawa (23). Several authors extended the classical lubrication-theory model of peristaltic pumping to non-Newtonian fluid flows (Shukla etal., (20); Shukla & Gupta (20); Bohme and Friedrich (3)) and different cross-sectional shapes (Rath (17)).

The effect of moving magnetic field on blood flow was studied by Sud et al., (22) and they have observed that the effect of suitable moving magnetic filed accelerates the speed of blood. Srivastava and Agrawal (21) and Prasad and Ramacharyulu (16) have observed that by considering the blood as an electrically conducting fluid constitutes a suspension of red cell in plasma. Agrawal and Anwaruddin (1) also studied the effect of magnetic field on the peristaltic flow of blood using long wavelength approximation method and observed for the flow of blood in arteries with arterial stenosis or arteriosclerosis, that the influence of magnetic field may be utilized as blood pump in carrying out cardiac operations. Li et al., (12) have studied an impulsive magnetic filed in the combined therapy of patients with stone fragments in the upper urinary tract. It was found that the Impulsive Magnetic Field activates the impulsive activity of the uretaral smooth muscles in 100% of cases. Mekheimer and Al-Arabi (15) investigated the non – linear peristaltic transport of MHD flow through a porous medium was studied in non – uniform channels. The peristaltic transport of blood under effect of a magnetic field in non uniform channels was studied by Mekheimer (14).

Most of the contributions in the study of peristaltic transport deal with the peristaltic flow in symmetric channels. Although a wide range of understanding about peristaltic pumping is possible by considering a channel to be a symmetric one, there are some physiological systems such as uterus in which asymmetry plays an important role in transporting the uterine (Eytan and Elad (4)). Mekheimer (13) studied non-linear peristaltic transport through a porous medium in an inclined channel considering the effect of gravity. Recently, Baber (2) has investigated the peristaltic transport of MHD flow through an asymmetric channel.

Therefore, the objective of the present paper is to investigate the effects of MHD on peristaltic flow of an incompressible viscous fluid in an inclined asymmetric channel under long wavelength and low Reynolds number assumptions.

Mathematical formulation and Solution

We consider the peristaltic transport of a conducting viscous fluid in an inclined asymmetric channel with an inclination angle α induced by sinusoidal wave trains propagating with same speed but with different amplitudes and phases along the channel walls. We assume that a uniform magnetic field strength B_0 is applied in the transverse direction to the direction of the flow (i. e., along the direction of the y-axis) and the induced magnetic field is assumed to be negligible. Fig 1.1 shows the physical model of the asymmetric channel.

We introduce a wave frame of reference (x, y) moving with velocity c in which the motion becomes independent of time when the channel length is an integral multiple of the wavelength and the pressure difference at the ends of the channel is a constant (Shapiro et al. 1969). The transformation from the fixed frame of reference (X, Y) to the wave frame of reference (x, y) is given by $x = X - ct,\ y = Y,\ u = U - c,\ v = V$ and $p^1(x) = P^1(X, t),$ where (u, v) and (U, V) are the velocity components, p^1 and P^1 are pressures in the wave and fixed frames of reference, respectively.

The channel walls are given by

$$Y = H_1(X,t) = a_1 + b_1 \cos \frac{2\pi}{\lambda}(X - ct) \qquad \text{(upper wall)} \qquad (1.1a)$$

$$Y = H_2(X,t) = -a_2 - b_2 \cos\left(\frac{2\pi}{\lambda}(X - ct) + \theta\right) \quad \text{(lower wall)} \quad (1.1b)$$
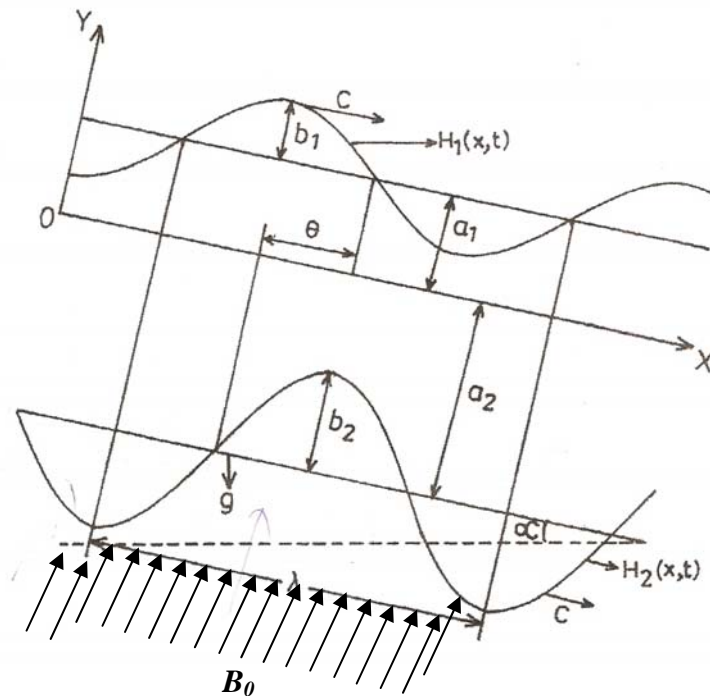
Fig 1.1. Physical Model

where b_1, b_2 are amplitudes of the waves, λ is the wavelength, $a_1 + a_2$ is the width of the channel, θ is the phase difference ($0 \leq \theta \leq \pi$) and t is the time.

The equations governing the flow in wave frame of reference are given by

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1.2}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p^1}{\partial x} + \frac{\mu}{\rho}\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) - \frac{\sigma_e B_0^2}{\rho}u + g\sin\alpha, \tag{1.3}$$

$$u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{1}{\rho}\frac{\partial p^1}{\partial y} + \frac{\mu}{\rho}\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) - g\cos\alpha. \tag{1.4}$$

where σ_e is the electrical conductivity of the fluid, ρ is the density and μ is the viscosity of the fluid.

The corresponding boundary conditions are

$u = -c$ at $y = h_1$ and $y = h_2$ (1.5)

Introducing the following non-dimensional variables

$$\bar{x} = \frac{x}{\lambda}, \bar{y} = \frac{y}{a_1}, \bar{u} = \frac{u}{c}, \bar{v} = \frac{v}{c\delta}, \delta = \frac{a_1}{\lambda}, d = \frac{a_2}{a_1}$$

$$\bar{p^1} = \frac{p^1 a_1^2}{\mu c \lambda}, h_1 = \frac{H_1}{a_1}, h_2 = \frac{H_2}{a_1}, \phi_1 = \frac{b_1}{a_1}, \phi_2 = \frac{b_2}{a_1}.$$

in the governing equations (1.1-1.4), and dropping the bars, we get

$$h_1 = 1 + \phi_1 \cos 2\pi x, \ h_2 = -d - \phi_2 \cos(2\pi x + \theta)$$ (1.6)

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0,$$ (1.7)

$$\text{Re}\,\delta\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} \right) = -\frac{\partial p^1}{\partial x} + \left(\delta^2 \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) - M^2 u + \eta \sin \alpha,$$ (1.8)

$$\text{Re}\,\delta^3\left(u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} \right) = -\frac{\partial p^1}{\partial y} + \delta^2\left(\delta^2 \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) - \eta_1 \cos \alpha.$$ (1.9)

where $\text{Re} = \frac{\rho a_1 c}{\mu}$ is the Reynolds number, $\eta = \frac{a_1^2 g}{\nu c}$, $\eta_1 = \frac{a_1^3 g}{\nu c \lambda}$, g is the

acceleration due to gravity $M = B_0 a_1 \sqrt{\frac{\sigma_e}{\mu}}$ is the Hartmann number and $\nu = \frac{\mu}{\rho}$ is

the kinematic viscosity of the fluid.

Using long wavelength (i.e., $\delta << 1$) and negligible inertia (i.e., $\text{Re} \to 0$) approximations, we have

$$\frac{\partial p^1}{\partial y} = -\eta_1 \cos \alpha, \qquad \frac{\partial^2 u}{\partial y^2} - M^2 u = \frac{\partial p^1}{\partial x} - \eta \sin \alpha.$$ (1.10)

Let $p^1 = p(x) - \eta_1 \cos \alpha\, y$ then (1.10) becomes

$$\frac{\partial p}{\partial y} = 0, \qquad \frac{\partial^2 u}{\partial y^2} - M^2 u = \frac{dp}{dx} - \eta \sin \alpha$$ (1.11)

The corresponding non-dimensional boundary conditions are given as

$u = -1$ at $y = h_1$ and $y = h_2$ (1.12)

Solving equation (1.11) using the boundary conditions (1.12), we get

$$u = c_1 \cosh My + c_2 \sinh My - \left(\frac{dp}{dx} - \eta \sin \alpha \right) / M^2 \qquad (1.13)$$

where $c_1 = \dfrac{\left\{ -1 + \left(\dfrac{dp}{dx} - \eta \sin \alpha \right) / M^2 \right\} \left[\sinh Mh_2 - \sinh Mh_1 \right]}{\left[\cosh Mh_1 \sinh Mh_2 - \cosh Mh_2 \sinh Mh_1 \right]}$ and

$$c_2 = \frac{\left\{ -1 + \left(\dfrac{dp}{dx} - \eta \sin \alpha \right) / M^2 \right\} \left[\cosh Mh_1 - \cosh Mh_2 \right]}{\left[\cosh Mh_1 \sinh Mh_2 - \cosh Mh_2 \sinh Mh_1 \right]}$$

The volume flow rate in the wave frame is given as

$$q = \int_{h_2}^{h_1} u \, dy$$

$$= \frac{c_1}{M} \left(\sinh Mh_1 - \sinh Mh_2 \right) + \frac{c_2}{M} \left(\cosh Mh_1 - \cosh Mh_2 \right) \qquad (1.14)$$

$$- \frac{\left(\dfrac{dp}{dx} - \eta \sin \alpha \right)}{M^2} (h_1 - h_2).$$

From (1.14), we have

$$\frac{dp}{dx} = \frac{qM^3 D_1 + D_2 M^2}{D_2 - (h_1 - h_2) M D_1} + \eta \sin \alpha \qquad (1.15)$$

where

$$D_1 = \cosh Mh_1 \sinh Mh_2 - \cosh Mh_2 \sinh Mh_1 \qquad \text{and}$$

$$D_2 = \left(\cosh Mh_1 - \cosh Mh_2 \right)^2 - \left(\sinh Mh_1 - \sinh Mh_2 \right)^2$$

The instantaneous flux at any axial station is given by

$$Q(x,t) = \int_{h_2}^{h_1} (u+1) dy = q + h_1 - h_2 \qquad (1.16)$$

The average volume flow rate over one wave period ($T = \lambda / c$) of the peristaltic wave is defined as

$$\overline{Q} = \frac{1}{T} \int_0^T Q \, dt = \frac{1}{T} \int_0^T (q + h_1 - h_2) dt = q + 1 + d. \qquad (1.17)$$

The pressure rise over one wave length of the peristaltic wave is given by

$$\Delta p = \int_0^1 \frac{dp}{dx}dx = \int_0^1 \left[\frac{qM^3 D_1 + D_2 M^2}{D_2 - (h_1 - h_2)MD_1} + \eta \sin\alpha \right] dx$$

(1.18)

$$= \int_0^1 \left[\frac{(\overline{Q} - 1 - d)M^3 D_1 + D_2 M^2}{D_2 - (h_1 - h_2)MD_1} + \eta \sin\alpha \right] = \overline{Q}I_1 + I_2 + \eta \sin\alpha.$$

where $I_1 = \int_0^1 \dfrac{M^3 D_1}{D_2 - (h_1 - h_2)MD_1}dx$ and $I_2 = \int_0^1 \dfrac{-(1+d)M^3 D_1 + D_2 M^2}{D_2 - (h_1 - h_2)MD_1}dx$.

The equation (1.17) can be rewritten as

$$\overline{Q} = \frac{\Delta p - I_2 - \eta \sin\alpha}{I_1}$$

(1.19)

Discussion of the Results

The variation of axial velocity u as a function of y at $x=0.25$ for different values of M with $\phi_1 = 0.7$, $\phi_2 = 1.2$, $d = 2$, $\eta = 0.5$, $\alpha = \pi/6$ and phase shift $\theta = 0$ for (i) $\dfrac{dp}{dx} = -0.5$ (ii) $\dfrac{dp}{dx} = 0$ and (iii) $\dfrac{dp}{dx} = 0.5$ as shown in Fig 1.2. The maximum velocity occurs at the centre of the channel and increases as M increases for $\dfrac{dp}{dx} = -0.5$, $\dfrac{dp}{dx} = 0$ and $\dfrac{dp}{dx} = 0.5$. Further for non conducting fluid (i.e., $M = 0$) the flow reverses when $\dfrac{dp}{dx} = 0.5$.
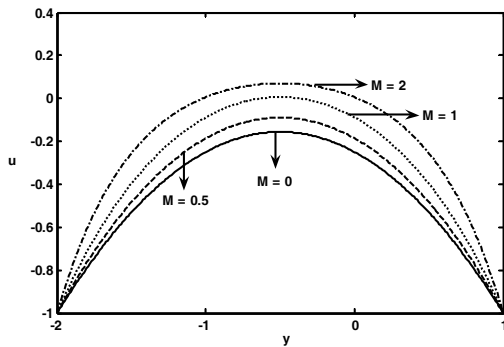
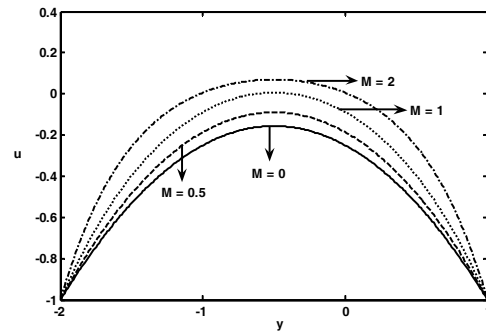Fig 1. 2(i).The variation of velocity u with y for different values of M

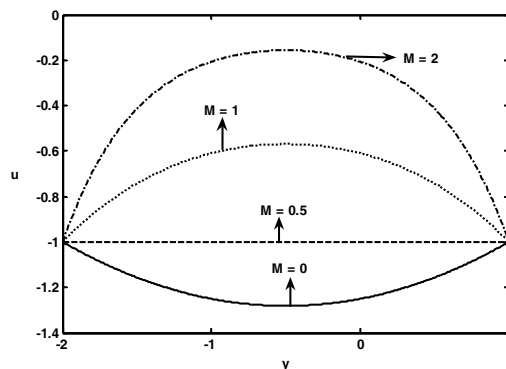Fig 1.2(ii). The variation of velocity u with y for different values of M

Fig 1.2(iii).The variation of velocity u with y for different values of M

Fig 1.3 shows that the variation of axial velocity u with y for different values of M at $x = 0.25$ with $\phi_1 = 0.7$, $\phi_2 = 1.2$, $d = 2$, $\eta = 0.5$, $\alpha = \pi / 6$ and $\theta = \dfrac{\pi}{4}$ for (i) $\dfrac{dp}{dx} = -0.5$, (ii) $\dfrac{dp}{dx} = 0$ (iii) $\dfrac{dp}{dx} = 0.5$ as shown in Fig 1.3. The maximum velocity increases as M increases for $\dfrac{dp}{dx} = -0.5$ $\dfrac{dp}{dx} = 0$ and $\dfrac{dp}{dx} = 0.5$. Further, for $\dfrac{dp}{dx} = 0.5$ the flow is reverse in the direction of wave proposition on the channel for $M = 0$.
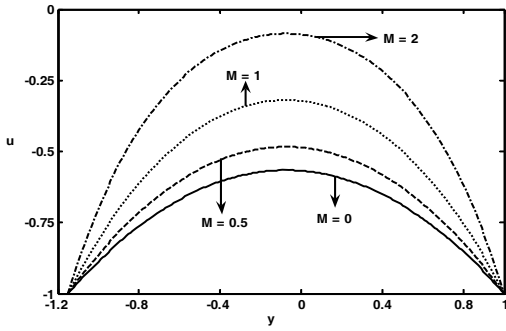
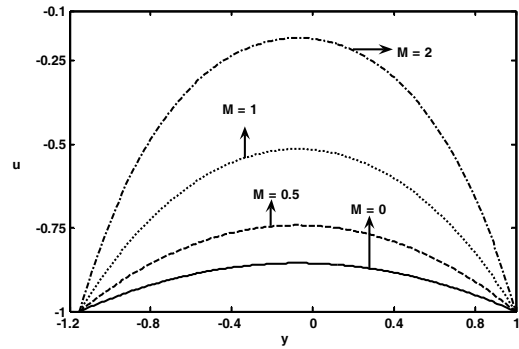Fig 1.3(i) .The variation of velocity u with y for different values of M.

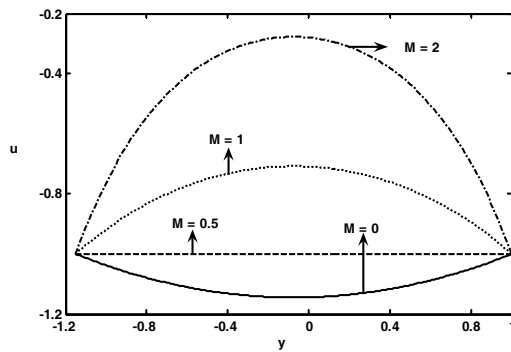Fig 1.3(ii). The variation of velocity u with y for different values of M.

Fig 1.3(iii).The variation of velocity u with y for different values of M with

The same behavior holds for $\theta = \dfrac{\pi}{2}$, as shown in Fig 1.4. Finally we conclude as phase shift increases the maximum velocity decrease (Fig 1.2 – 1.4). Further the maximum velocity is more when compared with that in the horizontal channel
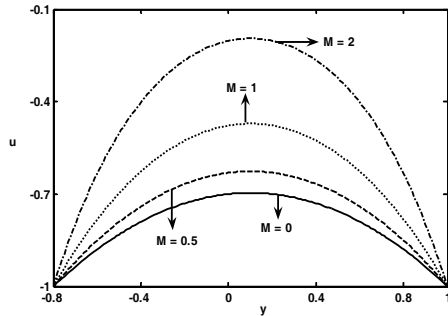
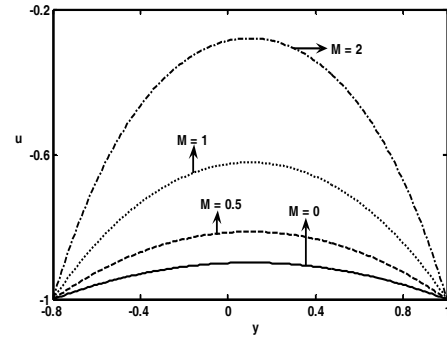Fig 1.4(i).   The variation of velocity u with y for different values of M.

Fig 1.4(ii).   The variation of velocity u with y for different values of M.
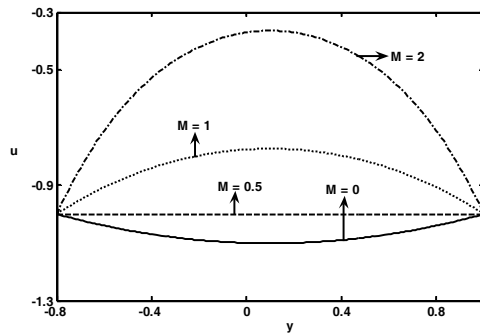

Fig 1.4(iii).The variation of velocity u with y for different values of M.

The variation of axial velocity u as a function of y at $x = 0.25$ for different values of α with $\phi_1 = 0.7$, $\phi_2 = 1.2$, $d = 2$, $\theta = \dfrac{\pi}{4}$, $\eta = 1$ and $M = 0.5$, for (i) $\dfrac{dp}{dx} = -0.5$, is shown in Fig 1.5. It is observed that the maximum velocity increases as η increases for all values of $\dfrac{dp}{dx}$.
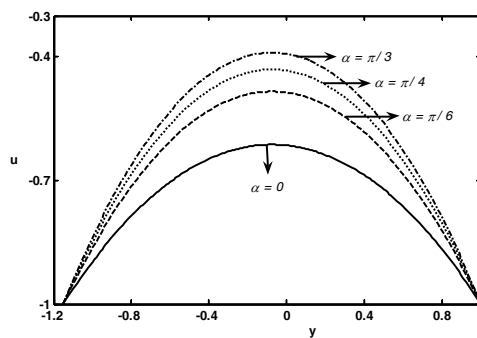

Fig 1.5(i).The variation of velocity u with y for different values of α

Acknowledgements:

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