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Peristaltic transport of a conducting fluid through a porous medium in an asymmetric vertical channel

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ABSTRACT

The aim of this problem is to study the peristaltic motion of a viscous conducting fluid through a porous medium in an asymmetric vertical channel by using Lubrication approach. The expressions for velocity and pressure rise are obtained. The effects of Darcy number, phase shipt and Hartmann number on flow characteristics are studied in detail.

Key words: Peristaltic transport, MHD, porous medium, Lubrication approach.

INTRODUCTION

The study of the mechanism of peristalsis in both mechanical and physiological situations has recently become the object of scientific research. Several theoretical and experimental attempts have been made to understand peristaltic action in different situations. A review of much of the early literature is presented in an article by Jaffrin and Shapiro (5). A summary of most of the experimental and theoretical investigations reported with details of the geometry, fluid Reynolds number, wavelength parameter, wave amplitude parameter and wave shape has been studied by Srivastava and Srivastava (16).

Flow through a porous medium has been studied by a number of workers employing Darcy's law is given by A.E. Scheidegger (14). Some studies about this point have been made by Varshney (18) and EL-Dave and EL-Mohendis (2). Elshehawey et al., (4) studied peristaltic motion of a generalized Newtonian fluid through a porous medium. Ramireddy *et.al.* (11) studied peristaltic transport of a conducting fluid in an inclined asymmetric channel. Peristaltic motion of a generalized Newtonian fluid under the effect of a transverse magnetic field is studied by Elshehawey et al.,(3). Satyanarayana *et a l(13)* studied Hall current effect on magnetohydro dynamics Free-convection flow past a semi-infinite vertical porous plate with mass transfer. Flow through a porous medium has been of considerable interest in recent years particularly among geophysical fluid dynamicists. Examples of natural porous media are beach sand, standstone, limestone, rye bread, wood, the human lung, bileduct, gall bladder with stones and in small blood vessels.

The first study of peristaltic flow through a porous medium is presented by Elshehawey et al., (3). The interaction of peristaltic flow with pulsatile fluid under the effect of a transverse magnetic field through a porous medium bounded by a two-dimensional channel is studied by Afifi and Gad (1). Mekheimer and Arabi (8) studied the non-linear peristaltic transport of MHD flow through a porous medium. Mekheimer (9) studied peristaltic flow of blood under effect of a magnetic field in non-uniform channels. He observed that the pressure rise for a couple stress fluids (as a blood model) is greater than for a Newtonian fluid and is smaller for a magnetohydrodynamic fluid than for a fluid without an effect of a magnetic field. Non-linear peristaltic transport through a porous medium in an inclined planar channel has been studied by Mekheimer (10) taking the gravity effect on pumping characteristics. Peristaltic transport of a viscous fluid in an inclined asymmetric channel has been studied by Subba Reddy (17). Recently the effects of heat transfer on MHD unsteady free convection flow past an infinite/semi infinite vertical plate was analyzed by [6,7, 12, 15].

This paper deals with the effects of Darcy number, phase shipt and Hartmann number in the peristaltic motion of a viscous conducting fluid through a porous medium in an asymmetric vertical channel in the presence of magnetic field..

Mathematical formulation and Solution

We consider the peristaltic transport of a viscous conducting fluid through an asymmetric channel with flexible walls and asymmetry being generated by the propagation of waves on the channel walls travelling with same speed c but with different amplitudes and phases. We assume that a uniform magnetic field strength B_0 is applied in the transverse direction to the direction of the flow (i. e., along the direction of the y-axis) and the induced magnetic field is assumed to be negligible. Fig 1. shows the physical model of the asymmetric channel.
The channel walls are given by

$$Y = H_1(X,t) = a_1 + b_1 \cos\frac{2\pi}{\lambda}(X - ct) \qquad (1a)$$

$$Y = H_2(X,t) = -a_2 - b_2 \cos\left(\frac{2\pi}{\lambda}(X - ct) + \theta\right) \qquad (1b)$$

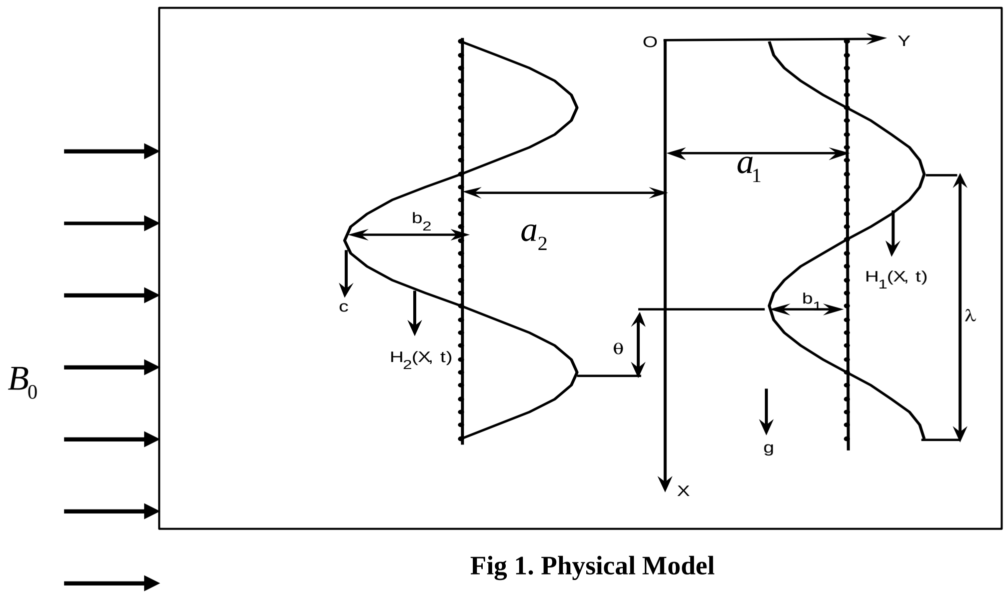

Fig 1. Physical Model

Where b_1 , b_2 are amplitudes of the waves, λ is the wavelength, $a_1 + a_2$ is the width of the channel, θ is the phase difference (0≤θ≤π) and t is the time.

We introduce a wave frame of reference (x, y) moving with velocity c in which the motion becomes independent of time when the channel length is an integral multiple of the wavelength and the pressure difference at the ends of the channel is a constant (Shapiro et al., (1969)). The transformation from the fixed frame of reference (X, Y) to the wave frame of reference (x, y) is given by

$x = X - ct,\ y = Y,\ u = U - c,\ v = V$ and $p(x) = P(X, t)$,

where (u, v) and (U, V) are the velocity components, p and P are pressures in the wave and fixed frames of reference, respectively.

The equations governing the flow in wave frame of reference are given by

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{2}$$

$$\rho\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=-\frac{\partial p}{\partial x}+\frac{\mu}{\varepsilon}\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)-\sigma_e B_0^2 u-\frac{\mu}{k}u, \tag{3}$$

$$\rho\left(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right)=-\frac{\partial p}{\partial y}+\frac{\mu}{\varepsilon}\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)-\frac{\mu}{k}v. \tag{4}$$

Where σ_e is the electrical conductivity of the fluid, ε and k are the porosity and permeability of the porous medium, ρ is the density and μ is the viscosity of the fluid.

Introducing the following non-dimensional variables

$$\bar{x}=\frac{x}{\lambda},\bar{y}=\frac{y}{a_1},\bar{u}=\frac{u}{c},\bar{v}=\frac{v}{c\delta},\delta=\frac{a_1}{\lambda},d=\frac{a_2}{a_1}$$

$$\bar{p}=\frac{pa_1^2}{\mu c\lambda},h_1=\frac{H_1}{a_1},h_2=\frac{H_2}{a_1},\phi_1=\frac{b_1}{a_1},\phi_2=\frac{b_2}{a_1}.$$

in the governing equations (1-4), and dropping the bars, we get

$$h_1=1+\phi_1\cos 2\pi x,\ h_2=-d-\phi_2\cos\left(2\pi x+\theta\right) \tag{5}$$

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{6}$$

$$\mathrm{Re}\,\delta\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=-\frac{\partial p}{\partial x}+\frac{1}{\varepsilon}\left(\delta^2\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)-M^2u-\frac{1}{Da}u, \tag{7}$$

$$\mathrm{Re}\,\delta^3\left(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right)=-\frac{\partial p}{\partial y}+\frac{\delta^2}{\varepsilon}\left(\delta^2\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)-\frac{\delta^2}{Da}v. \tag{8}$$

where $\mathrm{Re} = \dfrac{\rho a_1 c}{\mu}$ is the Reynolds number, $Da = \dfrac{k}{a_1^2}$ is the Darcy number and $M = B_0 a_1 \sqrt{\dfrac{\sigma_e}{\mu}}$ is the Hartmann number.

Using long wavelength (i.e., $\delta << 1$) and negligible inertia (i.e., $\mathrm{Re} \to 0$) approximations, we have

$$\frac{\partial p}{\partial y} = 0, \qquad \frac{\partial^2 u}{\partial y^2} - N^2 u = \frac{dp}{dx}. \tag{9}$$

where $N^2 = \varepsilon\left(\dfrac{1}{Da} + M^2\right)$.

The corresponding non-dimensional boundary conditions are given as

$$u = \text{-}1 \quad \text{at} \quad y = h_1 \text{ and } y = h_2 \tag{10}$$

Solving equation (9) using the boundary conditions (10), we get

$$u = c_1 \cosh Ny + c_2 \sinh Ny - \frac{dp}{dx} / N^2 \tag{11}$$

where $c_1 = \dfrac{\left(-1 + \dfrac{dp}{dx}/N^2\right)\left[\sinh Nh_2 - \sinh Nh_1\right]}{\left[\cosh Nh_1 \sinh Nh_2 - \cosh Nh_2 \sinh Nh_1\right]}$ and

$$c_2 = \frac{\left(-1 + \dfrac{dp}{dx}/N^2\right)\left[\cosh Nh_1 - \cosh Nh_2\right]}{\left[\cosh Nh_1 \sinh Nh_2 - \cosh Nh_2 \sinh Nh_1\right]}.$$

The volume flow rate in the wave frame is given as

$$q = \int_{h_2}^{h_1} u\,dy$$

$$= \frac{c_1}{M}\left(\sinh Nh_1 - \sinh Nh_2\right) + \frac{c_2}{M}\left(\cosh Nh_1 - \cosh Nh_2\right) - \frac{dp}{dx}\frac{\left(h_1 - h_2\right)}{N^2}. \tag{12}$$

From (12), we have

$$\frac{dp}{dx} = \frac{qN^3 D_1 + D_2 N^2}{D_2 - \left(h_1 - h_2\right) N D_1} \tag{13}$$

where

$D_1 = \cosh Nh_1 \sinh Nh_2 - \cosh Nh_2 \sinh Nh_1$ and

$D_2 = \left(\cosh Nh_1 - \cosh Nh_2\right)^2 - \left(\sinh Nh_1 - \sinh Nh_2\right)^2$.

The instantaneous flux at any axial station is given by

$$Q(x,t) = \int_{h_2}^{h_1} (u+1)dy = q + h_1 - h_2 . \quad (14)$$

The average volume flow rate over one wave period (T= λ / c) of the peristaltic wave is defined as

$$\overline{Q} = \frac{1}{T}\int_0^T Qdt = \frac{1}{T}\int_0^T (q + h_1 - h_2)dt = q + 1 + d . \quad (15)$$

The pressure rise over one wave length of the peristaltic wave is given by

$$\Delta p = \int_0^1 \frac{dp}{dx}dx = \int_0^1 \frac{qN^3 D_1 + D_2 N^2}{D_2 - (h_1 - h_2) ND_1} dx$$
$$= \int_0^1 \frac{(\overline{Q} - 1 - d) N^3 D_1 + D_2 N^2}{D_2 - (h_1 - h_2) ND_1} = \overline{Q} I_1 + I_2 . \quad (16)$$

where $I_1 = \int_0^1 \frac{N^3 D_1}{D_2 - (h_1 - h_2) ND_1} dx$ and $I_2 = \int_0^1 \frac{-(1+d)N^3 D_1 + D_2 N^2}{D_2 - (h_1 - h_2) ND_1} dx$.

The equation (4.16) can be rewritten as

$$\overline{Q} = \frac{\Delta p - I_2}{I_1} . \quad (17)$$

RESULTS AND DISCUSSION

The variation of velocity u with y for different values of M with $\phi_1 = 0.7$, $\phi_2 = 1.2$, d $=2$, $\delta = 0.5$, $x = 0.25$, $\varepsilon = 0.05$, $Da = 0.08$ *and* $\theta = 0$ for (i) $\frac{dp}{dx} = -0.5$; (*ii*) $\frac{dp}{dx} = 0$; (*iii*) $\frac{dp}{dx} = 1$ as depicted in Fig 2. It is observed that the maximum velocity U increases with the increase in Hartmann number M for all the three cases $\frac{dp}{dx}$ =- 0.5, $\frac{dp}{dx}$ = 0 and $\frac{dp}{dx}$ =1. The similar behaviour observed for phase shipt $\theta = \pi / 4$ as shown in Fig 3. The Fig 4 shows the variation of velocity μ with λ for different values of Darcy number *Da* with $\phi 1=0.7$, $\phi 2=1\text{-}2$, $d=2$, $n=0.5$, $x=0.25$ $\varepsilon=0.05$, $M=0.5$ and $\theta=\pi/4$ for (i) $\frac{dp}{dx} = 0$ and (ii) $\frac{dp}{dx} = 2$. It is observed that as the Darcy number *Da* decreases the maximum velocity increased. But when *Da* < 0.01, the change in maximum velocity is negligible.

Using equation (17), we have plotted the variation of time-averaged volume flow rate $\overline{Q}$ with pressure rise Δp for different values of phase shipt θ with $\phi_1=0.7$, $\phi_2 = 1.2$, $d=2$, $\delta=1$, $\varepsilon=0.1$ $M=0.5$ and $Da = 0.01$. The pumping increases as the phase shipt θ decreases, where as phase shipt θ increases. The free pumping as well as co-pumping both increases is show in Fig 5.

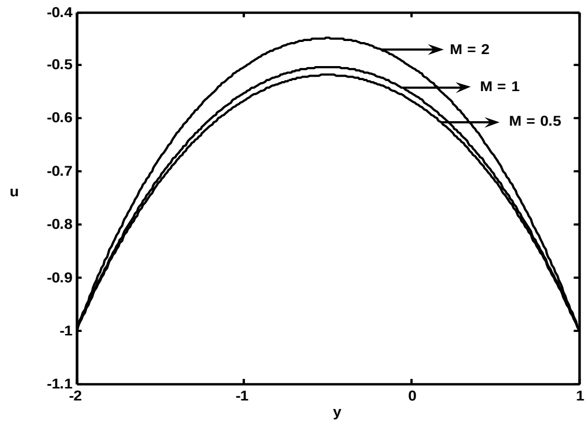

Fig 2(i).The variation of velocity u with y for different values of M with $\frac{dp}{dx} = -0.5$.

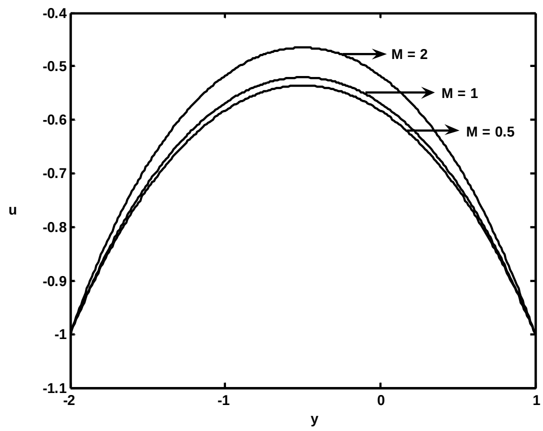

Fig 2(ii).The variation of velocity u with y for different values of M with $\frac{dp}{dx} = 0$.

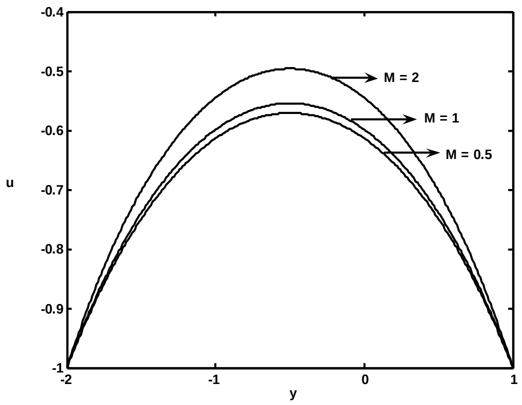

Fig 2(iii).The variation of velocity u with y for different values of M with $\frac{dp}{dx} = 1$.

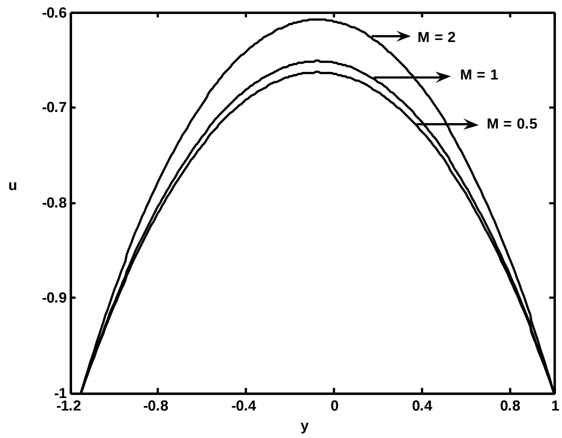

Fig 3(i).The variation of velocity u with y for different values of M with, $\frac{dp}{dx} = 2$.

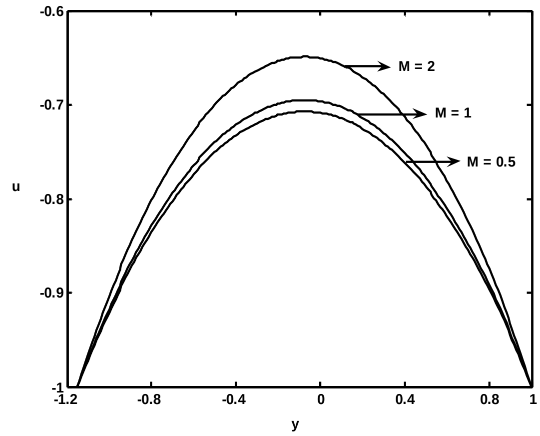

Fig 3(ii).The variation of velocity u with y for different values of M with $\frac{dp}{dx} = 0$.

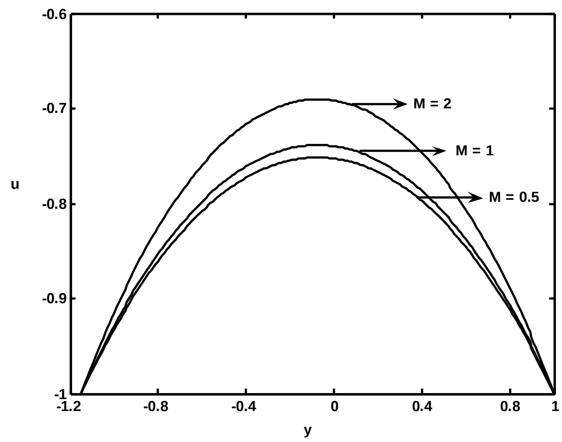

Fig 3(iii).The variation of velocity u with y for different values of M with, $\frac{dp}{dx} = 2$.

Fig 6 shows the variation of time-averaged volume flow rate $\overline{Q}$ with pressure rise Δp for different values of Hartmann number M with $\phi_1=0.7$, $\phi_2=1.2$, $d=2$, $\delta=1$, $\varepsilon=0.1$ $\theta=\pi/4$ and Da-0.01. As M increases the pumping is increases.

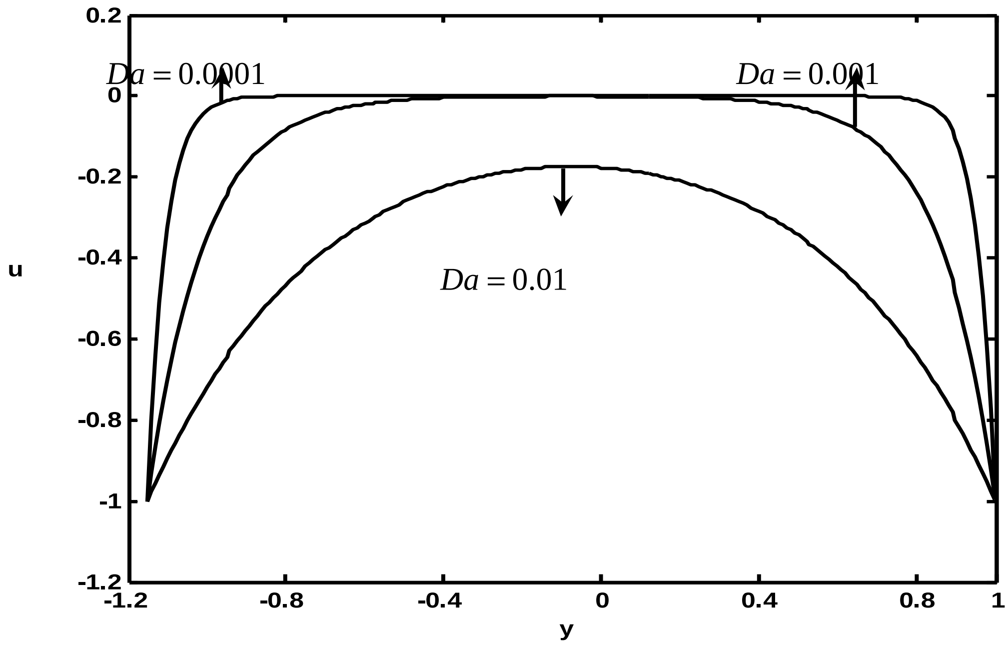

Fig 4(i). The variation of velocity u **with** y **for different values of** Da **with** $\frac{dp}{dx}=0$.

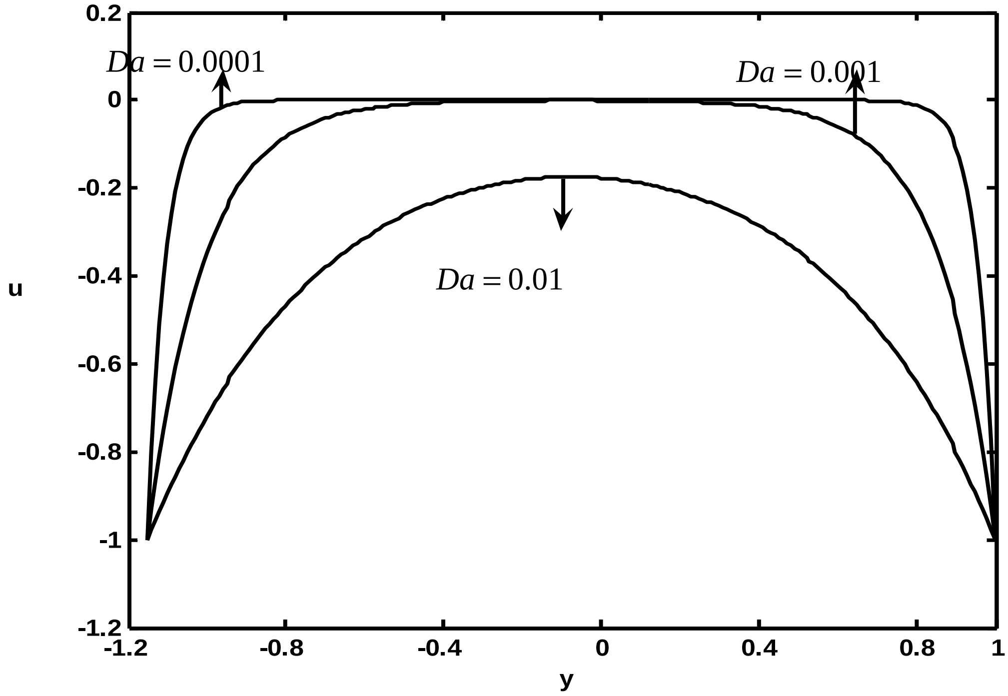

Fig 4(ii). The variation of velocity u **with** y **for different values of** Da **with** $\frac{dp}{dx}=2$.

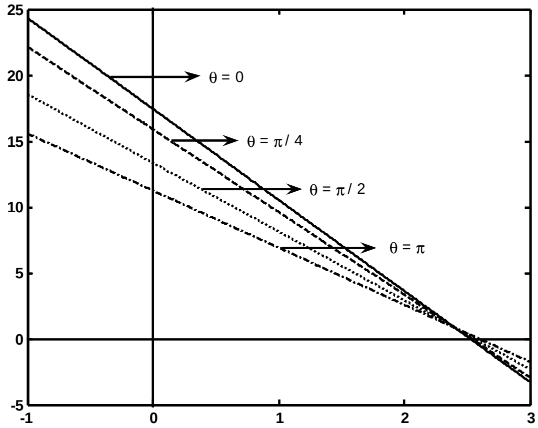

Fig 5. The variation of time-averaged volume flow rate $\overline{Q}$ with pressure rise Δp for different values of phase shift θ with $Da = 0.01$.

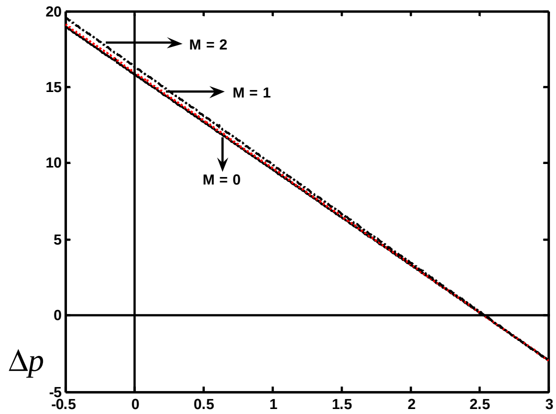

Fig 6.The variation of time-averaged volume flow rate $\overline{Q}$ with pressure rise Δp for different values of Hartmann number M with $Da=0.01$.

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