Network Theory and Transmission Lines

Course objectives:

1. To understand Network theorems and transient analysis.
2. To get knowledge about two port Networks.
3. To learn Locus diagrams, Resonance and Magnetic ckts.
4. To identify transmission line types & parameters
5. To describe Input Impedance relations of SC & OC circuit lines.

Course outcomes: After successful completion of the course students can

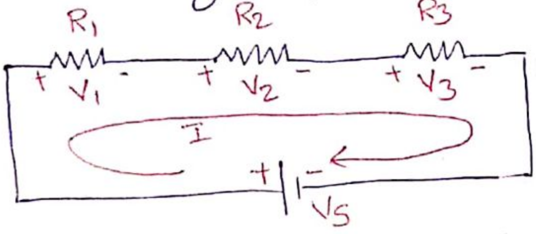
1. Able to understood Network theorems like Norton's, Reciprocity, Tellegen, Milliman and compensation theorems.
2. Get knowledge on Two port Network paremeters such as Admittance, Hybrid, ABCD. parameter. etc.
3. Learn locus diagrams, series/parallel resonance and magnetic circuits.
4. Identify transmission line types and paremets.
5. Describe Input Impedance relations of short circuit and open circuit lines

Kirchhoff's Voltage Law

Algebraic sum of all branch voltages around any closed path in a circuit is always zero at all instants of time.

NOTE 1: When the current passes through a resistor, there is a loss of energy and therefore, a voltage drop.

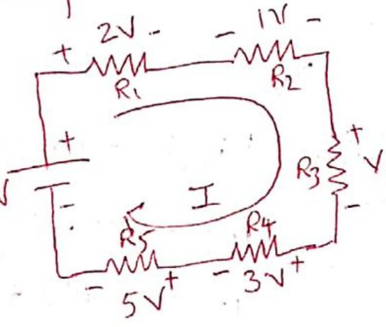
NOTE 2: Current always flows from higher potential to lower potential



As the current passes through the circuit, the sum of the voltages drop around the loop is equal to the total voltage in that loop.

$$\boxed{V_S = V_1 + V_2 + V_3 \text{ (or) } V_S - V_1 - V_2 - V_3 = 0}$$

(Q1) Determine the unknown voltage drop 'V_1'



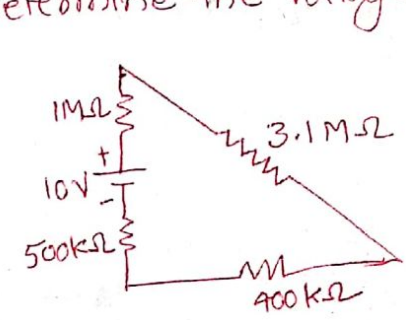
According to Kirchhoff's voltage law the sum of the potential drops is equal to the sum of the potential rises.

$30 = 2 + 1 + V_1 + 3 + 5$

$30 = 11 + V_1$; $V_1 = 30 - 11$

$\boxed{V_1 = 19V}$

(Q2) What is the current in the circuit? Determine the voltage across each resistor?



⇐

$V_{1M} = I$, $V_{3.1M\Omega} = 3.1I$, $V_{0.4} = 0.4I$, $V_{500K\Omega} = 0.5I$

Now by applying KVL $10 = I + 3.1I + 0.4I + 0.5I$

$10 = 5I$; $\boxed{I = \frac{10}{5M} = 2\mu A}$

Voltage across each resistor is

$\boxed{V_{1M} = 1 \times I = 1M \times 2\mu A = 2.0V}$

$\boxed{V_{3.1M} = 3.1 \times 10^{6} \times 2 \times 10^{-6} = 6.2V}$

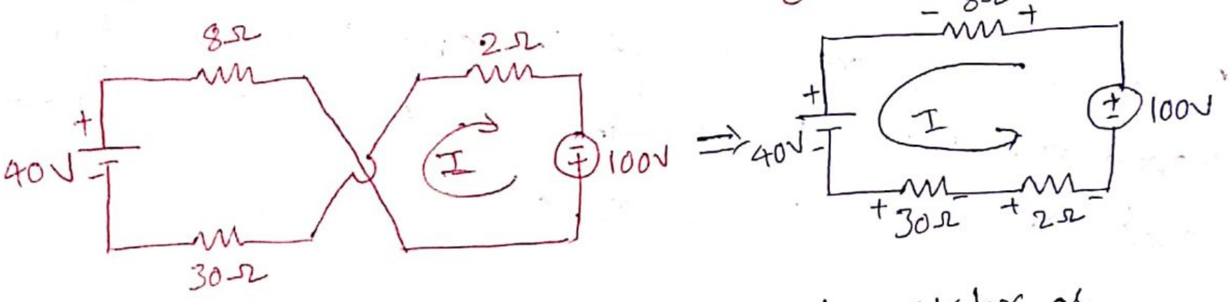
$\boxed{V_{400K} = 0.4 \times 10^{6} \times 2 \times 10^{-6} = 0.8V}$

$\boxed{V_{500K} = 0.5 \times 10^{6} \times 2 \times 10^{-6} = 1.0V}$

∴ KVL is $10 = 2 + 6.2 + 0.8 + 1$

$\boxed{10 = 10 \quad \text{or} \quad 10 - 10 = 0}$

(Q3) Find the current I and the voltage across 30Ω



By using Ohm's Law, voltage across each resistor as

$V_8 = 8I, \quad V_{30} = 30I, \quad V_2 = 2I$

By applying Kirchhoff's voltage Law

$100 = 8I + 30I + 2I + 40$

$100 = 40I + 40 \; ; \quad 40I = 60$

$I = \cancel{\frac{100}{40}} = \frac{60}{40} = 1.5A$

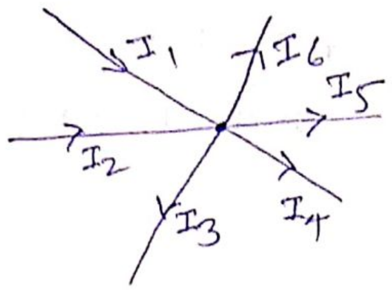
$I = 1.5A$

$\boxed{\text{Voltage drop across } 30\Omega = V_{30} = 30 \times 1.5A = 45V}$

Kirchhoff's Current Law

It states that the sum of the currents entering into any node is equal to the sum of the currents leaving that node.

NOTE: The node may be an interconnection of two or more branches.



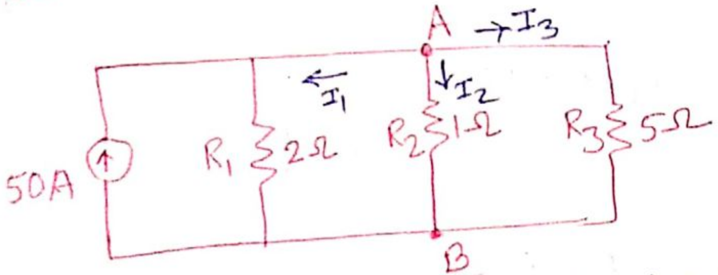
$$I_1+I_2 = I_3+I_4+I_5+I_6$$

(or)

$$I_1+I_2-I_3-I_4-I_5-I_6 = 0$$

This means, the algebraic sum of all the currents meeting at a junction is equal to zero.

Q.① Determine the current in all resistors in the following circuit



Single node 'A' with reference node 'B'. First step is to assume the voltage V at node 'A'. In parallel circuit the same voltage is applied across each element. According to Ohm's law, the currents passing through each element are $I_1 = \frac{V}{2}$; $I_2 = \frac{V}{1}$; $I_3 = \frac{V}{5}$

By applying Kirchhoff's current Law

$$I = I_1+I_2+I_3$$

$$I = \frac{V}{2}+\frac{V}{1}+\frac{V}{5}$$

$$50 = V\left[\frac{1}{2}+\frac{1}{1}+\frac{1}{5}\right] = V\left[\frac{5+10+2}{10}\right] = V\left[\frac{17}{10}\right].$$

$$\boxed{V = \frac{500}{17} = 29.41V}$$

Once we know the voltage V at node A, we can find the current in any element by using Ohm's law.

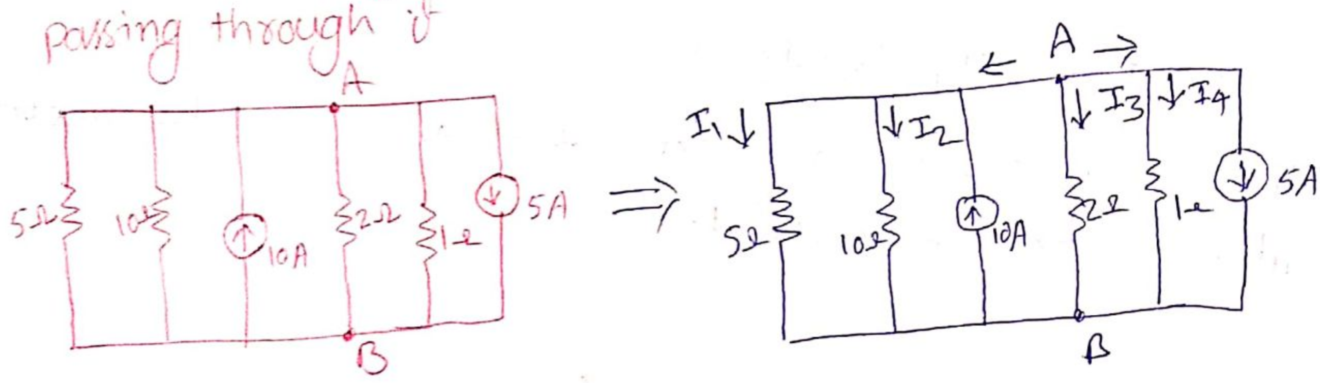
The current in the 2Ω resistor is $I_1 = \frac{V}{2} = \frac{29.41}{2} = 14.7A$

$I_2 = \frac{V}{1} = 29.41A$; $I_3 = \frac{V}{5} = \frac{29.41}{5} = 5.88A$

∴ $\boxed{I = I_1 + I_2 + I_3}$

50A = 14.7A + 29.41A + 5.88A.

(Q2) Find the voltage across the 10Ω resistor and current passing through it



$I_1 + I_2 + I_3 + I_4 + 5 = 10$

$I_1 = \frac{V}{5}$; $I_2 = \frac{V}{10}$; $I_3 = \frac{V}{2}$; $I_4 = \frac{V}{1}$

$10 = \frac{V}{5} + \frac{V}{10} + \frac{V}{2} + \frac{V}{1} + 5$

$10 = V\left[\frac{1}{5} + \frac{V}{10} + \frac{1}{2} + \frac{1}{1}\right] + 5$

$10 - 5 = V\left[\frac{2+1+5+10}{10}\right]$

$5 = V\left[\frac{18}{10}\right]$

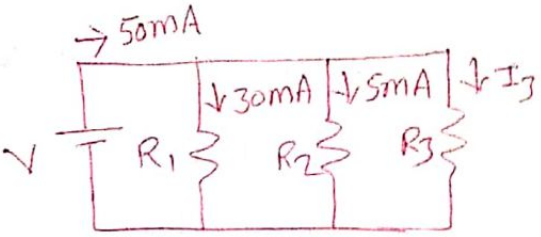
$V = \frac{50}{18} = 2.78V$ $\boxed{\therefore V = 2.78V}$

$\boxed{\text{Voltage across the 10Ω resistor is 2.78V}}$

current passing through 10Ω resistor is

$\boxed{I_2 = \frac{V}{10} = \frac{2.78}{10} = 0.278A}$

Q3 Determine the current through resistance R_3



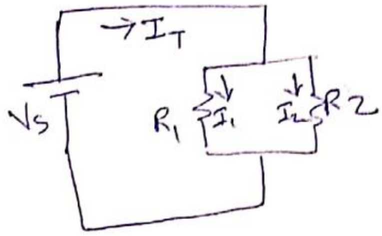
$$I_T = I_1 + I_2 + I_3$$

$$50 = 30 + 5 + I_3$$

$$I_3 = 50 - 35 = 15$$

current through resistance R_3 is 15mA

Current Division



$$R_T = \frac{R_1 R_2}{R_1 + R_2}$$

$$I_T = \frac{V_S}{R_T} = \frac{V_S (R_1 + R_2)}{R_1 R_2}$$

$$\therefore V_S = I_1 R_1$$

$$I_T = \frac{I_1 R_1 (R_1 + R_2)}{R_1 R_2}$$

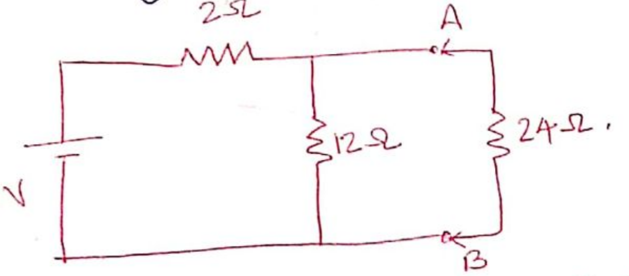
$$I_1 = \frac{I_T \cancel{R_1} R_2}{\cancel{R_1} (R_1 + R_2)}$$

$$I_1 = I_T \frac{R_2}{R_1 + R_2}$$

lly $I_2 = I_T \frac{R_1}{R_1 + R_2}$

Thevenin's Theorem

Thevenin's theorem states that any two terminal linear network having a no. of voltage, current sources and resistances can be replaced by a simple equivalent circuit consisting of a single voltage source in series with a resistance, where the value of the voltage source is equal to the openckt voltage across the two terminals of the network, and resistance is equal to the equivalent resistance measured between the terminals with all the energy sources are replaced by their Internal impedances.



Find current passing through 24Ω resistance and voltage across 24Ω resistance

I using current division rule $I_{24} = I_T \frac{12}{12+24}$

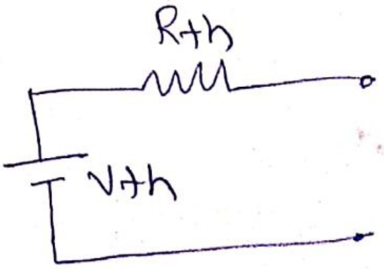
$$I_T = \frac{10}{2+\left[\frac{12\times24}{12+24}\right]} = \frac{10}{2+\frac{288}{36}} = \frac{10}{10} = 1A$$

$$I_{24} = 1\times\frac{12}{36} = 0.33A$$

voltage across 24Ω resistance $V_{24} = I_{24}\times 24 = 0.33 = 7.92V$

$$\boxed{\begin{aligned} I_{24} &= 0.33A \\ V_{24} &= 7.92V \end{aligned}}$$

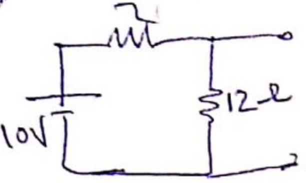
II using thevinin's theorem



The thevenin voltage is equal to the open circuit voltage across the terminals 'A B' i.e. voltage across the 12Ω resistor. when the load resistance is disconnected

using voltage division

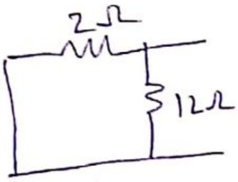
$$V_{th} = V \times \frac{\text{voltage measured resistance}}{\text{total resistance}}$$



$$V_{th} = 10 \times \frac{12}{12+2} = \frac{10\times 12}{14} = 8.57V$$

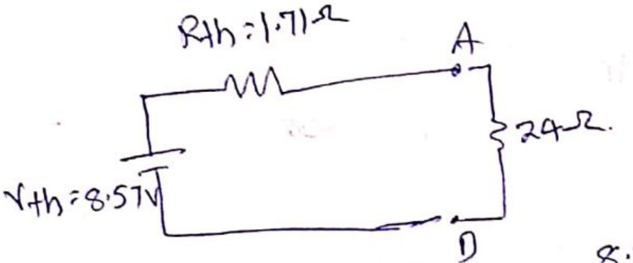
$$\boxed{V_{th} = 8.57V}$$

The resistance into the open circuit terminals is equal to the Thevenin resistance Rth



$$R_{th} = \frac{12\times 2}{12+2} = \frac{24}{14} = 1.71\Omega$$

$$\boxed{R_{th} = 1.71\Omega}$$



current through 24Ω resistance $I_{24} = \frac{V_{th}}{R_{th}+24} = \frac{8.57}{25.71} = 0.33A$

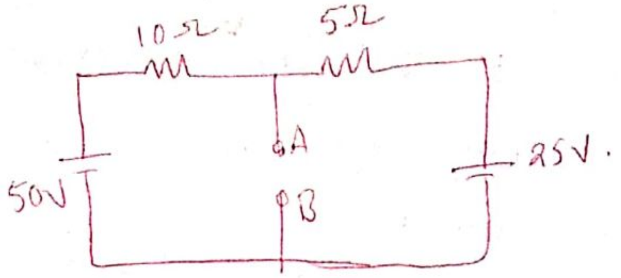
$$\boxed{I_{24} = 0.33A}$$

voltage across 24 resistance $V_{24} = I_{24} \times 24 = 0.33 \times 24 = 7.92$

$$\boxed{V_{24} = 7.92V}$$

NOTE: Load resistance $R_L = 24\Omega$ has the same values of current and voltage in the original circuit and Thevenin's equivalent ckt.

Q



$$50 - 25 = 10I + 5I$$

$$15I = 25$$

$$\boxed{I = \frac{25}{15} = 1.67A}$$

Voltage across 10Ω = I × 10Ω = 1.67 × 10 = 16.7A

$$\boxed{V_{10} = 16.7V}$$

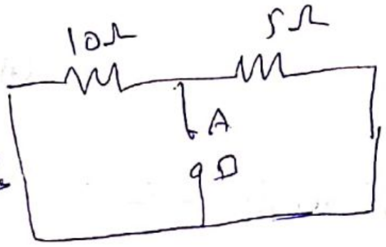
voltage drop across 5Ω = I × 5 = 1.67 × 5 = 8.35 V

$$\boxed{V_5 = 8.35 V}$$

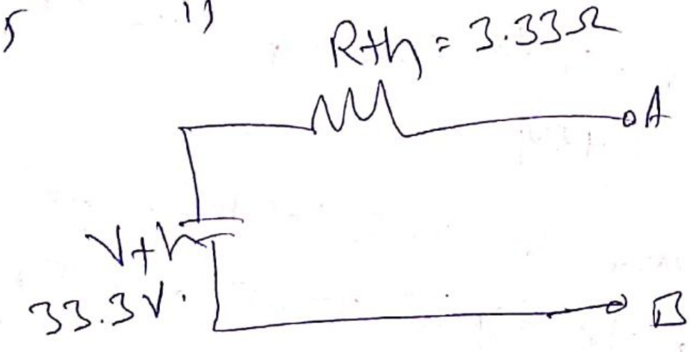
$$V_{th} = V_{AB} = 50 - V_{10} = 50 - 16.7 = 33.3V$$

$$\boxed{V_{th} = 33.3V}$$

Rth = voltage sources replaced with their internal impedances

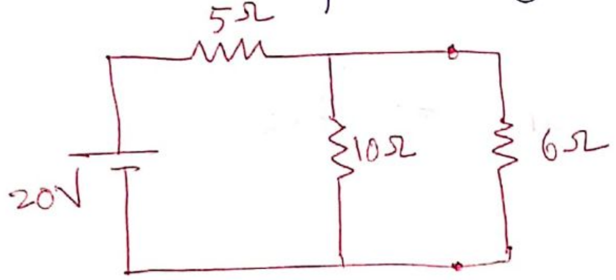


$$R_{th} = \frac{10 \times 5}{15} = \frac{50}{15} = 3.33Ω$$



NORTON'S Theorem

Any two terminal linear network with current sources, voltage sources and resistances can be replaced by an equivalent circuit consisting of a current source in parallel with a resistance. The value of the current source is the short circuit current between the two terminals of the network and the resistance is the equivalent resistance measured between the terminals of the network with all the energy sources are replaced by their internal resistance.



NOTE: If the load resistance 6Ω is connected to Norton's equivalent circuit, it will have the same current through it and the same voltage across its terminals as it experiences in the original circuit.

Proof: using original circuit

current passing through 6Ω is $I_6 = I_T \frac{10}{10+6}$

$$I_T = \frac{20}{5+\frac{10\times6}{16}} = \frac{20}{5+\frac{60}{16}} = \frac{20}{\frac{80+60}{16}} = \frac{20}{\frac{140}{16}} = \frac{20\times16}{140} = \frac{\overset{16}{\cancel{320}}}{\underset{7}{\cancel{140}}} = 2.281$$

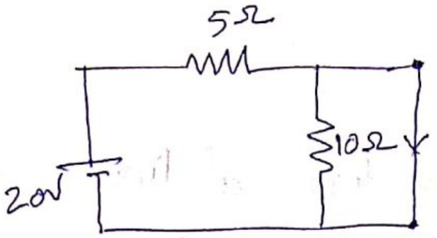
$$\boxed{I_T = 2.285A}$$

$$\boxed{I_{6\Omega} = 2.285 \times \frac{10}{16} = 1.43A}$$

voltage across 6Ω is $V_6 = I_{6\Omega} \times 6 = 1.43 \times 6 = 8.58V$

$$\boxed{V_6 = 8.58V}$$

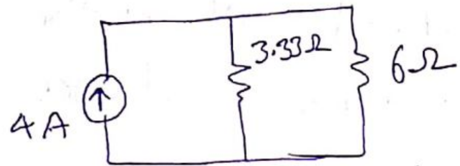
using Norton's theorem



The magnitude of the current in the Norton's equivalent circuit is equal to the current passing through short circuited terminals.

$$I_N = \frac{20}{5} = 4A$$

$$R_N = \frac{5 \times 10}{15} = \frac{50}{15} = 3.33\Omega$$



Norton's equivalent ckt.

current passing through 6Ω is $I_6 = I_T \frac{3.33}{3.33+6}$

$$I_6 = 4 \times \frac{3.33}{9.33} = 1.43A$$

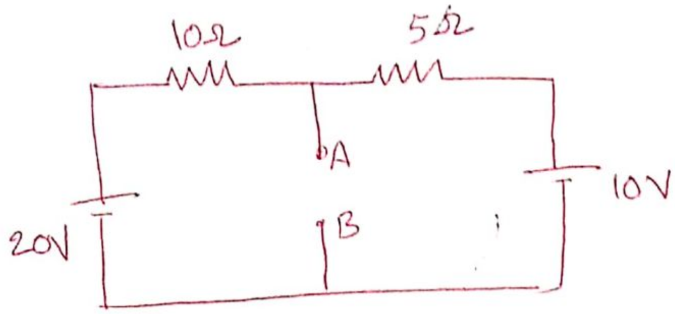
$$\boxed{I_{6\Omega} = 1.43A}$$

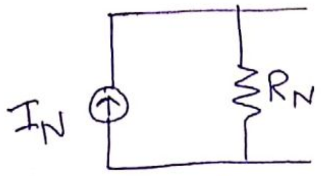
voltage drop across 6Ω is $V_{6\Omega} = I_{6\Omega} \times 6 = 1.43 \times 6 = 8.58V.$

$$\boxed{V_{6\Omega} = 8.58V}$$

Thus it is proved that $R_L (6\Omega)$ has the same values of current and voltage in both the original circuit and Norton's equivalent circuit.

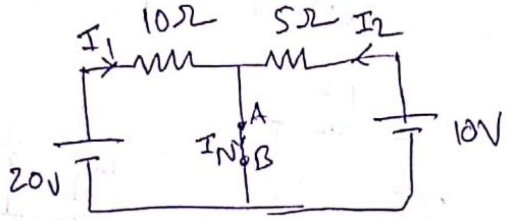
Q. Determine Norton's equivalent circuit at terminals A B.





I_N - current passing through the short circuited output terminals AB

R_N - Resistance as seen into the output terminals.



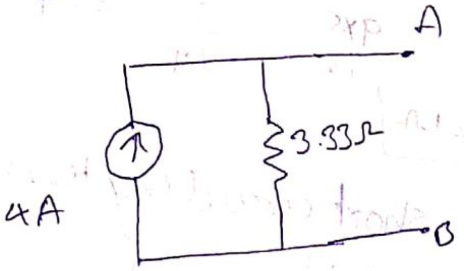
$I_N = I_1 + I_2$

$I_1 = \frac{20}{10} = 2A$

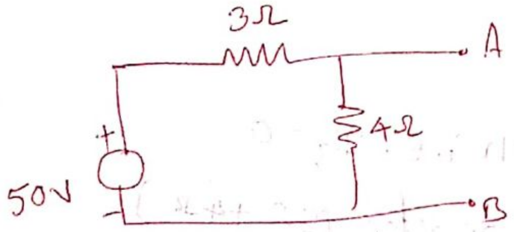
$I_2 = \frac{10}{5} = 2A$

$\boxed{I_N = 2+2 = 4A}$

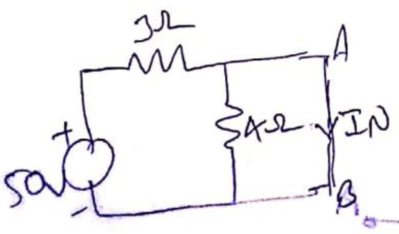
$R_N = \frac{10 \times 5}{15} = 3.33\Omega.$



Q.

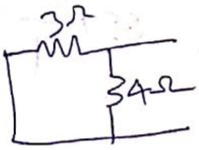


Determine Norton's equivalent circuit

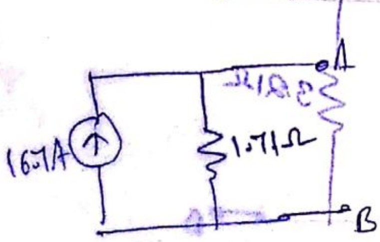


$I_N = \frac{50}{3} = 16.7A$

$R_N =$

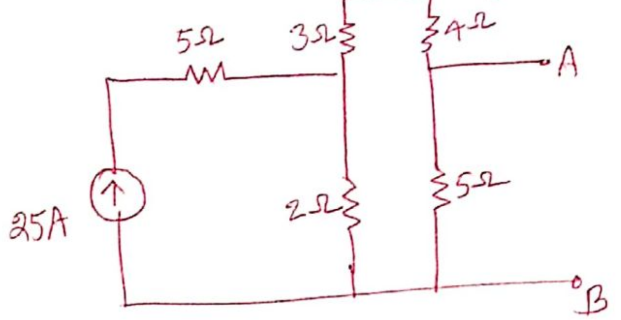


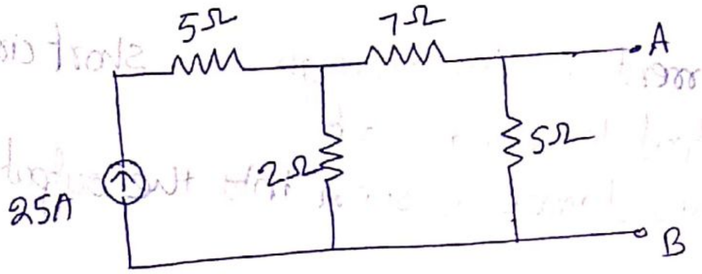
← $R_N = \frac{3 \times 4}{7} = \frac{12}{7}$ 1.71Ω.



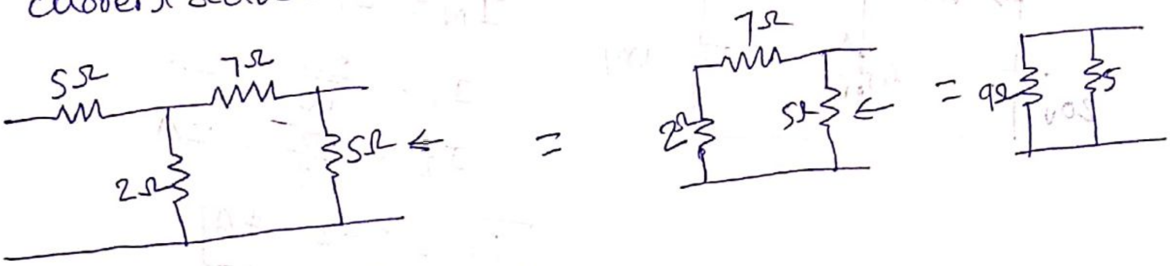
Norton's equivalent circuit.

Q Determine Norton's equivalent circuit





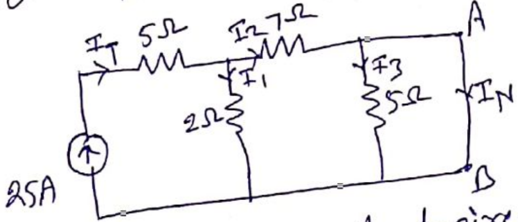
R_N – current source should be replaced with it's internal impedance.



$$\frac{9\times5}{14} = \frac{45}{14} = 3.21$$

$$\boxed{R_N = 3.21\Omega}$$

To find I_N short circuiting terminals A B

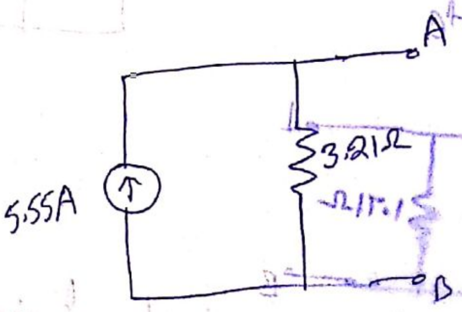


current flows through short circuit path i.e. $I_3 = 0$

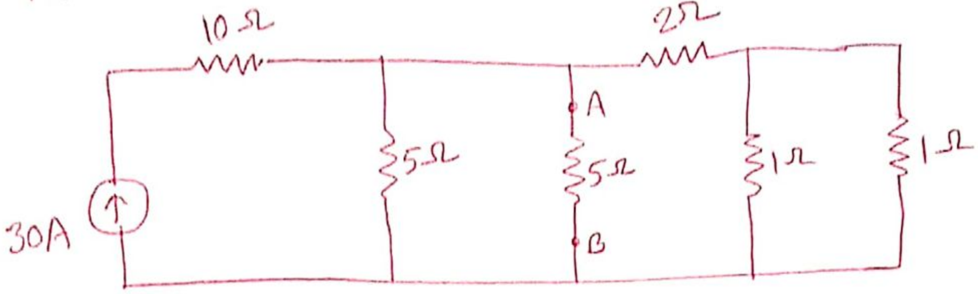
I_N is current flowing through 7Ω resistor (3Ω + 4Ω)

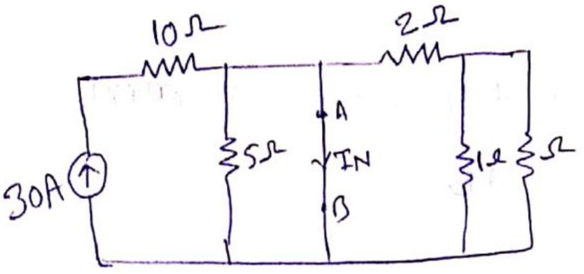
$$I_N = I_T \times \frac{2}{7+2} = 25\times\frac{2}{9} = 5.55\,A$$

$$\boxed{I_N = 5.55A}$$

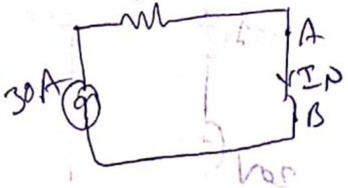


Q Determine the current flowing through 5Ω resistor in the circuit



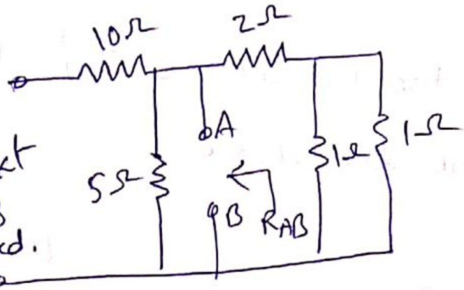


Current always flows through less resistance path. Here whole current 30A flows through short circuit terminals A B.



So $\boxed{I_N = 30A}$

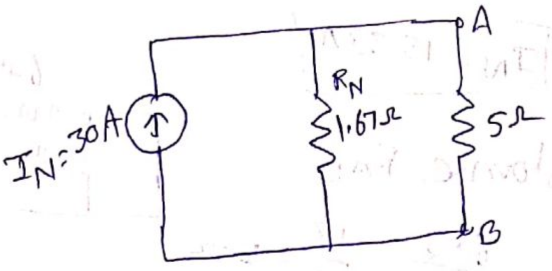
R_N is



openckt so 10Ω is not considered.

$R_{AB} = 5 \| 2 + \frac{1\times1}{2}$ = $5 \| 2 + \frac{1}{2}$ = $5 \| 2.5$ = $\frac{5\times2.5}{7.5} = \frac{12.5}{7.5}$ 1.66

$\boxed{R_{AB} = R_N = 1.66\Omega}$

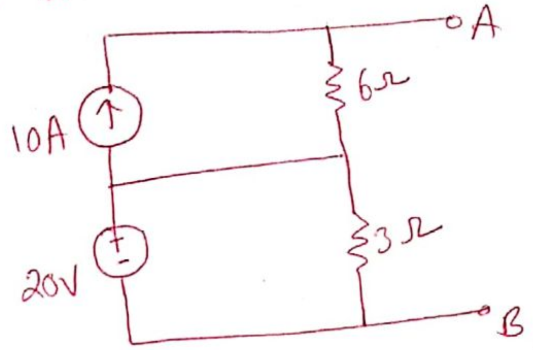


current flowing through 5Ω is

$I_5 = I_T \times \frac{1.67}{6.67} = 30 \times \frac{1.67}{6.67} = 7.51A$

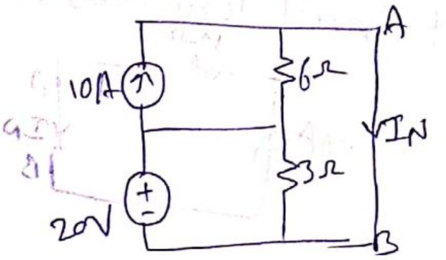
$\boxed{I_{5\Omega} = 7.51A}$

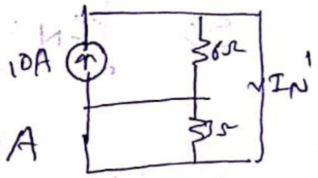
Q Replace the given network by a single current source in parallel with a resistance.



using superposition technique and Norton's theorem.

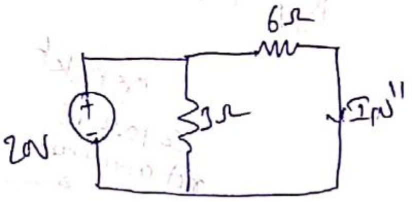
Short circuit current at terminals AB





The current I_N' due to the 10A source. $I_N' = 10A$

The current I_N'' due to the 20V source

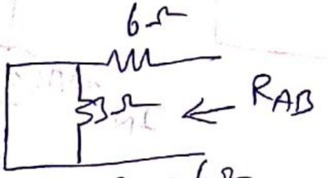


$I_N'' = \frac{20}{6} = 3.33A$.

$I_N = I_N' + I_N'' = 10 + 3.33 = 13.33A$

$\boxed{I_N = 13.33A}$

The resistance R_{AB}

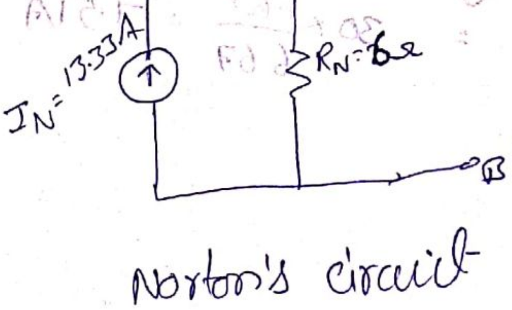


$\frac{0\times3}{0+3} + 6 = 6\Omega$

$R_N = 6\Omega$

[illegible] $= \frac{18}{9} = 8\Omega$

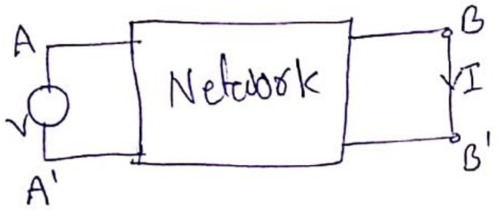
$\boxed{R_N = 8\Omega}$

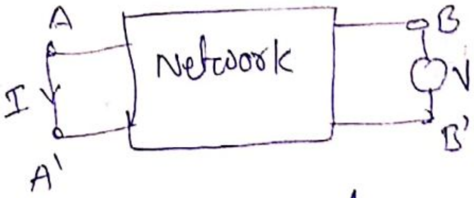


Norton's circuit

Reciprocity Theorem

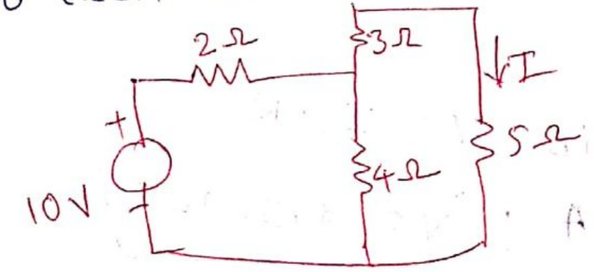
In any linear bilateral network, if a single voltage source Va in branch 'a' produces a current I_b in branch 'b', then if the voltage source Va is removed and inserted in branch 'b' will produce a current I_b in branch 'a'. The ratio of response to excitation is same for the two condition mentioned. This is called reciprocity theorem.





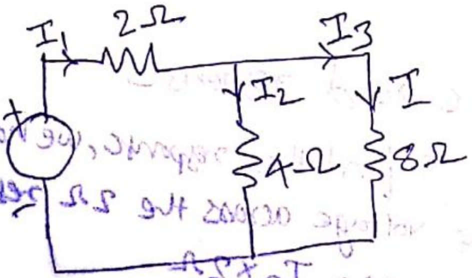
The application of voltage V across AA' produces current I at BB'. Now if the positions of the source and responses are Interchanged, by connecting the voltage source across BB', the resultant current I will be at terminals AA'. According to the reciprocity theorem, the ratio of response to excitation is the same in both cases.

Q



NOTE: For Reciprocity, consider resistance from response side only.

$\frac{8\times4}{8+4}+2$



$$I = I_T\frac{4}{8+4}$$

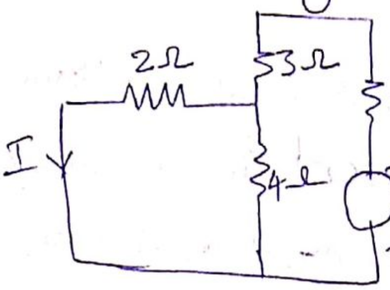
$$I_T = \frac{10}{R_T}$$

$$R_T = \frac{8\times4}{8+4}+2 = \frac{32}{12}+2 = 4.67$$

$$I_T = \frac{10}{4.67} = 2.14A, \quad I = 2.14\times\frac{4}{12} = \frac{8.56}{12} = 0.71A$$

$$\boxed{I = 0.71A}$$

we interchange the source and response



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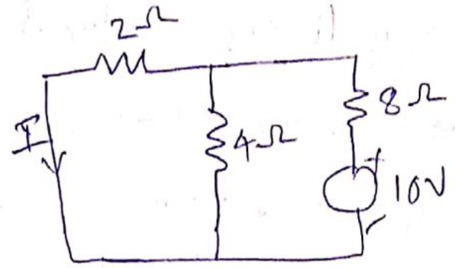
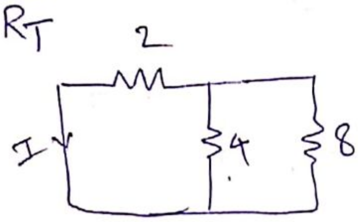


$I = I_T \times \frac{4}{4+2}$;

$I_T = \frac{10}{R_T}$

$R_T =$



$9.33\Omega ; \frac{2\times4}{2+4} + 8 \cdot \frac{8}{6} + 8 = \frac{56}{6}$

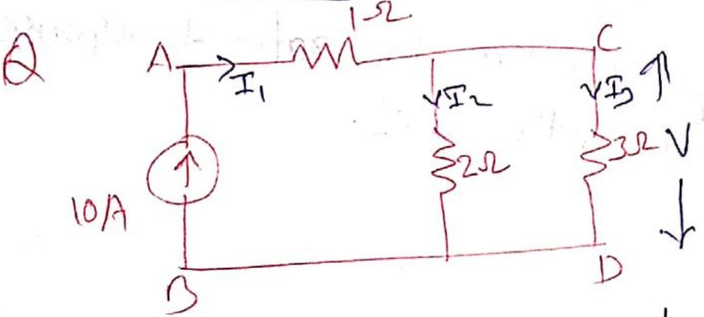
NOTE: consider resistance from response side

$I_T = \frac{10}{9.33} = 1.07A$

$I = 1.07\times\frac{4}{6} = 0.71A$

$\boxed{I = 0.71A}$

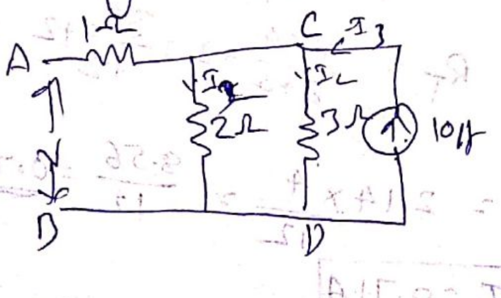
Q



Voltage V across 3Ω resistor is $V = I_3 \times R$

$I_3 = 10\times\frac{2}{2+3} = \frac{20}{5} = 4A$; $\boxed{V = 4\times3 = 12V}$.

we interchange the current source and response



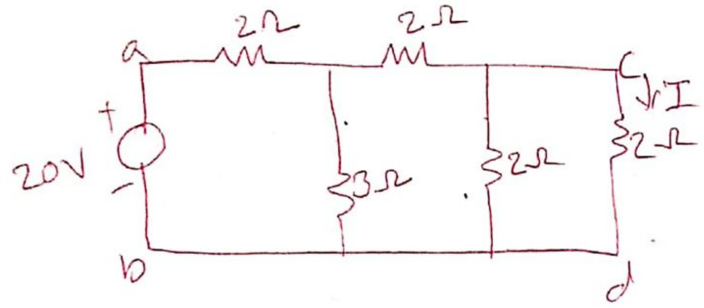
To find the response, we have to find the voltage across the 2Ω resistor.

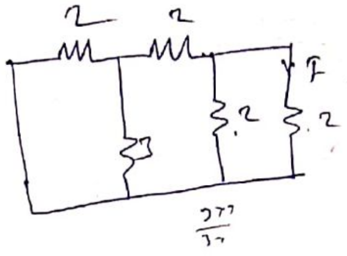
$V = I_2 \times 2\Omega$

$I_2 = 10\times\frac{3}{5} = 6A$

$\boxed{V = 6\times2 = 12V}$

Q3





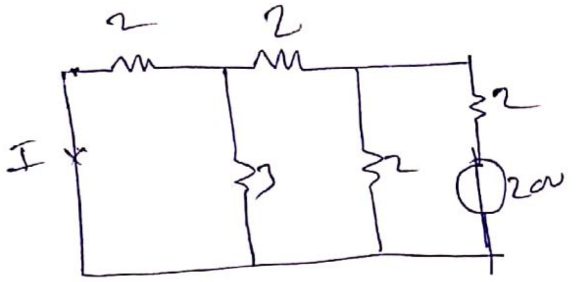
total resistance $R_T = 2 + [3 \| (2 + 2 \| 2)] = 2 + \frac{2 \times 2}{2+2} = \underline{3.5 \Omega}$

$I_T = \frac{20}{3.5} = 5.71$ A

$I = I_T \times \frac{2}{4} = 5.71 \times \frac{2}{4} = 2.855$

Answers

$\boxed{I = 1.43A}$



$R = [((2 \| 3) + 2) \| 2] + 2$

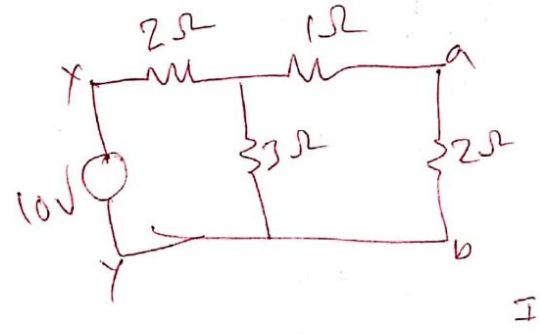
$\frac{6}{5} + \frac{2}{1} = \frac{6+10}{5} = \frac{\frac{16}{5} \times 2}{\frac{16}{5} + 2} = \frac{\frac{32}{5}}{\frac{16+10}{5}} = \frac{\frac{32}{5}}{\frac{26}{5}}$

$R = \frac{32}{5} \times \frac{5}{26} = 1.23$

$I_T = \frac{20}{1.23} = 16.26A$

$I = 16.26 \times \frac{3}{5} =$

Q



I = 1.43 A

Tellegen's Theorem

In an arbitrary lumped network, the algebraic sum of the powers in all branches at any instant is zero.

(or)

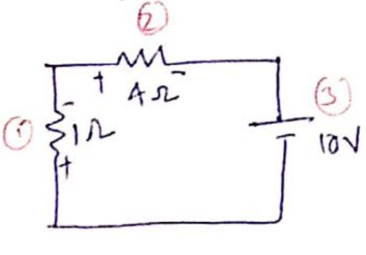
The algebraic sum of the powers delivered by all sources is equal to the algebraic sum of the powers absorbed by all elements.

NOTE: All branch currents and voltages in that network must satisfy Kirchhoff's Laws.

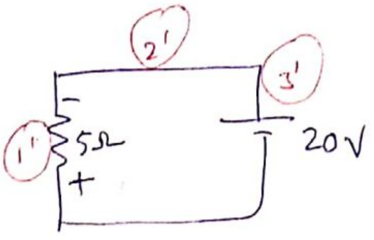
Consider two networks N_1 and N_2 having the same graph with different types of elements between the corresponding nodes.

$$\sum_{K=1}^{b} v_{1K} i_{2K} = 0 \quad \text{and} \quad \sum_{K=1}^{b} v_{2K} . i_{1K} = 0$$

Proof:



(a)



(b)

In fig(a) $i_1 = i_2 = i_3 = \frac{10}{5} = 2A$

$v_1 = i_1 \times 1\Omega = -2V$

$v_2 = i_2 \times 4 = -8V$

$v_3 =$ [illegible] $\times = 10V$

Now $\sum_{K=1}^{3} v_K i'_K = v_1 i'_1 + v_2 i'_2 + v_3 i'_3$

$= (-2)4 + (-8)(4) + 10 \times 4$

$= -8 - 32 + 40 = 0$

IIly $\sum_{K=1}^{3} v_K i_K = v_1 i_1 + v_2 i_2 + v_3 i_3$

$= -2\times2 + -8\times2 + 10\times2$

$= -4 - 16 + 20 = 0$

In fig(b)

$i'_1 = i'_2 = i'_3 = \frac{20}{5} = 4A$

$v'_1 = i'_1 \times 5 = -20V$

$v'_2 = i'_2 \times 0 = 0V$

$v'_3 =$ [illegible] $= 20V$

$\sum_{K=1}^{3} v'_K i_K = v'_1 i_1 + v'_2 i_2 + v'_3 i_3$

$= -20\times2 + 0 + 20\times2$

$= -40 + 40 = 0$

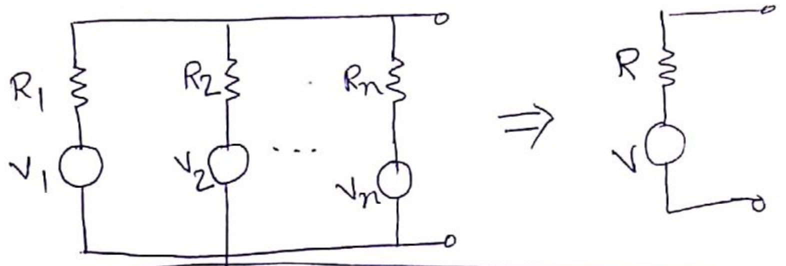
$\sum_{K=1}^{3} v'_K i'_K = v'_1 i'_1 + v'_2 i'_2 + v'_3 i'_3$

$= -20\times4 + 0 + 20\times4$

$= -80 + 80 = 0$

Millman's Theorem

In any network, if the voltage sources $V_1, V_2, \cdots V_n$ in series with internal resistances $R_1, R_2, \cdots R_n$ respectively, are in parallel then these sources may be replaced by a single voltage source V in series with R.



where
$$\boxed{V = \frac{V_1G_1 + V_2G_2 + \cdots + V_nG_n}{G_1 + G_2 + \cdots + G_n}}$$

G_n is the conductance of the n^{th} branch.

$$\boxed{R = \frac{1}{G_1 + G_2 + \cdots + G_n}}$$

$$G_1 = \frac{1}{R_1}, \ G_2 = \frac{1}{R_2}, \ G_n = \frac{1}{R_n}$$

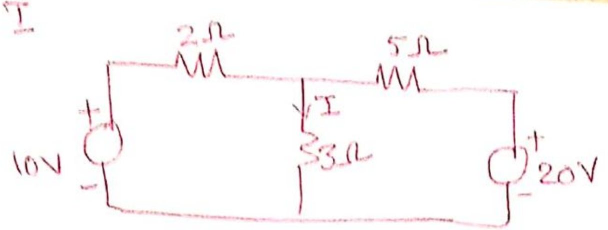
A similar theorem can be stated for n current sources having internal conductances which can be replaced by a single current source I in parallel with an equivalent conductance

where
$$\boxed{I = \frac{I_1R_1 + I_2R_2 + \cdots + I_nR_n}{R_1 + R_2 + \cdots + R_n}}$$

$$R_1 = \frac{1}{G_1}, \ R_2 = \frac{1}{G_2} \cdots R_n = \frac{1}{G_n}$$

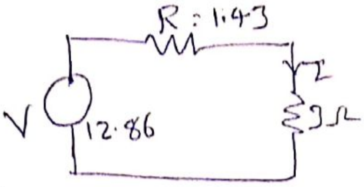
$$\boxed{G = \frac{1}{R_1 + R_2 + \cdots R_n}}$$

Q. Calculate the current I



$$V = \frac{V_1G_1 + V_2G_2}{G_1+G_2} = \frac{10\times\frac{1}{2} + 20\times\frac{1}{5}}{\frac{1}{2}+\frac{1}{5}} = 12.86\text{V}$$

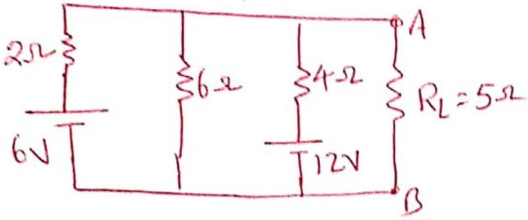
$$R = \frac{1}{G_1+G_2} = \frac{1}{\frac{1}{2}+\frac{1}{5}} = 1.43\Omega$$



current passing through the 3Ω resistor is

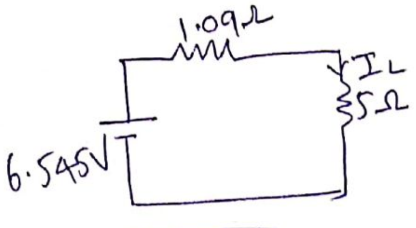
$$\boxed{I = \frac{12.86}{3+1.43} = 2.9A}$$

Q.



$$V_{AB} = \frac{\frac{6}{2}+\frac{0}{6}+\frac{12}{4}}{\frac{1}{2}+\frac{1}{6}+\frac{1}{4}} = \frac{\frac{6}{2}+\frac{12}{4}}{\frac{6+2+3}{12}} = \frac{\frac{12+12}{4}}{\frac{11}{12}} = \frac{24}{4}\times\frac{12}{11} = \frac{72}{11} = 6.545V$$

$$R = \frac{1}{\frac{1}{2}+\frac{1}{6}+\frac{1}{4}} = \frac{1}{\frac{6+2+3}{12}} = \frac{12}{11} = 1.09\Omega$$

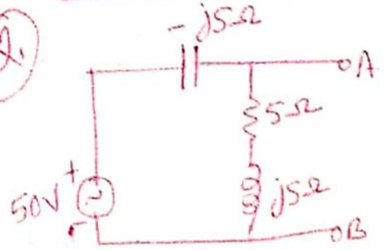


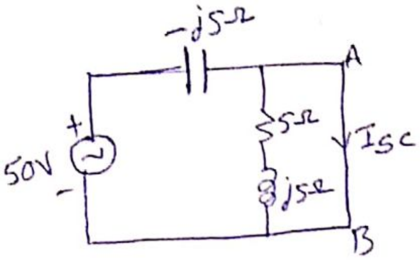
$$I_L = \frac{6.545}{6.09} = 1.07$$

$$\boxed{I_L = 1.07A}$$

Norton's theorem using A.C Source

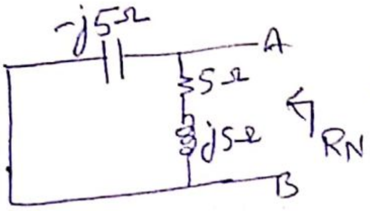
Q1.



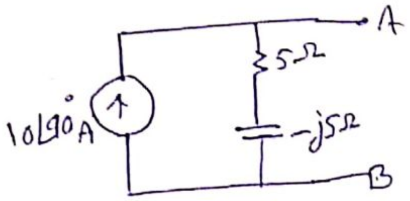


current doesn't flow through 5+j5Ω because a short ckt is there.

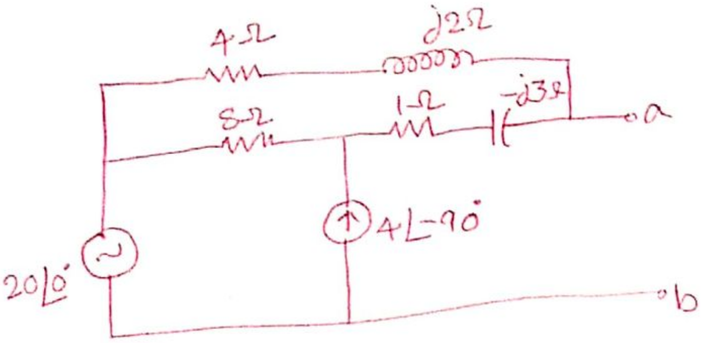
$$I_{sc} = \frac{50}{-j5} = 10\angle 90^\circ$$



$$R_N = \frac{-j5\,(5+j5)}{-j5+5+j5} = \frac{-j25+25}{5} = 5-j5$$

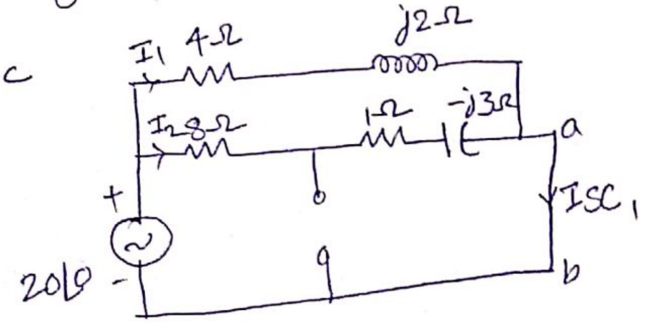


Q2



using superposition theorem, consider voltage source to find

Isc

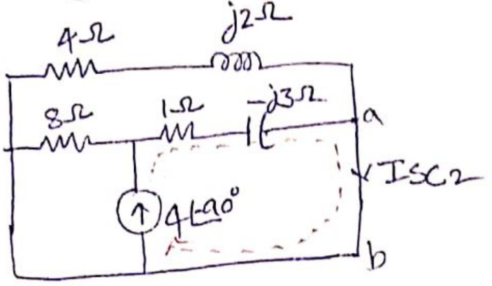


$$I_{sc_1} = I_1 + I_2$$

$$= \frac{20}{4+j2} + \frac{20}{9-j3}$$

$$I_{sc_1} = \frac{20(9-j3)+20(4+j2)}{(4+j2)(9-j3)} = \frac{180-j60+80+j40}{36-j12+j18+6} = \frac{260-j20}{42+j6}$$

Using current source $4\angle 90^\circ$

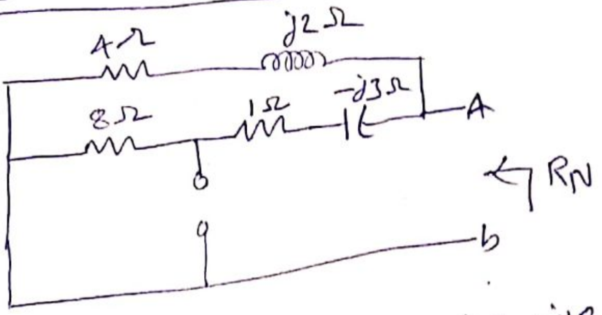


$$I_{SC2} = \frac{4\angle 90^\circ}{1-j3}$$

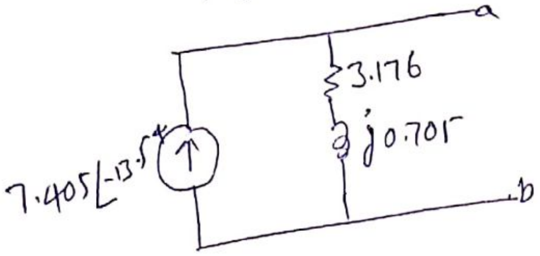
$I_{SC} = I_{SC_1} + I_{SC_2} = 6 - j1.334 + 1.2 - j0.4 = \underline{7.2 - j1.734} = \underline{7.405\angle -13.54}$

$$\boxed{I_{SC} = 7.405\angle -13.54}$$

$R_N =$

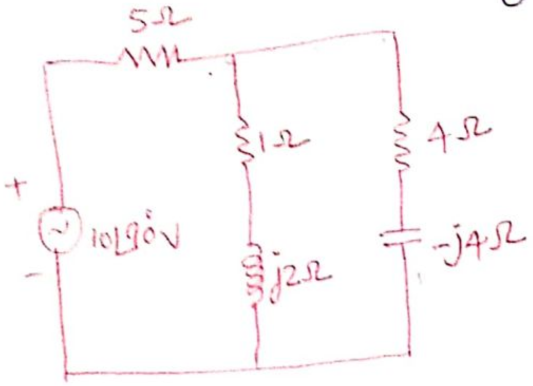


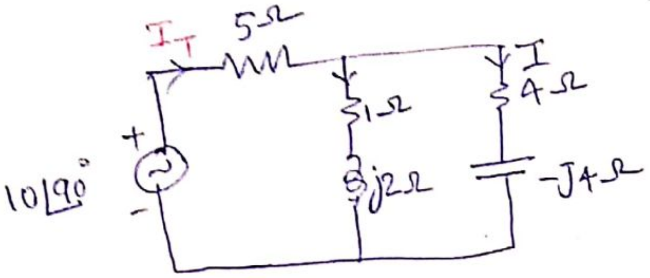
$$\frac{(4+j2)(9-j3)}{4+j2+9-j3} = \frac{36-j12+j18+6}{13-j1} = \frac{42+j6}{13-j1} = \underline{3.176+j0.705}$$



Reciprocity theorem using A.C. source

In case of a.c. source we are using Impedance instead of resistance and the voltage source is in its phasor form.





fig(a)

Impedance of the ckt across the voltage 10∠90°V is

$$Z_{in} = \frac{(4-j4)(1+j2)}{4-j4+1+j2} + 5$$

$$= \frac{(4-j4)(1+j2)+5(5-j2)}{5-j2}$$

$$= \frac{4+j8-j4+8+25-j10}{5-j2}$$

$$= \frac{37-j6}{5-j2} = \frac{37.48\angle -9.21^\circ}{5.385\angle -21.8^\circ}$$

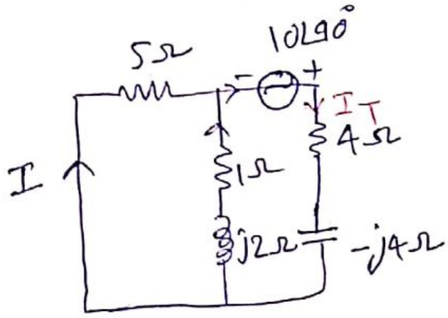
$$= 6.96\angle -9.21 - (-21.8)$$

$$\boxed{Z_{in} = 6.96\angle 12.59^\circ}$$

$$I_T = \frac{10\angle 90^\circ}{6.96\angle 12.59} = 1.437\angle 77.41$$

$$I = I_T \times \frac{1+j2}{(1+j2)+(4-j4)} = \frac{1.437\angle 77.41 \times 2.236\angle 63.43}{5.385\angle -21.8}$$

$$\boxed{I = 0.597\angle 77.41+63.43-(-21.8) = 0.597\angle 162.64}$$



fig(b)

$$Z_{in} = \frac{5(1+j2)}{5+1+j2} + (4-j4) = \frac{5+j10}{6+j2} + 4-j4$$

$$= \frac{5+j10+(4-j4)(6+j2)}{6+j2}$$

$$= \frac{5+j10+24+j8-j24+8}{6+j2} = \frac{37-j6}{6+j2}$$

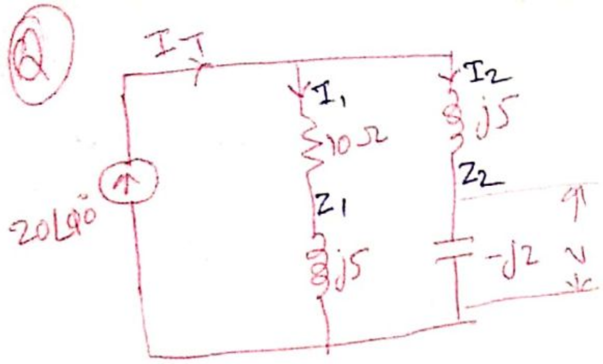
$$\boxed{Z_{in} = \frac{37.48\angle -9.21}{6.325\angle 18.43} = 5.926\angle -27.64^\circ}$$

$$I_T = \frac{10\angle 90^\circ}{5.926\angle -27.64} = 1.687\angle 117.64^\circ$$

$$I = I_T \times \frac{1+j2}{5+1+j2}$$

$$= \frac{1.687\angle 117.64 \times 2.236\angle 63.43^\circ}{6.325\angle 18.43}$$

$$\boxed{I = 0.597\angle 162.64^\circ}$$



$$I_2 = I_T \frac{(10+j5)\cancel{(j5-j2)}}{10+j5+j5-j2}$$

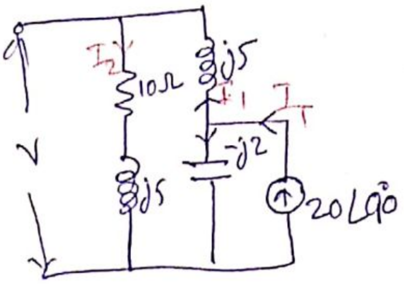
$$= I_T \frac{10+j5}{10+j8}$$

$$= \frac{20\angle 90^\circ \times 11.18\angle 26.56^\circ}{12.8\angle 38.65^\circ} = 17.46\angle 77.91$$

$$\boxed{I_2 = 17.46\angle 77.91}$$

$$V = I_2 \times (-j2) = 17.46\angle 77.91 \times 2\angle -90^\circ$$

$$\boxed{V = 34.92\angle -12.09^\circ}$$



$$I_2 = I_T \times \frac{-j2}{10+j5-j2+j5}$$

$$= I_T \times \frac{-j2}{10+j8}$$

$$= \frac{20\angle 90^\circ \times 2\angle -90^\circ}{12.80\angle 38.65^\circ} = 3.125\angle -38.65$$

$$V = I_2 \times (10+j5)$$

$$= 3.125\angle -38.65 \times 11.18\angle 26.56^\circ$$

$$\boxed{V = 34.93\angle -12.09^\circ}$$

Tellegen's Theorem using A.C. source

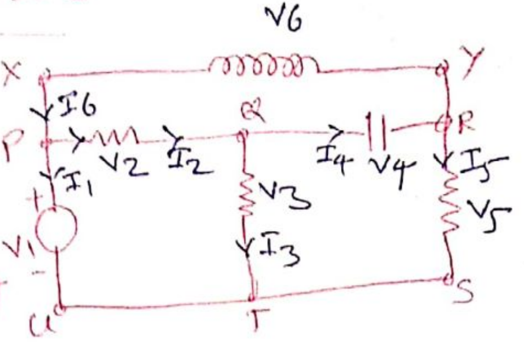
verify Tellegen's theorem for network

$v_1=4,\ v_2=2,\ v_3=2$

$v_4=3,\ v_5=-1,\ v_6=-5$

$i_1=2,\ i_2=2,\ i_3=4$

$i_4=-2,\ i_5=-6,\ i_6=4$



KVL - Loop PQTU

$-V_1+V_2+V_3=0$

$-4+2+2=0$

Loop PXYR $= V_2+V_4+V_6=0$

$2+3-5=0$

Loop QRST $V_4+V_5-V_3=0$

$3-1-2=0$

KCL. node P $I_1+I_2+I_6 = 2+2-4=0$

node Q $I_3+I_4-I_2= 4-2-2=0$

node R $I_5+I_6-I_4=0 = -6+4+2=0$

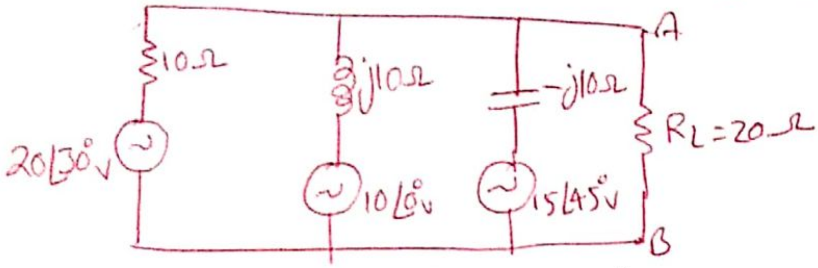
$$\sum_{K=1}^{6} V_K i_k=0$$

$V_1I_1+V_2i_2+V_3i_3+V_4i_4+V_5i_5+V_6i_6=0$

$4\times2+2\times2+2\times4+3\times(-2)+(-1)(-6)+(-5)\times4=0$

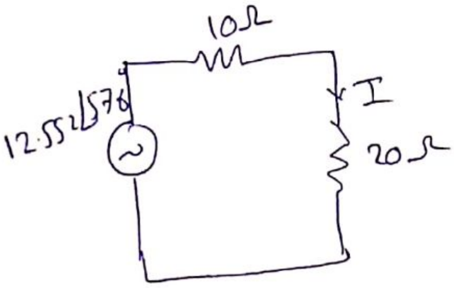
$8+4+8+6+6-20=0$

Millman's theorem using A.C.source



$$V_{AB} = \frac{\frac{20\angle 30^\circ}{10} + \frac{10\angle 0^\circ}{j10} + \frac{15\angle 45^\circ}{-j10}}{\frac{1}{10} + \frac{1}{j10} + \frac{1}{-j10}}$$

$$\boxed{V_{AB} = 12.552\angle 57.66^\circ}$$



$$R_{AB} = \frac{1}{\frac{1}{10} + \cancel{\frac{1}{j10}} - \cancel{\frac{1}{j10}}} = 10\Omega$$

$$I = \frac{12.552\angle 57.66}{30}$$

$$\boxed{I = 0.418\angle 57.66}$$

Compensation Theorem

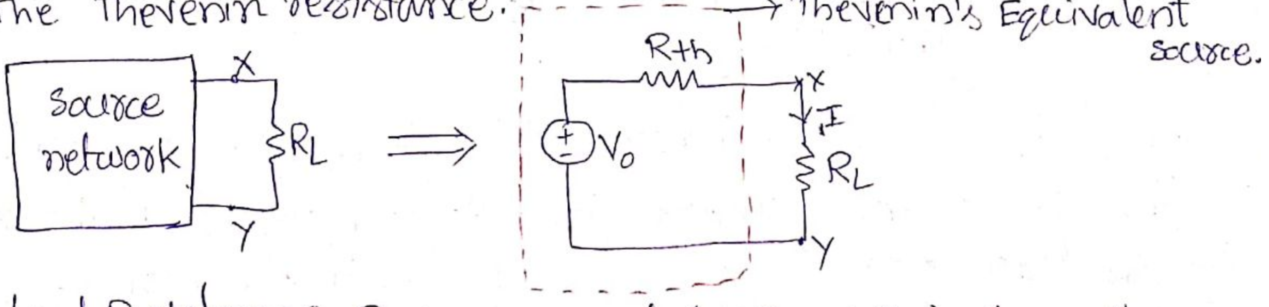
If a circuit has certain distribution of voltages & currents across & through various branches and due to some reason if the resistance of one of the branch changes, the voltages & currents in other branch affect.

To balance this affect the voltage source should be changed accordingly.

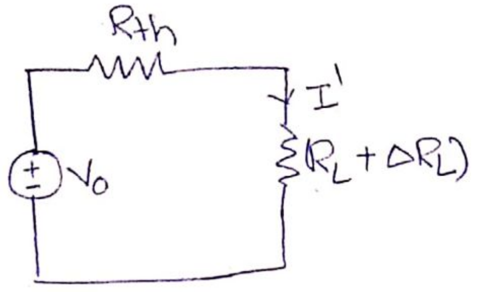
Statement: A linear time invarient circuit having a voltage source 'V' with internal resistance R_{th} delivers a current I to load R_b. If the resistance R_b changes to $(R+\Delta R)$, the change in current 'ΔI' can be found by replacing the voltage source by it's internal resistance & placing a compensation voltage source of magnitude $V_c = I.\Delta R$ in series with the resistance $(R+\Delta R)$ and it's polarity opposes the flow of current I".

Explanation:

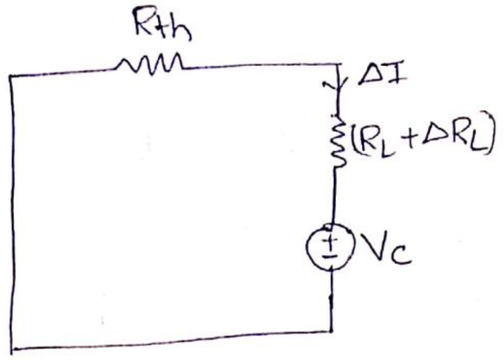
Let us assume a load of R_L be connected to a D.C. source network whose Thevenin's equivalent gives V_0 as the Thevenin voltage and R_{th} as the Thevenin resistance.



Let the Load Resistance R_L be changed to $(R_L+\Delta R_L)$. Since the rest of the circuit remains same, the thevenin equivalent network remains the same as



Thevenin equivalent of source.



Source network with source replaced by it's internal resistance.

Here $I' = \frac{V_0}{R_{th}+(R_L+\Delta R_L)}$, $I = \frac{V_0}{R_{th}+R_L}$

The change of current being as $\Delta I = I' - I$

$$\Delta I = \frac{V_0}{R_{th}+(R_L+\Delta R_L)} - \frac{V_0}{R_{th}+R_L}$$

$$= \frac{V_0[R_{th}+R_L] - V_0[R_{th}+R_L+\Delta R_L]}{[R_{th}+(R_L+\Delta R_L)][R_{th}+R_L]}$$

$$= \frac{V_0[\cancel{R_{th}}+\cancel{R_L}-\cancel{R_{th}}-\cancel{R_L}-\Delta R_L]}{(R_{th}+R_L)[R_{th}+R_L+\Delta R_L]}$$

$$= \frac{V_0}{R_{th}+R_L}\left[\frac{-\Delta R_L}{R_{th}+R_L+\Delta R_L}\right]$$

$$\Delta I = -I\frac{\Delta R_L}{R_{th}+R_L+\Delta R_L}$$

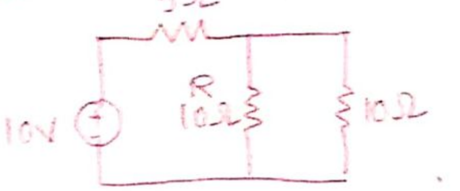
$\boxed{\Delta I = \frac{-V_c}{R_{th}+R_L+\Delta R_L}}$ $\boxed{V_c = I\Delta R_L}$ is compensation voltage.

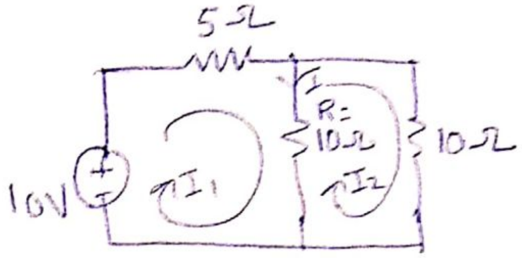
Applications This theorem is particularly useful in determing the incremental changes in voltages & currents in the branches of a circuit due to a change in resistance in one branch.

Limitations: Not applicable to the circuit consisting of only dependent sources.

→ Not applicable to the non-linear circuits i.e. circuits consisting of non-linear elements like, diode, transistor etc.

Q1. In the circuit, the resistance R is changed from 10Ω to 5Ω. Verify the compensation theorem.





Applying KVL

$$\begin{bmatrix} Z_{11} & -Z_{12} \\ -Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$$\begin{bmatrix} 15 & -10 \\ -10 & 20 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 10 \\ 0 \end{bmatrix}$$

$15I_1 - 10I_2 = 10$ — ①

$-10I_1 + 20I_2 = 0$ — ②

Multiplying ① by 2

$30I_1 - \cancel{20I_2} = 20$

$-10I_1 + \cancel{20I_2} = 0$

$20I_1 = 20$

$\boxed{I_1 = 1A}$

Substitute in ①

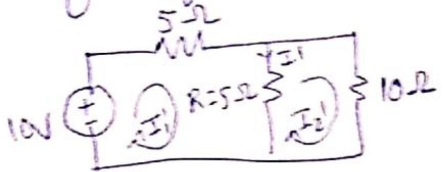
$15 - 10I_2 = 10$

$-10I_2 = -5$

$\boxed{I_2 = 0.5A}$

$\boxed{I = I_1 - I_2 = 0.5A}$

when the resistance R is changed from 10Ω to 5Ω



Applying KVL

$$\begin{bmatrix} 10 & -5 \\ -5 & 15 \end{bmatrix} \begin{bmatrix} I_1' \\ I_2' \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$10I_1' - 5I_2' = 10$ — ①

$-5I_1' + 15I_2' = 0$ — ②

Multiplying ① with 3

$30I_1' - \cancel{15I_2'} = 30$

$-5I_1' + \cancel{15I_2'} = 0$

$25I_1' = 30 \quad I_1' = \frac{\cancel{30}^{6}}{\cancel{25}_{5}}$

$\boxed{I_1' = 1.2A}$

Substitute in ①

$12 - 5I_2' = 10$

$-5I_2' = -2 \quad I_2' = \frac{2}{5}$

$\boxed{I_2' = 0.4A}$

$I' = I_1' - I_2' = 1.2 - 0.4 = 0.8$

$\boxed{I' = 0.8A}$

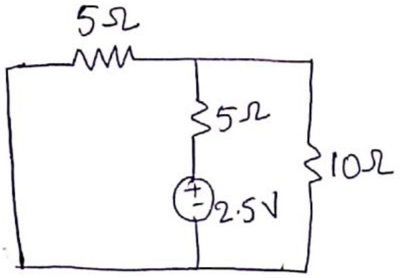
Change in current $\Delta I = I' - I = 0.8 - 0.5 = 0.3A$

$\boxed{\Delta I = 0.3A}$

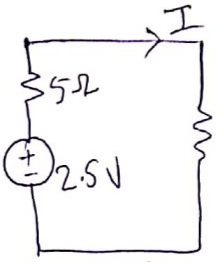
Using compensation theorem

$$V_c = I\Delta R_L = \cancel{I\Delta R} = 0.5(10-5) = -2.5V$$

$$\boxed{V_c = -2.5V}$$



⇒



$$\frac{5\times10}{5+10} = \frac{50}{15} = \frac{10}{3}$$

The compensation voltage with a new circuit is shown above.

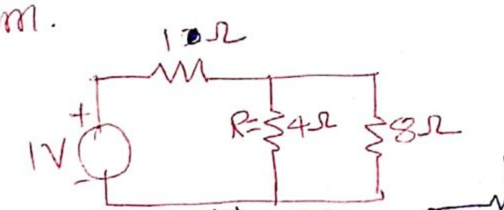
The current flowing in above ckt is change in current

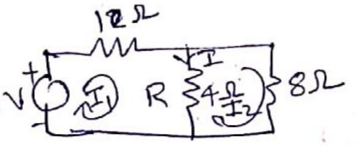
$$\Delta I = \frac{V_c}{R_{th}+R_L} = \frac{2.5}{5+\frac{10}{3}} = \frac{2.5}{\frac{15+10}{3}} = \frac{\cancel{2.5}^{1}}{1}\times\frac{3}{\cancel{25}_{10}} = 0.3A$$

$$\boxed{\Delta I = 0.3A}$$

Hence compensation theorem proved.

Q2. In the network, the resistance R is changed from 4Ω to 2Ω verify compensation theorem.





$5I_1 - 4I_2 = 1$ — ① ×4

$-4I_1 + 12I_2 = 0$ — ② ×5

$\cancel{20}I_1 - 16I_2 = 4$

$-\cancel{20}I_1 + 60I_2 = 0$

$44I_2 = 4$ $\quad\boxed{I_2 = \frac{1}{11} = 0.09A}$

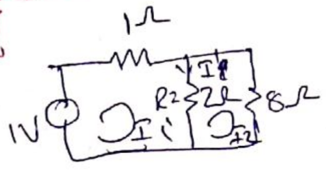
Substitute in ①

$5I_1 - 4(0.09) = 1 \quad I_1 = \frac{1.36}{5} = 0.272$

$\boxed{I_1 = 0.272A}$ $\quad I = I_1 - I_2 = 0.272 - 0.09$

$\boxed{I = 0.182A}$

change in resistance



$3I_1' - 2I_2' = 1$ — ① ×5

$-2I_1' + 10I_2' = 0$

$15I_1' - \cancel{10I_2'} = 5$

$-2I_1' + \cancel{10I_2'} = 0$

$13I_1' = 5$

$$\boxed{I_1' = \frac{5}{13} = 0.384}$$

Substitute in ②

$-0.769 + 10I_2' = 0$

$$\boxed{I_2' = \frac{0.769}{10} = 0.0769}$$

$I_0'' = I_1' - I_2' = 0.384 - 0.0769$

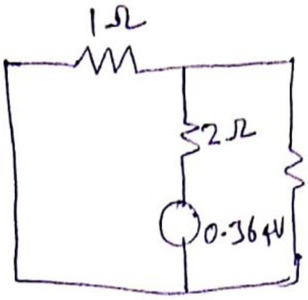
$\boxed{I_0' = 0.308A}$

Change in current $\Delta I = I' - I = 0.308 - 0.182 = 0.126$

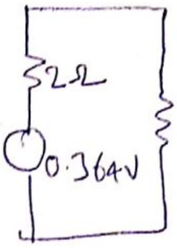
$\boxed{\Delta I = 0.126A}$

Compensation voltage $V_C = I\Delta R = 0.182 \times (-2) = 0.364$

$\boxed{V_C = -0.364V}$



$\Rightarrow$

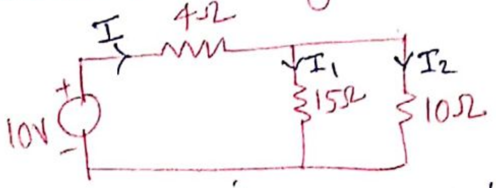


$\frac{1\times8}{1+8} = \frac{8}{9} = 0.88$

$$\Delta I = \frac{V_C}{R_{th}+R_L} = \frac{0.364}{2+0.88} = \frac{0.364}{2.88} = 0.126A$$

$\boxed{\Delta I = 0.126A}$ Hence compensation theorem proved.

Q3. In the circuit shown, 10Ω resistor is changed to 15Ω. Find current through 4Ω resistance before & after change in resistance. Determine change in current ΔI through 4Ω resistance.

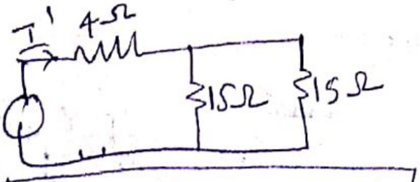


current through 4Ω resistance before change in resistance.

$I = \frac{V}{R_T}$ $\quad R_T = \frac{15\times10}{15} = \frac{150}{15} = 10\Omega$

$\boxed{I = \frac{10}{10} = 1A}$

current through 4Ω resistance after change in resistance



$R_T = \frac{15\times15}{30} = \frac{225}{30} + 4 = 11.5\Omega$

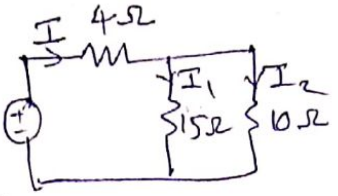
$\boxed{I' = \frac{10}{11.5} = 0.869A}$

change in current $\Delta I = I' - I = 0.869 - 1 = -0.13$

$\Delta I = -0.13$ $\quad \boxed{\Delta I = -0.13A}$

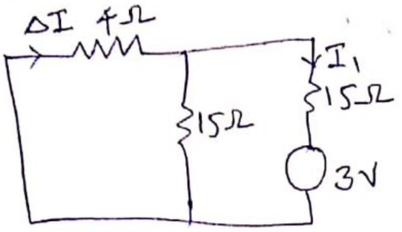
using compensation theorem

$V_c = I_2 \Delta R =$



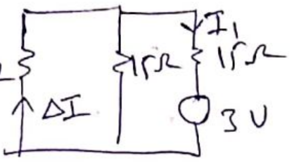
$I_2 = I \times \frac{15}{25} = \frac{1\times 15}{25} = \frac{3}{5} = 0.6A$

$\boxed{V_c = 0.6\times 5 = 3V}$



$I_1 = \frac{3}{\cancel{15}R_T} = \quad R_T = \frac{15\times 4}{19}+15 = \frac{60}{19}+15 = 18.15$

$\boxed{I_1 = \frac{3}{18.15} = 0.165A}$

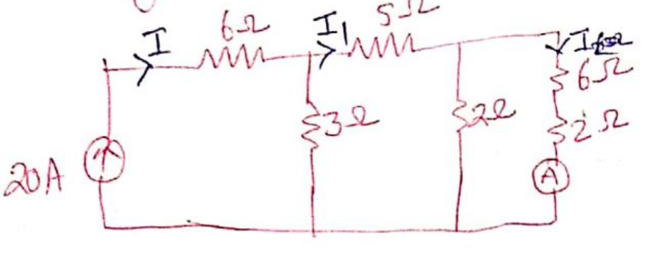


$\Delta I = -I_1 \frac{15}{19} = -0.165\times\frac{15}{19}$

$\Delta I = 0.130 \quad \boxed{\Delta I = -0.130A}$

Hence compensation theorem proved

Q4. Using the compensation theorem, determine the Ammeter reading when it is connected to the 6Ω resistance as shown in fig. The internal resistance of ammeter is 2Ω.

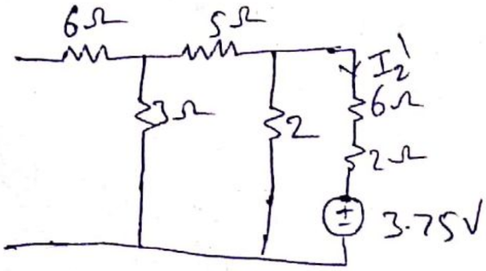


$I_2 = I_1 \times \frac{2}{8}$

$I_1 = I\times\frac{3}{8} = 20\times\frac{3}{8} = 7.5A$

$\boxed{I_2 = \frac{7.5\times 2}{8} = 1.87A}$

$\boxed{V_c = I_2 \Delta R = 1.87\times 2 = 3.75V}$



$R_T = \frac{3\times 5}{3+5} \| 2 + 6 + 2$

$= \frac{\frac{15}{8}\times 2}{\frac{15}{8}+2} + 8 = \frac{\frac{30}{8}}{\frac{15+16}{8}} + 8 = \frac{30}{8}\times\frac{8}{31}+8$

$R_T = \frac{30}{31}+8 = 8.96; \; I_2' = \frac{3.75}{8.96} = 0.41A$

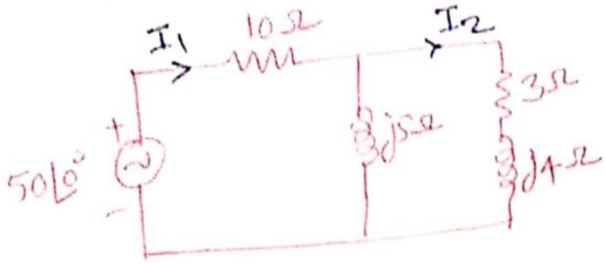
Ammeter reading is $I_2 - I_2' = 1.87 - 0.41 = 1.46A$

Compensation theorem using A.C. Source

In the circuit given find

(a) current in 10Ω resistance

(b) The (3+j4)Ω resistance is changed to (4+j4)Ω. Find the new current in 10Ω resistance



(a) current through 10Ω resistance is $I_1 = \frac{V}{\cancel{R}Z_T}$

$$\cancel{R}Z_T = 10 + \frac{(3+j4)j5}{3+j4+j5} = 10 + \frac{j15+20}{3+j9} = \frac{10(3+j9)+20+j15}{3+j9}$$

$$R_T = \frac{30+j90+j15-20}{3+j9} = \frac{10+j115}{3+j9} =$$

$$\boxed{Z_T = 11.1\angle 13^\circ}$$

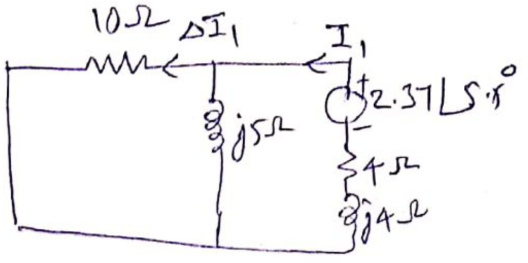
$$\boxed{I_1 = \frac{50\angle 0}{11.1\angle 13} = 4.5\angle -13^\circ}$$

(b) The initial current in (3+j4)Ω branch is

$$I_2 = I_1 \times \frac{j5}{3+j9} = 4.5\angle -13 \times \frac{j5}{3+j9} = \underline{\underline{2.37\angle 5.5^\circ}}$$

Compensation voltage $V_C = I_2 \Delta Z = 2.37\angle 5.5\,(4+j4-3-j4)$

$$\boxed{V_C = 2.37\angle 5.5^\circ \text{ V}}$$



$$\Delta I_1 = I_1 \times \frac{j5}{10+j5}$$

$$I_1 = \frac{2.37\angle 5.5^\circ}{4+j4+\frac{10\times j5}{10+j5}} = \frac{2.37\angle 5.5^\circ}{\frac{(4+j4)(10+j5)+j50}{10+j5}} = \frac{2.37\angle 5.5^\circ}{\frac{40+j20+j40-20+j50}{10+j5}}$$

$$= \frac{2.37\angle 5.5}{\frac{20+j110}{10+j5}} =$$

$$\Delta I_1 = 0.106\angle 15.8^\circ$$

$$I_1' = I_1 - \Delta I_1 = 4.5\angle -13^\circ - 0.106\angle 15.8^\circ$$

$$\boxed{I_1' = 4.39\angle -12.93}$$

Transient Analysis:

Transient means sudden changes.

In RLC circuits transient currents are produced due to sudden ON & OFF switching actions from the supply voltage.

→ The transient currents are not driven by any part of the applied voltage but are entirely associated with the changes in the stored energy in Inductor (L) (or) capacitor (C).

NOTE: There are no transients in pure resistors.

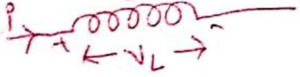
→ The behaviour of the voltage (or) current when it is changed from one state to another is called the "transient state".

→ Response of the storage elements changes with time by delivering their energy to the resistors, gets saturated after some time, and is refered as "transient response".

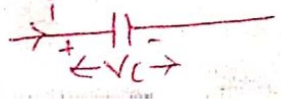
→ When we consider sources acting on a circuit, the response depends on the nature of the source or sources. This response is called "forced response."

→ When we consider a differential equation, the complete solution consists of two parts - the complementary function and the particular solution. The complementary function dies out after short interval, and is referred to as the transient response (or) source free response. The particular solution is the steady state response or the forced response.

$$V_R = iR, \qquad i = \frac{V}{R}$$

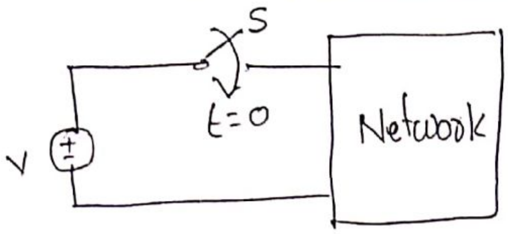


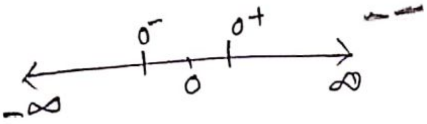
$$V_L = L.\frac{di}{dt} \qquad i = \frac{1}{L}\int_{-\infty}^{t} V_L dt$$



$$V_C = \frac{1}{C}\int_{-\infty}^{t} i\,dt \qquad i = C.\frac{dv}{dt}$$

Transients





Behaviour of L and C elements

$t = 0$ switch is closed just

$t = 0^+$ After some time (switch closed)

$t = \infty$ steady state (Long time)

$t = 0^-$ switch is open

Inductor, $Z_L = SL = j\omega L$

Capacitor, $Z_C = \frac{1}{j\omega C} = \frac{1}{SC}$

For $Z_L = SL\,\Omega$, $Z_C = \frac{1}{SC}\,\Omega$

$t = 0^+ \Rightarrow S = \infty \Rightarrow Z_L = \infty \Rightarrow$ L is open ckt

$\Rightarrow Z_c = 0 \Rightarrow$ C is short ckt

$t = \infty \Rightarrow S = 0 \Rightarrow Z_L = 0 \Rightarrow$ L is short ckt

$Z_L = \infty \Rightarrow$ C is open ckt

The steady state (or) Drunked state:

Whenever the independent source is connected to the Network for a long time (up t 5τ) then the network said to be in the steady state.

→ In steady state the energy stored in the memory element is maximum & constant i.e., the energy stored in L & C elements is maximum and constant.

i.e., $\frac{1}{2}Li_L^2$: max & constant i_L

Therefore in steady state inductor acts as constant current source.

since $v_L = L\frac{di}{dt}$, $v_L = 0 \Rightarrow$ L - short ckt.

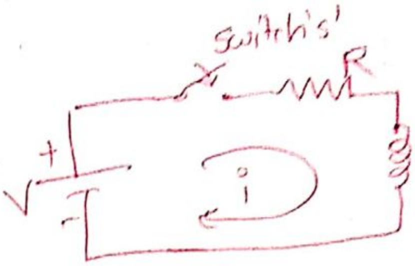
IIly $\frac{1}{2}Cv_c^2$: v_c is maximum and constant

Therefore in steady state capacitor acts as constant voltage source

since $i_c = C\frac{dv}{dt}$: $i_c = 0 \Rightarrow$ C - open ckt

NOTE: Transients are more serious for DC as compared to AC and the transient free condition is possible to a.c. excitation only.

DC Response of R-L circuit



The inductor in the circuit is initially uncharged and is in series with the R. When the switch 's' is closed, we can find the complete solution for the current.

→ Application of Kirchhoff's voltage law to the circuit results first order differential equation as

$$V = Ri + L\frac{di}{dt}$$

$$\boxed{\frac{di}{dt} + \frac{R}{L}i = \frac{V}{L}} \text{ — ①}$$

first order differential equation.

→ the current i is the solution to be found and V is the applied constant voltage.

→ The voltage V is applied to the circuit only when the switch 's' is closed.

~~Comparing ① with a non-homogeneous differential equation~~

$\frac{dy}{dx} + ay = b$ ~~$\boxed{\frac{dx}{dt} + Px = k}$ — ②~~ ~~$\boxed{x = i \,;\, P = \frac{R}{L} \,;\, k = \frac{V}{L}}$~~

~~whose solution is $x = e^{-Pt}\int ke^{+Pt}dt + ce^{-Pt}$ — ③~~

~~where 'c' is an arbitrary constant.~~

~~Ily current equation for ① is $i = e^{-\frac{R}{L}t}\int \frac{V}{L}e^{\frac{R}{L}t}dt + ce^{-\frac{R}{L}t}$~~

when above equation compare with the first order differential equation $\boxed{\frac{dy}{dx} + a.y = b}$ solution for that is $y = \frac{b}{a} + k.e^{-ax}$

Ily $\frac{di}{dt} + \frac{R}{L}i = \frac{V}{L}$ $y = i$; $a = \frac{R}{L}$; $b = \frac{V}{L}$; $x = t$

Hence solution for current is $i = \frac{b}{a} + ke^{-ax} = \frac{\frac{V}{L}}{\frac{R}{L}} + ke^{-\frac{R}{L}t}$

$$\boxed{i = \frac{V}{R} + k.e^{-\left(\frac{R}{L}\right)t}} \text{ — ②} \quad k - \text{constant}$$

complete solution = Particular integral + complementary function
(Steady state) + (transient)

$$i = \frac{V}{R} + K \cdot e^{-\left(\frac{R}{L}\right)t}$$

Let γ time constant $= \frac{L}{R}$ i.e. $\frac{R}{L} = \frac{1}{\gamma}$

$$\boxed{\text{transient response} = K \cdot e^{-\left(\frac{R}{L}\right)t} = K \cdot e^{-t/\gamma}} \quad -③$$

To find K from initial conditions at t(0⁻) (or) t(0⁺), S = closed

Inductor does not allow sudden changes in current so $i = 0$

$$\boxed{i = \frac{V}{R} + K \cdot e^{-t/\gamma}} \quad -④$$

$$0 = \frac{V}{R} + Ke^{0}$$

$$\boxed{K = -\frac{V}{R}}$$

substitute in ④ $\quad i = \frac{V}{R} - \frac{V}{R} e^{-t/\gamma}$

$$\boxed{i = \frac{V}{R}\left[1 - e^{-t/\gamma}\right]} \quad -⑤ ✓$$

$$i = \underbrace{\frac{V}{R}}_{\text{steady state part}} - \underbrace{\frac{V}{R} e^{-t/\gamma}}_{\text{transient part}}$$

. The transient part of the solution is $i = -\frac{V}{R} e^{-t/\gamma}$

one γ $\quad i(\gamma) = -\frac{V}{R} e^{-t/\gamma} = -\frac{V}{R} e^{-1} = -0.368 \frac{V}{R}$

Two γ $\quad i(2\gamma) = -\frac{V}{R} e^{-2\gamma/\gamma} = -\frac{V}{R} e^{-2} = -0.135 \frac{V}{R}$

$3\gamma = \quad i(3\gamma) = -\frac{V}{R} e^{-3\gamma/\gamma} = -\frac{V}{R} e^{-3} = -0.0498 \frac{V}{R}$

$4\gamma = \quad i(4\gamma) = -\frac{V}{R} e^{-4\gamma/\gamma} = -\frac{V}{R} e^{-4} =$

$5\gamma = \quad i(5\gamma) = -\frac{V}{R} e^{-5\gamma/\gamma} = -\frac{V}{R} e^{-5} = -0.0067 \frac{V}{R}$ ⑤

$2\gamma = 1 - 0.135 = 0.865$

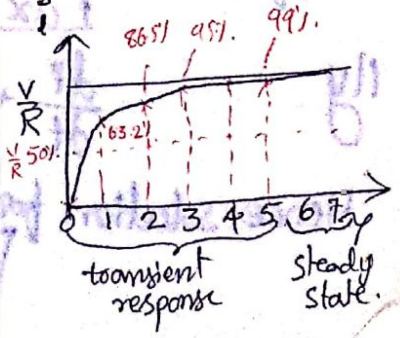
$3\gamma = 1 - 0.0498 = 0.9502$

$4\gamma =$

$5\gamma = 1 - 0.0067 = 0.99$

$\frac{V}{R} - 0.368 \frac{V}{R} =$

$\frac{V}{R}[1 - 0.368] = 0.632 \frac{V}{R}$



After 5 timeconstant, the transient part reaches more than 99% of its final value.

To find voltages across R and L

Voltage across the resistor is

$$V_R = Ri$$
$$= R \times \frac{V}{R}\left[1 - e^{-\left(\frac{R}{L}\right)t}\right]$$

$$\boxed{V_R = V\left[1 - e^{-t/\gamma}\right]}$$

$\gamma = \frac{L}{R}$, time constant for inductor.

voltage across the inductor is

$$V_L = L \cdot \frac{di}{dt} = L \times \frac{d\left(\frac{V}{R}(1 - e^{-t/\gamma})\right)}{dt}$$

$$= L \times \frac{d\ \frac{V}{R}(1 - e^{-t/\gamma})}{dt} = L \times \frac{V}{R}$$

$$= L \times \frac{R}{L}$$

$$\frac{d\frac{V}{R}}{dt} = 0$$
$$\frac{de^{-t/\gamma}}{dt} = -t\,e^{-t/\gamma}$$

$$V_L = L \times \left[\frac{d\frac{V}{R}}{dt} - \frac{V}{R}\frac{de^{-t/\gamma}}{dt}\right]$$

$$= L \times \left[0 - \frac{V}{R}\left(-\frac{1}{\gamma}e^{-t/\gamma}\right)\right] \qquad \gamma = \frac{L}{R}$$

$$= \cancel{L} \times \frac{V}{\cancel{R}} \times \frac{\cancel{R}}{\cancel{L}} e^{-t/\gamma}$$

$$\boxed{V_L = V e^{-t/\gamma}}$$

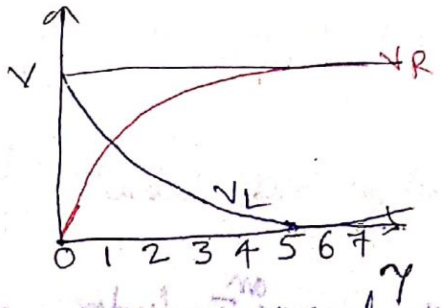
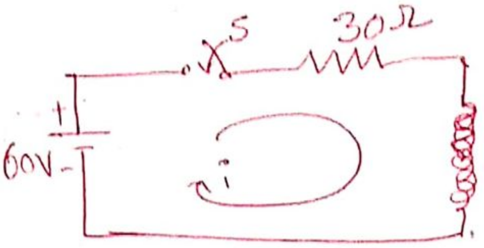


fig. voltage responses of Resistor and Inductor.

Q1. A Series RL circuit with $R = 30\Omega$ and $L = 15H$ has a constant voltage $V = 60V$ applied at $t=0$ as shown in figure. Determine the current i, the voltage across resistor and the voltage across the Inductor?



By applying KVL

$$60 = 30i + 15\frac{di}{dt}$$

$$15\frac{di}{dt} + 30i = 60 \Rightarrow \boxed{\frac{di}{dt} + 2i = 4}$$

General solution for a linear first order differential equation is

$$\frac{dy}{dx} + ay = b \quad ; \quad y = \frac{b}{a} + ke^{-ax}$$

Ily $\frac{di}{dt} + 2i = 4$; $i = y$; $x = t$; $a = 2$; $b = 4$

Solution for $i = \frac{4}{2} + ke^{-2t}$

$$\boxed{i = 2 + ke^{-2t}}$$

At $t=0$, the switch S is closed, since the inductor never allows sudden changes in currents. At $t = 0^+$ the current in the circuit is zero.

Therefore at $t = 0^+$, $i = 0$; $i = 2 + ke^{-2t}$

$$0 = 2 + ke^{-2(0)}$$

$$\boxed{K = -2}$$

Substitute K value in the current equation

$$\boxed{i = 2 + (-2)e^{-2t} = 2(1 - e^{-2t})A}$$

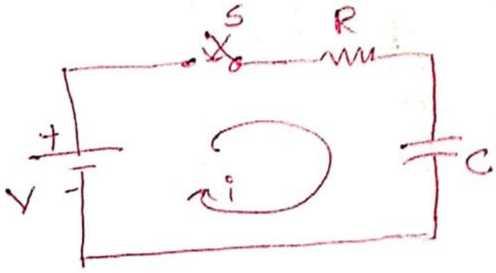
Voltage across Resistor $\boxed{V_R = iR = 2(1 - e^{-2t}) \times 30 = 60(1 - e^{-2t})V}$

voltage across an Inductor, $V_L = L\cdot\frac{di}{dt} = 15 \times \frac{d(2(1 - e^{-2t}))}{dt}$

$$V_L = 15 \times \left[\frac{d2}{dt} - \left[\frac{de^{-2t}}{dt}\right]\right] = 30\left[0 - (-2e^{-2t})\right] = 60e^{-2t}V$$

$$\boxed{V_L = 60e^{-2t}V}$$

DC Response of an R-C Circuit



Capacitor in the circuit is Initially uncharged and is in series with a R. When the switch S is closed at t=0, we can determine the complete solution for the current.

Applying KVL $\boxed{V = Ri + \frac{1}{C}\int i\,dt}$ —①

By differentating

$$0 = R\frac{di}{dt} + \frac{i}{C}$$

$$R\left(\frac{di}{dt} + \frac{i}{RC}\right) = 0$$

$\boxed{\frac{di}{dt} + \frac{i}{RC} = 0}$ —② It is linear first order differential equation with only the complementary function. The particular solution is zero.

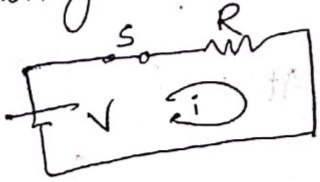
$\frac{dy}{dx} + ax = 0$

Solution is $y = k\cdot e^{-ax}$

The solution for this type of differential equation is

from ② $\boxed{i = ke^{-t/RC}}$ —③

To find k value, consider switch S is closed at t=0, capacitor acts as short circuit for sudden changes in voltages i.e. at $t=0^+$, the current $\boxed{i = \frac{V}{R}}$ —④



substitute ④ in ③ at t=0

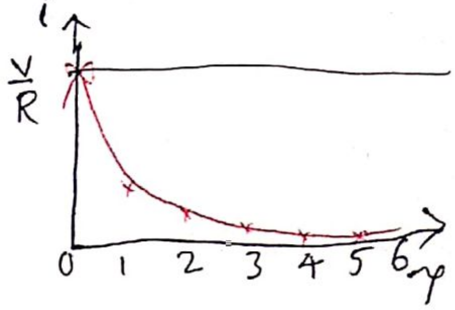
$\frac{V}{R} = ke^{0}$ ∴ $\boxed{k = \frac{V}{R}}$ —⑤

The current equation becomes $\boxed{i = \frac{V}{R}e^{-t/RC}}$

RC - Time constant of a capacitor $\tau = RC$

$$\boxed{i = \frac{V}{R}e^{-t/\tau}}$$

$i = \frac{V}{R} e^{-t/\tau}$



at $t=0$ $\quad i = \frac{V}{R}$

$t = 1\tau \quad i = \frac{V}{R} e^{-1} = 0.368 \frac{V}{R}$

$t = 2\tau \quad i = \frac{V}{R} e^{-2} = 0.135 \frac{V}{R}$

$t = 3\tau \quad i = \frac{V}{R} e^{-3} = 0.0498 \frac{V}{R}$

$t = 5\tau \quad i = \frac{V}{R} e^{-5} = 0.0067 \frac{V}{R}$

After 5 time constant the curve reaches to zero value.

Voltage across Resistor is

$$V_R = Ri = \cancel{R} \times \frac{V}{\cancel{R}} e^{-t/RC}$$

$$\boxed{V_R = Ve^{-t/RC} = Ve^{-t/\tau}}$$

lly voltage across the capacitor is

$$V_C = \frac{1}{C}\int i\,dt = \frac{1}{C}\int \frac{V}{R} e^{-t/RC} dt$$

$$= \frac{V}{RC}\int e^{-t/RC} dt = \frac{V}{\cancel{RC}} \times -RC\, e^{-t/RC} + K$$

$$\boxed{V_C = -Ve^{-t/RC} + K}$$

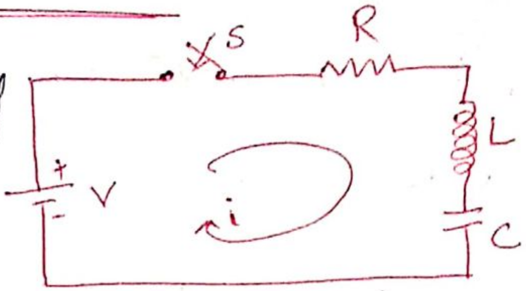
At $t=0$, voltage across capacitor is zero

$$0 = -Ve^0 + K \qquad \therefore K = V.$$

Therefore $V_C = -Ve^{-t/RC} + V$

$$\boxed{V_C = V(1 - e^{-t/RC})}$$

Transient Response of RLC series Circuit



The capacitor and inductor are initially uncharged and are in series with 'R'

When switch 's' is closed, using KVL

$$V = Ri + L\frac{di}{dt} + \frac{1}{C}\int i\,dt$$

Differentiating equation

$$0 = R\frac{di}{dt} + L\frac{d^2i}{dt^2} + \frac{1}{C}i$$

$$\boxed{\frac{d^2i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{1}{LC}i = 0} \quad \text{— ①}$$

second order linear differential equation.

with only complementary function.

→ The particular solution for the above equation is zero.

the second order differential equation as we knew that the second order

$$\frac{d^2y}{dx^2} + a\frac{dy}{dx} + b.y = 0$$

and solution $\boxed{y = K_1e^{\lambda_1 x} + K_2e^{\lambda_2 x}}$ — ② λ_1 & λ_2; are constants.

$$\lambda_1, \lambda_2 = -\frac{a}{2} \pm \sqrt{\left[\left(\frac{a}{2}\right)^2 - b\right]}$$

From ① $\frac{di}{dt} = D$, then $\boxed{D^2 + \frac{R}{L}D + \frac{1}{LC} = 0}$ — ③ , roots $\frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$a = 1$, $b = \frac{R}{L}$, $c = \frac{1}{LC}$

The roots are $D_1, D_2 = \frac{-\frac{R}{L} \pm \sqrt{\left(\frac{R}{L}\right)^2 - 4\times\frac{1}{LC}}}{2}$

$$\boxed{D_1, D_2 = \frac{-R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}}$$

Assume $\alpha = -\frac{R}{2L}$, $\beta = \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$; $D_1 = \alpha + \beta$, $D_2 = \underset{\text{real}}{\alpha} - \overset{\text{imaginary}}{\beta}$

α is damping co-efficient, which decides how well the circuit is able to damping the oscillations.

β is damping factor, whose value decides how well the circuit responses to the different excitations.

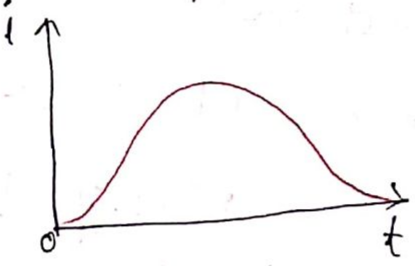
Here β may be +ve, -ve or zero.

case-1 β is +ve when $\left(\frac{R}{2L}\right)^2 > \frac{1}{LC}$, β = positive real.

Roots are real and unequal, it gives the over damped response.

The solution for i is

$$\boxed{i = C_1 e^{(\alpha+\beta)t} + C_2 e^{(\alpha-\beta)t}}$$ ✓



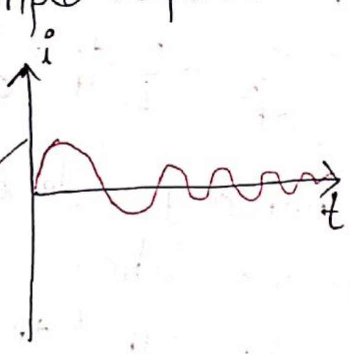
over damped responsonse

Case-2: β is -ve, $\left(\frac{R}{2L}\right)^2 < \frac{1}{LC}$, β = negative

Roots are complex conjugate, it give under damped response.

$D_1 = \alpha + j\beta$; $D_2 = \alpha - j\beta$

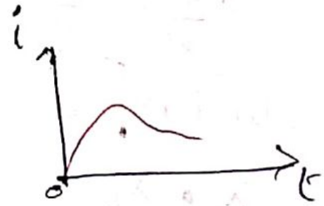
The solution is $\boxed{i = e^{\alpha t}\left(C_1\cos(\beta t) + C_2\sin(\beta t)\right)}$ ✓



Case-3: β is zero when $\left(\frac{R}{2L}\right)^2 = \frac{1}{LC}$, β = 0

critically damped response.

$$\boxed{i = e^{\alpha t}(C_1 + C_2 t)}$$ ✓



NOTE: For critical damping and over damping no oscillations are produced. For Under damped, oscillations are produced but die after some time.

→ Normally Inductor, capacitor induce the oscillations and resistor supress (or) damping out the oscillations.

→ RLC circuits useful in applications like tuning circuits, passive filters.

Initial conditions:

→ The conditions obtained in the network immediately after the closing (or) opening the switch, are known as "initial conditions" in the network.

→ Integration represents 'memory' in a circuit. The capacitor, Inductor are energy storage memory elements.

→ The capacitance. remember the charges which have been stored in it, and inductor remembers the flux(ϕ) linkages in it. The response of the elements depends on their stored values at $t=0$.

→ Derivative represents. Prediction of the future. If we know derivative of current in an inductance, we can calculate the current at feuture instant.

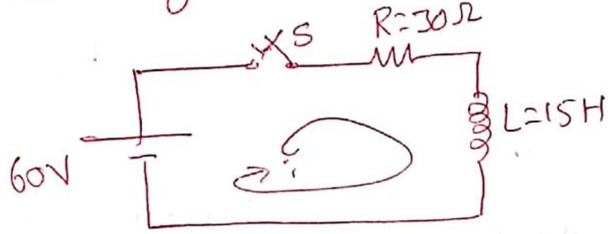
→ lly if we know the derivative of voltage in capacitance, we can know the voltage at a feuture instant. But this requires the values of currents (or) voltage at the initial instant.

→ Therefore, the Initial conditions in the network play a very important role in determining the response of a network when Energy storage elements are present in the network.

component	Initial condition	$t=0^+$	Steadystate $t=\infty$	Time constant τ	Remarks.
Inductor (L)	$I(0^-) = I(0^+)$	open	Short & constant current source	$\frac{L}{R}$	Doesn't allow sudden changes in current.
capacitor	$V(0^-) = V(0^+)$	short	open & constant voltage source	RC	Doesn't allow sudden changes in voltages

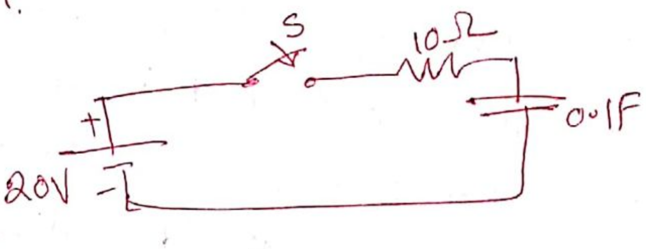
Solutions using Differential Equations approach

(Q1.) A series RL circuit with R=30Ω and L=15H has a constant voltage V=60V applied at t=0 as shown. Determine the current i, the voltage across resistor and the voltage across the Inductor.



Solution is at 1.21(b).

(Q2). A series RC circuit consists of resistor of 10Ω and capacitor of 0.1F as shown. A constant voltage of 20V is applied to the circuit at t=0. Obtain current equation & Determine voltage across Resistor and capacitor.?



Applying KVL

$$20 = 10i + \frac{1}{0.1}\int i\,dt$$

$$20 = 10i + 10\int i\,dt$$

$$20 = 10\left(i + \int i\,dt\right)$$

$$i + \int i\,dt = 2$$

Differentiate w.r.t 't'

$$\frac{di}{dt} + i = 0$$

Solution for i

$$i = \frac{V}{R} e^{-t/RC}$$

$$i = \frac{20}{10} e^{-t/10\times0.1}$$

$$\boxed{i = 2e^{-t}\,A}$$

Voltage across Resistor is V_R

$$V_R = Ri$$

$$= \cancel{R}\frac{V}{\cancel{R}} e^{-t/RC} = V e^{-t/RC}$$

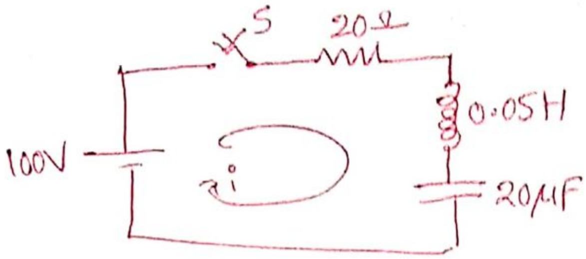
$$\boxed{V_R = 20e^{-t}\,V}$$

Voltage across capacitor is V_C

$$V_C = V\left(1 - e^{-t/RC}\right)$$

$$\boxed{V_C = 20\left(1 - e^{-t}\right)V}$$

Q3 Apply KVL



$$100 = 20i + 0.05\frac{di}{dt} + \frac{1}{20\times10^{-6}}\int i\,dt$$

$$0.05\frac{di}{dt} + 20i + \frac{1}{20\times10^{-6}}\int i\,dt = 100$$

Differentiate w.r.t. 't'

$$0.05\frac{d^2i}{dt^2} + 20\frac{di}{dt} + \frac{i}{20\times10^{-6}} = 0$$

Devide by 0.05

$$\frac{d^2i}{dt^2} + \frac{20}{0.05}\frac{di}{dt} + \frac{i}{20\times10^{-6}\times0.05} = 0$$

$$\frac{d^2i}{dt^2} + 400\frac{di}{dt} + \frac{100}{100\times10^{-6}}i = 0$$

$$\boxed{\frac{d^2i}{dt^2} + 400\frac{di}{dt} + 10^6 i = 0}$$

Second order differential eqn.

$$D^2 + 400D + 10^6 = 0$$

$$D_1, D_2 = -\frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$$

$$D_1, D_2 = -\frac{20}{2\times0.05} \pm \sqrt{\left(\frac{20}{2\times0.05}\right)^2 - \frac{1}{0.05\times20\times10^{-6}}}$$

$$D_1, D_2 = -200 \pm \sqrt{(200)^2 - 10^6}$$

$$D_1 = -200 + j979.8 = \alpha + j\beta$$

$$D_2 = -200 - j979.8 = \alpha - j\beta$$

Here roots are complex conjugate under damped response solution.

Therefor the current is

$$i = e^{\alpha t}\left[c_1\cos(\beta t) + c_2\sin(\beta t)\right]$$

$$\boxed{i = e^{-200t}\left[c_1\cos(979.8t) + c_2\sin(979.8t)\right]}$$

Find c_1 & c_2, From initial conditions, $t=0$ the current flowing through the circuit is '0'.

$$i = 0 = e^{-200(0)}\left[c_1\cos 0 + c_2\sin 0\right]$$

$$c_1\cos 0 + c_2\sin 0 = 0$$

$$\boxed{c_1 = 0}$$

$$i = e^{-200t}\left[c_2\sin(979.8t)\right]$$

To find c_2 differentiate above eqn. w.r.t. t

$$\frac{di}{dt} = c_2\left[e^{-200t}(979.8)\cos(979.8)t + (-200)e^{-200t}\sin 979.8t\right]$$

At $t=0$, $V_L = 100V$

$$V_L = L\cdot\frac{di}{dt} = 100 = 0.05\frac{di}{dt}$$

$$\frac{di}{dt} = 2000$$

At $t=0$

$$2000 = c_2(979.8)\cos 0$$

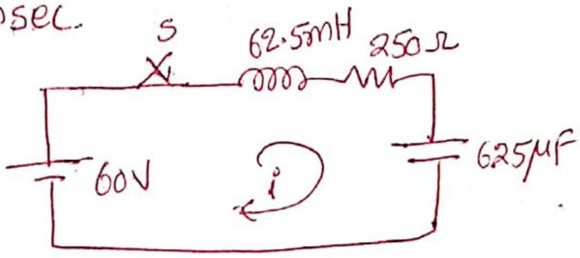
$$c_2 = \frac{2000}{979.8} = 2.04$$

The final current equation is

$$\boxed{i = e^{-200t}\left[2.04\sin(979.8t)\right] A}$$

1.25

Q4) For the given series RLC circuit, determine transient current at $t=0$, $t=1msec$, $t=5msec$ & $t=10msec$. Assume switch is closed for $t>0msec$.



Apply KVL $t=0$

$$60 = 62.5\times10^{-3}\frac{di}{dt} + 250i + \frac{1}{6.25\times10^{-6}}\int i\,dt$$

Differentiate w.r.t. t.

$$0 = 62.5\times10^{-3}\frac{d^2i}{dt^2} + 250\frac{di}{dt} + \frac{i}{6.25\times10^{-6}}$$

Divide by 62.5×10^{-3}

$$\frac{d^2i}{dt^2} + \frac{250}{62.5\times10^{-3}}\frac{di}{dt} + \frac{i}{62.5\times6.25\times10^{-9}} = 0$$

$$\frac{d^2i}{dt^2} + 4000\frac{di}{dt} + 2.56\times10^{6}i = 0$$

let $\frac{di}{dt} = D$; $D^2 + 4000D + 2.56\times10^6 = 0$

$$D_1, D_2 = \frac{-R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$$

$$= -2000 \pm \sqrt{(2000)^2 - 2.56\times10^6}$$

$D_1, D_2 = -2000 \pm 1200$

$D_1 = -2000+1200 = -800 \quad = \alpha+\beta$

$D_2 = -2000-1200 = -3200 = \alpha-\beta$

Roots are real and unequal overdamped response.

The solution is

$$i = c_1 e^{(\alpha+\beta)t} + c_2 e^{(\alpha-\beta)t}$$

$$\boxed{i = c_1 e^{-800t} + c_2 e^{-3200t}} \text{ —①}$$

$t=0$, $i=0$; $\boxed{c_1 = -c_2}$ —②

$t=0$ $V_L = 60V$

$V_L = L\cdot\frac{di}{dt}$

$60 = 62.5\times10^{-3}\cdot\frac{di}{dt}$

$$\frac{di}{dt} = \frac{60}{62.5\times10^{-3}} = 960$$

to find c_1 & c_2, differentiate ① w.r.t. t at 0^+

$$\frac{di}{dt}(0^+) = -800c_1\cdot e^{-800(0)} - 3200c_2\cdot e^{-3200(0)}$$

$$\boxed{960 = -800c_1 - 3200c_2} \text{ —③}$$

put ② in ③

$960 = -800(-c_2) - 3200c_2$

$960 = 800c_2 - 3200c_2$

$960 = c_2(800-3200)$

$960 = -2400c_2$

$c_2 = \frac{\cancel{960}^{4}}{-\cancel{2400}}\times\frac{10}{10} = \frac{4}{10}\ 0.4$

$c_2 = -0.4$, then $c_1 = +0.4$

Therefore $i = 0.4e^{-800t} + 0.4e^{-3200t}$

$$\boxed{i = 0.4\left(e^{-800t} - e^{-3200t}\right)}$$

For $t=0$, $\boxed{i=0}$

$t = 1$ msec, $i = 0.4\left(e^{-800\times10^{-3}} - e^{-3200\times10^{-3}}\right)$

$\boxed{i = 0.574A}$

$t = 5$ msec

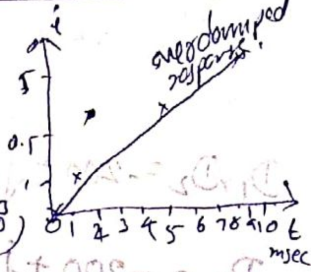
$i = 0.4\left(e^{-800\times5\times10^{-3}} - e^{-3200\times5\times10^{-3}}\right)$

$\boxed{i = 4.802A}$

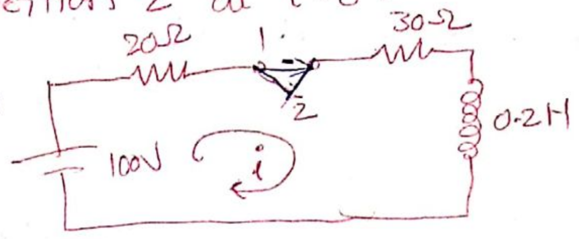
$t = 10$ msec

$i = 0.4\left(e^{-800\times10\times10^{-3}} - e^{-3200\times10\times10^{-3}}\right) = 5.48A$

$\boxed{i = 5.48A}$



(Q5) For the circuit find the current equation when the switch 'S' is changed from position 1 to position 2 at $t=0$.



1~~00~~= 20Ω resistor is consider before $t=0$ before closing switch i.e. switch is open.

so Apply KVL at $t=0$ $\cancel{10}0 = 30i + 0.2\frac{di}{dt}$

$$0.2\frac{di}{dt} + 30i = 0$$

divide with 0.2

$$\frac{di}{dt} + \frac{30}{0.2}i = 0$$

$$\boxed{\frac{di}{dt} + 150i = 0}$$

solution is $i = Ke^{-t/\gamma}$

$$\gamma = \frac{L}{R} = \frac{0.2}{30} = 0.0066$$

$$\boxed{i = Ke^{-150t}}$$

→ at $t=0$, $i(0) = K$

→ at $t=0^-$, switch is in 1 position inductor is short circuit.

For constant current source

$$i = \frac{100}{20+30} = 2A$$

→ at $t=0^+$ inductor doesn't allow sudden changes in the current

so $i(0^+) = 2A = i(0) = K$

$K = 2A$

$$\boxed{i = 2 \cdot e^{-150t}}$$

Solution using Laplace transform Method

- Laplace transform (LT) is used to solve differential equations and corresponding initial and final value problems.
- L.T is widely used in engineering particularly when the source has discontinuities and appear for a short period only.
- This method is used to find out transient currents in circuits containing energy storage elements.

$$L[e^{-at}] = \frac{1}{s+a}$$

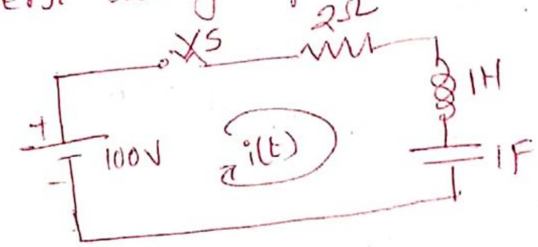
$$L[u(t)] = \frac{1}{s}$$

$$L[t.e^{-at}] = \frac{1}{(s+a)^2}$$

$$L[\int i\,dt] = \frac{I(s)}{s}$$

$$L\left[\frac{di}{dt}\right] = s.I(s)$$

Q1) For the given circuit find i(t) current using Laplace Transform



Apply KVL

$$100 = 2i(t) + 1\frac{di}{dt} + \frac{1}{1}\int i\,dt$$

$$2i(t) + \frac{di}{dt} + \int i\,dt = 100$$

Apply Laplace transform

$$2I(s) + s.I(s) + \frac{I(s)}{s} = \frac{100}{s}$$

$$I(s)\left[2 + s + \frac{1}{s}\right] = \frac{100}{s}$$

$$I(s)\left[\frac{2s+s^2+1}{\cancel{s}}\right] = \frac{100}{\cancel{s}}$$

$$I(s) = \frac{100}{s^2+2s+1} = \frac{100}{(s+1)^2}$$

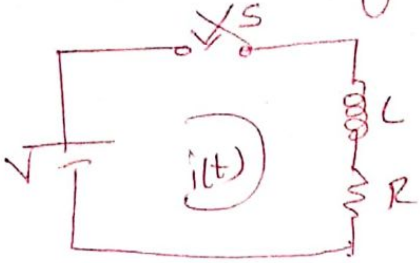
Apply Inverse Laplace Transform

$$L^{-1}I(s) = 100\,L^{-1}\frac{1}{(s+1)^2}$$

$$L^{-1}I(s) = 100\,L^{-1}\frac{1}{(s+1)^2}$$

$$\boxed{i(t) = 100\,t e^{-t}}$$

Q2 Determine current i(t) for the ciruit using L.T



Apply KVL

$$V = L\frac{di}{dt} + Ri$$

Apply L.T

$$\frac{V}{S} = LS \cdot I(S) + RI(S)$$

$$\frac{V}{S} = I(S)\left[L.S + R\right]$$

$$I(S) = \frac{V}{S(R+SL)}$$

Divide withL

$$I(S) = \frac{\frac{V}{L}}{S(\frac{R}{L}+S)}$$

Apply ILT

$$i(t) = L^{-1}I(S) = \frac{V}{L}\cdot L^{-1}\left[\frac{1}{S(S+\frac{R}{L})}\right]$$

using partial fractions method

$$\frac{1}{S(S+\frac{R}{L})} = \frac{A}{S} + \frac{B}{S+\frac{R}{L}}$$

$$A = \frac{1}{S(S+\frac{R}{L})}\cdot S\Big|_{S=0} \; ; A = \frac{L}{R}$$

$$B = \frac{1}{S(S+\frac{R}{L})}\cdot\left(S+\frac{R}{L}\right)\Big|_{S=-\frac{R}{L}} \quad B = -\frac{L}{R}$$

$$\therefore \; i(t) = \frac{V}{L}L^{-1}\left[\frac{L}{RS} - \frac{L}{R(S+\frac{R}{L})}\right]$$

$$i(t) = \frac{V}{L}\left[\frac{L}{R}\cdot 1 - \frac{L}{R}e^{-(\frac{R}{L})t}\right]$$

$$i(t) = \frac{V}{\cancel{L}}\times\frac{\cancel{L}}{R}\left[1 - e^{-(\frac{R}{L})t}\right]$$

$$\boxed{i(t) = \frac{V}{R}\left[1 - e^{-t/\gamma}\right]} \quad \gamma = \frac{L}{R}$$

Chapter 19, Problem 1.

Obtain the z parameters for the network in Fig. 19.65.

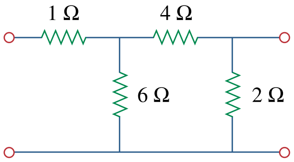
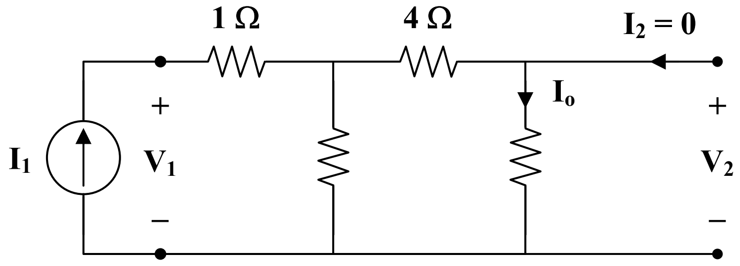


Figure 19.65

For Prob. 19.1 and 19.28.

Chapter 19, Solution 1.

To get $\mathbf{z}_{11}$ and $\mathbf{z}_{21}$, consider the circuit in Fig. (a).



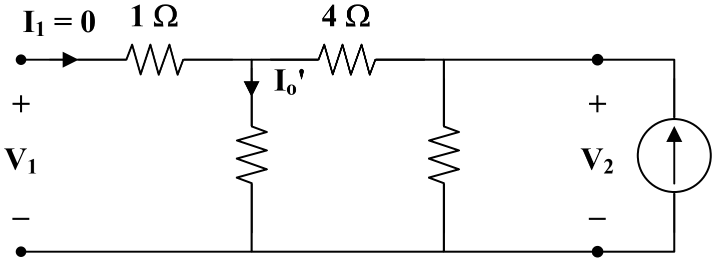
(a)

$$\mathbf{z}_{11} = \frac{\mathbf{V}_1}{\mathbf{I}_1} = 1 + 6 \,||\, (4+2) = 4\,\Omega$$

$$\mathbf{I}_o = \frac{1}{2}\mathbf{I}_1, \qquad \mathbf{V}_2 = 2\mathbf{I}_o = \mathbf{I}_1$$

$$\mathbf{z}_{21} = \frac{\mathbf{V}_2}{\mathbf{I}_1} = 1\,\Omega$$

To get $\mathbf{z}_{22}$ and $\mathbf{z}_{12}$, consider the circuit in Fig. (b).



(b)

$$\mathbf{z}_{22} = \frac{\mathbf{V}_2}{\mathbf{I}_2} = 2 \,||\, (4+6) = 1.667\,\Omega$$

$$\mathbf{I}_o{}' = \frac{2}{2+10}\mathbf{I}_2 = \frac{1}{6}\mathbf{I}_2, \qquad \mathbf{V}_1 = 6\mathbf{I}_o{}' = \mathbf{I}_2$$

$$\mathbf{z}_{12} = \frac{\mathbf{V}_1}{\mathbf{I}_2} = 1\,\Omega$$

Hence, $$\underline{[\mathbf{z}] = \begin{bmatrix} \mathbf{4} & \mathbf{1} \\ \mathbf{1} & \mathbf{1.667} \end{bmatrix}\Omega}$$

Chapter 19, Problem 2.

* Find the impedance parameter equivalent of the network in Fig. 19.66.

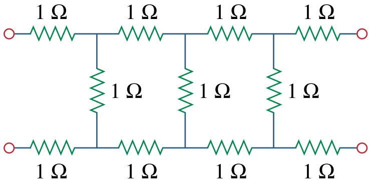


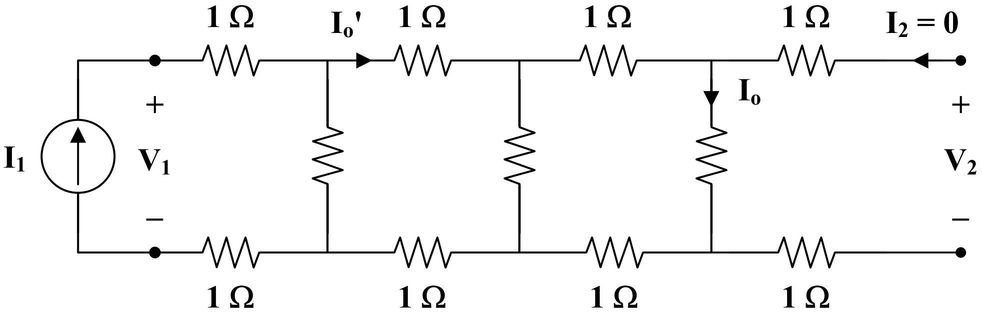
Figure 19.66

For Prob. 19.2.

* An asterisk indicates a challenging problem.

Chapter 19, Solution 2.

Consider the circuit in Fig. (a) to get $\mathbf{z}_{11}$ and $\mathbf{z}_{21}$.



(a)

$$\mathbf{z}_{11} = \frac{\mathbf{V}_1}{\mathbf{I}_1} = 2 + 1 \,||\, [\,2 + 1 \,||\, (2+1)\,]$$

$$\mathbf{z}_{11} = 2 + 1 \,||\left(2 + \frac{3}{4}\right) = 2 + \frac{(1)(11/4)}{1 + 11/4} = 2 + \frac{11}{15} = 2.733$$

$$\mathbf{I}_o = \frac{1}{1+3}\mathbf{I}_o^{'} = \frac{1}{4}\mathbf{I}_o^{'}$$

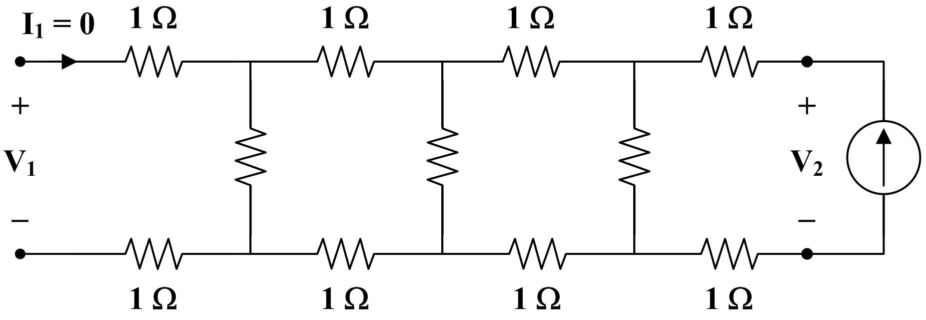
$$\mathbf{I}_o^{'} = \frac{1}{1 + 11/4}\mathbf{I}_1 = \frac{4}{15}\mathbf{I}_1$$

$$\mathbf{I}_o = \frac{1}{4}\cdot\frac{4}{15}\mathbf{I}_1 = \frac{1}{15}\mathbf{I}_1$$

$$\mathbf{V}_2 = \mathbf{I}_o = \frac{1}{15}\mathbf{I}_1$$

$$\mathbf{z}_{21} = \frac{\mathbf{V}_2}{\mathbf{I}_1} = \frac{1}{15} = \mathbf{z}_{12} = 0.06667$$

To get $\mathbf{z}_{22}$, consider the circuit in Fig. (b).



(b)

$$\mathbf{z}_{22} = \frac{\mathbf{V}_2}{\mathbf{I}_2} = 2 + 1 \,||\, (2 + 1 \,||\, 3) = \mathbf{z}_{11} = 2.733$$

Thus,

$$[\mathbf{z}] = \underline{\begin{bmatrix} \mathbf{2.733} & \mathbf{0.06667} \\ \mathbf{0.06667} & \mathbf{2.733} \end{bmatrix}\Omega}$$

Chapter 19, Problem 3.

Find the z parameters of the circuit in Fig. 19.67.

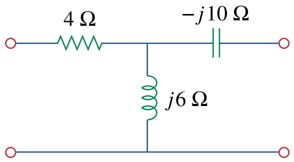


Figure 19.67

For Prob. 19.3.

Chapter 19, Solution 3.

$$z_{12} = j6 = z_{21}$$

$$z_{11} - z_{12} = 4 \quad \longrightarrow \quad z_{11} = z_{12} + 4 = 4 + j6\ \Omega$$

$$z_{22} - z_{12} = -j10 \quad \longrightarrow \quad z_{22} = z_{12} - j10 = -j4\ \Omega$$

$$[z] = \underline{\begin{bmatrix} 4+j6 & j6 \\ j6 & -j4 \end{bmatrix} \Omega} = \begin{bmatrix} 4+\mathrm{j}6 & \mathrm{j}6 \\ \mathrm{j}6 & -\mathrm{j}4 \end{bmatrix} \Omega$$

Chapter 19, Problem 4.

Calculate the *z* parameters for the circuit in Fig. 19.68.

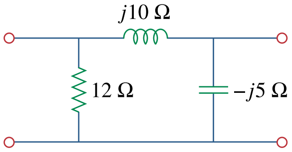
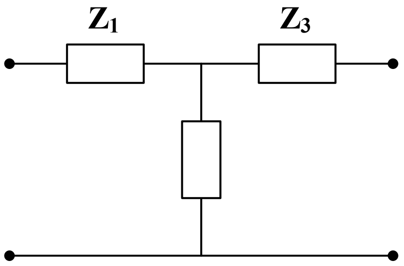


Figure 19.68

For Prob. 19.4.

Chapter 19, Solution 4.

Transform the Π network to a T network.



$$\mathbf{Z}_1 = \frac{(12)(j10)}{12 + j10 - j5} = \frac{j120}{12 + j5}$$

$$\mathbf{Z}_2 = \frac{-\,j60}{12 + j5}$$

$$\mathbf{Z}_3 = \frac{50}{12 + j5}$$

The z parameters are

$$\mathbf{z}_{12} = \mathbf{z}_{21} = \mathbf{Z}_2 = \frac{(-j60)(12 - j5)}{144 + 25} = -1.775 - j4.26$$

$$\mathbf{z}_{11} = \mathbf{Z}_1 + \mathbf{z}_{12} = \frac{(j120)(12 - j5)}{169} + \mathbf{z}_{12} = 1.775 + j4.26$$

$$\mathbf{z}_{22} = \mathbf{Z}_3 + \mathbf{z}_{21} = \frac{(50)(12 - j5)}{169} + \mathbf{z}_{21} = 1.7758 - j5.739$$

Thus,

$$\underline{[\mathbf{z}] = \begin{bmatrix} \mathbf{1.775 + j4.26} & \mathbf{-1.775 - j4.26} \\ \mathbf{-1.775 - j4.26} & \mathbf{1.775 - j5.739} \end{bmatrix} \Omega}$$

Chapter 19, Problem 5.

Obtain the *z* parameters for the network in Fig. 19.69 as functions of *s*.

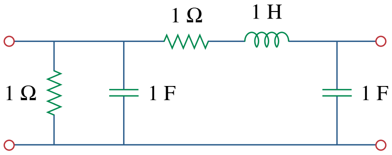
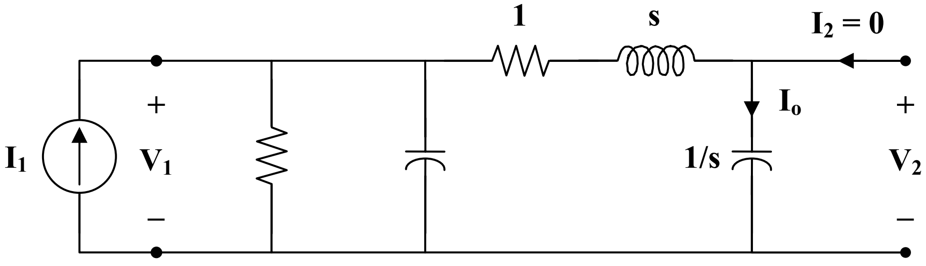


Figure 19.69

For Prob. 19.5.

Chapter 19, Solution 5.

Consider the circuit in Fig. (a).



(a)

$$\mathbf{z}_{11} = 1 \,\|\, \frac{1}{s} \,\|\, \left(1+s+\frac{1}{s}\right) = \frac{\frac{1}{s}}{1+\frac{1}{s}} \,\|\, \left(1+s+\frac{1}{s}\right) = \frac{\left(\frac{1}{s+1}\right)\left(1+s+\frac{1}{s}\right)}{\left(\frac{1}{s+1}\right)+1+s+\frac{1}{s}}$$

$$\mathbf{z}_{11} = \frac{s^2+s+1}{s^3+2s^2+3s+1}$$

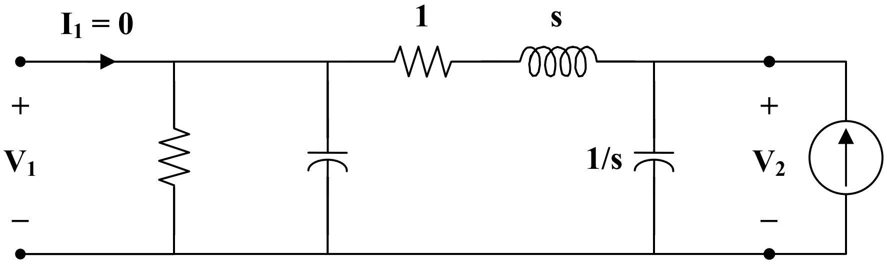
$$\mathbf{I}_o = \frac{1 \,\|\, \frac{1}{s}}{1 \,\|\, \frac{1}{s}+1+s+\frac{1}{s}}\mathbf{I}_1 = \frac{\frac{1}{s+1}}{\frac{1}{s+1}+1+s+\frac{1}{s}}\mathbf{I}_1 = \frac{\frac{s}{s+1}}{\frac{s}{s+1}+s^2+s+1}\mathbf{I}_1$$

$$\mathbf{I}_o = \frac{s}{s^3+2s^2+3s+1}\mathbf{I}_1$$

$$\mathbf{V}_2 = \frac{1}{s}\mathbf{I}_o = \frac{\mathbf{I}_1}{s^3+2s^2+3s+1}$$

$$\mathbf{z}_{21} = \frac{\mathbf{V}_2}{\mathbf{I}_1} = \frac{1}{s^3+2s^2+3s+1}$$

Consider the circuit in Fig. (b).



(b)

$$\mathbf{z}_{22} = \frac{\mathbf{V}_2}{\mathbf{I}_2} = \frac{1}{s} \,||\, \left(1 + s + 1 \,||\, \frac{1}{s}\right) = \frac{1}{s} \,||\, \left(1 + s + \frac{1}{s+1}\right)$$

$$\mathbf{z}_{22} = \frac{\left(\frac{1}{s}\right)\left(1 + s + \frac{1}{s+1}\right)}{\frac{1}{s} + 1 + s + \frac{1}{s+1}} = \frac{1 + s + \frac{1}{s+1}}{1 + s + s^2 + \frac{s}{s+1}}$$

$$\mathbf{z}_{22} = \frac{s^2 + 2s + 2}{s^3 + 2s^2 + 3s + 1}$$

$$\mathbf{z}_{12} = \mathbf{z}_{21}$$

Hence,

$$\underline{[\mathbf{z}] = \begin{bmatrix} \dfrac{\mathbf{s^2 + s + 1}}{\mathbf{s^3 + 2s^2 + 3s + 1}} & \dfrac{\mathbf{1}}{\mathbf{s^3 + 2s^2 + 3s + 1}} \\ \dfrac{\mathbf{1}}{\mathbf{s^3 + 2s^2 + 3s + 1}} & \dfrac{\mathbf{s^2 + 2s + 2}}{\mathbf{s^3 + 2s^2 + 3s + 1}} \end{bmatrix}}$$

Chapter 19, Problem 6.

Compute the *z* parameters of the circuit in Fig. 19.70.

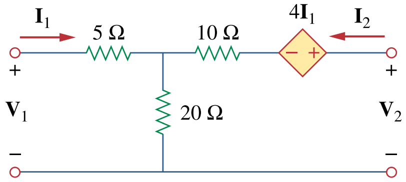
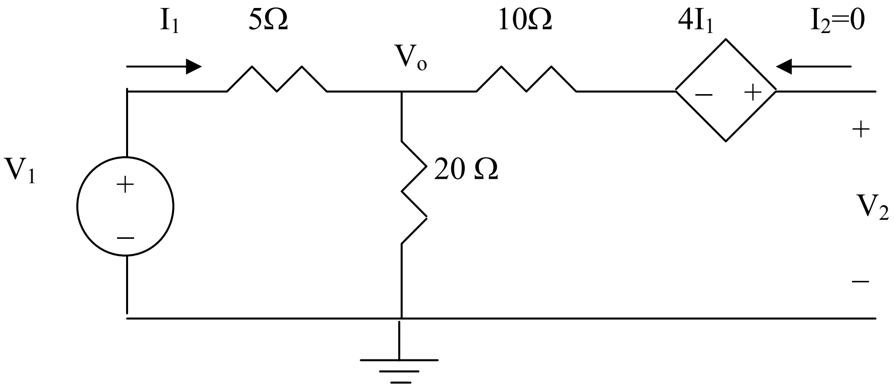


Figure 19.70

For Prob. 19.6 and 19.73.

Chapter 19, Solution 6.

To find z_{11} and z_{21} , consider the circuit below.



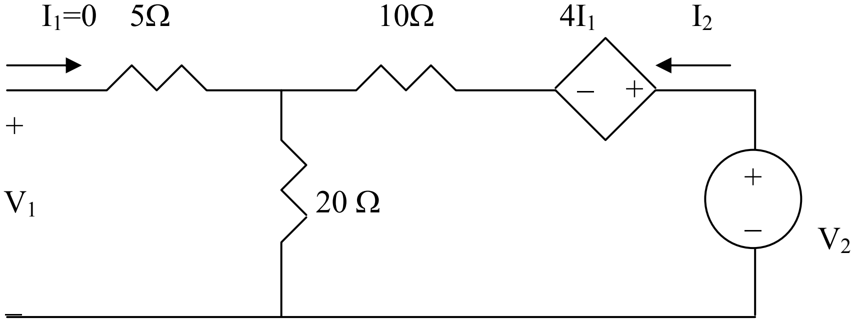
$$z_{11} = \frac{V_1}{I_1} = \frac{(20+5)I_1}{I_1} = 25\ \Omega$$

$$V_o = \frac{20}{25}V_1 = 20I_1$$

$$-V_o - 4I_2 + V_2 = 0 \qquad \longrightarrow \qquad V_2 = V_o + 4I_1 = 20I_1 + 4I_1 = 24I_1$$

$$z_{21} = \frac{V_2}{I_1} = 24\ \Omega$$

To find z_{12} and z_{22}, consider the circuit below.



$$V_2 = (10+20)I_2 = 30I_2$$

$$z_{22} = \frac{V_2}{I_1} = 30\ \Omega$$

$$V_1 = 20I_2$$

$$z_{12} = \frac{V_1}{I_2} = 20\ \Omega$$

Thus,

$$\underline{[z] = \begin{bmatrix} 25 & 20 \\ 24 & 30 \end{bmatrix}\Omega}$$

Chapter 19, Problem 7.

Calculate the impedance-parameter equivalent of the circuit in Fig. 19.71.

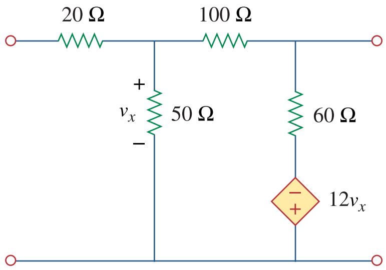
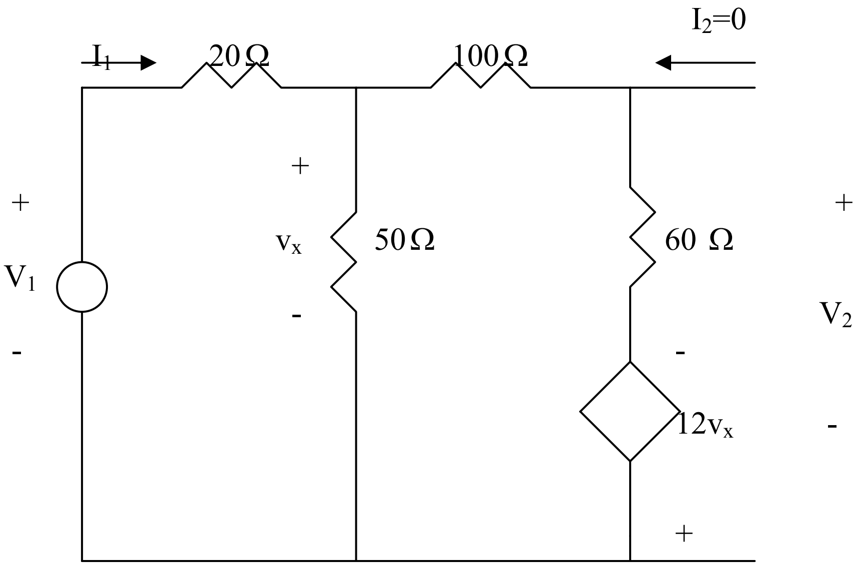


Figure 19.71

For Prob. 19.7 and 19.80.

Chapter 19, Solution 7.

To get z_{11} and z_{21}, we consider the circuit below.



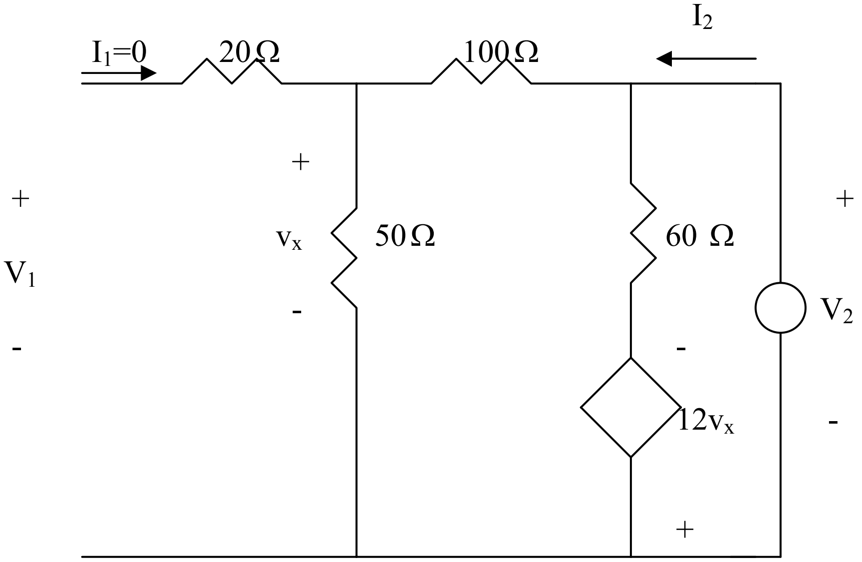
$$\frac{V_1 - V_x}{20} = \frac{V_x}{50} + \frac{V_x + 12V_x}{160} \longrightarrow V_x = \frac{40}{121}V_1$$

$$I_1 = \frac{V_1 - V_x}{20} = \frac{81}{121}(\frac{V_1}{20}) \longrightarrow z_{11} = \frac{V_1}{I_1} = 29.88$$

$$V_2 = 60(\frac{13V_x}{160}) - 12V_x = -\frac{57}{8}V_x = -\frac{57}{8}(\frac{40}{121})V_1 = -\frac{57}{8}(\frac{40}{121})\frac{20x121}{81}I_1$$

$$= -70.37I_1 \longrightarrow z_{21} = \frac{V_2}{I_1} = -70.37$$

To get z_{12} and z_{22}, we consider the circuit below.



$$V_x = \frac{50}{100+50}V_2 = \frac{1}{3}V_2, \quad I_2 = \frac{V_2}{150} + \frac{V_2 + 12V_x}{60} = 0.09V_2$$

$$z_{22} = \frac{V_2}{I_2} = 1/0.09 = 11.11$$

$$V_1 = V_x = \frac{1}{3}V_2 = \frac{11.11}{3}I_2 = 3.704I_2 \longrightarrow z_{12} = \frac{V_1}{I_2} = 3.704$$

Thus,

$$\underline{[z] = \begin{bmatrix} 29.88 & 3.704 \\ -70.37 & 11.11 \end{bmatrix}\Omega}$$

Chapter 19, Problem 8.

Find the z parameters of the two-port in Fig. 19.72.

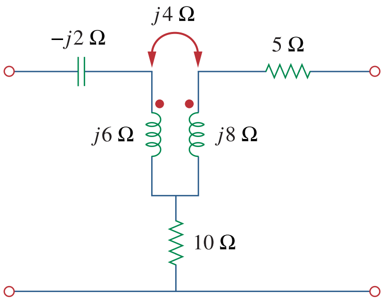
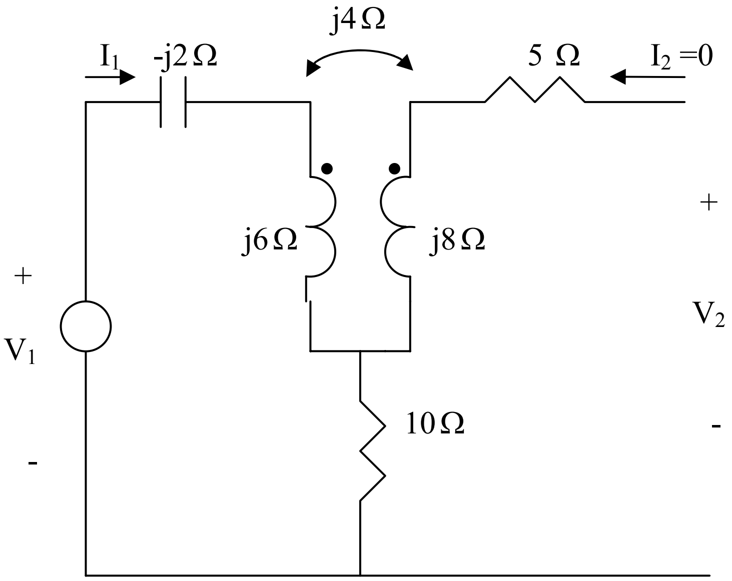


Figure 19.72

For Prob. 19.8.

Chapter 19, Solution 8.

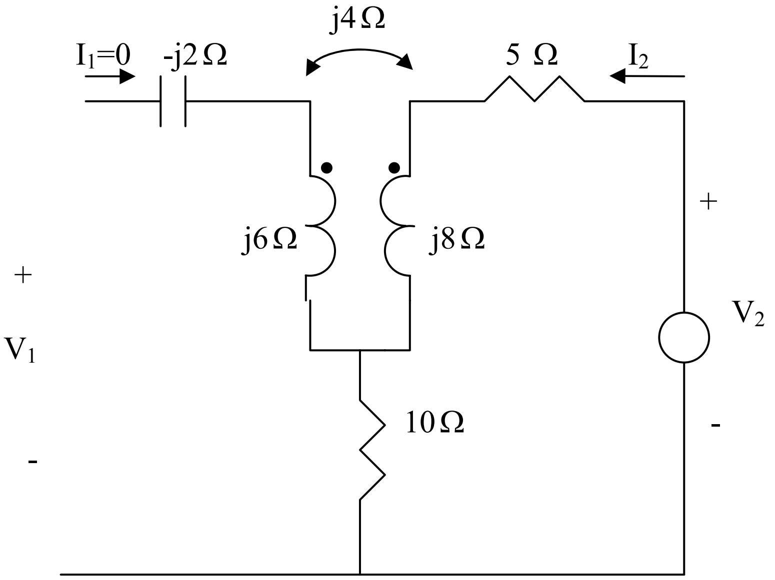
To get z_{11} and z_{21}, consider the circuit below.



$$V_1 = (10 - j2 + j6)I_1 \quad \longrightarrow \quad z_{11} = \frac{V_1}{I_1} = 10 + j4$$

$$V_2 = -10I_1 - j4I_1 \quad \longrightarrow \quad z_{21} = \frac{V_2}{I_1} = -(10 + j4)$$

To get z_{22} and z_{12}, consider the circuit below.



$$V_2 = (5 + 10 + j8)I_2 \quad \longrightarrow \quad z_{22} = \frac{V_2}{I_2} = 15 + j8$$

$$V_1 = -(10 + j4)I_2 \quad \longrightarrow \quad z_{12} = \frac{V_1}{I_2} = -(10 + j4)$$

Thus,

$$\underline{[z] = \begin{bmatrix} (10 + j4) & -(10 + j4) \\ -(10 + j4) & (15 + j8) \end{bmatrix} \Omega}$$

Chapter 19, Problem 9.

The *y* parameters of a network are:

$$[\mathbf{y}] = \begin{bmatrix} 0.5 & -0.2 \\ -0.2 & 0.4 \end{bmatrix}$$

Determine the *z* parameters for the network.

Chapter 19, Solution 9.

$$z_{11} = \frac{y_{22}}{\Delta y} = \frac{0.4}{0.16} = 2.5, \quad \Delta y = y_{11}y_{22} - y_{21}y_{12} = 05x0.4 - 0.2x0.2 = 0.16$$

$$z_{12} = \frac{-y_{12}}{\Delta y} = \frac{0.2}{0.16} = 1.25 = z_{21}$$

$$z_{22} = \frac{y_{11}}{\Delta y} = \frac{0.5}{0.16} = 3.125$$

Thus,

$$[z] = \underline{\begin{bmatrix} 2.5 & 1.25 \\ 1.25 & 3.125 \end{bmatrix}\Omega} \begin{bmatrix} 2.5 & 1.25 \\ 1.25 & 3.125 \end{bmatrix}\Omega$$

Chapter 19, Problem 10.

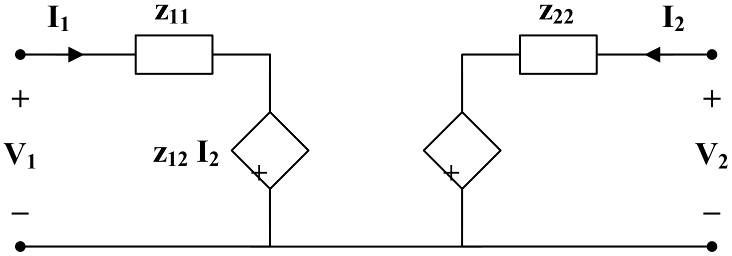
Construct a two-port that realizes each of the following z parameters.

(a) $[\mathbf{z}] = \begin{bmatrix} 25 & 20 \\ 5 & 10 \end{bmatrix} \Omega$

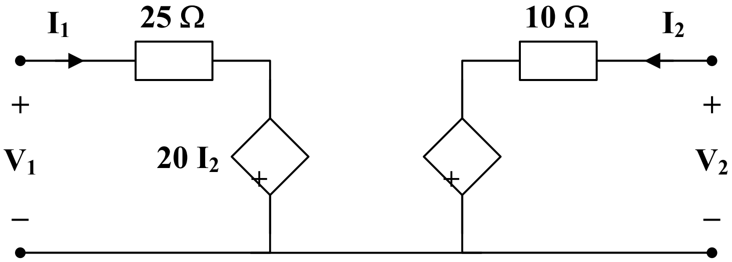
(b) $[\mathbf{z}] = \begin{bmatrix} 1+\dfrac{3}{s} & \dfrac{1}{s} \\ \dfrac{1}{s} & 2s+\dfrac{1}{s} \end{bmatrix} \Omega$

Chapter 19, Solution 10.

(a) This is a non-reciprocal circuit so that **the two-port looks like the one shown in Figs. (a) and (b)**.

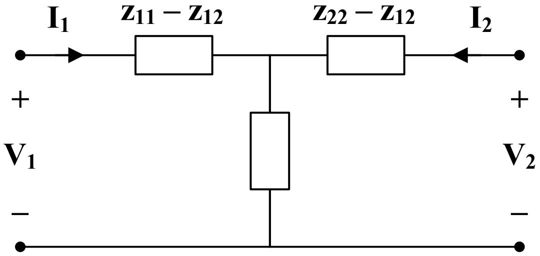


(a)



(b)

(b) This is a reciprocal network and **the two-port look like the one shown in Figs. (c) and (d)**.

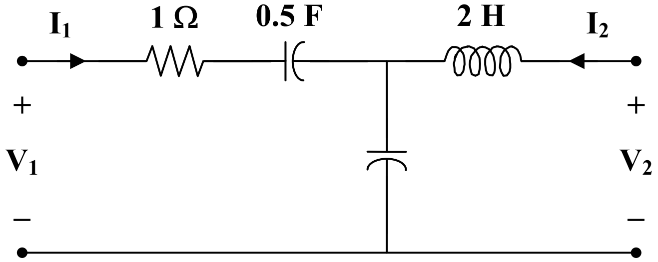


(c)

$$\mathbf{z}_{11} - \mathbf{z}_{12} = 1 + \frac{2}{s} = 1 + \frac{1}{0.5s}$$

$$\mathbf{z}_{22} - \mathbf{z}_{12} = 2s$$

$$\mathbf{z}_{12} = \frac{1}{s}$$



(d)

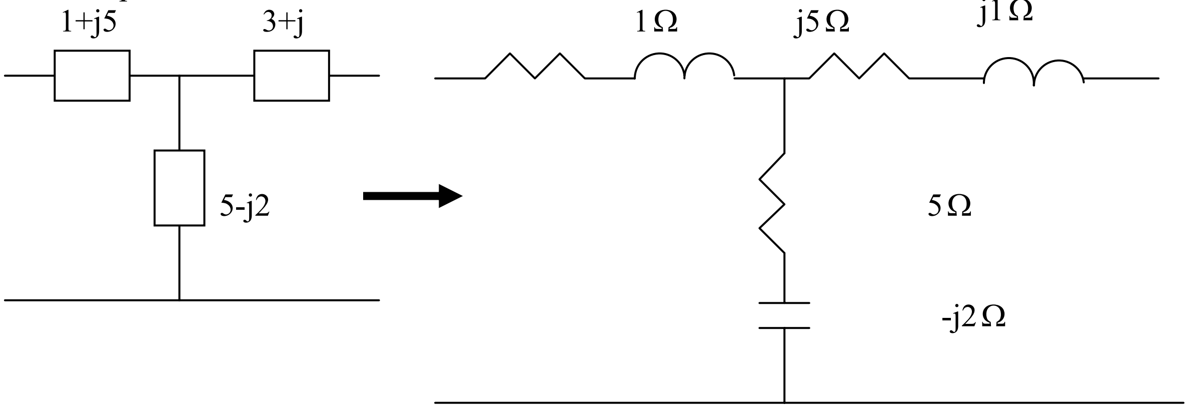
Chapter 19, Problem 11.

Determine a two-port network that is represented by the following z parameters:

$$[\mathbf{z}] = \begin{bmatrix} 6+j3 & 5-j2 \\ 5-j2 & 8-j \end{bmatrix}\ \Omega$$

Chapter 19, Solution 11.

This is a reciprocal network, as shown below.



Chapter 19, Problem 12.

For the circuit shown in Fig. 19.73, let

$$[\mathbf{z}] = \begin{bmatrix} 10 & -6 \\ -4 & 12 \end{bmatrix}$$

Find I_1, I_2, V_1, and V_2.

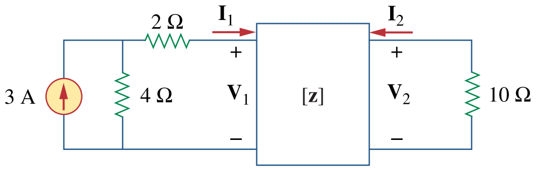


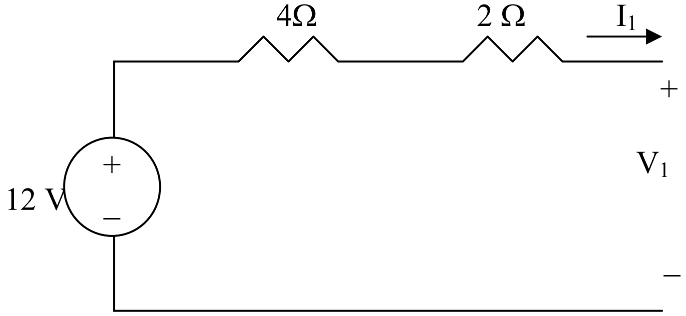
Figure 19.73

For Prob. 19.12.

Chapter 19, Solution 12.

$$V_1 = 10I_1 - 6I_2 \qquad (1)$$
$$V_2 = -4I_2 + 12I_2 \qquad (2)$$
$$V_2 = -10I_2 \qquad (3)$$

If we convert the current source to a voltage source, that portion of the circuit becomes what is shown below.



$$-12 + 6I_1 + V_1 = 0 \quad \longrightarrow \quad V_1 = 12 - 6I_1 \qquad (4)$$

Substituting (3) and (4) into (1) and (2), we get

$$12 - 6I_1 = 10I_1 - 6I_2 \quad \longrightarrow \quad 12 = 16I_1 - 6I_2 \qquad (5)$$
$$-10I_2 = -4I_1 + 12I_2 \quad \longrightarrow \quad 0 = -4I_1 + 22I_2 \quad \longrightarrow \quad I_1 = 5.5I_2 \qquad (6)$$

From (5) and (6),

$$12 = 88I_2 - 6I_2 = 82I_2 \quad \longrightarrow \quad I_2 = \underline{0.1463\text{ A}}$$
$$I_1 = 5.5I_2 = \underline{0.8049\text{ A}}$$
$$V_2 = -10I_2 = \underline{-1.463\text{ V}}$$
$$V_1 = 12 - 6I_1 = \underline{7.1706\text{ V}}$$

Chapter 19, Problem 13.

Determine the average power delivered to $Z_L = 5 + j4$ in the network of Fig. 19.74. *Note:* The voltage is rms.

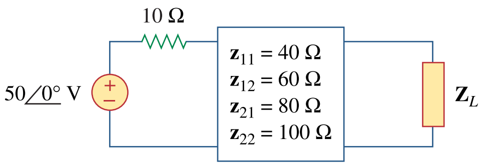
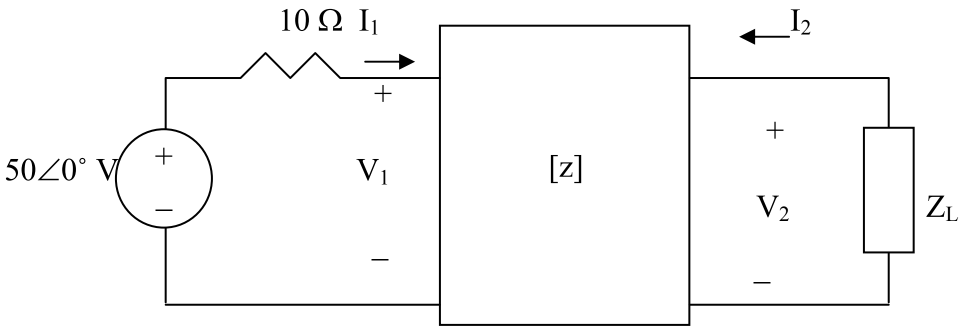


Figure 19.74

For Prob. 19.13.

Chapter 19, Solution 13.

Consider the circuit as shown below.



$V_1 = 40I_1 + 60I_2$ (1)

$V_2 = 80I_1 + 100I_2$ (2)

$V_2 = -I_2 Z_L = -I_2(5 + j4)$ (3)

$50 = V_1 + 10I_1 \longrightarrow V_1 = 50 - 10I_1$ (4)

Substituting (4) in (1)

$50 - 10I_1 = 40I_1 + 60I_2 \longrightarrow 5 = 5I_1 + 6I_2$ (5)

Substituting (3) into (2),

$-I_2(5 + j4) = 80I_1 + 100I_2 \longrightarrow 0 = 80I_1 + (105 + j4)I_2$ (6)

Solving (5) and (6) gives

$I_2 = -7.423 + j3.299$ A

We can check the answer using MATLAB.

First we need to rewrite equations 1-4 as follows,

$$\begin{bmatrix} 1 & 0 & -40 & -60 \\ 0 & 1 & -80 & -100 \\ 0 & 1 & 0 & 5+j4 \\ 1 & 0 & 10 & 0 \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \\ I_1 \\ I_2 \end{bmatrix} = A * X = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 50 \end{bmatrix} = U$$

```
>> A=[1,0,-40,-60;0,1,-80,-100;0,1,0,(5+4i);1,0,10,0]
A =
  1.0e+002 *
   0.0100            0          -0.4000           -0.6000
        0       0.0100          -0.8000           -1.0000
        0       0.0100                0       0.0500 + 0.0400i
   0.0100            0           0.1000                0
>> U=[0;0;0;50]
U =
     0
     0
     0
    50
>> X=inv(A)*U
X =
 -49.0722 +39.5876i
  50.3093 +13.1959i
   9.9072 - 3.9588i
  -7.4227 + 3.2990i
```

$P = |I_2|^2 5 =$ **329.9 W**.

Chapter 19, Problem 14.

For the two-port network shown in Fig. 19.75, show that at the output terminals,

$$\mathbf{Z}_{\text{Th}} = \mathbf{z}_{22} - \frac{\mathbf{z}_{12}\mathbf{z}_{21}}{\mathbf{z}_{11} + \mathbf{Z}_s}$$

and

$$\mathbf{V}_{\text{Th}} = \frac{\mathbf{z}_{21}}{\mathbf{z}_{11} + \mathbf{Z}_s}\mathbf{V}s$$

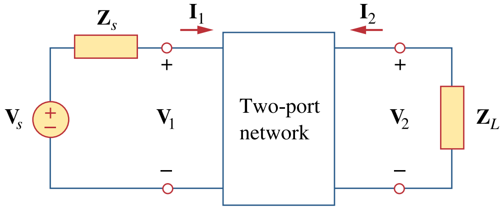
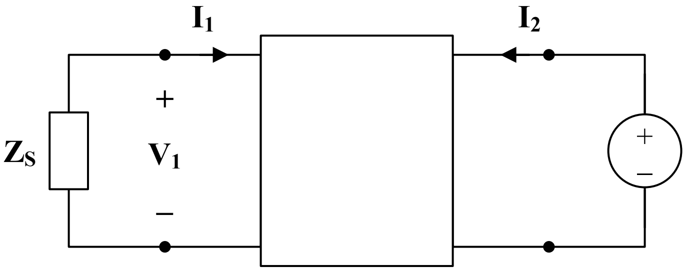


Figure 19.75

For Prob. 19.14 and 19.41.

Chapter 19, Solution 14.

To find $\mathbf{Z}_{\text{Th}}$, consider the circuit in Fig. (a).



(a)

$$\mathbf{V}_1 = \mathbf{z}_{11}\mathbf{I}_1 + \mathbf{z}_{12}\mathbf{I}_2 \qquad (1)$$

$$\mathbf{V}_2 = \mathbf{z}_{21}\mathbf{I}_1 + \mathbf{z}_{22}\mathbf{I}_2 \qquad (2)$$

But

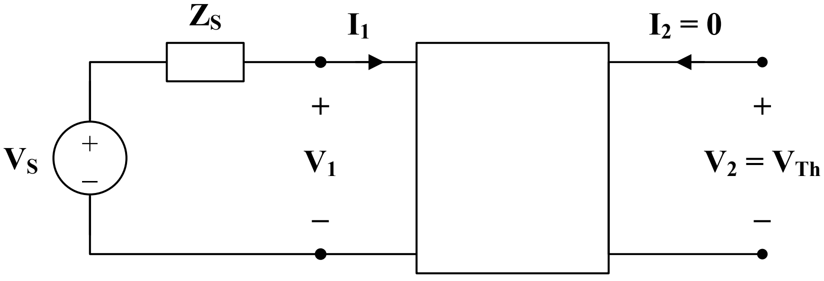
$$\mathbf{V}_2 = 1, \qquad \mathbf{V}_1 = -\mathbf{Z}_s\,\mathbf{I}_1$$

Hence, $0 = (\mathbf{z}_{11} + \mathbf{Z}_s)\mathbf{I}_1 + \mathbf{z}_{12}\,\mathbf{I}_2 \longrightarrow \mathbf{I}_1 = \dfrac{-\mathbf{z}_{12}}{\mathbf{z}_{11} + \mathbf{Z}_s}\mathbf{I}_2$

$$1 = \left(\frac{-\mathbf{z}_{21}\,\mathbf{z}_{12}}{\mathbf{z}_{11} + \mathbf{Z}_s} + \mathbf{z}_{22}\right)\mathbf{I}_2$$

$$\mathbf{Z}_{\text{Th}} = \frac{\mathbf{V}_2}{\mathbf{I}_2} = \frac{1}{\mathbf{I}_2} = \underline{\mathbf{z}_{22} - \frac{\mathbf{z}_{21}\,\mathbf{z}_{12}}{\mathbf{z}_{11} + \mathbf{Z}_s}}$$

To find $\mathbf{V}_{\text{Th}}$, consider the circuit in Fig. (b).



(b)

$$\mathbf{I}_2 = 0, \qquad \mathbf{V}_1 = \mathbf{V}_s - \mathbf{I}_1\,\mathbf{Z}_s$$

Substituting these into (1) and (2),

$$\mathbf{V}_s - \mathbf{I}_1\,\mathbf{Z}_s = \mathbf{z}_{11}\,\mathbf{I}_1 \longrightarrow \mathbf{I}_1 = \frac{\mathbf{V}_s}{\mathbf{z}_{11} + \mathbf{Z}_s}$$

$$\mathbf{V}_2 = \mathbf{z}_{21}\,\mathbf{I}_1 = \frac{\mathbf{z}_{21}\,\mathbf{V}_s}{\mathbf{z}_{11} + \mathbf{Z}_s}$$

$$\mathbf{V}_{\text{Th}} = \mathbf{V}_2 = \underline{\frac{\mathbf{z}_{21}\,\mathbf{V}_s}{\mathbf{z}_{11} + \mathbf{Z}_s}}$$

Chapter 19, Problem 15.

For the two-port circuit in Fig. 19.76,

$$[\mathbf{z}] = \begin{bmatrix} 40 & 60 \\ 80 & 120 \end{bmatrix} \Omega$$

(a) Find $\mathbf{Z}_L$ for maximum power transfer to the load.
(b) Calculate the maximum power delivered to the load.

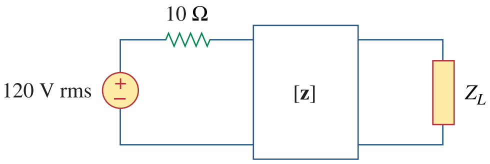


Figure 19.76
For Prob. 19.15.

Chapter 19, Solution 15.

(a) From Prob. 18.12,

$$Z_{Th} = z_{22} - \frac{z_{12}z_{21}}{z_{11} + Z_s} = 120 - \frac{80x60}{40+10} = 24$$

$$\underline{Z_L = Z_{Th} = 24\Omega}$$

(b) $V_{Th} = \dfrac{z_{21}}{z_{11} + Z_s} V_s = \dfrac{80}{40+10}(120) = 192$

$$P_{max} = \left(\frac{V_{Th}}{2R_{Th}}\right)^2 R_{Th} = 4^2 x24 = \underline{\mathbf{384W}}$$

Chapter 19, Problem 16.

For the circuit in Fig. 19.77, at $\omega = 2$ rad/s, $\mathbf{z}_{11} = 10\Omega$, $\mathbf{z}_{12} = \mathbf{z}_{21} = j6\Omega$, $\mathbf{z}_{22} = 4\Omega$. Obtain the Thevenin equivalent circuit at terminals *a*-*b* and calculate v_o.

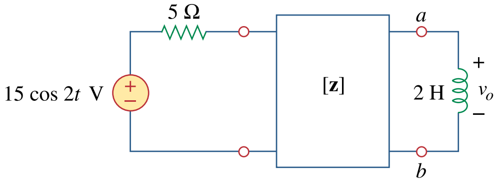
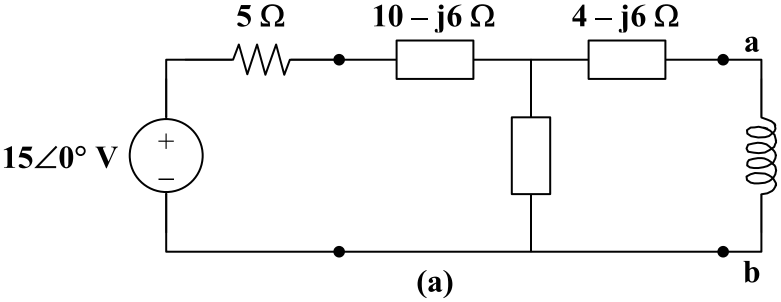


Figure 19.77

For Prob. 19.16.

Chapter 19, Solution 16.

As a reciprocal two-port, the given circuit can be represented as shown in Fig. (a).



(a)

At terminals a-b,

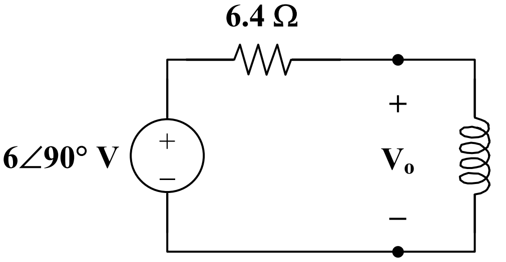
$$\mathbf{Z}_{Th} = (4 - j6) + j6 \,||\, (5 + 10 - j6)$$

$$\mathbf{Z}_{Th} = 4 - j6 + \frac{j6(15 - j6)}{15} = 4 - j6 + 2.4 + j6$$

$$\mathbf{Z}_{Th} = \underline{\mathbf{6.4\,\Omega}}$$

$$\mathbf{V}_{Th} = \frac{j6}{j6 + 5 + 10 - j6}(15\angle 0°) = j6 = \underline{\mathbf{6\angle 90°\ V}}$$

The Thevenin equivalent circuit is shown in Fig. (b).



(b)

From this,

$$\mathbf{V}_o = \frac{j4}{6.4 + j4}(j6) = 3.18\angle 148°$$

$$v_o(t) = \underline{\mathbf{3.18\cos(2t + 148°)\ V}}$$

Chapter 19, Problem 17.

* Determine the *z* and *y* parameters for the circuit in Fig. 19.78.

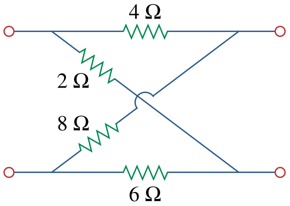


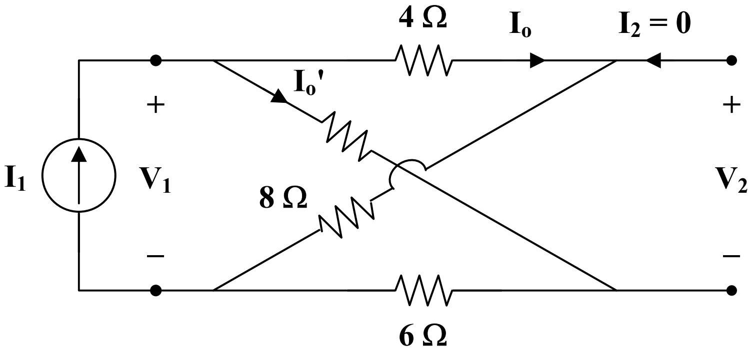
Figure 19.78

For Prob. 19.17.

* An asterisk indicates a challenging problem.

Chapter 19, Solution 17.

To obtain $\mathbf{z}_{11}$ and $\mathbf{z}_{21}$, consider the circuit in Fig. (a).



(a)

In this case, the 4-Ω and 8-Ω resistors are in series, since the same current, $\mathbf{I}_o$, passes through them. Similarly, the 2-Ω and 6-Ω resistors are in series, since the same current, $\mathbf{I}_o^{'}$, passes through them.

$$\mathbf{z}_{11} = \frac{\mathbf{V}_1}{\mathbf{I}_1} = (4+8)\,||\,(2+6) = 12\,||\,8 = \frac{(12)(8)}{20} = 4.8\,\Omega$$

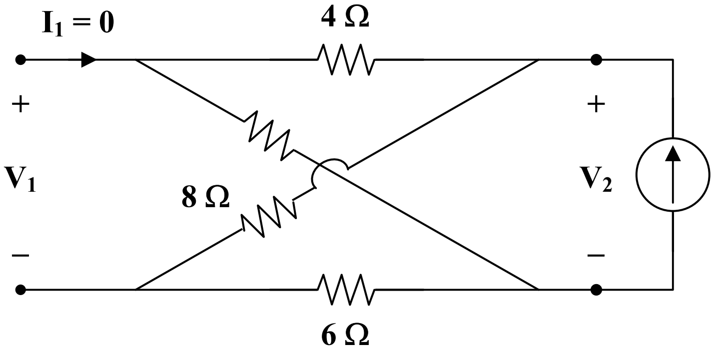
$$\mathbf{I}_o = \frac{8}{8+12}\mathbf{I}_1 = \frac{2}{5}\mathbf{I}_1 \qquad \mathbf{I}_o^{'} = \frac{3}{5}\mathbf{I}_1$$

But $$-\mathbf{V}_2 - 4\mathbf{I}_o + 2\mathbf{I}_o^{'} = 0$$

$$\mathbf{V}_2 = -4\mathbf{I}_o + 2\mathbf{I}_o^{'} = \frac{-8}{5}\mathbf{I}_1 + \frac{6}{5}\mathbf{I}_1 = \frac{-2}{5}\mathbf{I}_1$$

$$\mathbf{z}_{21} = \frac{\mathbf{V}_2}{\mathbf{I}_1} = \frac{-2}{5} = -0.4\,\Omega$$

To get $\mathbf{z}_{22}$ and $\mathbf{z}_{12}$, consider the circuit in Fig. (b).



(b)

$$\mathbf{z}_{22} = \frac{\mathbf{V}_2}{\mathbf{I}_2} = (4+2)\,||\,(8+6) = 6\,||\,14 = \frac{(6)(14)}{20} = 4.2\,\Omega$$

$$\mathbf{z}_{12} = \mathbf{z}_{21} = -0.4\,\Omega$$

Thus,

$$\underline{[\mathbf{z}] = \begin{bmatrix} \mathbf{4.8} & \mathbf{-0.4} \\ \mathbf{-0.4} & \mathbf{4.2} \end{bmatrix}\Omega}$$

We may take advantage of Table 18.1 to get [**y**] from [**z**].

$$\Delta_z = (4.8)(4.2) - (0.4)^2 = 20$$

$$\mathbf{y}_{11} = \frac{\mathbf{z}_{22}}{\Delta_z} = \frac{4.2}{20} = 0.21 \qquad \mathbf{y}_{12} = \frac{-\mathbf{z}_{12}}{\Delta_z} = \frac{0.4}{20} = 0.02$$

$$\mathbf{y}_{21} = \frac{-\mathbf{z}_{21}}{\Delta_z} = \frac{0.4}{20} = 0.02 \qquad \mathbf{y}_{22} = \frac{\mathbf{z}_{11}}{\Delta_z} = \frac{4.8}{20} = 0.24$$

Thus,

$$\underline{[\mathbf{y}] = \begin{bmatrix} \mathbf{0.21} & \mathbf{0.02} \\ \mathbf{0.02} & \mathbf{0.24} \end{bmatrix}\mathbf{S}}$$

Chapter 19, Problem 18.

Calculate the *y* parameters for the two-port in Fig. 19.79.

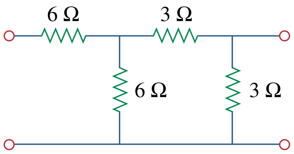
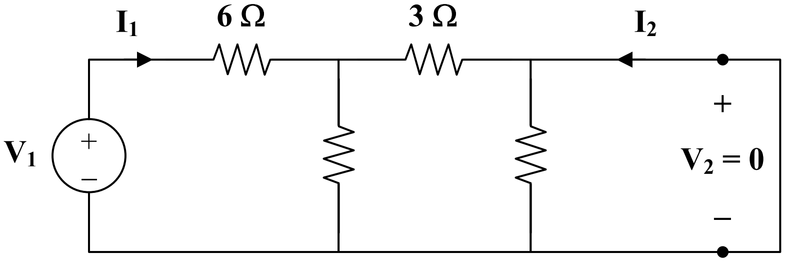


Figure 19.79

For Prob. 19.18 and 19.37.

Chapter 19, Solution 18.

To get $\mathbf{y}_{11}$ and $\mathbf{y}_{21}$, consider the circuit in Fig.(a).



(a)

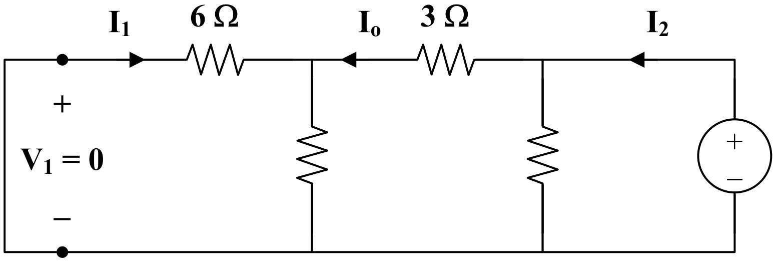
$$\mathbf{V}_1 = (6 + 6 \,||\, 3)\mathbf{I}_1 = 8\mathbf{I}_1$$

$$\mathbf{y}_{11} = \frac{\mathbf{I}_1}{\mathbf{V}_1} = \frac{1}{8}$$

$$\mathbf{I}_2 = \frac{-6}{6+3}\mathbf{I}_1 = \frac{-2}{3}\frac{\mathbf{V}_1}{8} = \frac{-\mathbf{V}_1}{12}$$

$$\mathbf{y}_{21} = \frac{\mathbf{I}_2}{\mathbf{V}_1} = \frac{-1}{12}$$

To get $\mathbf{y}_{22}$ and $\mathbf{y}_{12}$, consider the circuit in Fig.(b).



(b)

$$\mathbf{y}_{22} = \frac{\mathbf{I}_2}{\mathbf{V}_2} = \frac{1}{3 \,||\, (3+6 \,||\, 6)} = \frac{1}{3 \,||\, 6} = \frac{1}{2}$$

$$\mathbf{I}_1 = \frac{-\mathbf{I}_o}{2}, \qquad \mathbf{I}_o = \frac{3}{3+6}\mathbf{I}_2 = \frac{1}{3}\mathbf{I}_2$$

$$\mathbf{I}_1 = \frac{-\mathbf{I}_2}{6} = \left(\frac{-1}{6}\right)\left(\frac{1}{2}\mathbf{V}_2\right) = \frac{-\mathbf{V}_2}{12}$$

$$\mathbf{y}_{12} = \frac{\mathbf{I}_1}{\mathbf{V}_2} = \frac{-1}{12} = \mathbf{y}_{21}$$

Thus,

$$[\mathbf{y}] = \underline{\begin{bmatrix} \mathbf{\frac{1}{8}} & \mathbf{\frac{-1}{12}} \\ \mathbf{\frac{-1}{12}} & \mathbf{\frac{1}{2}} \end{bmatrix} \mathbf{S}}$$

Chapter 19, Problem 19.

Find the *y* parameters of the two-port in Fig. 19.80 in terms of *s*.

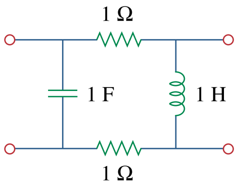
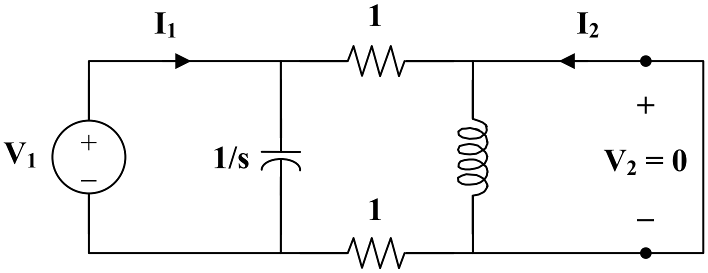


Figure 19.80

For Prob. 19.19.

Chapter 19, Solution 19.

Consider the circuit in Fig.(a) for calculating $\mathbf{y}_{11}$ and $\mathbf{y}_{21}$.



(a)

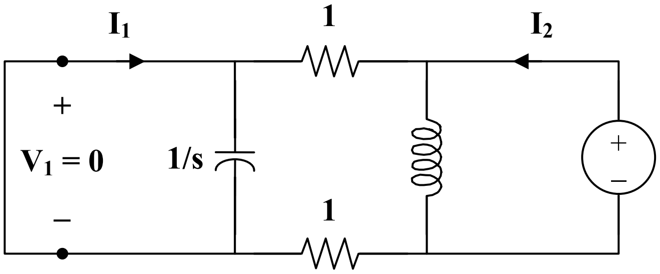
$$\mathbf{V}_1 = \left(\frac{1}{s} \parallel 2\right)\mathbf{I}_1 = \frac{2/s}{2+(1/s)}\mathbf{I}_1 = \frac{2}{2s+1}\mathbf{I}_1$$

$$\mathbf{y}_{11} = \frac{\mathbf{I}_1}{\mathbf{V}_1} = \frac{2s+1}{2} = s + 0.5$$

$$\mathbf{I}_2 = \frac{(-1/s)}{(1/s)+2}\mathbf{I}_1 = \frac{-\mathbf{I}_1}{2s+1} = \frac{-\mathbf{V}_1}{2}$$

$$\mathbf{y}_{21} = \frac{\mathbf{I}_2}{\mathbf{V}_1} = -0.5$$

To get $\mathbf{y}_{22}$ and $\mathbf{y}_{12}$, refer to the circuit in Fig.(b).



(b)

$$\mathbf{V}_2 = (\text{s} \parallel 2)\mathbf{I}_2 = \frac{2\text{s}}{\text{s}+2}\mathbf{I}_2$$

$$\mathbf{y}_{22} = \frac{\mathbf{I}_2}{\mathbf{V}_2} = \frac{\text{s}+2}{2\text{s}} = 0.5 + \frac{1}{\text{s}}$$

$$\mathbf{I}_1 = \frac{-\text{s}}{\text{s}+2}\mathbf{I}_2 = \frac{-\text{s}}{\text{s}+2}\cdot\frac{\text{s}+2}{2\text{s}}\mathbf{V}_2 = \frac{-\mathbf{V}_2}{2}$$

$$\mathbf{y}_{12} = \frac{\mathbf{I}_1}{\mathbf{V}_2} = -0.5$$

Thus,

$$[\mathbf{y}] = \underline{\begin{bmatrix} \mathbf{s + 0.5} & \mathbf{-0.5} \\ \mathbf{-0.5} & \mathbf{0.5 + 1/s} \end{bmatrix}\mathbf{S}}$$

Chapter 19, Problem 20.

Find the *y* parameters for the circuit in Fig. 19.81.

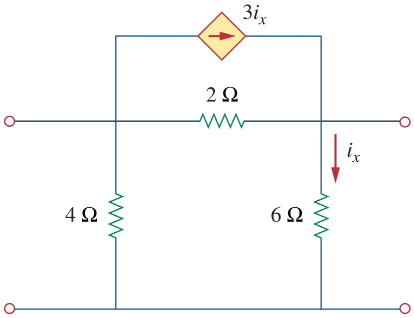
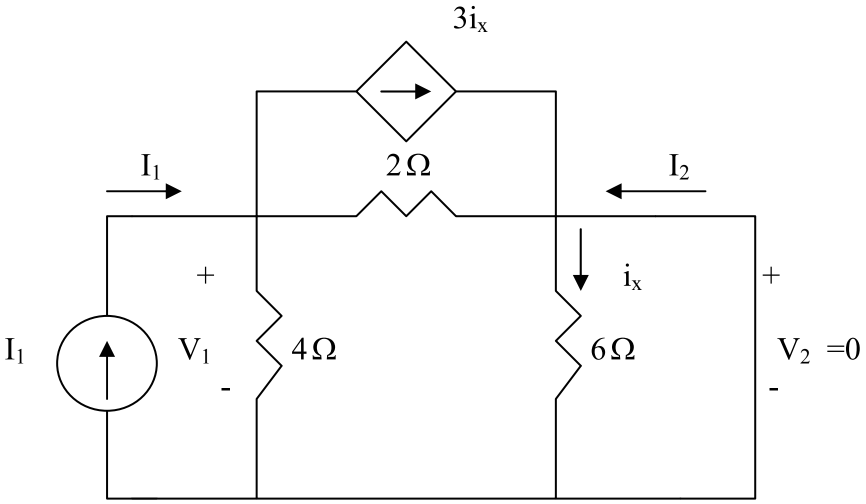


Figure 19.81

For Prob. 19.20.

Chapter 19, Solution 20.

To get y_{11} and y_{21}, consider the circuit below.

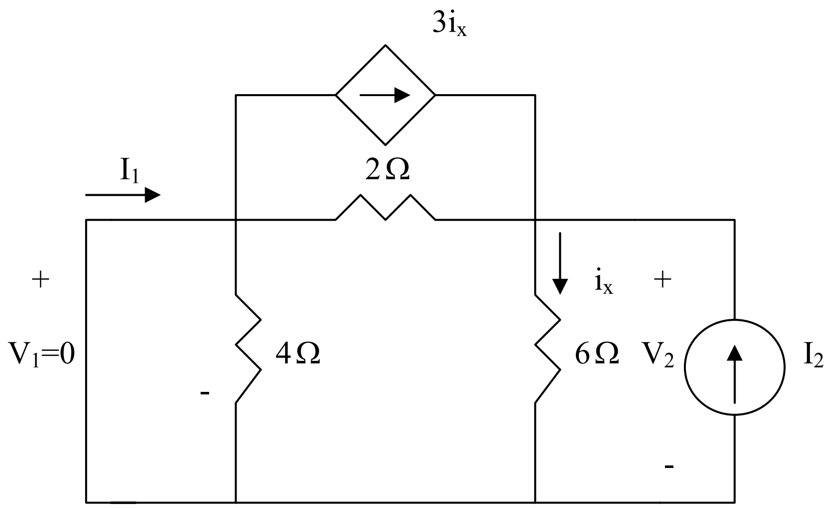


Since 6-ohm resistor is short-circuited, $i_x = 0$

$$V_1 = I_1(4//2) = \frac{8}{6}I_1 \longrightarrow y_{11} = \frac{I_1}{V_1} = 0.75$$

$$I_2 = -\frac{4}{4+2}I_1 = -\frac{2}{3}(\frac{6}{8}V_1) = -\frac{1}{2}V_1 \longrightarrow y_{21} = \frac{I_2}{V_1} = -0.5$$

To get y_{22} and y_{12}, consider the circuit below.



$$i_x = \frac{V_2}{6}, \quad I_2 = i_x - 3i_x + \frac{V_2}{2} = \frac{V_2}{6} \quad \longrightarrow \quad y_{22} = \frac{I_2}{V_2} = \frac{1}{6} = 0.1667$$

$$I_1 = 3i_x - \frac{V_2}{2} = 0 \quad \longrightarrow \quad y_{12} = \frac{I_1}{V_2} = 0$$

Thus,

$$\underline{[y] = \begin{bmatrix} 0.75 & 0 \\ -0.5 & 0.1667 \end{bmatrix} \text{S}}$$

Chapter 19, Problem 21.

Obtain the admittance parameter equivalent circuit of the two-port in Fig. 19.82.

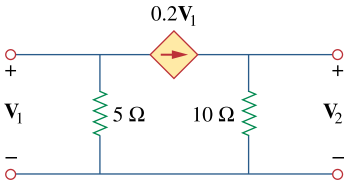
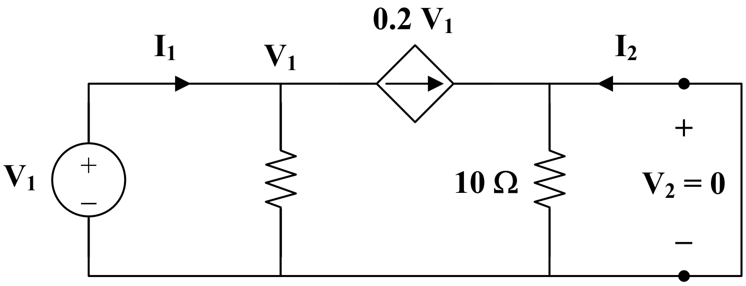


Figure 19.82

For Prob. 19.21.

Chapter 19, Solution 21.

To get $\mathbf{y}_{11}$ and $\mathbf{y}_{21}$, refer to Fig. (a).



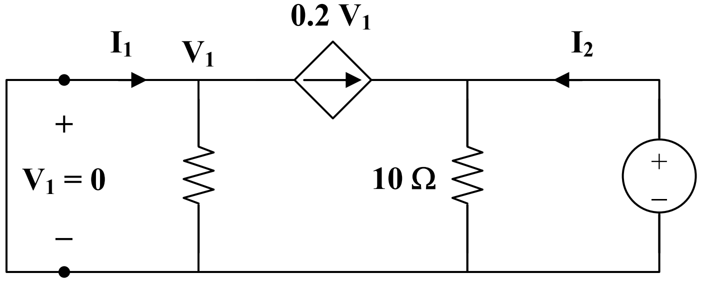
(a)

At node 1,

$$\mathbf{I}_1 = \frac{\mathbf{V}_1}{5} + 0.2\,\mathbf{V}_1 = 0.4\,\mathbf{V}_1 \longrightarrow \mathbf{y}_{11} = \frac{\mathbf{I}_1}{\mathbf{V}_1} = 0.4$$

$$\mathbf{I}_2 = -0.2\,\mathbf{V}_1 \longrightarrow \mathbf{y}_{21} = \frac{\mathbf{I}_2}{\mathbf{V}_1} = -0.2$$

To get $\mathbf{y}_{22}$ and $\mathbf{y}_{12}$, refer to the circuit in Fig. (b).



(b)

Since $\mathbf{V}_1 = 0$, the dependent current source can be replaced with an open circuit.

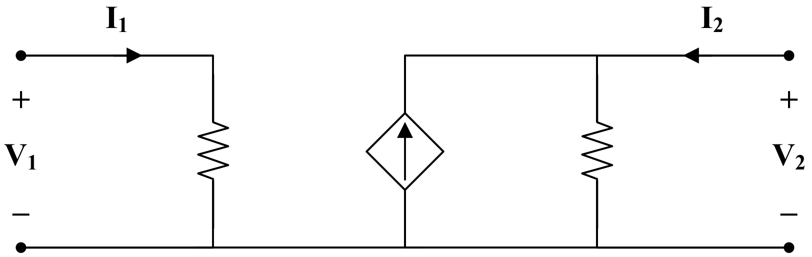
$$\mathbf{V}_2 = 10\mathbf{I}_2 \quad \longrightarrow \quad \mathbf{y}_{22} = \frac{\mathbf{I}_2}{\mathbf{V}_2} = \frac{1}{10} = 0.1$$

$$\mathbf{y}_{12} = \frac{\mathbf{I}_1}{\mathbf{V}_2} = 0$$

Thus,

$$[\mathbf{y}] = \begin{bmatrix} 0.4 & 0 \\ -0.2 & 0.1 \end{bmatrix} \text{S}$$

Consequently, **the y parameter equivalent circuit is shown in Fig. (c)**.



(c)

Chapter 19, Problem 22.

Obtain the *y* parameters of the two-port network in Fig. 19.83.

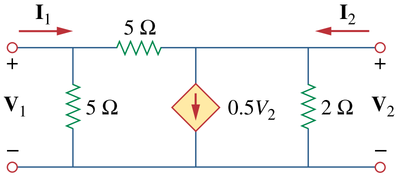
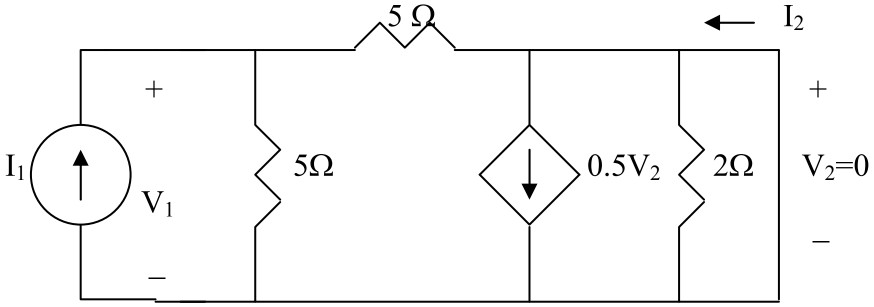


Figure 19.83

For Prob. 19.22.

Chapter 19, Solution 22.

To obtain y_{11} and y_{21}, consider the circuit below.

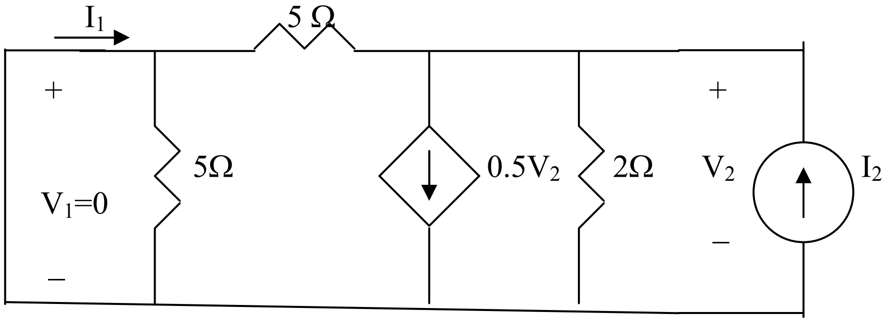


The 2-Ω resistor is short-circuited.

$$V_1 = 5\frac{I_1}{2} \longrightarrow y_{11} = \frac{I_1}{V_1} = \frac{2}{5} = 0.4$$

$$I_2 = \frac{1}{2}I_1 \longrightarrow y_{21} = \frac{I_2}{V_1} = \frac{\frac{1}{2}I_1}{2.5I_1} = 0.2$$

To obtain y_{12} and y_{22}, consider the circuit below.



At the top node, KCL gives

$$I_2 = 0.5V_2 + \frac{V_2}{2} + \frac{V_2}{5} = 1.2V_2 \longrightarrow y_{22} = \frac{I_2}{V_2} = 1.2$$

$$I_1 = -\frac{V_2}{5} = -0.2V_2 \longrightarrow y_{12} = \frac{I_1}{V_2} = -0.2$$

Hence,

$$\underline{[y] = \begin{bmatrix} 0.4 & -0.2 \\ 0.2 & 1.2 \end{bmatrix} \text{S}}$$

Chapter 19, Problem 23.

(a) Find the *y* parameters of the two-port in Fig. 19.84.
(b) Determine $\mathbf{V}_2(s)$ for $v_s = 2u(t)$V.

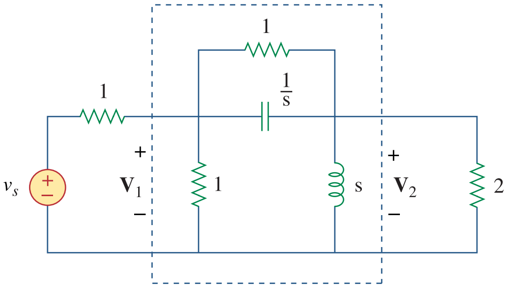


Figure 19.84
For Prob. 19.23.

Chapter 19, Solution 23.

(a)

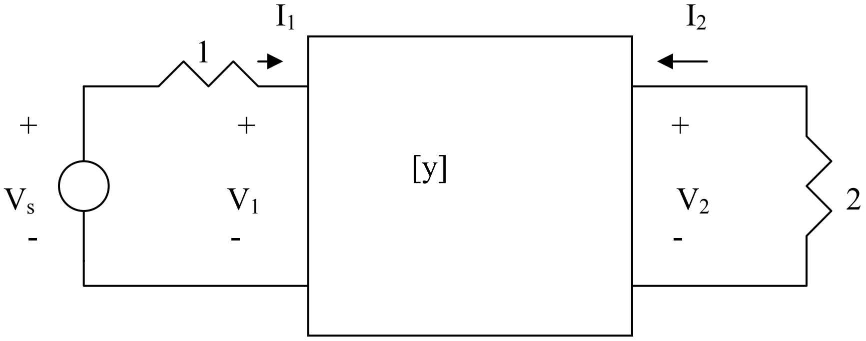
$$-y_{12} = 1/\left(1//\frac{1}{s}\right) = 1 + s \longrightarrow y_{12} = -(s+1)$$

$$y_{11} + y_{12} = 1 \longrightarrow y_{11} = 1 - y_{12} = 1 + s + 1 = s + 2$$

$$y_{22} + y_{12} = s \longrightarrow y_{22} = \frac{1}{s} - y_{12} = \frac{1}{s} + s + 1 = \frac{s^{2s} + s + 1}{s}$$

$$\underline{[y] = \begin{bmatrix} s+2 & -(s+1) \\ -(s+1) & \dfrac{s^2 + s + 1}{s} \end{bmatrix}}$$

(b) Consider the network below.



$$V_s = I_1 + V_1 \text{ or } V_s - V_1 = I_1 \qquad (1)$$

$$V_2 = -2I_2 \qquad (2)$$

$$I_1 = y_{11}V_1 + y_{12}V_2 \qquad (3)$$

$$I_2 = y_{21}V_1 + y_{22}V_2 \qquad (4)$$

From (1) and (3)

$$V_s - V_1 = y_{11}V_1 + y_{12}V_2 \quad \longrightarrow \quad V_s = (1+y_{11})V_1 + y_{12}V_2 \qquad (5)$$

From (2) and (4),

$$-0.5V_2 = y_{21}V_1 + y_{22}V_2 \quad \longrightarrow \quad V_1 = -\frac{1}{y_{21}}(0.5+y_{22})V_2 \qquad (6)$$

Substituting (6) into (5),

$$V_s = -\frac{(1+y_{11})(0.5+y_{22})}{y_{21}}V_2 + y_{12}V_2$$

$$= \frac{2}{s} \quad \longrightarrow \quad V_2 = \frac{2/s}{\left[y_{12} - \frac{1}{y_{21}}(1+y_{11})(0.5+y_{22})\right]}$$

$$V_2 = \frac{2/s}{-(s+1) + \frac{1}{s+1}(1+s+2)\left(\frac{1}{2} + \frac{s^2+s+1}{s}\right)} = \frac{0.8(s+1)}{(s^2+1.8s+1.2)}$$

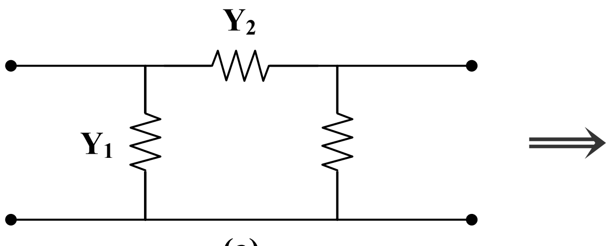
Chapter 19, Problem 24.

Find the resistive circuit that represents these *y* parameters:

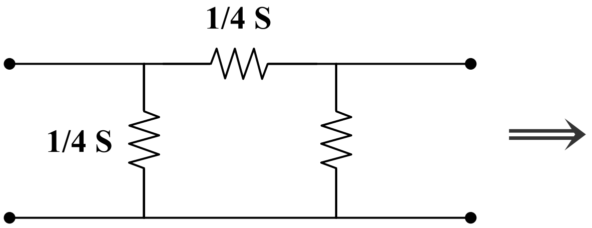
$$[\mathbf{y}] = \begin{bmatrix} \frac{1}{2} & -\frac{1}{4} \\ -\frac{1}{4} & \frac{3}{8} \end{bmatrix}$$

Chapter 19, Solution 24.

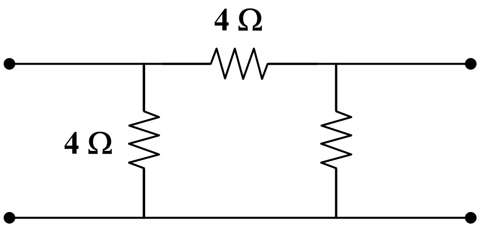
Since this is a reciprocal network, **a Π network is appropriate, as shown below**.



(a)



(b)



(c)

$$\mathbf{Y}_1 = \mathbf{y}_{11} + \mathbf{y}_{12} = \frac{1}{2} - \frac{1}{4} = \frac{1}{4}\,\text{S}, \qquad \mathbf{Z}_1 = \underline{\mathbf{4}\,\Omega}$$

$$\mathbf{Y}_2 = \text{-}\mathbf{y}_{12} = \frac{1}{4}\,\text{S}, \qquad \mathbf{Z}_2 = \underline{\mathbf{4}\,\Omega}$$

$$\mathbf{Y}_3 = \mathbf{y}_{22} + \mathbf{y}_{21} = \frac{3}{8} - \frac{1}{4} = \frac{1}{8}\,\text{S}, \qquad \mathbf{Z}_3 = \underline{\mathbf{8}\,\Omega}$$

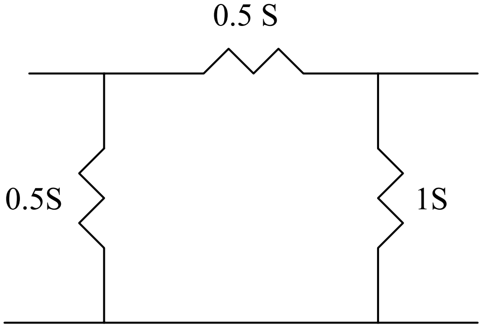
Chapter 19, Problem 25.

Draw the two-port network that has the following *y* parameters:

$$[\mathbf{y}] = \begin{bmatrix} 1 & -0.5 \\ -0.5 & 1.5 \end{bmatrix} \text{S}$$

Chapter 19, Solution 25.

This is a reciprocal network and is shown below.



Chapter 19, Problem 26.

Calculate [**y**] for the two-port in Fig. 19.85.

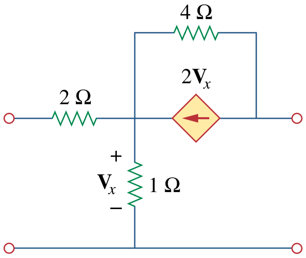
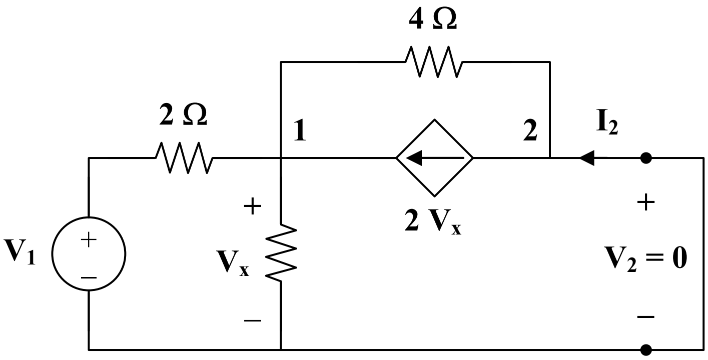


Figure 19.85

For Prob. 19.26.

Chapter 19, Solution 26.

To get $\mathbf{y}_{11}$ and $\mathbf{y}_{21}$, consider the circuit in Fig. (a).



(a)

At node 1,

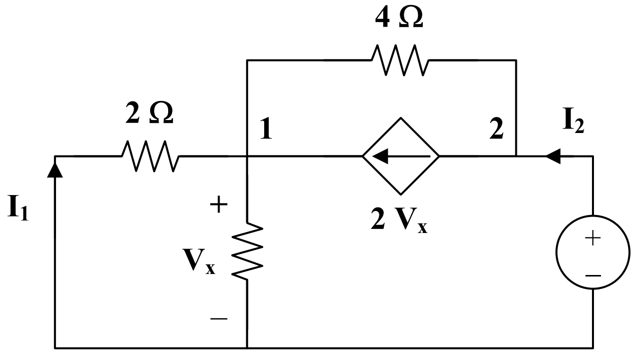
$$\frac{\mathbf{V}_1 - \mathbf{V}_x}{2} + 2\mathbf{V}_x = \frac{\mathbf{V}_x}{1} + \frac{\mathbf{V}_x}{4} \longrightarrow 2\mathbf{V}_1 = \text{-}\mathbf{V}_x \qquad (1)$$

But $$\mathbf{I}_1 = \frac{\mathbf{V}_1 - \mathbf{V}_x}{2} = \frac{\mathbf{V}_1 + 2\mathbf{V}_1}{2} = 1.5\mathbf{V}_1 \longrightarrow \mathbf{y}_{11} = \frac{\mathbf{I}_1}{\mathbf{V}_1} = 1.5$$

Also, $$\mathbf{I}_2 + \frac{\mathbf{V}_x}{4} = 2\mathbf{V}_x \longrightarrow \mathbf{I}_2 = 1.75\mathbf{V}_x = \text{-}3.5\mathbf{V}_1$$

$$\mathbf{y}_{21} = \frac{\mathbf{I}_2}{\mathbf{V}_1} = \text{-}3.5$$

To get $\mathbf{y}_{22}$ and $\mathbf{y}_{12}$, consider the circuit in Fig.(b).



(b)

At node 2,

$$\mathbf{I}_2 = 2\mathbf{V}_x + \frac{\mathbf{V}_2 - \mathbf{V}_x}{4} \qquad (2)$$

At node 1,

$$2\mathbf{V}_x + \frac{\mathbf{V}_2 - \mathbf{V}_x}{4} = \frac{\mathbf{V}_x}{2} + \frac{\mathbf{V}_x}{1} = \frac{3}{2}\mathbf{V}_x \longrightarrow \mathbf{V}_2 = \text{-}\mathbf{V}_x \qquad (3)$$

Substituting (3) into (2) gives

$$\mathbf{I}_2 = 2\mathbf{V}_x - \frac{1}{2}\mathbf{V}_x = 1.5\mathbf{V}_x = \text{-}1.5\mathbf{V}_2$$

$$\mathbf{y}_{22} = \frac{\mathbf{I}_2}{\mathbf{V}_2} = \text{-}1.5$$

$$\mathbf{I}_1 = \frac{\text{-}\mathbf{V}_x}{2} = \frac{\mathbf{V}_2}{2} \longrightarrow \mathbf{y}_{12} = \frac{\mathbf{I}_1}{\mathbf{V}_2} = 0.5$$

Thus,

$$\underline{[\mathbf{y}] = \begin{bmatrix} \mathbf{1.5} & \mathbf{0.5} \\ \mathbf{-3.5} & \mathbf{-1.5} \end{bmatrix} \mathbf{S}}$$

Chapter 19, Problem 27.

Find the *y* parameters for the circuit in Fig. 19.86.

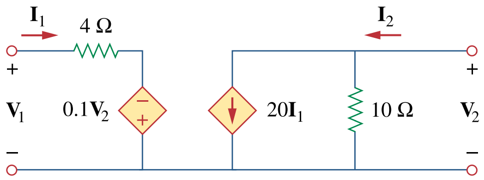
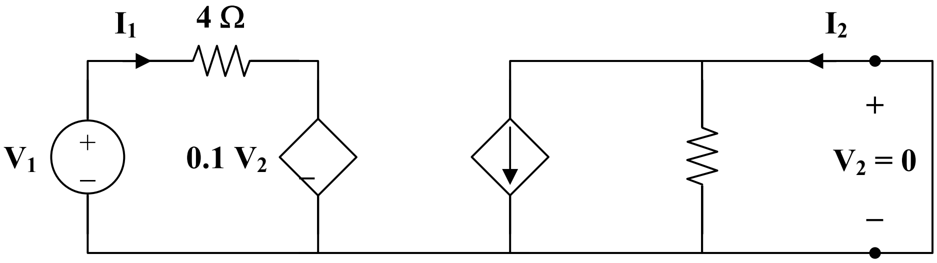


Figure 19.86

For Prob. 19.27.

Chapter 19, Solution 27.

Consider the circuit in Fig. (a).

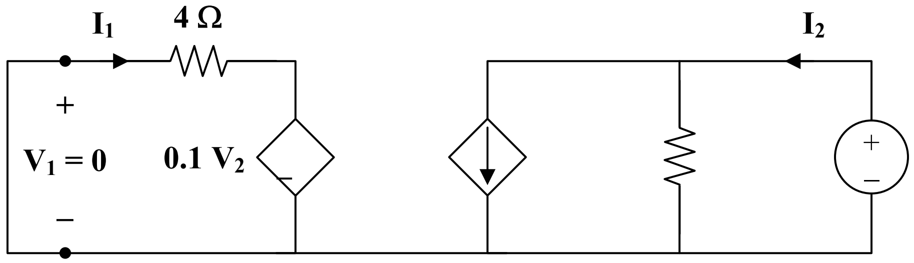


(a)

$$\mathbf{V}_1 = 4\mathbf{I}_1 \quad \longrightarrow \quad \mathbf{y}_{11} = \frac{\mathbf{I}_1}{\mathbf{V}_1} = \frac{\mathbf{I}_1}{4\mathbf{I}_1} = 0.25$$

$$\mathbf{I}_2 = 20\mathbf{I}_1 = 5\mathbf{V}_1 \quad \longrightarrow \quad \mathbf{y}_{21} = \frac{\mathbf{I}_2}{\mathbf{V}_1} = 5$$

Consider the circuit in Fig. (b).



(b)

$$4\mathbf{I}_1 = 0.1\mathbf{V}_2 \quad \longrightarrow \quad \mathbf{y}_{12} = \frac{\mathbf{I}_1}{\mathbf{V}_2} = \frac{0.1}{4} = 0.025$$

$$\mathbf{I}_2 = 20\mathbf{I}_1 + \frac{\mathbf{V}_2}{10} = 0.5\mathbf{V}_2 + 0.1\mathbf{V}_2 = 0.6\mathbf{V}_2 \quad \longrightarrow \quad \mathbf{y}_{22} = \frac{\mathbf{I}_2}{\mathbf{V}_2} = 0.6$$

Thus,

$$[\mathbf{y}] = \underline{\begin{bmatrix} \mathbf{0.25} & \mathbf{0.025} \\ \mathbf{5} & \mathbf{0.6} \end{bmatrix} \mathbf{S}}$$

Alternatively, from the given circuit,

$$\mathbf{V}_1 = 4\mathbf{I}_1 - 0.1\mathbf{V}_2$$
$$\mathbf{I}_2 = 20\mathbf{I}_1 + 0.1\mathbf{V}_2$$

Comparing these with the equations for the h parameters show that

$$\mathbf{h}_{11} = 4, \qquad \mathbf{h}_{12} = \text{-}0.1, \qquad \mathbf{h}_{21} = 20, \qquad \mathbf{h}_{22} = 0.1$$

Using Table 18.1,

$$\mathbf{y}_{11} = \frac{1}{\mathbf{h}_{11}} = \frac{1}{4} = 0.25, \qquad \mathbf{y}_{12} = \frac{\text{-}\mathbf{h}_{12}}{\mathbf{h}_{11}} = \frac{0.1}{4} = 0.025$$

$$\mathbf{y}_{21} = \frac{\mathbf{h}_{21}}{\mathbf{h}_{11}} = \frac{20}{4} = 5, \qquad \mathbf{y}_{22} = \frac{\Delta_h}{\mathbf{h}_{11}} = \frac{0.4 + 2}{4} = 0.6$$

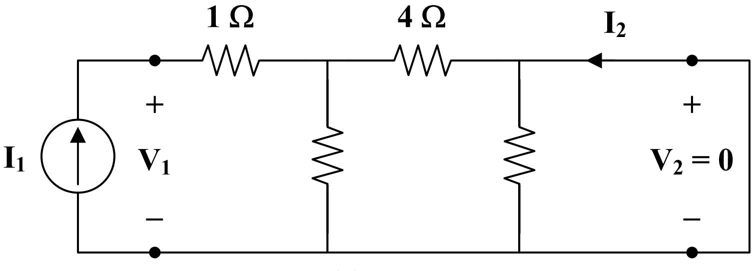
as above.

Chapter 19, Problem 28.

In the circuit of Fig. 19.65, the input port is connected to a 1-A dc current source. Calculate the power dissipated by the $2\text{-}\Omega$ resistor by using the *y* parameters. Confirm your result by direct circuit analysis.

Chapter 19, Solution 28.

We obtain $\mathbf{y}_{11}$ and $\mathbf{y}_{21}$ by considering the circuit in Fig.(a).



(a)

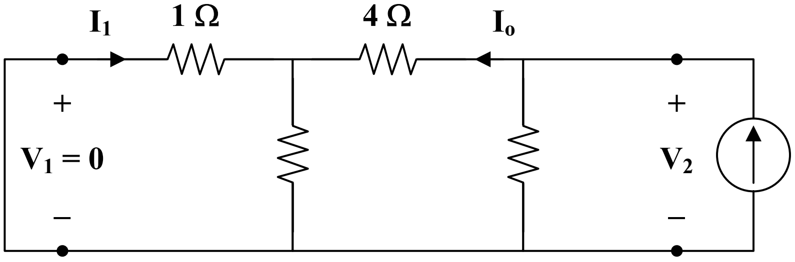
$$\mathbf{Z}_{in} = 1 + 6 \,||\, 4 = 3.4$$

$$\mathbf{y}_{11} = \frac{\mathbf{I}_1}{\mathbf{V}_1} = \frac{1}{\mathbf{Z}_{in}} = 0.2941$$

$$\mathbf{I}_2 = \frac{-6}{10}\mathbf{I}_1 = \left(\frac{-6}{10}\right)\left(\frac{\mathbf{V}_1}{3.4}\right) = \frac{-6}{34}\mathbf{V}_1$$

$$\mathbf{y}_{21} = \frac{\mathbf{I}_2}{\mathbf{V}_1} = \frac{-6}{34} = -0.1765$$

To get $\mathbf{y}_{22}$ and $\mathbf{y}_{12}$, consider the circuit in Fig. (b).



(b)

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$$\frac{1}{\mathbf{y}_{22}} = 2 \,||\, (4 + 6 \,||\, 1) = 2 \,||\left(4 + \frac{6}{7}\right) = \frac{(2)(34/7)}{2 + (34/7)} = \frac{34}{24} = \frac{V_2}{I_2}$$

$$\mathbf{y}_{22} = \frac{24}{34} = 0.7059$$

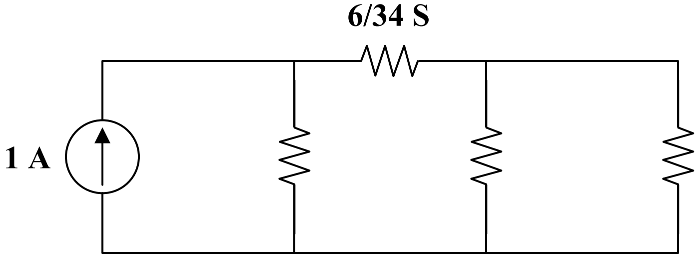
$$\mathbf{I}_1 = \frac{-6}{7}\mathbf{I}_o \qquad \mathbf{I}_o = \frac{2}{2 + (34/7)}\mathbf{I}_2 = \frac{14}{48}\mathbf{I}_2 = \frac{7}{34}\mathbf{V}_2$$

$$\mathbf{I}_1 = \frac{-6}{34}\mathbf{V}_2 \longrightarrow \mathbf{y}_{12} = \frac{\mathbf{I}_1}{\mathbf{V}_2} = \frac{-6}{34} = -0.1765$$

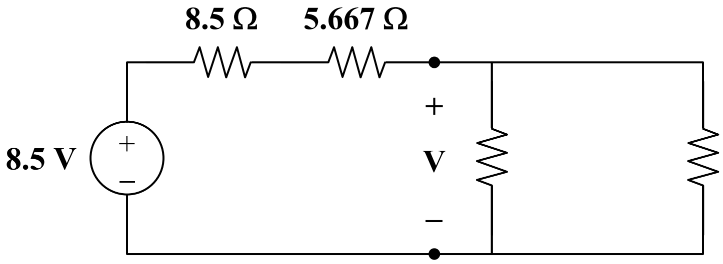
Thus,

$$[\mathbf{y}] = \begin{bmatrix} 0.2941 & -0.1765 \\ -0.1765 & 0.7059 \end{bmatrix} \mathrm{S}$$

The equivalent circuit is shown in Fig. (c). After transforming the current source to a voltage source, we have the circuit in Fig. (d).



(c)



(d)

$$V = \frac{(2 \,||\, 1.889)(8.5)}{2 \,||\, 1.889 + 8.5 + 5.667} = \frac{(0.9714)(8.5)}{0.9714 + 14.167} = 0.5454$$

$$P = \frac{V^2}{R} = \frac{(0.5454)^2}{2} = \underline{\mathbf{0.1487\ W}}$$

Chapter 19, Problem 29.

In the bridge circuit of Fig. 19.87, $I_1 = 10$ A and $I_2 = -4$ A

(a) Find V_1 and V_2 using *y* parameters.
(b) -Confirm the results in part (a) by direct circuit analysis.

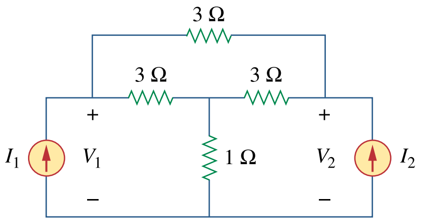
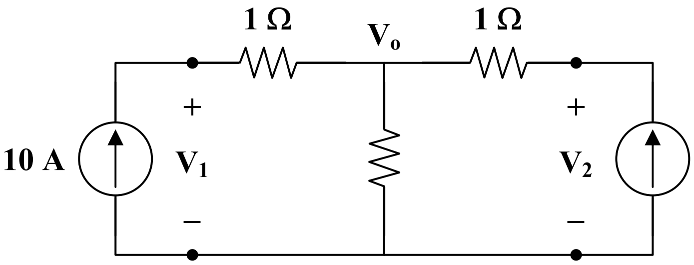


Figure 19.87

For Prob. 19.29.

Chapter 19, Solution 29.

(a) Transforming the Δ subnetwork to Y gives the circuit in Fig. (a).



(a)

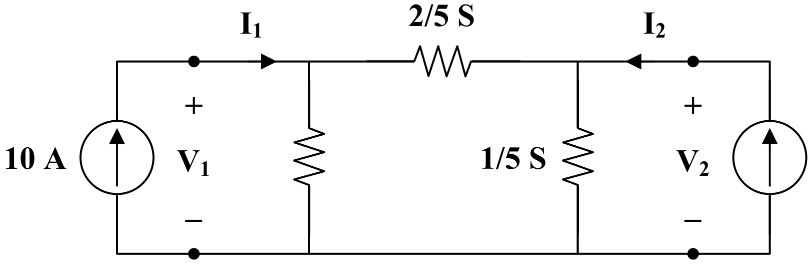
It is easy to get the z parameters

$$\mathbf{z}_{12} = \mathbf{z}_{21} = 2, \quad \mathbf{z}_{11} = 1 + 2 = 3, \quad \mathbf{z}_{22} = 3$$

$$\Delta_z = \mathbf{z}_{11}\mathbf{z}_{22} - \mathbf{z}_{12}\mathbf{z}_{21} = 9 - 4 = 5$$

$$\mathbf{y}_{11} = \frac{\mathbf{z}_{22}}{\Delta_z} = \frac{3}{5} = \mathbf{y}_{22}, \qquad \mathbf{y}_{12} = \mathbf{y}_{21} = \frac{-\mathbf{z}_{12}}{\Delta_z} = \frac{-2}{5}$$

Thus, the equivalent circuit is as shown in Fig. (b).



(b)

$$\mathbf{I}_1 = 10 = \frac{3}{5}\mathbf{V}_1 - \frac{2}{5}\mathbf{V}_2 \longrightarrow 50 = 3\mathbf{V}_1 - 2\mathbf{V}_2 \qquad (1)$$

$$\mathbf{I}_2 = \text{-}4 = \frac{\text{-}2}{5}\mathbf{V}_1 + \frac{3}{5}\mathbf{V}_2 \longrightarrow \text{-}20 = \text{-}2\mathbf{V}_1 + 3\mathbf{V}_2$$

$$10 = \mathbf{V}_1 - 1.5\mathbf{V}_2 \longrightarrow \mathbf{V}_1 = 10 + 1.5\mathbf{V}_2 \qquad (2)$$

Substituting (2) into (1),

$$50 = 30 + 4.5\mathbf{V}_2 - 2\mathbf{V}_2 \longrightarrow \mathbf{V}_2 = \underline{\mathbf{8\ V}}$$

$$\mathbf{V}_1 = 10 + 1.5\mathbf{V}_2 = \underline{\mathbf{22\ V}}$$

(b) For direct circuit analysis, consider the circuit in Fig. (a).

For the main non-reference node,

$$10 - 4 = \frac{\mathbf{V}_o}{2} \longrightarrow \mathbf{V}_o = 12$$

$$10 = \frac{\mathbf{V}_1 - \mathbf{V}_o}{1} \longrightarrow \mathbf{V}_1 = 10 + \mathbf{V}_o = \underline{\mathbf{22\ V}}$$

$$\text{-}4 = \frac{\mathbf{V}_2 - \mathbf{V}_o}{1} \longrightarrow \mathbf{V}_2 = \mathbf{V}_o - 4 = \underline{\mathbf{8\ V}}$$

Chapter 19, Problem 30.

Find the *h* parameters for the networks in Fig. 19.88.

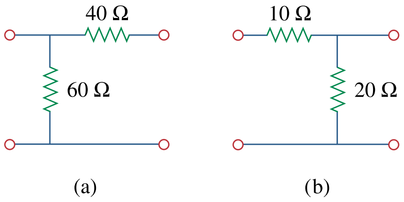


Figure 19.88

For Prob. 19.30.

Chapter 19, Solution 30.

(a) Convert to z parameters; then, convert to h parameters using Table 18.1.

$$\mathbf{z}_{11} = \mathbf{z}_{12} = \mathbf{z}_{21} = 60\,\Omega, \qquad \mathbf{z}_{22} = 100\,\Omega$$

$$\Delta_z = \mathbf{z}_{11}\mathbf{z}_{22} - \mathbf{z}_{12}\mathbf{z}_{21} = 6000 - 3600 = 2400$$

$$\mathbf{h}_{11} = \frac{\Delta_z}{\mathbf{z}_{22}} = \frac{2400}{100} = 24, \qquad \mathbf{h}_{12} = \frac{\mathbf{z}_{12}}{\mathbf{z}_{22}} = \frac{60}{100} = 0.6$$

$$\mathbf{h}_{21} = \frac{\text{-}\,\mathbf{z}_{21}}{\mathbf{z}_{22}} = \text{-}0.6, \qquad \mathbf{h}_{22} = \frac{1}{\mathbf{z}_{22}} = 0.01$$

Thus,

$$\underline{[\mathbf{h}] = \begin{bmatrix} \mathbf{24\,\Omega} & \mathbf{0.6} \\ \mathbf{\text{-}0.6} & \mathbf{0.01\,S} \end{bmatrix}}$$

(b) Similarly,

$$\mathbf{z}_{11} = 30\,\Omega \qquad \mathbf{z}_{12} = \mathbf{z}_{21} = \mathbf{z}_{22} = 20\,\Omega$$

$$\Delta_z = 600 - 400 = 200$$

$$\mathbf{h}_{11} = \frac{200}{20} = 10 \qquad \mathbf{h}_{12} = \frac{20}{20} = 1$$

$$\mathbf{h}_{21} = \text{-}1 \qquad \mathbf{h}_{22} = \frac{1}{20} = 0.05$$

Thus,

$$\underline{[\mathbf{h}] = \begin{bmatrix} \mathbf{10\,\Omega} & \mathbf{1} \\ \mathbf{\text{-}1} & \mathbf{0.05\,S} \end{bmatrix}}$$

Chapter 19, Problem 31.

Determine the hybrid parameters for the network in Fig. 19.89.

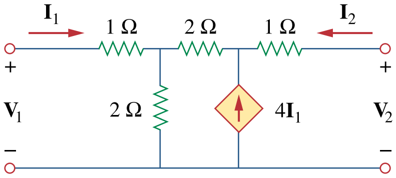
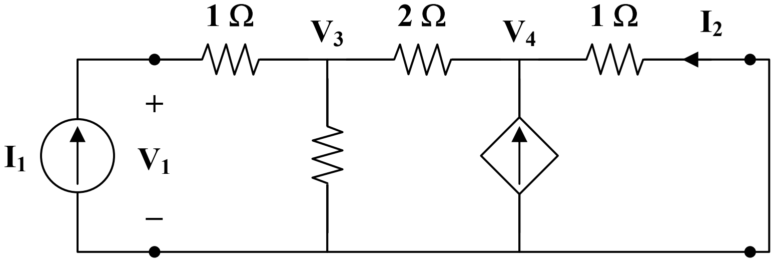


Figure 19.89

For Prob. 19.31.

Chapter 19, Solution 31.

We get $\mathbf{h}_{11}$ and $\mathbf{h}_{21}$ by considering the circuit in Fig. (a).



(a)

At node 1,

$$\mathbf{I}_1 = \frac{\mathbf{V}_3}{2} + \frac{\mathbf{V}_3 - \mathbf{V}_4}{2} \quad \longrightarrow \quad 2\mathbf{I}_1 = 2\mathbf{V}_3 - \mathbf{V}_4 \qquad (1)$$

At node 2,

$$\frac{\mathbf{V}_3 - \mathbf{V}_4}{2} + 4\mathbf{I}_1 = \frac{\mathbf{V}_4}{1}$$

$$8\mathbf{I}_1 = \text{-}\mathbf{V}_3 + 3\mathbf{V}_4 \quad \longrightarrow \quad 16\mathbf{I}_1 = \text{-}2\mathbf{V}_3 + 6\mathbf{V}_4 \qquad (2)$$

Adding (1) and (2),

$$18\mathbf{I}_1 = 5\mathbf{V}_4 \longrightarrow \mathbf{V}_4 = 3.6\mathbf{I}_1$$

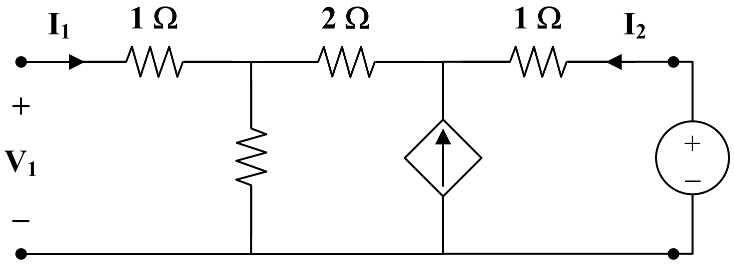
$$\mathbf{V}_3 = 3\mathbf{V}_4 - 8\mathbf{I}_1 = 2.8\mathbf{I}_1$$

$$\mathbf{V}_1 = \mathbf{V}_3 + \mathbf{I}_1 = 3.8\mathbf{I}_1$$

$$\mathbf{h}_{11} = \frac{\mathbf{V}_1}{\mathbf{I}_1} = 3.8\,\Omega$$

$$\mathbf{I}_2 = \frac{\text{-}\mathbf{V}_4}{1} = \text{-}3.6\mathbf{I}_1 \longrightarrow \mathbf{h}_{21} = \frac{\mathbf{I}_2}{\mathbf{I}_1} = \text{-}3.6$$

To get $\mathbf{h}_{22}$ and $\mathbf{h}_{12}$, refer to the circuit in Fig. (b). The dependent current source can be replaced by an open circuit since $4\mathbf{I}_1 = 0$.



(b)

$$\mathbf{V}_1 = \frac{2}{2+2+1}\mathbf{V}_2 = \frac{2}{5}\mathbf{V}_2 \longrightarrow \mathbf{h}_{12} = \frac{\mathbf{V}_1}{\mathbf{V}_2} = 0.4$$

$$\mathbf{I}_2 = \frac{\mathbf{V}_2}{2+2+1} = \frac{\mathbf{V}_2}{5} \longrightarrow \mathbf{h}_{22} = \frac{\mathbf{I}_2}{\mathbf{V}_2} = \frac{1}{5} = 0.2\text{ S}$$

Thus,

$$[\mathbf{h}] = \underline{\begin{bmatrix} \mathbf{38\,\Omega} & \mathbf{0.4} \\ \mathbf{\text{-}3.6} & \mathbf{0.2\,S} \end{bmatrix}}$$

Chapter 19, Problem 32.

Find the *h* and *g* parameters of the two-port network in Fig. 19.90 as functions of *s*.

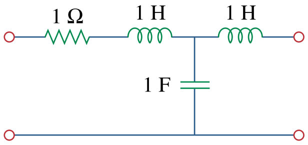
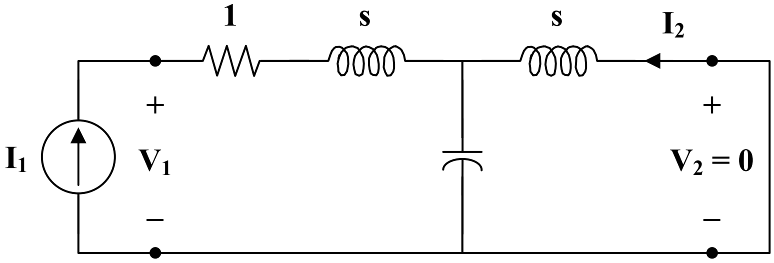


Figure 19.90

For Prob. 19.32.

Chapter 19, Solution 32.

(a) We obtain $\mathbf{h}_{11}$ and $\mathbf{h}_{21}$ by referring to the circuit in Fig. (a).



(a)

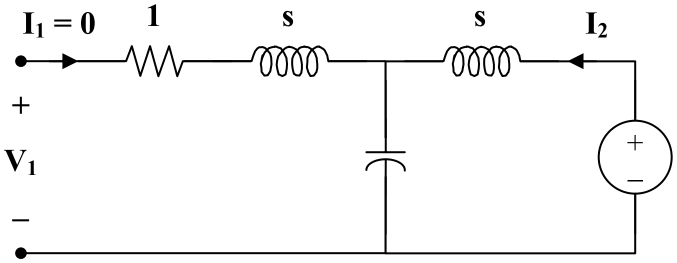
$$\mathbf{V}_1 = \left(1 + s + s \,\|\, \frac{1}{s}\right)\mathbf{I}_1 = \left(1 + s + \frac{s}{s^2+1}\right)\mathbf{I}_1$$

$$\mathbf{h}_{11} = \frac{\mathbf{V}_1}{\mathbf{I}_1} = s + 1 + \frac{s}{s^2+1}$$

By current division,

$$\mathbf{I}_2 = \frac{-1/s}{s+1/s}\mathbf{I}_1 = \frac{-\mathbf{I}_1}{s+1} \longrightarrow \mathbf{h}_{21} = \frac{\mathbf{I}_2}{\mathbf{I}_1} = \frac{-1}{s^2+1}$$

To get $\mathbf{h}_{22}$ and $\mathbf{h}_{12}$, refer to Fig. (b).



(b)

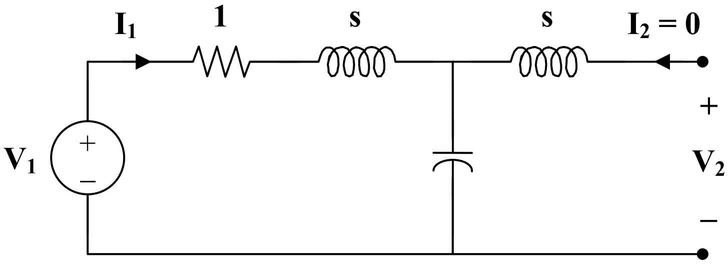
$$\mathbf{V}_1 = \frac{1/s}{s+1/s}\mathbf{V}_2 = \frac{\mathbf{V}_2}{s^2+1} \longrightarrow \mathbf{h}_{12} = \frac{\mathbf{V}_1}{\mathbf{V}_2} = \frac{1}{s^2+1}$$

$$\mathbf{V}_2 = \left(s + \frac{1}{s}\right)\mathbf{I}_2 \longrightarrow \mathbf{h}_{22} = \frac{\mathbf{I}_2}{\mathbf{V}_2} = \frac{1}{s+1/s} = \frac{s}{s^2+1}$$

Thus,

$$\underline{[\mathbf{h}] = \begin{bmatrix} \mathbf{s+1+\dfrac{s}{s^2+1}} & \mathbf{\dfrac{1}{s^2+1}} \\ \mathbf{\dfrac{-1}{s^2+1}} & \mathbf{\dfrac{s}{s^2+1}} \end{bmatrix}}$$

(b) To get $\mathbf{g}_{11}$ and $\mathbf{g}_{21}$, refer to Fig. (c).

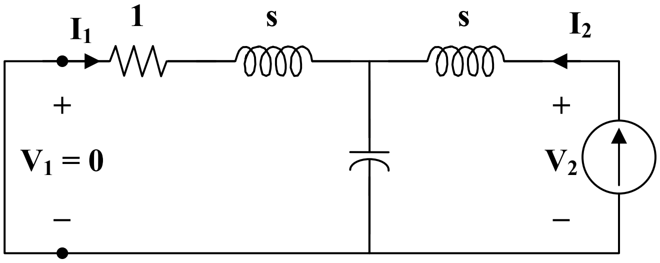


(c)

$$\mathbf{V}_1 = \left(1 + s + \frac{1}{s}\right)\mathbf{I}_1 \quad \longrightarrow \quad \mathbf{g}_{11} = \frac{\mathbf{I}_1}{\mathbf{V}_1} = \frac{1}{1 + s + 1/s} = \frac{s}{s^2 + s + 1}$$

$$\mathbf{V}_2 = \frac{1/s}{1 + s + 1/s}\mathbf{V}_1 = \frac{\mathbf{V}_1}{s^2 + s + 1} \quad \longrightarrow \quad \mathbf{g}_{21} = \frac{\mathbf{V}_2}{\mathbf{V}_1} = \frac{1}{s^2 + s + 1}$$

To get $\mathbf{g}_{22}$ and $\mathbf{g}_{12}$, refer to Fig. (d).



(d)

$$\mathbf{V}_2 = \left(s + \frac{1}{s} \,\|\, (s+1)\right)\mathbf{I}_2 = \left(s + \frac{(s+1)/s}{1 + s + 1/s}\right)\mathbf{I}_2$$

$$\mathbf{g}_{22} = \frac{\mathbf{V}_2}{\mathbf{I}_2} = s + \frac{s+1}{s^2 + s + 1}$$

$$\mathbf{I}_1 = \frac{-1/s}{1 + s + 1/s}\mathbf{I}_2 = \frac{-\mathbf{I}_2}{s^2 + s + 1} \quad \longrightarrow \quad \mathbf{g}_{12} = \frac{\mathbf{I}_1}{\mathbf{I}_2} = \frac{-1}{s^2 + s + 1}$$

Thus,

$$\underline{[\mathbf{g}] = \begin{bmatrix} \dfrac{\mathbf{s}}{\mathbf{s}^2 + \mathbf{s} + \mathbf{1}} & \dfrac{-\mathbf{1}}{\mathbf{s}^2 + \mathbf{s} + \mathbf{1}} \\ \dfrac{\mathbf{1}}{\mathbf{s}^2 + \mathbf{s} + \mathbf{1}} & \mathbf{s} + \dfrac{\mathbf{s} + \mathbf{1}}{\mathbf{s}^2 + \mathbf{s} + \mathbf{1}} \end{bmatrix}}$$

Chapter 19, Problem 33.

Obtain the *h* parameters for the two-port of Fig. 19.91.

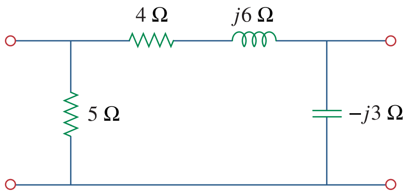
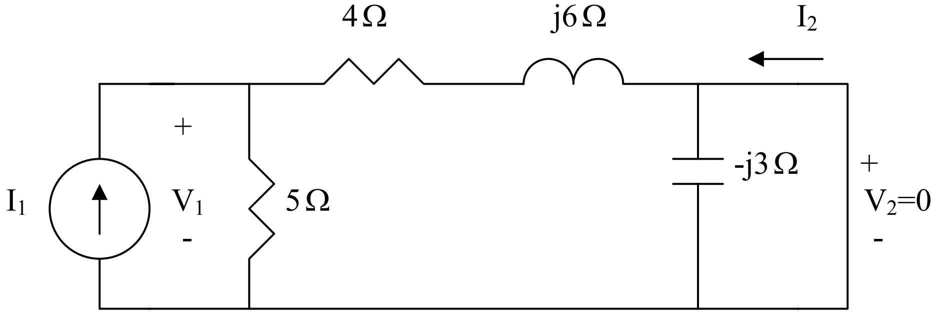


Figure 19.91

For Prob. 19.33.

Chapter 19, Solution 33.

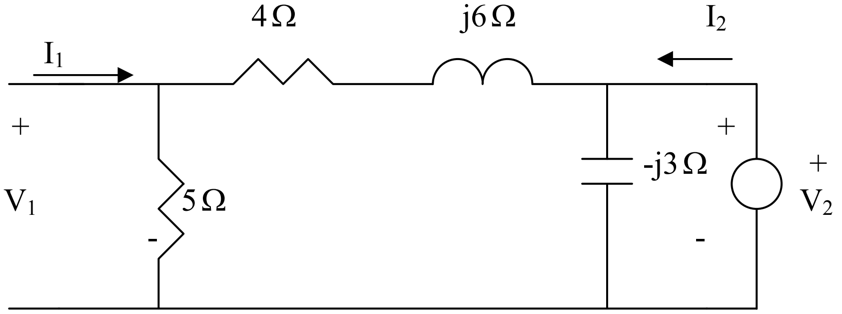
To get h_{11} and h_{21}, consider the circuit below.



$$V_1 = 5//(4+j6)I_1 = \frac{5(4+j6)I_1}{9+j6} \qquad h_{11} = \frac{V_1}{I_1} = 3.0769 + j1.2821$$

Also, $I_2 = -\frac{5}{9+j6}I_1 \quad \longrightarrow \quad h_{21} = \frac{I_2}{I_1} = -0.3846 + j0.2564$

To get h_{22} and h_{12}, consider the circuit below.



$$V_1 = \frac{5}{9+j6}V_2 \quad \longrightarrow \quad h_{12} = \frac{V_1}{V_2} = \frac{5}{9+j6} = 0.3846 - j0.2564$$

$$V_2 = -j3//(9+j6)I_2 \quad \longrightarrow \quad h_{22} = \frac{I_2}{V_2} = \frac{1}{-j3//(9+j6)} = \frac{9+j3}{-j3(9+j6)}$$
$$= 0.0769 + j0.2821$$

Thus,

$$[h] = \begin{bmatrix} 3.077 + j1.2821 & 0.3846 - j0.2564 \\ -0.3846 + j0.2564 & 0.0769 + j0.2821 \end{bmatrix}$$

Chapter 19, Problem 34.

Obtain the *h* and *g* parameters of the two-port in Fig. 19.92.

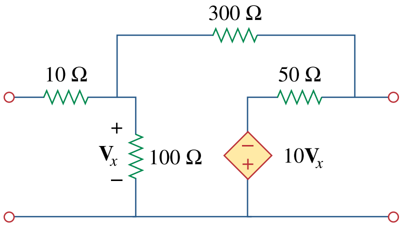
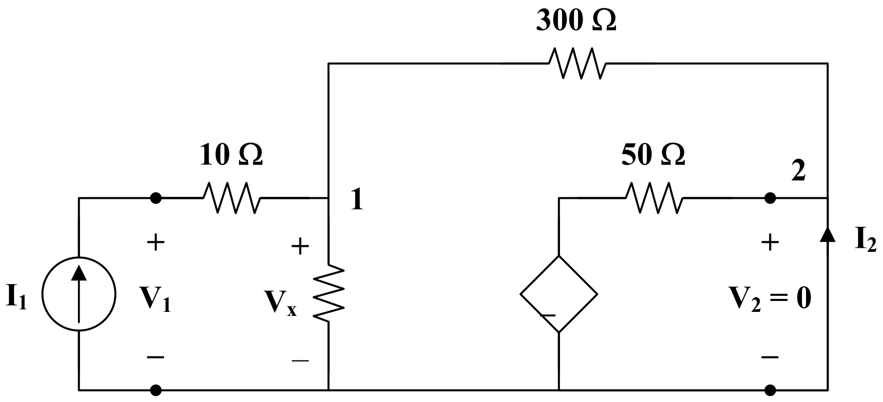


Figure 19.92

For Prob. 19.34.

Chapter 19, Solution 34.

Refer to Fig. (a) to get $\mathbf{h}_{11}$ and $\mathbf{h}_{21}$.



(a)

At node 1,

$$\mathbf{I}_1 = \frac{\mathbf{V}_x}{100} + \frac{\mathbf{V}_x - 0}{300} \longrightarrow 300\mathbf{I}_1 = 4\mathbf{V}_x \qquad (1)$$

$$\mathbf{V}_x = \frac{300}{4}\mathbf{I}_1 = 75\mathbf{I}_1$$

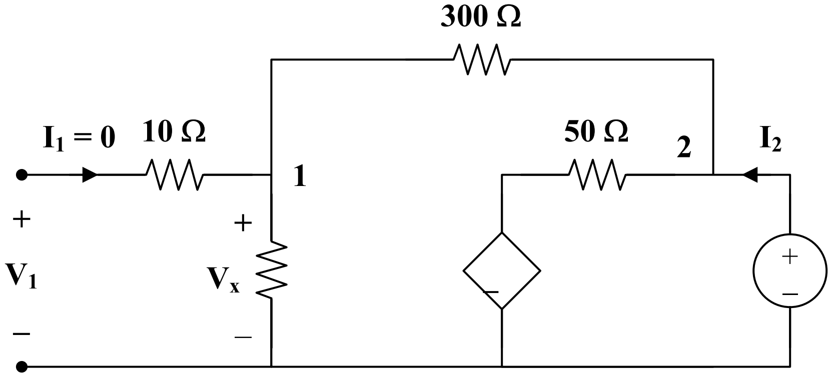
But $\mathbf{V}_1 = 10\mathbf{I}_1 + \mathbf{V}_x = 85\mathbf{I}_1 \longrightarrow \mathbf{h}_{11} = \frac{\mathbf{V}_1}{\mathbf{I}_1} = 85\,\Omega$

At node 2,

$$\mathbf{I}_2 = \frac{0+10\mathbf{V}_x}{50} - \frac{\mathbf{V}_x}{300} = \frac{\mathbf{V}_x}{5} - \frac{\mathbf{V}_x}{300} = \frac{75}{5}\mathbf{I}_1 - \frac{75}{300}\mathbf{I}_1 = 14.75\mathbf{I}_1$$

$$\mathbf{h}_{21} = \frac{\mathbf{I}_2}{\mathbf{I}_1} = 14.75$$

To get $\mathbf{h}_{22}$ and $\mathbf{h}_{12}$, refer to Fig. (b).



(b)

At node 2,

$$\mathbf{I}_2 = \frac{\mathbf{V}_2}{400} + \frac{\mathbf{V}_2 + 10\mathbf{V}_x}{50} \quad \longrightarrow \quad 400\mathbf{I}_2 = 9\mathbf{V}_2 + 80\mathbf{V}_x$$

But $$\mathbf{V}_x = \frac{100}{400}\mathbf{V}_2 = \frac{\mathbf{V}_2}{4}$$

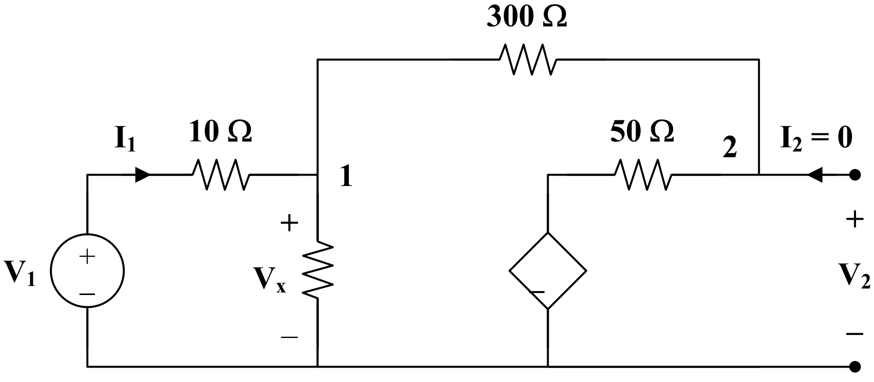
Hence, $$400\mathbf{I}_2 = 9\mathbf{V}_2 + 20\mathbf{V}_2 = 29\mathbf{V}_2$$

$$\mathbf{h}_{22} = \frac{\mathbf{I}_2}{\mathbf{V}_2} = \frac{29}{400} = 0.0725 \text{ S}$$

$$\mathbf{V}_1 = \mathbf{V}_x = \frac{\mathbf{V}_2}{4} \quad \longrightarrow \quad \mathbf{h}_{12} = \frac{\mathbf{V}_1}{\mathbf{V}_2} = \frac{1}{4} = 0.25$$

$$[\mathbf{h}] = \underline{\begin{bmatrix} \mathbf{85\,\Omega} & \mathbf{0.25} \\ \mathbf{14.75} & \mathbf{0.0725\ S} \end{bmatrix}}$$

To get $\mathbf{g}_{11}$ and $\mathbf{g}_{21}$, refer to Fig. (c).



(c)

At node 1,

$$\mathbf{I}_1 = \frac{\mathbf{V}_x}{100} + \frac{\mathbf{V}_x + 10\mathbf{V}_x}{350} \longrightarrow 350\mathbf{I}_1 = 14.5\mathbf{V}_x \qquad (2)$$

But
$$\mathbf{I}_1 = \frac{\mathbf{V}_1 - \mathbf{V}_x}{10} \longrightarrow 10\mathbf{I}_1 = \mathbf{V}_1 - \mathbf{V}_x$$

or
$$\mathbf{V}_x = \mathbf{V}_1 - 10\mathbf{I}_1 \qquad (3)$$

Substituting (3) into (2) gives

$$350\mathbf{I}_1 = 14.5\mathbf{V}_1 - 145\mathbf{I}_1 \longrightarrow 495\mathbf{I}_1 = 14.5\mathbf{V}_1$$

$$\mathbf{g}_{11} = \frac{\mathbf{I}_1}{\mathbf{V}_1} = \frac{14.5}{495} = 0.02929 \text{ S}$$

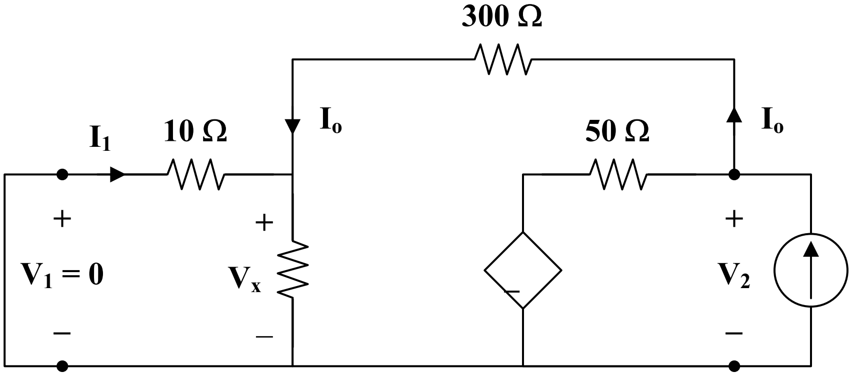
At node 2,

$$\mathbf{V}_2 = (50)\left(\frac{11}{350}\mathbf{V}_x\right) - 10\mathbf{V}_x = \text{-}8.4286\mathbf{V}_x$$

$$= \text{-}8.4286\mathbf{V}_1 + 84.286\mathbf{I}_1 = \text{-}8.4286\mathbf{V}_1 + (84.286)\left(\frac{14.5}{495}\right)\mathbf{V}_1$$

$$\mathbf{V}_2 = \text{-}5.96\mathbf{V}_1 \longrightarrow \mathbf{g}_{21} = \frac{\mathbf{V}_2}{\mathbf{V}_1} = \text{-}5.96$$

To get $\mathbf{g}_{22}$ and $\mathbf{g}_{12}$, refer to Fig. (d).



(d)

$$10 \| 100 = 9.091$$

$$\mathbf{I}_2 = \frac{\mathbf{V}_2 + 10\mathbf{V}_x}{50} + \frac{\mathbf{V}_2}{300 + 9.091}$$

$$309.091\mathbf{I}_2 = 7.1818\mathbf{V}_2 + 61.818\mathbf{V}_x \qquad (4)$$

But $$\mathbf{V}_x = \frac{9.091}{309.091}\mathbf{V}_2 = 0.02941\mathbf{V}_2 \qquad (5)$$

Substituting (5) into (4) gives

$$309.091\mathbf{I}_2 = 9\mathbf{V}_2$$

$$\mathbf{g}_{22} = \frac{\mathbf{V}_2}{\mathbf{I}_2} = 34.34\,\Omega$$

$$\mathbf{I}_o = \frac{\mathbf{V}_2}{309.091} = \frac{34.34\mathbf{I}_2}{309.091}$$

$$\mathbf{I}_1 = \frac{-100}{110}\mathbf{I}_o = \frac{-34.34\mathbf{I}_2}{(1.1)(309.091)}$$

$$\mathbf{g}_{12} = \frac{\mathbf{I}_1}{\mathbf{I}_2} = -0.101$$

Thus,

$$\underline{[\mathbf{g}] = \begin{bmatrix} \mathbf{0.02929\,S} & \mathbf{-0.101} \\ \mathbf{-5.96} & \mathbf{34.34\,\Omega} \end{bmatrix}}$$

Chapter 19, Problem 35.

Determine the *h* parameters for the network in Fig. 19.93.

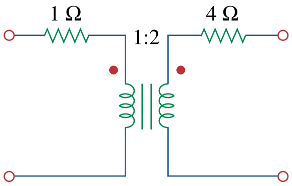
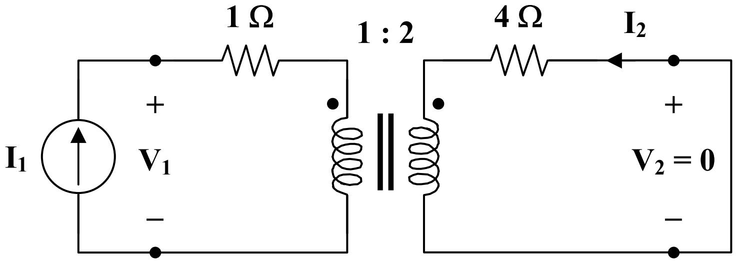


Figure 19.93

For Prob. 19.35.

Chapter 19, Solution 35.

To get $\mathbf{h}_{11}$ and $\mathbf{h}_{21}$ consider the circuit in Fig. (a).



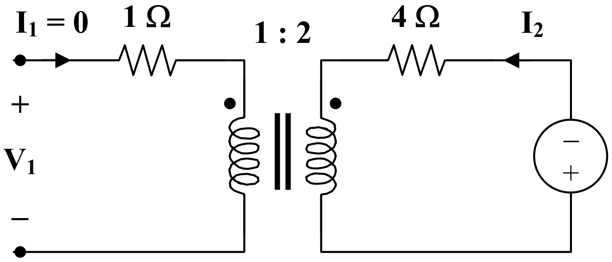
(a)

$$Z_R = \frac{4}{n^2} = \frac{4}{4} = 1$$

$$\mathbf{V}_1 = (1+1)\mathbf{I}_1 = 2\mathbf{I}_1 \longrightarrow \mathbf{h}_{11} = \frac{\mathbf{V}_1}{\mathbf{I}_1} = 2\,\Omega$$

$$\frac{\mathbf{I}_1}{\mathbf{I}_2} = \frac{-N_2}{N_1} = -2 \longrightarrow \mathbf{h}_{21} = \frac{\mathbf{I}_2}{\mathbf{I}_1} = \frac{-1}{2} = -0.5$$

To get $\mathbf{h}_{22}$ and $\mathbf{h}_{12}$, refer to Fig. (b).



(b)

Since $\mathbf{I}_1 = 0$, $\mathbf{I}_2 = 0$.

Hence, $\mathbf{h}_{22} = 0$.

At the terminals of the transformer, we have $\mathbf{V}_1$ and $\mathbf{V}_2$ which are related as

$$\frac{\mathbf{V}_2}{\mathbf{V}_1} = \frac{N_2}{N_1} = n = 2 \longrightarrow \mathbf{h}_{12} = \frac{\mathbf{V}_1}{\mathbf{V}_2} = \frac{1}{2} = 0.5$$

Thus,

$$[\mathbf{h}] = \underline{\begin{bmatrix} \mathbf{2\,\Omega} & \mathbf{0.5} \\ \mathbf{-0.5} & \mathbf{0} \end{bmatrix}}$$

Chapter 19, Problem 36.

For the two-port in Fig. 19.94,

$$[\mathbf{h}]\begin{bmatrix} 16\Omega & 3 \\ -2 & 0.01\text{S} \end{bmatrix}$$

Find:

(a) V_2 / V_1 (b) I_2 / I_1

(c) I_1 / V_1 (d) V_2 / I_1

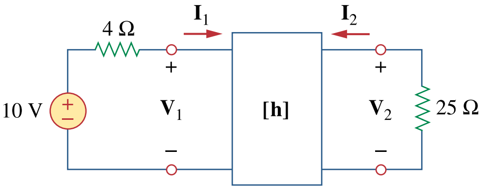
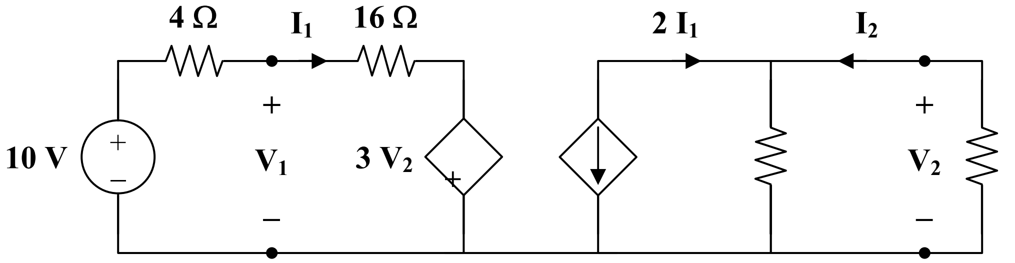


Figure 19.94

For Prob. 19.36.

Chapter 19, Solution 36.

We replace the two-port by its equivalent circuit as shown below.



$$100 \,||\, 25 = 20\,\Omega$$

$$\mathbf{V}_2 = (20)(2\mathbf{I}_1) = 40\mathbf{I}_1 \qquad (1)$$

$$-10 + 20\mathbf{I}_1 + 3\mathbf{V}_2 = 0$$
$$10 = 20\mathbf{I}_1 + (3)(40\mathbf{I}_1) = 140\mathbf{I}_1$$

$$\mathbf{I}_1 = \frac{1}{14}, \qquad \mathbf{V}_2 = \frac{40}{14}$$

$$\mathbf{V}_1 = 16\mathbf{I}_1 + 3\mathbf{V}_2 = \frac{136}{14}$$

$$\mathbf{I}_2 = \left(\frac{100}{125}\right)(2\mathbf{I}_1) = \frac{-8}{70}$$

(a) $\frac{\mathbf{V}_2}{\mathbf{V}_1} = \frac{40}{136} = \underline{\mathbf{0.2941}}$

(b) $\frac{\mathbf{I}_2}{\mathbf{I}_1} = \underline{\mathbf{-1.6}}$

(c) $\frac{\mathbf{I}_1}{\mathbf{V}_1} = \frac{1}{136} = \underline{\mathbf{7.353 \times 10^{-3}\ S}}$

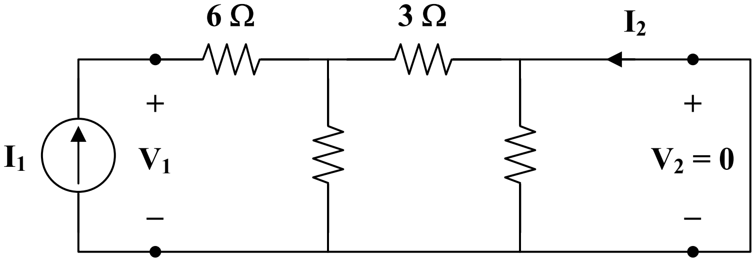
(d) $\frac{\mathbf{V}_2}{\mathbf{I}_1} = \frac{40}{1} = \underline{\mathbf{40\,\Omega}}$

Chapter 19, Problem 37.

The input port of the circuit in Fig. 19.79 is connected to a 10-V dc voltage source while the output port is terminated by a $5\text{-}\Omega$ resistor. Find the voltage across the $5\text{-}\Omega$ resistor by using *h* parameters of the circuit. Confirm your result by using direct circuit analysis.

Chapter 19, Solution 37.

(a) We first obtain the h parameters. To get $\mathbf{h}_{11}$ and $\mathbf{h}_{21}$ refer to Fig. (a).



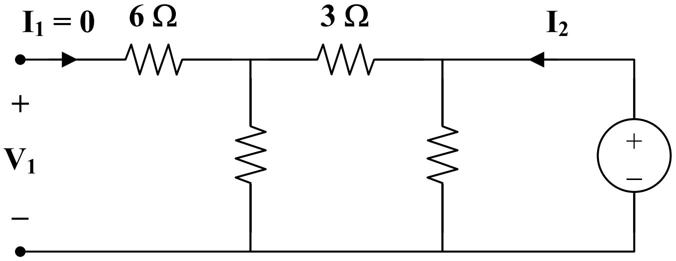
(a)

$$3 \,||\, 6 = 2$$

$$\mathbf{V}_1 = (6+2)\mathbf{I}_1 = 8\mathbf{I}_1 \longrightarrow \mathbf{h}_{11} = \frac{\mathbf{V}_1}{\mathbf{I}_1} = 8\,\Omega$$

$$\mathbf{I}_2 = \frac{-6}{3+6}\mathbf{I}_1 = \frac{-2}{3}\mathbf{I}_1 \longrightarrow \mathbf{h}_{21} = \frac{\mathbf{I}_2}{\mathbf{I}_1} = \frac{-2}{3}$$

To get $\mathbf{h}_{22}$ and $\mathbf{h}_{12}$, refer to the circuit in Fig. (b).



(b)

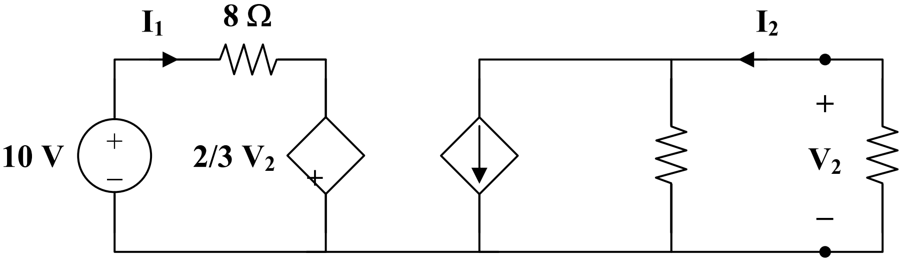
$$3 \| 9 = \frac{9}{4}$$

$$\mathbf{V}_2 = \frac{9}{4}\mathbf{I}_2 \longrightarrow \mathbf{h}_{22} = \frac{\mathbf{I}_2}{\mathbf{V}_2} = \frac{4}{9}$$

$$\mathbf{V}_1 = \frac{6}{6+3}\mathbf{V}_2 = \frac{2}{3}\mathbf{V}_2 \longrightarrow \mathbf{h}_{12} = \frac{\mathbf{V}_1}{\mathbf{V}_2} = \frac{2}{3}$$

$$[\mathbf{h}] = \begin{bmatrix} 8\,\Omega & \frac{2}{3} \\ \frac{-2}{3} & \frac{4}{9}\,\text{S} \end{bmatrix}$$

The equivalent circuit of the given circuit is shown in Fig. (c).



(c)

$$8\mathbf{I}_1 + \frac{2}{3}\mathbf{V}_2 = 10 \qquad (1)$$

$$\mathbf{V}_2 = \frac{2}{3}\mathbf{I}_1\left(5 \| \frac{9}{4}\right) = \frac{2}{3}\mathbf{I}_1\left(\frac{45}{29}\right) = \frac{30}{29}\mathbf{I}_1$$

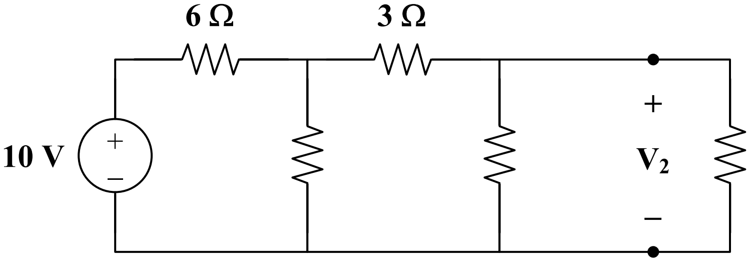
$$\mathbf{I}_1 = \frac{29}{30}\mathbf{V}_2 \qquad (2)$$

Substituting (2) into (1),

$$(8)\left(\frac{29}{30}\right)\mathbf{V}_2 + \frac{2}{3}\mathbf{V}_2 = 10$$

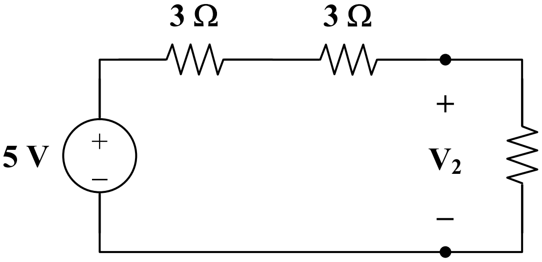
$$\mathbf{V}_2 = \frac{300}{252} = \underline{\mathbf{1.19\ V}}$$

(b) By direct analysis, refer to Fig.(d).



(d)

Transform the 10-V voltage source to a $\frac{10}{6}$-A current source. Since $6 \,||\, 6 = 3\ \Omega$, we combine the two 6-Ω resistors in parallel and transform the current source back to $\frac{10}{6} \times 3 = 5\ \text{V}$ voltage source shown in Fig. (e).



(e)

$$3 \,||\, 5 = \frac{(3)(5)}{8} = \frac{15}{8}$$

$$\mathbf{V}_2 = \frac{15/8}{6+15/8}(5) = \frac{75}{63} = \underline{\mathbf{1.1905\ V}}$$

Chapter 19, Problem 38.

The *h* parameters of the two-port of Fig. 19.95 are:

$$[\mathbf{h}] = \begin{bmatrix} 600\Omega & 0.04 \\ 30 & 2\,\mathrm{mS} \end{bmatrix}$$

Given the $Z_s = 2\,\mathrm{k}\Omega$ and $Z_L = 400\Omega$, find Z_{in} and Z_{out}.

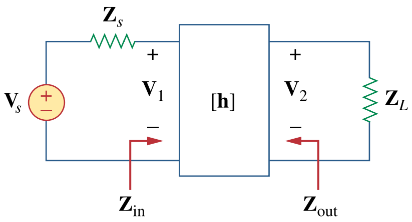


Figure 19.95

For Prob. 19.38.

Chapter 19, Solution 38.

From eq. (19.75),

$$Z_{in} = h_{ie} - \frac{h_{re}h_{fe}R_L}{1+h_{oe}R_L} = h_{11} - \frac{h_{12}h_{21}R_L}{1+h_{22}R_L} = 600 - \frac{0.04x30x400}{1+2x10^{-3}x400} = \underline{333.33\ \Omega}$$

From eq. (19.79),

$$Z_{out} = \frac{R_s + h_{ie}}{(R_s + h_{ie})h_{0e} - h_{re}h_{fe}} = \frac{R_s + h_{11}}{(R_s + h_{11})h_{22} - h_{21}h_{12}} = \frac{2,000+600}{2600x2x10^{-3} - 30x0.04} = \underline{650\ \Omega}$$

Chapter 19, Problem 39.

Obtain the *g* parameters for the wye circuit of Fig. 19.96.

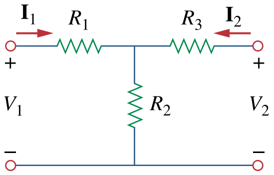
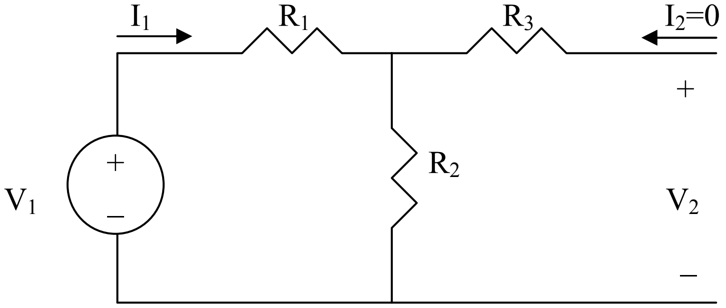


Figure 19.96

For Prob. 19.39.

Chapter 19, Solution 39.

We obtain g_{11} and g_{21} using the circuit below.

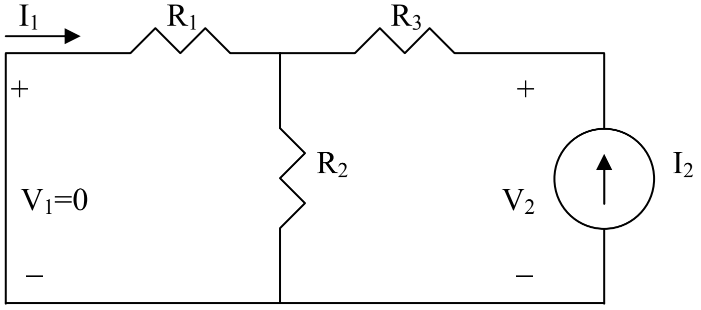


$$I_1 = \frac{V_1}{R_1+R_2} \qquad \longrightarrow \qquad g_{11} = \frac{I_1}{V_1} = \underline{\frac{1}{R_1+R_2}}$$

By voltage division,

$$V_2 = \frac{R_2}{R_1+R_2}V_1 \qquad \longrightarrow \qquad g_{21} = \frac{V_2}{V_1} = \underline{\frac{R_2}{R_1+R_2}}$$

We obtain g_{12} and g_{22} using the circuit below.



By current division,

$$I_1 = -\frac{R_2}{R_1+R_2}I_2 \qquad \longrightarrow \qquad g_{12} = \frac{I_1}{I_2} = \underline{-\frac{R_2}{R_1+R_2}}$$

Also,

$$V_2 = I_2(R_3 + R_1 // R_2) = I_2\left(R_3 + \frac{R_1R_2}{R_1+R_2}\right) \quad g_{22} = \frac{V_2}{I_2} = \underline{R_3 + \frac{R_1R_2}{R_1+R_2}}$$

$$g_{11} = \frac{1}{R_1+R_2}, g_{12} = -\frac{R_2}{R_1+R_2}$$

$$\underline{g_{21} = \frac{R_2}{R_1+R_2}, g_{22} = R_3 + \frac{R_1R_2}{R_1+R_2}}$$

Chapter 19, Problem 40.

Find the *g* parameters for the circuit in Fig. 19.97.

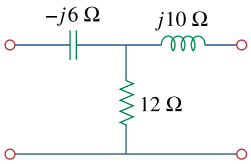
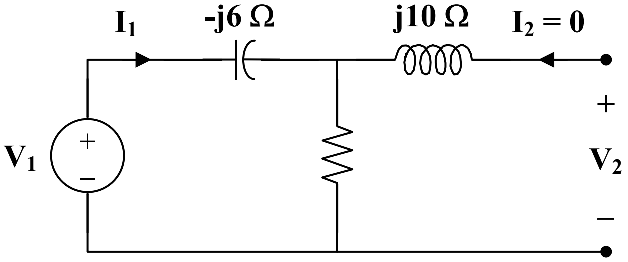


Figure 19.97

For Prob. 19.40.

Chapter 19, Solution 40.

To get $\mathbf{g}_{11}$ and $\mathbf{g}_{21}$, consider the circuit in Fig. (a).

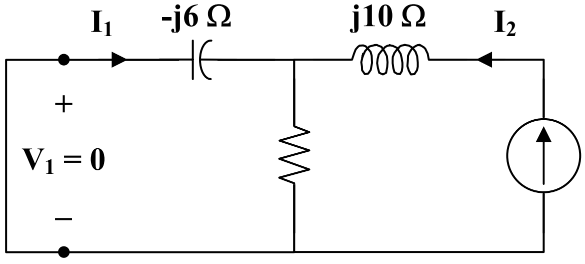


(a)

$$\mathbf{V}_1 = (12 - j6)\mathbf{I}_1 \quad \longrightarrow \quad \mathbf{g}_{11} = \frac{\mathbf{I}_1}{\mathbf{V}_1} = \frac{1}{12 - j6} = 0.0667 + j0.0333 \text{ S}$$

$$\mathbf{g}_{21} = \frac{\mathbf{V}_2}{\mathbf{V}_1} = \frac{12\mathbf{I}_1}{(12 - j6)\mathbf{I}_1} = \frac{2}{2 - j} = 0.8 + j0.4$$

To get $\mathbf{g}_{12}$ and $\mathbf{g}_{22}$, consider the circuit in Fig. (b).



(b)

$$\mathbf{I}_1 = \frac{-12}{12 - j6}\mathbf{I}_2 \quad \longrightarrow \quad \mathbf{g}_{12} = \frac{\mathbf{I}_1}{\mathbf{I}_2} = \frac{-12}{12 - j6} = -\mathbf{g}_{21} = -0.8 - j0.4$$

$$\mathbf{V}_2 = (j10 + 12 \,||\, -j6)\mathbf{I}_2$$

$$\mathbf{g}_{22} = \frac{\mathbf{V}_2}{\mathbf{I}_2} = j10 + \frac{(12)(-j6)}{12 - j6} = 2.4 + j5.2\,\Omega$$

$$[\mathbf{g}] = \underline{\begin{bmatrix} \mathbf{0.0667 + j0.0333 \text{ S}} & \mathbf{-0.8 - j0.4} \\ \mathbf{0.8 + j0.4} & \mathbf{2.4 + j5.2\,\Omega} \end{bmatrix}}$$

Chapter 19, Problem 41.

For the two-port in Fig. 19.75, show that

$$\frac{\mathbf{I}_2}{\mathbf{I}_1} = \frac{-\mathbf{g}_{21}}{\mathbf{g}_{11}\mathbf{Z}_L + \Delta_g}$$

$$\frac{\mathbf{V}_2}{\mathbf{V}_s} = \frac{\mathbf{g}_{21}\mathbf{Z}_L}{(1+\mathbf{g}_{11}\mathbf{Z}_s)(\mathbf{g}_{22}+\mathbf{Z}_L) - \mathbf{g}_{21}\mathbf{g}_{12}\mathbf{Z}_s}$$

where Δ_g is the determinant of [**g**] matrix.

Chapter 19, Solution 41.

For the g parameters

$$\mathbf{I}_1 = \mathbf{g}_{11}\mathbf{V}_1 + \mathbf{g}_{12}\mathbf{I}_2 \qquad (1)$$

$$\mathbf{V}_2 = \mathbf{g}_{21}\mathbf{V}_1 + \mathbf{g}_{22}\mathbf{I}_2 \qquad (2)$$

But $\mathbf{V}_1 = \mathbf{V}_s - \mathbf{I}_1\mathbf{Z}_s$ and

$$\mathbf{V}_2 = \text{-}\mathbf{I}_2\mathbf{Z}_L = \mathbf{g}_{21}\mathbf{V}_1 + \mathbf{g}_{22}\mathbf{I}_2$$

$$0 = \mathbf{g}_{21}\mathbf{V}_1 + (\mathbf{g}_{22} + \mathbf{Z}_L)\mathbf{I}_2$$

or

$$\mathbf{V}_1 = \frac{\text{-}(\mathbf{g}_{22} + \mathbf{Z}_L)}{\mathbf{g}_{21}}\mathbf{I}_2$$

Substituting this into (1),

$$\mathbf{I}_1 = \frac{(\mathbf{g}_{22}\mathbf{g}_{11} + \mathbf{Z}_L\mathbf{g}_{11} - \mathbf{g}_{21}\mathbf{g}_{12})}{\text{-}\mathbf{g}_{21}}\mathbf{I}_2$$

or

$$\underline{\frac{\mathbf{I_2}}{\mathbf{I_1}} = \frac{\text{-}\mathbf{g_{21}}}{\mathbf{g_{11}}\mathbf{Z_L} + \Delta_\mathbf{g}}}$$

Also,

$$\mathbf{V}_2 = \mathbf{g}_{21}(\mathbf{V}_s - \mathbf{I}_1\mathbf{Z}_s) + \mathbf{g}_{22}\mathbf{I}_2$$

$$= \mathbf{g}_{21}\mathbf{V}_s - \mathbf{g}_{21}\mathbf{Z}_s\mathbf{I}_1 + \mathbf{g}_{22}\mathbf{I}_2$$

$$= \mathbf{g}_{21}\mathbf{V}_s + \mathbf{Z}_s(\mathbf{g}_{11}\mathbf{Z}_L + \Delta_g)\mathbf{I}_2 + \mathbf{g}_{22}\mathbf{I}_2$$

But

$$\mathbf{I}_2 = \frac{\text{-}\mathbf{V}_2}{\mathbf{Z}_L}$$

$$\mathbf{V}_2 = \mathbf{g}_{21}\mathbf{V}_s - [\mathbf{g}_{11}\mathbf{Z}_s\mathbf{Z}_L + \Delta_g\mathbf{Z}_s + \mathbf{g}_{22}]\left[\frac{\mathbf{V}_2}{\mathbf{Z}_L}\right]$$

$$\frac{\mathbf{V}_2[\mathbf{Z}_L + \mathbf{g}_{11}\mathbf{Z}_s\mathbf{Z}_L + \Delta_g\mathbf{Z}_s + \mathbf{g}_{22}]}{\mathbf{Z}_L} = \mathbf{g}_{21}\mathbf{V}_s$$

$$\frac{\mathbf{V}_2}{\mathbf{V}_s} = \frac{\mathbf{g}_{21}\mathbf{Z}_L}{\mathbf{Z}_L + \mathbf{g}_{11}\mathbf{Z}_s\mathbf{Z}_L + \Delta_g\mathbf{Z}_s + \mathbf{g}_{22}}$$

$$\frac{\mathbf{V}_2}{\mathbf{V}_s} = \frac{\mathbf{g}_{21}\mathbf{Z}_L}{\mathbf{Z}_L + \mathbf{g}_{11}\mathbf{Z}_s\mathbf{Z}_L + \mathbf{g}_{11}\mathbf{g}_{22}\mathbf{Z}_s - \mathbf{g}_{21}\mathbf{g}_{12}\mathbf{Z}_s + \mathbf{g}_{22}}$$

$$\underline{\frac{\mathbf{V_2}}{\mathbf{V_s}} = \frac{\mathbf{g_{21}}\mathbf{Z_L}}{(1 + \mathbf{g_{11}}\mathbf{Z_s})(\mathbf{g_{22}} + \mathbf{Z_L}) - \mathbf{g_{12}}\mathbf{g_{21}}\mathbf{Z_s}}}$$

Chapter 19, Problem 42.

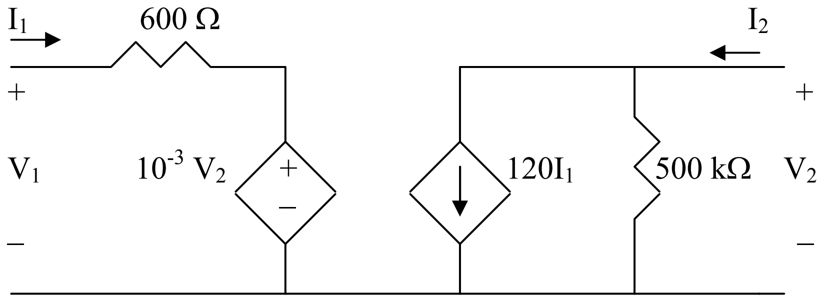
The *h* parameters of a two-port device are given by

$\mathbf{h}_{11} = 600\Omega$, $\mathbf{h}_{12} = 10^{-3}$, $\mathbf{h}_{21} = 120$, $\mathbf{h}_{22} = 2\times10^{-6}$ S

Draw a circuit model of the device including the value of each element.

Chapter 19, Solution 42.

With the help of Fig. 19.20, we obtain the circuit model below.



Chapter 19, Problem 43.

Find the transmission parameters for the single-element two-port networks in Fig. 19.98.

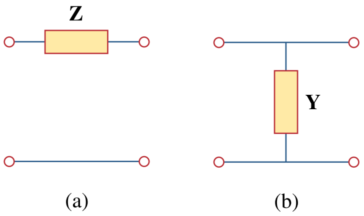
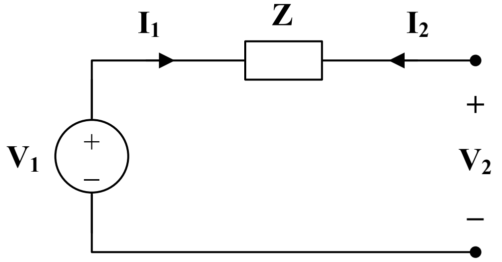


Figure 19.98

For Prob. 19.43.

Chapter 19, Solution 43.

(a) To find **A** and **C**, consider the network in Fig. (a).

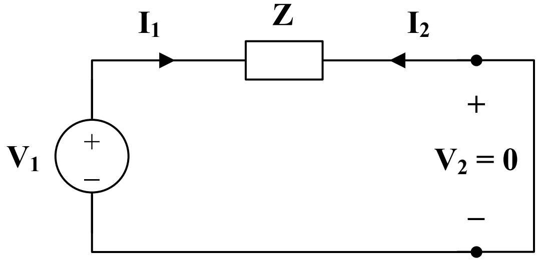


(a)

$$\mathbf{V}_1 = \mathbf{V}_2 \longrightarrow \mathbf{A} = \frac{\mathbf{V}_1}{\mathbf{V}_2} = 1$$

$$\mathbf{I}_1 = 0 \longrightarrow \mathbf{C} = \frac{\mathbf{I}_1}{\mathbf{V}_2} = 0$$

To get **B** and **D**, consider the circuit in Fig. (b).



(b)

$$\mathbf{V}_1 = \mathbf{Z}\mathbf{I}_1, \qquad \mathbf{I}_2 = -\mathbf{I}_1$$

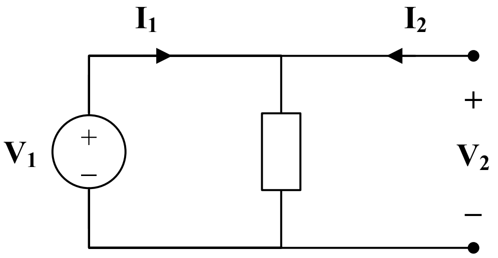
$$\mathbf{B} = \frac{-\mathbf{V}_1}{\mathbf{I}_2} = \frac{-\mathbf{Z}\mathbf{I}_1}{-\mathbf{I}_1} = \mathbf{Z}$$

$$\mathbf{D} = \frac{-\mathbf{I}_1}{\mathbf{I}_2} = 1$$

Hence,

$$[\mathbf{T}] = \underline{\begin{bmatrix} \mathbf{1} & \mathbf{Z} \\ \mathbf{0} & \mathbf{1} \end{bmatrix}}$$

(b) To find **A** and **C**, consider the circuit in Fig. (c).

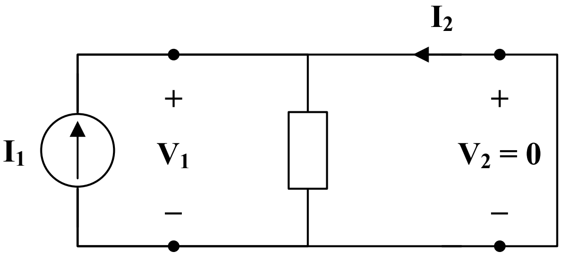


(c)

$$\mathbf{V}_1 = \mathbf{V}_2 \quad \longrightarrow \quad \mathbf{A} = \frac{\mathbf{V}_1}{\mathbf{V}_2} = 1$$

$$\mathbf{V}_1 = \mathbf{Z}\mathbf{I}_1 = \mathbf{V}_2 \quad \longrightarrow \quad \mathbf{C} = \frac{\mathbf{I}_1}{\mathbf{V}_2} = \frac{1}{\mathbf{Z}} = \mathbf{Y}$$

To get **B** and **D**, refer to the circuit in Fig.(d).



(d)

$$\mathbf{V}_1 = \mathbf{V}_2 = 0 \qquad \mathbf{I}_2 = \text{-}\mathbf{I}_1$$

$$\mathbf{B} = \frac{\text{-}\mathbf{V}_1}{\mathbf{I}_2} = 0, \qquad \mathbf{D} = \frac{\text{-}\mathbf{I}_1}{\mathbf{I}_2} = 1$$

Thus,

$$\underline{[\mathbf{T}] = \begin{bmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{Y} & \mathbf{1} \end{bmatrix}}$$

Chapter 19, Problem 44.

Determine the transmission parameters of the circuit in Fig. 19.99.

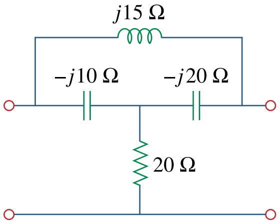
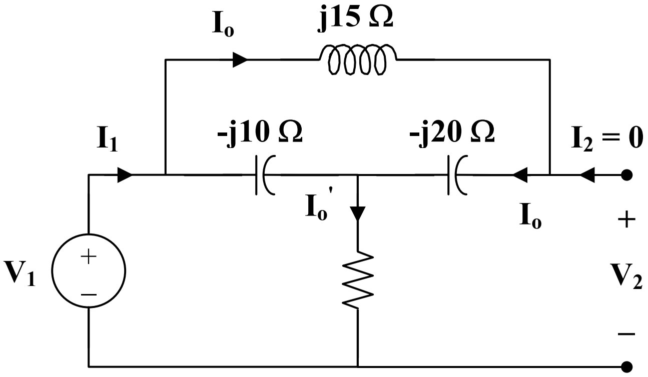


Figure 19.99

For Prob. 19.44.

Chapter 19, Solution 44.

To determine $\mathbf{A}$ and $\mathbf{C}$, consider the circuit in Fig.(a).



(a)

$$\mathbf{V}_1 = [\,20 + (\text{-j10}) \,||\, (\text{j15} - \text{j20})\,]\mathbf{I}_1$$

$$\mathbf{V}_1 = \left[\,20 + \frac{(\text{-j10})(\text{-j5})}{\text{- j15}}\right]\mathbf{I}_1 = \left[\,20 - \text{j}\frac{10}{3}\right]\mathbf{I}_1$$

$$\mathbf{I}_o^{'} = \mathbf{I}_1$$

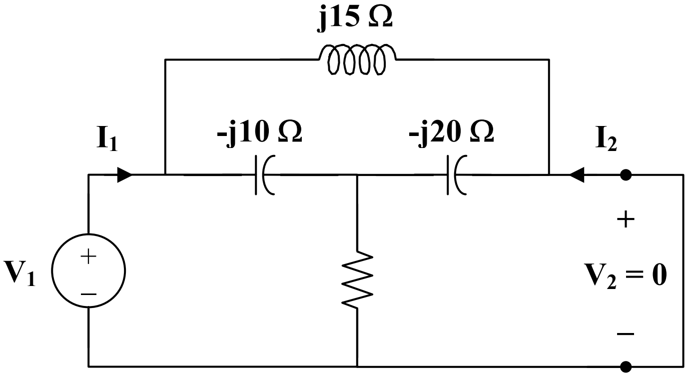
$$\mathbf{I}_o = \left(\frac{\text{- j10}}{\text{- j10} - \text{j5}}\right)\mathbf{I}_1 = \left(\frac{2}{3}\right)\mathbf{I}_1$$

$$\mathbf{V}_2 = (\text{-j20})\mathbf{I}_o + 20\mathbf{I}_o{}' = -\text{j}\frac{40}{3}\text{I}_1 + 20\text{I}_1 = \left(20 - \text{j}\frac{40}{3}\right)\text{I}_1$$

$$\mathbf{A} = \frac{\mathbf{V}_1}{\mathbf{V}_2} = \frac{(20 - j10/3)\,\mathbf{I}_1}{\left(20 - j\frac{40}{3}\right)I_1} = 0.7692 + j0.3461$$

$$\mathbf{C} = \frac{\mathbf{I}_1}{\mathbf{V}_2} = \frac{1}{20 - j\frac{40}{3}} = 0.03461 + j0.023$$

To find **B** and **D**, consider the circuit in Fig. (b).



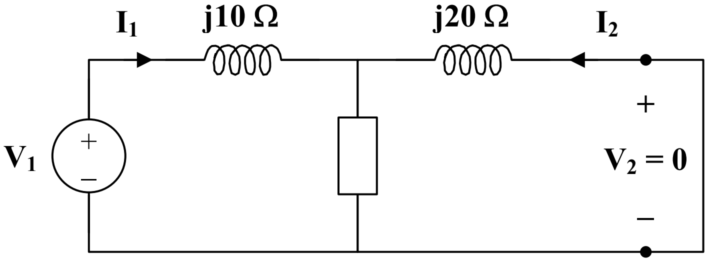
(b)

We may transform the Δ subnetwork to a T as shown in Fig. (c).

$$\mathbf{Z}_1 = \frac{(j15)(-j10)}{j15 - j10 - j20} = j10$$

$$\mathbf{Z}_2 = \frac{(-j10)(-j20)}{-j15} = -j\frac{40}{3}$$

$$\mathbf{Z}_3 = \frac{(j15)(-j20)}{-j15} = j20$$



(c)

$$-\mathbf{I}_2 = \frac{20 - j40/3}{20 - j40/3 + j20}\mathbf{I}_1 = \frac{3 - j2}{3 + j}\mathbf{I}_1$$

$$\mathbf{D} = \frac{-\mathbf{I}_1}{\mathbf{I}_2} = \frac{3 + j}{3 - j2} = 0.5385 + j0.6923$$

$$\mathbf{V}_1 = \left[j10 + \frac{(j20)(20 - j40/3)}{20 - j40/3 + j20} \right]\mathbf{I}_1$$

$$\mathbf{V}_1 = [\, j10 + 2(9 + j7)\,]\mathbf{I}_1 = j\mathbf{I}_1(24 - j18)$$

$$\mathbf{B} = \frac{-\mathbf{V}_1}{\mathbf{I}_2} = \frac{-j\mathbf{I}_1(24 - j18)}{\frac{-(3 - j2)}{3 + j}\mathbf{I}_1} = \frac{6}{13}(-15 + j55)$$

$$\mathbf{B} = -6.923 + j25.385\,\Omega$$

$$\underline{[\mathbf{T}] = \begin{bmatrix} \mathbf{0.7692 + j0.3461} & \mathbf{-6.923 + j25.385\,\Omega} \\ \mathbf{0.03461 + j0.023\ S} & \mathbf{0.5385 + j0.6923} \end{bmatrix}}$$

Chapter 19, Problem 45.

Find the **ABCD** parameters for the circuit in Fig. 19.100.

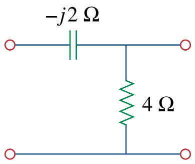
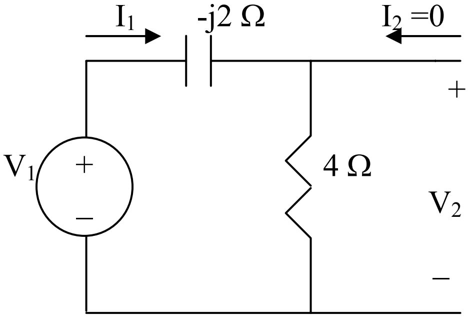


Figure 19.100

For Prob. 19.45.

Chapter 19, Solution 45.

To determine A and C, consider the circuit below.

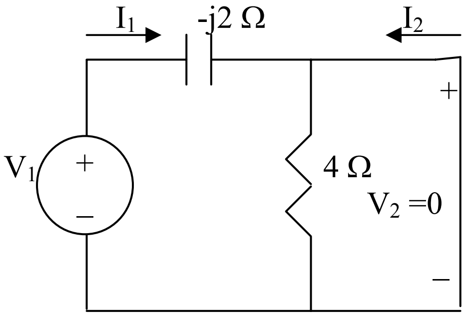


$$V_1 = (4 - j2)I_1, \qquad V_2 = 4I_1$$

$$A = \frac{V_1}{V_2} = \frac{4 - j2}{4} = 1 - j0.5$$

$$C = \frac{I_1}{V_2} = \frac{I_1}{4I_1} = 0.25$$

To determine B and D, consider the circuit below.



The 4-Ω resistor is short-circuited. Hence,

$$I_2 = -I_1, \qquad D = -\frac{I_1}{I_2} = 1$$

$$V_1 = -j2I_1 = j2I_2 \qquad B = -\frac{V_1}{I_2} = -\frac{j2I_2}{I_2} = -2j\Omega$$

Hence,

$$\underline{\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 - j0.5 & -j2\Omega \\ 0.25S & 1 \end{bmatrix}} = \begin{bmatrix} 1 - j0.5 & -j2\Omega \\ 0.25S & 1 \end{bmatrix}$$

Chapter 19, Problem 46.

Find the transmission parameters for the circuit in Fig. 19.101.

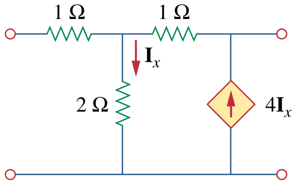
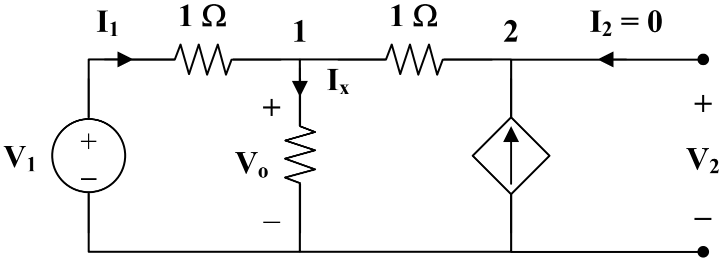


Figure 19.101

For Prob. 19.46.

Chapter 19, Solution 46.

To get **A** and **C**, refer to the circuit in Fig.(a).



(a)

At node 1,

$$\mathbf{I}_1 = \frac{\mathbf{V}_o}{2} + \frac{\mathbf{V}_o - \mathbf{V}_2}{1} \longrightarrow 2\mathbf{I}_1 = 3\mathbf{V}_o - 2\mathbf{V}_2 \qquad (1)$$

At node 2,

$$\frac{\mathbf{V}_o - \mathbf{V}_2}{1} = 4\mathbf{I}_x = \frac{4\mathbf{V}_o}{2} = 2\mathbf{V}_o \longrightarrow \mathbf{V}_o = \text{-}\mathbf{V}_2 \qquad (2)$$

From (1) and (2),

$$2\mathbf{I}_1 = \text{-}5\mathbf{V}_2 \longrightarrow \mathbf{C} = \frac{\mathbf{I}_1}{\mathbf{V}_2} = \frac{\text{-}5}{2} = \text{-}2.5\text{ S}$$

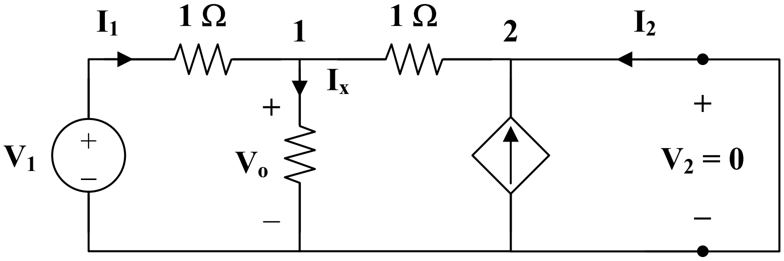
But

$$\mathbf{I}_1 = \frac{\mathbf{V}_1 - \mathbf{V}_o}{1} = \mathbf{V}_1 + \mathbf{V}_2$$

$$\text{-}2.5\mathbf{V}_2 = \mathbf{V}_1 + \mathbf{V}_2 \longrightarrow \mathbf{V}_1 = \text{-}3.5\mathbf{V}_2$$

$$\mathbf{A} = \frac{\mathbf{V}_1}{\mathbf{V}_2} = \text{-}3.5$$

To get **B** and **D**, consider the circuit in Fig. (b).



(b)

At node 1,

$$\mathbf{I}_1 = \frac{\mathbf{V}_o}{2} + \frac{\mathbf{V}_o}{1} \longrightarrow 2\mathbf{I}_1 = 3\mathbf{V}_o \qquad (3)$$

At node 2,

$$\mathbf{I}_2 + \frac{\mathbf{V}_o}{1} + 4\mathbf{I}_x = 0$$

$$-\mathbf{I}_2 = \mathbf{V}_o + 2\mathbf{V}_o = 0 \longrightarrow \mathbf{I}_2 = \text{-}3\mathbf{V}_o \qquad (4)$$

Adding (3) and (4),

$$2\mathbf{I}_1 + \mathbf{I}_2 = 0 \longrightarrow \mathbf{I}_1 = \text{-}0.5\mathbf{I}_2 \qquad (5)$$

$$\mathbf{D} = \frac{\text{-}\mathbf{I}_1}{\mathbf{I}_2} = 0.5$$

But
$$\mathbf{I}_1 = \frac{\mathbf{V}_1 - \mathbf{V}_o}{1} \longrightarrow \mathbf{V}_1 = \mathbf{I}_1 + \mathbf{V}_o \qquad (6)$$

Substituting (5) and (4) into (6),

$$\mathbf{V}_1 = \frac{\text{-}1}{2}\mathbf{I}_2 + \frac{\text{-}1}{3}\mathbf{I}_2 = \frac{\text{-}5}{6}\mathbf{I}_2$$

$$\mathbf{B} = \frac{\text{-}\mathbf{V}_1}{\mathbf{I}_2} = \frac{5}{6} = 0.8333\,\Omega$$

Thus,

$$[\mathbf{T}] = \underline{\begin{bmatrix} \mathbf{-3.5} & \mathbf{0.8333\,\Omega} \\ \mathbf{-2.5\,S} & \mathbf{-0.5} \end{bmatrix}}$$

Chapter 19, Problem 47.

Obtain the **ABCD** parameters for the network in Fig. 19.102.

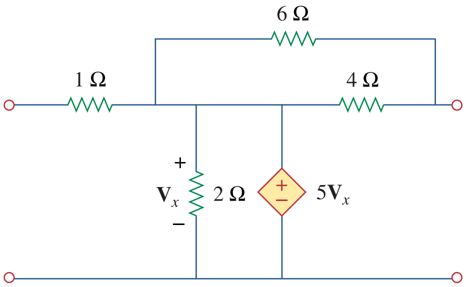
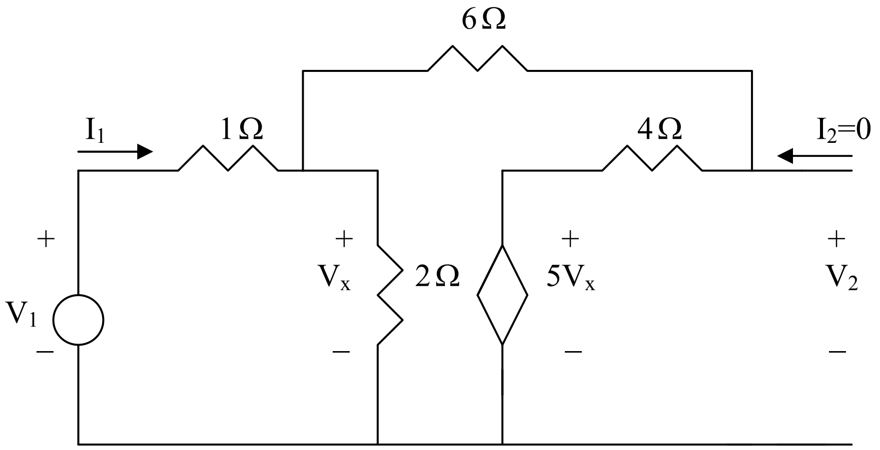


Figure 19.102

For Prob. 19.47.

Chapter 19, Solution 47.

To get A and C, consider the circuit below.

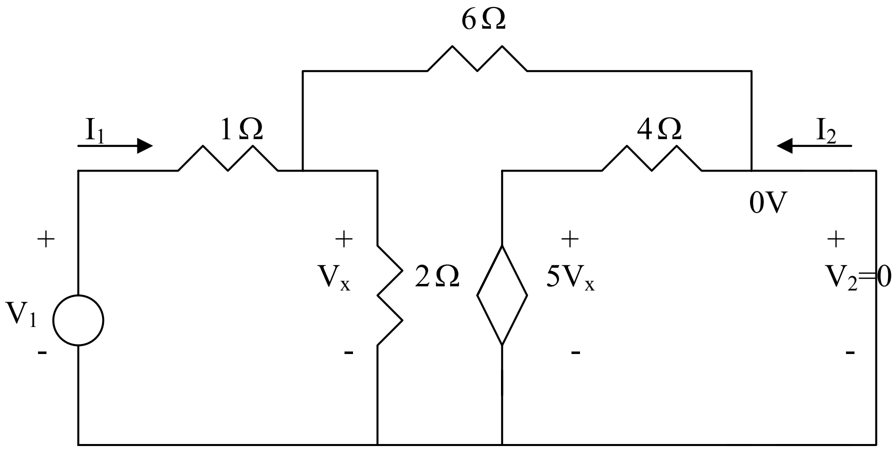


$$\frac{V_1 - V_x}{1} = \frac{V_x}{2} + \frac{V_x - 5V_x}{10} \longrightarrow V_1 = 1.1V_x$$

$$V_2 = 4(-0.4V_x) + 5V_x = 3.4V_x \longrightarrow A = \frac{V_1}{V_2} = 1.1/3.4 = 0.3235$$

$$I_1 = \frac{V_1 - V_x}{1} = 1.1V_x - V_x = 0.1V_x \quad \longrightarrow \quad C = \frac{I_1}{V_2} = 0.1/3.4 = 0.02941$$

To get B and D, consider the circuit below.



$$\frac{V_1 - V_x}{1} = \frac{V_x}{6} + \frac{V_x}{2} \quad \longrightarrow \quad V_1 = \frac{10}{6}V_x \qquad (1)$$

$$I_2 = -\frac{5V_x}{4} - \frac{V_x}{6} = -\frac{17}{12}V_x \qquad (2)$$

$$V_1 = I_1 + V_x \qquad (3)$$

From (1) and (3)

$$I_1 = V_1 - V_x = \frac{4}{6}V_x \quad \longrightarrow \quad D = -\frac{I_1}{I_2} = \frac{4}{6}(\frac{12}{17}) = 0.4706$$

$$B = -\frac{V_1}{I_2} = \frac{10}{6}(\frac{12}{17}) = 1.176$$

$$\underline{[T] = \begin{bmatrix} 0.3235 & 1.176 \\ 0.02941 & 0.4706 \end{bmatrix}}$$

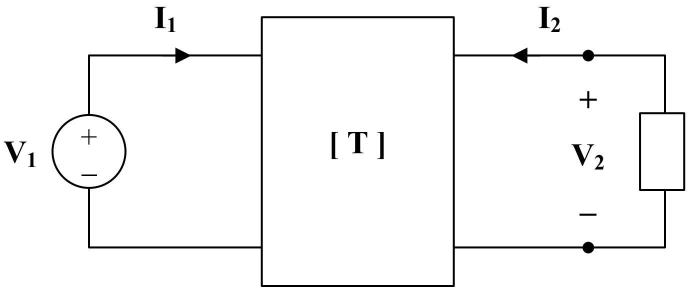
Chapter 19, Problem 48.

For a two-port, let $\mathbf{A} = 4$, $\mathbf{B} = 30\,\Omega$, $\mathbf{C} = 0.1$ S, and $\mathbf{D} = 1.5$. Calculate the input impedance, $\mathbf{Z}_{\text{in}} = \mathbf{V}_1 / \mathbf{I}_1$ when:

(a) the output terminals are short-circuited,
(b) the output port is open-circuited,
(c) the output port is terminated by a $10\text{-}\Omega$ load.

Chapter 19, Solution 48.

(a) Refer to the circuit below.



$$\mathbf{V}_1 = 4\mathbf{V}_2 - 30\mathbf{I}_2 \quad (1)$$
$$\mathbf{I}_1 = 0.1\mathbf{V}_2 - \mathbf{I}_2 \quad (2)$$

When the output terminals are shorted, $\mathbf{V}_2 = 0$.
So, (1) and (2) become

$\mathbf{V}_1 = \text{-}30\mathbf{I}_2$ and $\mathbf{I}_1 = \text{-}\mathbf{I}_2$

Hence,

$$\mathbf{Z}_{in} = \frac{\mathbf{V}_1}{\mathbf{I}_1} = \underline{\mathbf{30\,\Omega}}$$

(b) When the output terminals are open-circuited, $\mathbf{I}_2 = 0$.
So, (1) and (2) become

$\mathbf{V}_1 = 4\mathbf{V}_2$

$\mathbf{I}_1 = 0.1\mathbf{V}_2$ or $\mathbf{V}_2 = 10\mathbf{I}_1$

$\mathbf{V}_1 = 40\mathbf{I}_1$

$$\mathbf{Z}_{in} = \frac{\mathbf{V}_1}{\mathbf{I}_1} = \underline{\mathbf{40\,\Omega}}$$

(c) When the output port is terminated by a 10-Ω load, $\mathrm{V}_2 = \text{-}10\mathrm{I}_2$.
So, (1) and (2) become

$$\mathbf{V}_1 = \text{-}40\mathbf{I}_2 - 30\mathbf{I}_2 = \text{-}70\mathbf{I}_2$$
$$\mathbf{I}_1 = \text{-}\mathbf{I}_2 - \mathbf{I}_2 = \text{-}2\mathbf{I}_2$$
$$\mathbf{V}_1 = 35\mathbf{I}_1$$
$$\mathbf{Z}_{in} = \frac{\mathbf{V}_1}{\mathbf{I}_1} = \underline{\mathbf{35\,\Omega}}$$

Alternatively, we may use $\mathbf{Z}_{in} = \dfrac{\mathbf{A}\mathbf{Z}_L + \mathbf{B}}{\mathbf{C}\mathbf{Z}_L + \mathbf{D}}$

Chapter 19, Problem 49.

Using impedances in the *s* domain, obtain the transmission parameters for the circuit in Fig. 19.103.

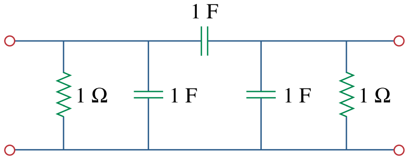
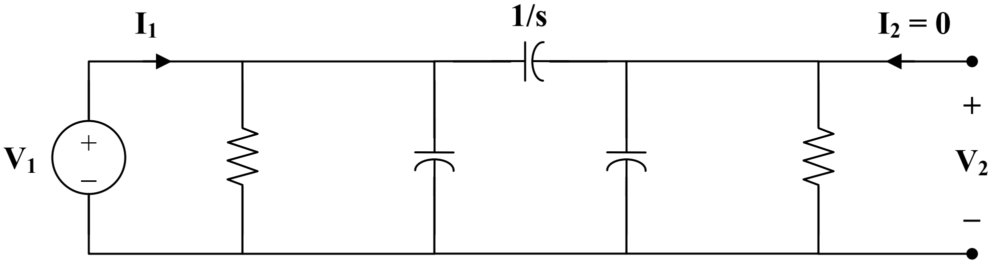


Figure 19.103

For Prob. 19.49.

Chapter 19, Solution 49.

To get $\mathbf{A}$ and $\mathbf{C}$, refer to the circuit in Fig.(a).



(a)

$$1\,||\,\frac{1}{s}=\frac{1/s}{1+1/s}=\frac{1}{s+1}$$

$$\mathbf{V}_2=\frac{1\,||\,1/s}{1/s+1\,||\,1/s}\mathbf{V}_1$$

$$\mathbf{A}=\frac{\mathbf{V}_1}{\mathbf{V}_2}=\frac{\frac{1}{s}+\frac{1}{s+1}}{\frac{1}{s+1}}=\frac{2s+1}{s}$$

$$\mathbf{V}_1=\mathbf{I}_1\left(\frac{1}{s+1}\right)||\left(\frac{1}{s}+\frac{1}{s+1}\right)=\mathbf{I}_1\left(\frac{1}{s+1}\right)||\left(\frac{2s+1}{s(s+1)}\right)$$

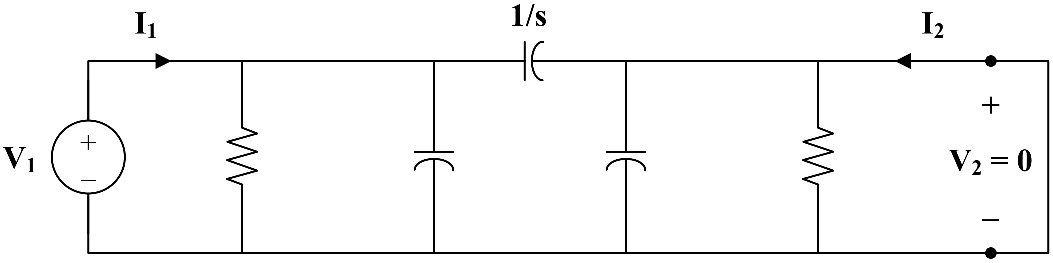
$$\frac{\mathbf{V}_1}{\mathbf{I}_1}=\frac{\left(\frac{1}{s+1}\right)\cdot\left(\frac{2s+1}{s(s+1)}\right)}{\frac{1}{s+1}+\frac{2s+1}{s(s+1)}}=\frac{2s+1}{(s+1)(3s+1)}$$

But $$V_1 = V_2 \cdot \frac{2s+1}{s}$$

Hence, $$\frac{V_2}{I_1} \cdot \frac{2s+1}{s} = \frac{2s+1}{(s+1)(3s+1)}$$

$$\mathbf{C} = \frac{\mathbf{V}_2}{\mathbf{I}_1} = \frac{(s+1)(3s+1)}{s}$$

To get **B** and **D**, consider the circuit in Fig. (b).



(b)

$$\mathbf{V}_1 = \mathbf{I}_1\left(1 \,||\, \frac{1}{s} \,||\, \frac{1}{s}\right) = \mathbf{I}_1\left(1 \,||\, \frac{1}{2s}\right) = \frac{\mathbf{I}_1}{2s+1}$$

$$\mathbf{I}_2 = \frac{\frac{-1}{s+1}\mathbf{I}_1}{\frac{1}{s+1} + \frac{1}{s}} = \frac{-s}{2s+1}\mathbf{I}_1$$

$$\mathbf{D} = \frac{-\mathbf{I}_1}{\mathbf{I}_2} = \frac{2s+1}{s} = 2 + \frac{1}{s}$$

$$\mathbf{V}_1 = \left(\frac{1}{2s+1}\right)\left(\frac{2s+1}{-s}\right)\mathbf{I}_2 = \frac{\mathbf{I}_2}{-s} \quad \longrightarrow \quad \mathbf{B} = \frac{-\mathbf{V}_1}{\mathbf{I}_2} = \frac{1}{s}$$

Thus,

$$\underline{[\mathbf{T}] = \begin{bmatrix} \frac{2s+1}{s} & \frac{1}{s} \\ \frac{(s+1)(3s+1)}{s} & 2+\frac{1}{s} \end{bmatrix}}$$

Chapter 19, Problem 50.

Derive the *s*-domain expression for the *t* parameters of the circuit in Fig. 19.104.

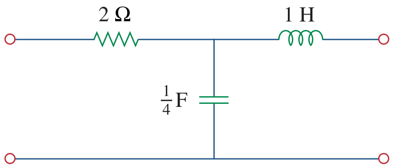
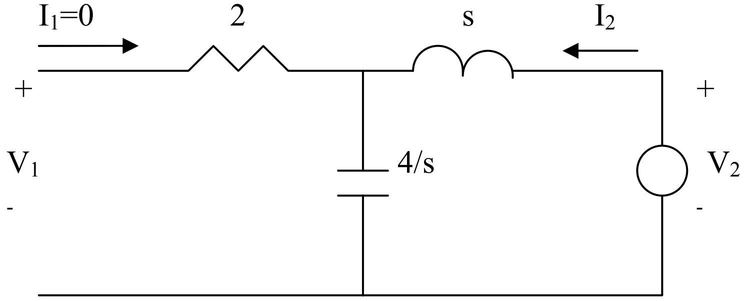


Figure 19.104

For Prob. 19.50.

Chapter 19, Solution 50.

To get a and c, consider the circuit below.

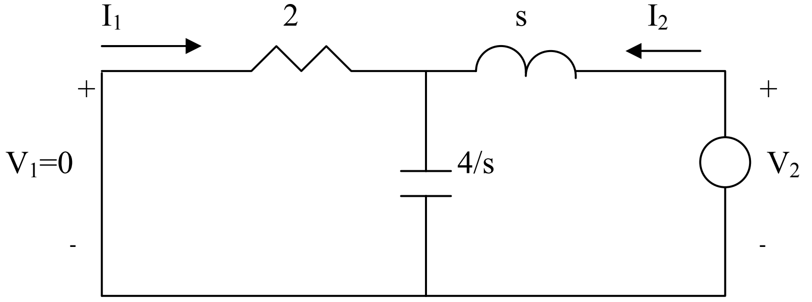


$$V_1 = \frac{4/s}{s+4/s}V_2 = \frac{4}{s^2+4}V_2 \quad \longrightarrow \quad a = {}^{V_2}\!\!\diagup_{V_1} = 1+0.25s^2$$

$$V_2 = (s+4/s)I_2 \text{ or}$$

$$I_2 = \frac{V_2}{s+4/s} = \frac{(1+0.25s^2)V_1}{s+4/s} \quad \longrightarrow \quad c = \frac{I_2}{V_1} = \frac{s+0.25s^3}{s^2+4}$$

To get b and d, consider the circuit below.



$$I_1 = \frac{-4/s}{2+4/s}I_2 = -\frac{2I_2}{s+2} \quad \longrightarrow \quad d = -\frac{I_2}{I_1} = 1+0.5s$$

$$V_2 = (s+2//\frac{4}{s})I_2 = \frac{(s^2+2s+4)}{s+2}I_2$$

$$= -\frac{(s^2+2s+4)(\ s+2)}{s+2}\frac{}{2}I_1 \quad \longrightarrow \quad b = -\frac{V_2}{I_1} = 0.5s^2+s+2$$

$$[t] = \begin{bmatrix} 0.25s^2+1 & 0.5s^2+s+2 \\ \dfrac{0.25s^2+s}{s^2+4} & 0.5s+1 \end{bmatrix}$$

Chapter 19, Problem 51.

Obtain the *t* parameters for the network in Fig. 19.105.

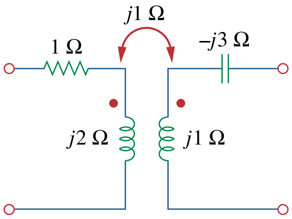
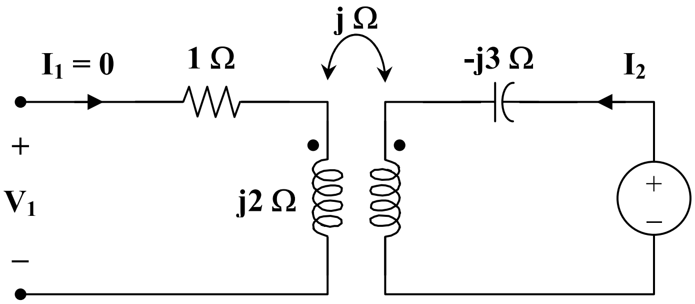


Figure 19.105
For Prob. 19.51.

Chapter 19, Solution 51.

To get **a** and **c**, consider the circuit in Fig. (a).



(a)

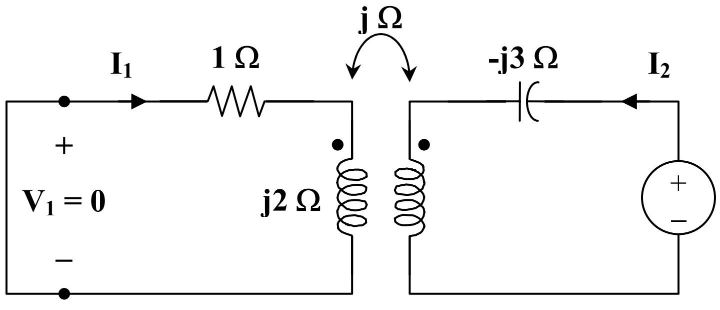
$$\mathbf{V}_2 = \mathbf{I}_2\,(\mathrm{j} - \mathrm{j}3) = \text{-j}2\,\mathbf{I}_2$$

$$\mathbf{V}_1 = \text{-j}\mathbf{I}_2$$

$$\mathbf{a} = \frac{\mathbf{V}_2}{\mathbf{V}_1} = \frac{-\,\mathrm{j}2\,\mathbf{I}_2}{-\,\mathrm{j}\mathbf{I}_2} = 2$$

$$\mathbf{c} = \frac{\mathbf{I}_2}{\mathbf{V}_1} = \frac{1}{-\,\mathrm{j}} = \mathrm{j}$$

To get **b** and **d**, consider the circuit in Fig. (b).



(b)

For mesh 1,

$$0 = (1 + j2)\mathbf{I}_1 - j\mathbf{I}_2$$

or
$$\frac{\mathbf{I}_2}{\mathbf{I}_1} = \frac{1 + j2}{j} = 2 - j$$

$$\mathbf{d} = \frac{-\mathbf{I}_2}{\mathbf{I}_1} = -2 + j$$

For mesh 2,

$$\mathbf{V}_2 = \mathbf{I}_2 (j - j3) - j\mathbf{I}_1$$
$$\mathbf{V}_2 = \mathbf{I}_1 (2 - j)(-j2) - j\mathbf{I}_1 = (-2 - j5)\mathbf{I}_1$$

$$\mathbf{b} = \frac{-\mathbf{V}_2}{\mathbf{I}_1} = 2 + j5$$

Thus,

$$[\mathbf{t}] = \underline{\begin{bmatrix} \mathbf{2} & \mathbf{2 + j5} \\ \mathbf{j} & \mathbf{-2 + j} \end{bmatrix}}$$

Chapter 19, Problem 52.

(a) For the *T* network in Fig. 19.106, show that the *h* parameters are:

$$\mathbf{h}_{11} = R_1 + \frac{R_2 R_3}{R_1 + R3}, \qquad \mathbf{h}_{12} = \frac{R_2}{R_2 + R_3}$$

$$\mathbf{h}_{21} = -\frac{R_2}{R_2 + R_3}, \qquad \mathbf{h}_{22} = \frac{1}{R_2 + R_3}$$

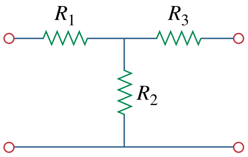


Figure 19.106

For Prob. 19.52.

(b) For the same network, show that the transmission parameters are:

$$\mathbf{A} = 1 + \frac{R_1}{R_2}, \qquad \mathbf{B} = R_3 + \frac{R_1}{R_2}\left(R_2 + R_3\right)$$

$$\mathbf{C} = \frac{1}{R_2}, \qquad \mathbf{D} = 1 + \frac{R_3}{R_2}$$

Chapter 19, Solution 52.

It is easy to find the z parameters and then transform these to h parameters and T parameters.

$$[\mathbf{z}] = \begin{bmatrix} R_1 + R_2 & R_2 \\ R_2 & R_2 + R_3 \end{bmatrix}$$

$$\Delta_z = (R_1 + R_2)(R_2 + R_3) - R_2^2$$
$$= R_1R_2 + R_2R_3 + R_3R_1$$

(a) $$[\mathbf{h}] = \begin{bmatrix} \dfrac{\Delta_z}{\mathbf{z}_{22}} & \dfrac{\mathbf{z}_{12}}{\mathbf{z}_{22}} \\ \dfrac{-\mathbf{z}_{21}}{\mathbf{z}_{22}} & \dfrac{1}{\mathbf{z}_{22}} \end{bmatrix} = \begin{bmatrix} \dfrac{R_1R_2 + R_2R_3 + R_3R_1}{R_2 + R_3} & \dfrac{R_2}{R_2 + R_3} \\ \dfrac{-R_2}{R_2 + R_3} & \dfrac{1}{R_2 + R_3} \end{bmatrix}$$

Thus,

$$\underline{\mathbf{h}_{11} = \mathbf{R}_1 + \frac{\mathbf{R}_2\mathbf{R}_3}{\mathbf{R}_2 + \mathbf{R}_3}}, \quad \underline{\mathbf{h}_{12} = \frac{\mathbf{R}_2}{\mathbf{R}_2 + \mathbf{R}_3} = -\mathbf{h}_{21}}, \quad \underline{\mathbf{h}_{22} = \frac{1}{\mathbf{R}_2 + \mathbf{R}_3}}$$

as required.

(b) $$[\mathbf{T}] = \begin{bmatrix} \dfrac{\mathbf{z}_{11}}{\mathbf{z}_{21}} & \dfrac{\Delta_z}{\mathbf{z}_{21}} \\ \dfrac{1}{\mathbf{z}_{21}} & \dfrac{\mathbf{z}_{22}}{\mathbf{z}_{21}} \end{bmatrix} = \begin{bmatrix} \dfrac{R_1 + R_2}{R_2} & \dfrac{R_1R_2 + R_2R_3 + R_3R_1}{R_2} \\ \dfrac{1}{R_2} & \dfrac{R_2 + R_3}{R_2} \end{bmatrix}$$

Hence,

$$\underline{\mathbf{A} = 1 + \frac{\mathbf{R}_1}{\mathbf{R}_2}}, \quad \underline{\mathbf{B} = \mathbf{R}_3 + \frac{\mathbf{R}_1}{\mathbf{R}_2}(\mathbf{R}_2 + \mathbf{R}_3)}, \quad \underline{\mathbf{C} = \frac{1}{\mathbf{R}_2}}, \quad \underline{\mathbf{D} = 1 + \frac{\mathbf{R}_3}{\mathbf{R}_2}}$$

as required.

Chapter 19, Problem 53.

Through derivation, express the z parameters in terms of the **ABCD** parameters.

Chapter 19, Solution 53.

For the z parameters,

$$\mathbf{V}_1 = \mathbf{z}_{11}\,\mathbf{I}_1 + \mathbf{z}_{12}\,\mathbf{I}_2 \tag{1}$$

$$\mathbf{V}_2 = \mathbf{z}_{12}\,\mathbf{I}_1 + \mathbf{z}_{22}\,\mathbf{I}_2 \tag{2}$$

For **ABCD** parameters,

$$\mathbf{V}_1 = \mathbf{A}\,\mathbf{V}_2 - \mathbf{B}\,\mathbf{I}_2 \tag{3}$$

$$\mathbf{I}_1 = \mathbf{C}\,\mathbf{V}_2 - \mathbf{D}\,\mathbf{I}_2 \tag{4}$$

From (4),

$$\mathbf{V}_2 = \frac{\mathbf{I}_1}{\mathbf{C}} + \frac{\mathbf{D}}{\mathbf{C}}\mathbf{I}_2 \tag{5}$$

Comparing (2) and (5),

$$\mathbf{z}_{21} = \frac{1}{\mathbf{C}}, \qquad \mathbf{z}_{22} = \frac{\mathbf{D}}{\mathbf{C}}$$

Substituting (5) into (3),

$$\mathbf{V}_1 = \frac{\mathbf{A}}{\mathbf{C}}\mathbf{I}_1 + \left(\frac{\mathbf{AD}}{\mathbf{C}} - \mathbf{B}\right)\mathbf{I}_2$$

$$= \frac{\mathbf{A}}{\mathbf{C}}\mathbf{I}_1 + \frac{\mathbf{AD} - \mathbf{BC}}{\mathbf{C}}\mathbf{I}_2 \tag{6}$$

Comparing (6) and (1),

$$\mathbf{z}_{11} = \frac{\mathbf{A}}{\mathbf{C}} \qquad \mathbf{z}_{12} = \frac{\mathbf{AD} - \mathbf{BC}}{\mathbf{C}} = \frac{\Delta_T}{\mathbf{C}}$$

Thus,

$$\underline{[Z] = \begin{bmatrix} \dfrac{\mathbf{A}}{\mathbf{C}} & \dfrac{\Delta_T}{\mathbf{C}} \\ \dfrac{1}{\mathbf{C}} & \dfrac{\mathbf{D}}{\mathbf{C}} \end{bmatrix}}$$

Chapter 19, Problem 54.

Show that the transmission parameters of a two-port may be obtained from the *y* parameters as:

$$A = -\frac{\mathbf{y}_{22}}{\mathbf{y}_{21}}, \qquad B = -\frac{1}{\mathbf{y}_{21}}$$

$$C = -\frac{\Delta_y}{\mathbf{y}_{21}}, \qquad D = -\frac{\mathbf{y}_{11}}{\mathbf{y}_{21}}$$

Chapter 19, Solution 54.

For the y parameters

$$\mathbf{I}_1 = \mathbf{y}_{11}\mathbf{V}_1 + \mathbf{y}_{12}\mathbf{V}_2 \qquad (1)$$

$$\mathbf{I}_2 = \mathbf{y}_{21}\mathbf{V}_1 + \mathbf{y}_{22}\mathbf{V}_2 \qquad (2)$$

From (2),

$$\mathbf{V}_1 = \frac{\mathbf{I}_2}{\mathbf{y}_{21}} - \frac{\mathbf{y}_{22}}{\mathbf{y}_{21}}\mathbf{V}_2$$

or

$$\mathbf{V}_1 = \frac{-\mathbf{y}_{22}}{\mathbf{y}_{12}}\mathbf{V}_2 + \frac{1}{\mathbf{y}_{21}}\mathbf{I}_2 \qquad (3)$$

Substituting (3) into (1) gives

$$\mathbf{I}_1 = \frac{-\mathbf{y}_{11}\mathbf{y}_{22}}{\mathbf{y}_{21}}\mathbf{V}_2 + \mathbf{y}_{12}\mathbf{V}_2 + \frac{\mathbf{y}_{11}}{\mathbf{y}_{21}}\mathbf{I}_2$$

or

$$\mathbf{I}_1 = \frac{-\Delta_y}{\mathbf{y}_{21}}\mathbf{V}_2 + \frac{\mathbf{y}_{11}}{\mathbf{y}_{21}}\mathbf{I}_2 \qquad (4)$$

Comparing (3) and (4) with the following equations

$$\mathbf{V}_1 = \mathbf{A}\mathbf{V}_2 - \mathbf{B}\mathbf{I}_2$$

$$\mathbf{I}_1 = \mathbf{C}\mathbf{V}_2 - \mathbf{D}\mathbf{I}_2$$

clearly shows that

$$\underline{\mathbf{A} = \frac{-\mathbf{y}_{22}}{\mathbf{y}_{21}}}, \quad \underline{\mathbf{B} = \frac{-\mathbf{1}}{\mathbf{y}_{21}}}, \quad \underline{\mathbf{C} = \frac{-\Delta_y}{\mathbf{y}_{21}}}, \quad \underline{\mathbf{D} = \frac{-\mathbf{y}_{11}}{\mathbf{y}_{21}}}$$

as required.

Chapter 19, Problem 55.

Prove that the *g* parameters can be obtained from the *z* parameters as

$$\mathbf{g}_{11} = \frac{1}{\mathbf{z}_{11}}, \qquad \mathbf{g}_{12} = -\frac{\mathbf{z}_{12}}{\mathbf{z}_{11}}$$

$$\mathbf{g}_{21} = \frac{\mathbf{z}_{21}}{\mathbf{z}_{11}}, \qquad \mathbf{g}_{22} = \frac{\Delta_z}{\mathbf{z}_{11}}$$

Chapter 19, Solution 55.

For the z parameters

$$\mathbf{V}_1 = \mathbf{z}_{11}\mathbf{I}_1 + \mathbf{z}_{12}\mathbf{I}_2 \qquad (1)$$

$$\mathbf{V}_2 = \mathbf{z}_{21}\mathbf{I}_1 + \mathbf{z}_{22}\mathbf{I}_2 \qquad (2)$$

From (1),

$$\mathbf{I}_1 = \frac{1}{\mathbf{z}_{11}}\mathbf{V}_1 - \frac{\mathbf{z}_{12}}{\mathbf{z}_{11}}\mathbf{I}_2 \qquad (3)$$

Substituting (3) into (2) gives

$$\mathbf{V}_2 = \frac{\mathbf{z}_{21}}{\mathbf{z}_{11}}\mathbf{V}_1 + \left(\mathbf{z}_{22} - \frac{\mathbf{z}_{21}\mathbf{z}_{12}}{\mathbf{z}_{11}}\right)\mathbf{I}_2$$

or

$$\mathbf{V}_2 = \frac{\mathbf{z}_{21}}{\mathbf{z}_{11}}\mathbf{V}_1 + \frac{\Delta_z}{\mathbf{z}_{11}}\mathbf{I}_2 \qquad (4)$$

Comparing (3) and (4) with the following equations

$$\mathbf{I}_1 = \mathbf{g}_{11}\mathbf{V}_1 + \mathbf{g}_{12}\mathbf{I}_2$$

$$\mathbf{V}_2 = \mathbf{g}_{21}\mathbf{V}_1 + \mathbf{g}_{22}\mathbf{I}_2$$

indicates that

$$\underline{\mathbf{g}_{11} = \frac{1}{\mathbf{z}_{11}}}, \quad \underline{\mathbf{g}_{12} = \frac{-\mathbf{z}_{12}}{\mathbf{z}_{11}}}, \quad \underline{\mathbf{g}_{21} = \frac{\mathbf{z}_{21}}{\mathbf{z}_{11}}}, \quad \underline{\mathbf{g}_{22} = \frac{\Delta_z}{\mathbf{z}_{11}}}$$

as required.

Chapter 19, Problem 56.

For the network of Fig. 19.107, obtain $\mathbf{V}_o/\mathbf{V}_s$.

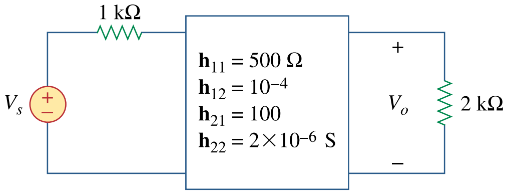
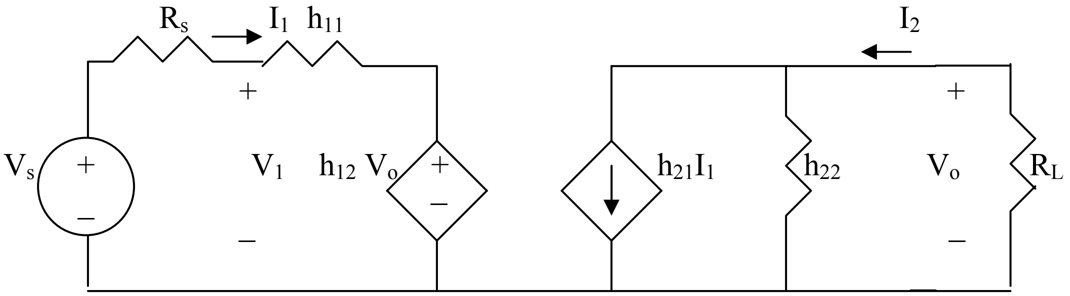


Figure 19.107
For Prob. 19.56.

Chapter 19, Solution 56.

Using Fig. 19.20, we obtain the equivalent circuit as shown below.



We can solve this using MATLAB. First, we generate 4 equations from the given circuit. It may help to let $V_s = 10$ V.

$-10 + R_sI_1 + V_1 = 0$ or $V_1 + 1000I_1 = 10$
$-10 + R_sI_1 + h_{11}I_1 + h_{12}V_o = 0$ or $0.0001V_s + 1500 = 10$
$I_2 = -V_o/R_L$ or $V_o + 2000I_2 = 0$
$h_{21}I_1 + h_{22}V_o - I_2 = 0$ or $2x10^{-6}V_o + 100I_1 - I_2 = 0$

```
>> A=[1,0,1000,0;0,0.0001,1500,0;0,1,0,2000;0,(2*10^-6),100,-1]
A =
  1.0e+003 *
    0.0010         0    1.0000         0
         0    0.0000    1.5000         0
         0    0.0010         0    2.0000
         0    0.0000    0.1000   -0.0010
>> U=[10;10;0;0]
U =
    10
    10
     0
     0
>> X=inv(A)*U
X =
  1.0e+003 *
    0.0032
   -1.3459
    0.0000
    0.0007
```

Gain = V_o /V_s = –1,345.9/10 = **<u>–134.59</u>**.

There is a second approach we can take to check this problem. First, the resistive value of h_{22} is quite large, 500 kΩ versus R_L so can be ignored. Working on the right side of the circuit we obtain the following,

$I_2 = 100I_1$ which leads to $V_o = -I_2x2k = -2x10^5I_1$.

Now the left hand loop equation becomes,

$-V_s + (1000 + 500 + 10^{-4}(-2x10^5))I_1 = 1480I_1$.

Solving for V_o/V_s we get,

$V_o/V_s = -200,000/1480 =$ **<u>–134.14</u>**.

Our answer checks!

Chapter 19, Problem 57.

Given the transmission parameters

$$[\mathbf{T}]=\begin{bmatrix}3 & 20\\ 1 & 7\end{bmatrix}$$

obtain the other five two-port parameters.

Chapter 19, Solution 57.

$$\Delta_T = (3)(7) - (20)(1) = 1$$

$$[\mathbf{z}] = \begin{bmatrix}\dfrac{\mathbf{A}}{\mathbf{C}} & \dfrac{\Delta_T}{\mathbf{C}}\\ \dfrac{1}{\mathbf{C}} & \dfrac{\mathbf{D}}{\mathbf{C}}\end{bmatrix} = \underline{\begin{bmatrix}\mathbf{3} & \mathbf{1}\\ \mathbf{1} & \mathbf{7}\end{bmatrix}\Omega}$$

$$[\mathbf{y}] = \begin{bmatrix}\dfrac{\mathbf{D}}{\mathbf{B}} & \dfrac{-\Delta_T}{\mathbf{B}}\\ \dfrac{-1}{\mathbf{B}} & \dfrac{\mathbf{A}}{\mathbf{B}}\end{bmatrix} = \underline{\begin{bmatrix}\dfrac{\mathbf{7}}{\mathbf{20}} & \dfrac{\mathbf{-1}}{\mathbf{20}}\\ \dfrac{\mathbf{-1}}{\mathbf{20}} & \dfrac{\mathbf{3}}{\mathbf{20}}\end{bmatrix}\mathbf{S}}$$

$$[\mathbf{h}] = \begin{bmatrix}\dfrac{\mathbf{B}}{\mathbf{D}} & \dfrac{\Delta_T}{\mathbf{D}}\\ \dfrac{-1}{\mathbf{D}} & \dfrac{\mathbf{C}}{\mathbf{D}}\end{bmatrix} = \underline{\begin{bmatrix}\dfrac{\mathbf{20}}{\mathbf{7}}\Omega & \dfrac{\mathbf{1}}{\mathbf{7}}\\ \dfrac{\mathbf{-1}}{\mathbf{7}} & \dfrac{\mathbf{1}}{\mathbf{7}}\mathbf{S}\end{bmatrix}}$$

$$[\mathbf{g}] = \begin{bmatrix}\dfrac{\mathbf{C}}{\mathbf{A}} & \dfrac{-\Delta_T}{\mathbf{A}}\\ \dfrac{1}{\mathbf{A}} & \dfrac{\mathbf{B}}{\mathbf{A}}\end{bmatrix} = \underline{\begin{bmatrix}\dfrac{\mathbf{1}}{\mathbf{3}}\mathbf{S} & \dfrac{\mathbf{-1}}{\mathbf{3}}\\ \dfrac{\mathbf{1}}{\mathbf{3}} & \dfrac{\mathbf{20}}{\mathbf{3}}\Omega\end{bmatrix}}$$

$$[\mathbf{t}] = \begin{bmatrix}\dfrac{\mathbf{D}}{\Delta_T} & \dfrac{\mathbf{B}}{\Delta_T}\\ \dfrac{\mathbf{C}}{\Delta_T} & \dfrac{\mathbf{A}}{\Delta_T}\end{bmatrix} = \underline{\begin{bmatrix}\mathbf{7} & \mathbf{20}\,\Omega\\ \mathbf{1\,S} & \mathbf{3}\end{bmatrix}}$$

Chapter 19, Problem 58.

A two-port is described by

$\mathbf{V}_1 = \mathbf{I}_1 + 2\mathbf{V}_2, \qquad \mathbf{I}_2 = -2\mathbf{I}_1 + 0.4\mathbf{V}_2$

Find: (a) the *y* parameters, (b) the transmission parameters.

Chapter 19, Solution 58.

The given set of equations is for the h parameters.

$$[\mathbf{h}] = \begin{bmatrix} 1\,\Omega & 2 \\ -2 & 0.4\text{ S} \end{bmatrix} \qquad \Delta_h = (1)(0.4) - (2)(-2) = 4.4$$

(a) $$[\mathbf{y}] = \begin{bmatrix} \dfrac{1}{\mathbf{h}_{11}} & \dfrac{-\mathbf{h}_{12}}{\mathbf{h}_{11}} \\ \dfrac{\mathbf{h}_{21}}{\mathbf{h}_{11}} & \dfrac{\Delta_h}{\mathbf{h}_{11}} \end{bmatrix} = \underline{\begin{bmatrix} \mathbf{1} & \mathbf{-2} \\ \mathbf{-2} & \mathbf{4.4} \end{bmatrix}\mathbf{S}}$$

(b) $$[\mathbf{T}] = \begin{bmatrix} \dfrac{-\Delta_h}{\mathbf{h}_{21}} & \dfrac{-\mathbf{h}_{11}}{\mathbf{h}_{21}} \\ \dfrac{-\mathbf{h}_{22}}{\mathbf{h}_{21}} & \dfrac{-1}{\mathbf{h}_{21}} \end{bmatrix} = \underline{\begin{bmatrix} \mathbf{2.2} & \mathbf{0.5\,\Omega} \\ \mathbf{0.2\text{ S}} & \mathbf{0.5} \end{bmatrix}}$$

Chapter 19, Problem 59.

Given that

$$[\mathbf{g}] = \begin{bmatrix} 0.06\,\text{S} & -0.4 \\ 0.2 & 2\Omega \end{bmatrix}$$

determine:

(a) [**z**]   (b) [**y**]   (c) [**h**]   (d) [**T**]

Chapter 19, Solution 59.

$$\Delta_g = (0.06)(2) - (\text{-}0.4)(0.2) = 0.12 + 0.08 = 0.2$$

(a) $$[\mathbf{z}] = \begin{bmatrix} \dfrac{1}{g_{11}} & \dfrac{-g_{12}}{g_{11}} \\ \dfrac{g_{21}}{g_{11}} & \dfrac{\Delta_g}{g_{11}} \end{bmatrix} = \underline{\begin{bmatrix} \mathbf{16.667} & \mathbf{6.667} \\ \mathbf{3.333} & \mathbf{3.333} \end{bmatrix} \Omega}$$

(b) $$[\mathbf{y}] = \begin{bmatrix} \dfrac{\Delta_g}{g_{22}} & \dfrac{g_{12}}{g_{22}} \\ \dfrac{-g_{21}}{g_{22}} & \dfrac{1}{g_{22}} \end{bmatrix} = \underline{\begin{bmatrix} \mathbf{0.1} & \mathbf{-0.2} \\ \mathbf{-0.1} & \mathbf{0.5} \end{bmatrix} \text{S}}$$

(c) $$[\mathbf{h}] = \begin{bmatrix} \dfrac{g_{22}}{\Delta_g} & \dfrac{-g_{12}}{\Delta_g} \\ \dfrac{-g_{21}}{\Delta_g} & \dfrac{g_{11}}{\Delta_g} \end{bmatrix} = \underline{\begin{bmatrix} \mathbf{10\,\Omega} & \mathbf{2} \\ \mathbf{-1} & \mathbf{0.3\,S} \end{bmatrix}}$$

(d) $$[\mathbf{T}] = \begin{bmatrix} \dfrac{1}{g_{21}} & \dfrac{g_{22}}{g_{21}} \\ \dfrac{g_{11}}{g_{21}} & \dfrac{\Delta_g}{g_{21}} \end{bmatrix} = \underline{\begin{bmatrix} \mathbf{5} & \mathbf{10\,\Omega} \\ \mathbf{0.3\,S} & \mathbf{1} \end{bmatrix}}$$

Chapter 19, Problem 60.

Design a **T** network necessary to realize the following *z* parameters at $\omega = 10^6$ rad/s

$$[\mathbf{z}] = \begin{bmatrix} 4 + j3 & 2 \\ 2 & 5 - j \end{bmatrix} \text{k}\Omega$$

Chapter 19, Solution 60.

Comparing this with Fig. 19.5,

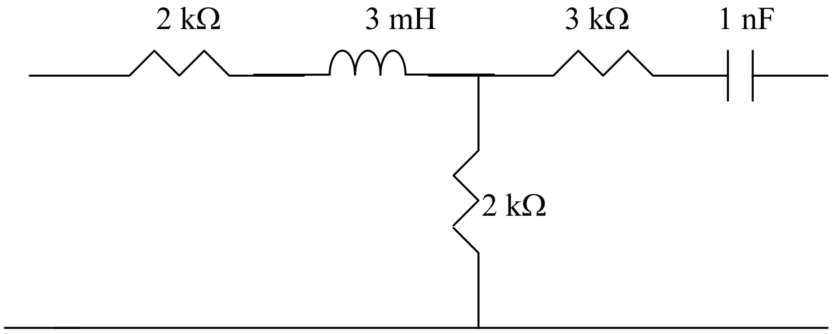
$$z_{11} - z_{12} = 4 + j3 - 2 = 2 + j3k\Omega$$

$z_{22} - z_{12} = 5 - j - 2 = 3 - j$ kΩ

$$X_L = 3x10^3 = \omega L \quad \longrightarrow \quad L = \frac{3x10^3}{10^6} = 3mH$$

$X_C = 1x10^3 = 1/(\omega C)$ or $C = 1/(10^3 x 10^6) = 1$ nF

Hence, the resulting T network is shown below.



Chapter 19, Problem 61.

For the bridge circuit in Fig. 19.108, obtain:

(a) the *z* parameters
(b) the *h* parameters
(c) the transmission parameters

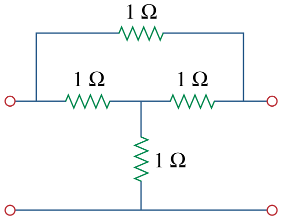
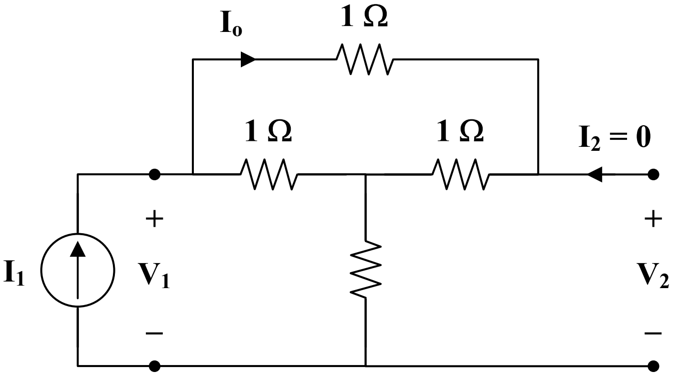


Figure 19.108

For Prob. 19.61.

Chapter 19, Solution 61.

(a) To obtain $\mathbf{z}_{11}$ and $\mathbf{z}_{21}$, consider the circuit in Fig. (a).



(a)

$$\mathbf{V}_1 = \mathbf{I}_1[1 + 1 \| (1+1)] = \mathbf{I}_1\left(1 + \frac{2}{3}\right) = \frac{5}{3}\mathbf{I}_1$$

$$\mathbf{z}_{11} = \frac{\mathbf{V}_1}{\mathbf{I}_1} = \frac{5}{3}$$

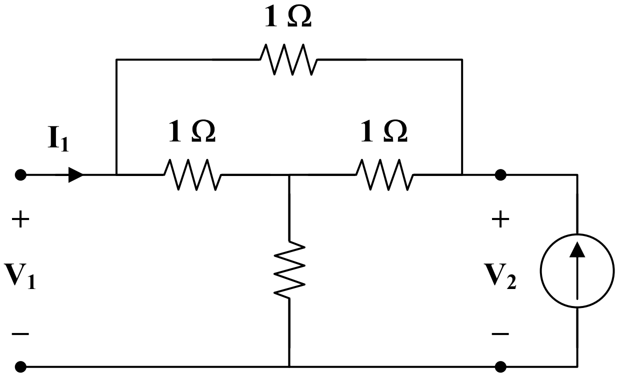
$$\mathbf{I}_o = \frac{1}{1+2}\mathbf{I}_1 = \frac{1}{3}\mathbf{I}_1$$

$$-\mathbf{V}_2 + \mathbf{I}_o + \mathbf{I}_1 = 0$$

$$\mathbf{V}_2 = \frac{1}{3}\mathbf{I}_1 + \mathbf{I}_1 = \frac{4}{3}\mathbf{I}_1$$

$$\mathbf{z}_{21} = \frac{\mathbf{V}_2}{\mathbf{I}_1} = \frac{4}{3}$$

To obtain $\mathbf{z}_{22}$ and $\mathbf{z}_{12}$, consider the circuit in Fig. (b).



(b)

Due to symmetry, this is similar to the circuit in Fig. (a).

$$\mathbf{z}_{22} = \mathbf{z}_{11} = \frac{5}{3}, \qquad \mathbf{z}_{21} = \mathbf{z}_{12} = \frac{4}{3}$$

$$\underline{[\mathbf{z}] = \begin{bmatrix} \frac{5}{3} & \frac{4}{3} \\ \frac{4}{3} & \frac{5}{3} \end{bmatrix} \Omega}$$

(b) $$[\mathbf{h}] = \begin{bmatrix} \frac{\Delta_z}{\mathbf{z}_{22}} & \frac{\mathbf{z}_{12}}{\mathbf{z}_{22}} \\ \frac{-\mathbf{z}_{21}}{\mathbf{z}_{22}} & \frac{1}{\mathbf{z}_{22}} \end{bmatrix} = \underline{\begin{bmatrix} \frac{3}{5}\Omega & \frac{4}{5} \\ \frac{-4}{5} & \frac{3}{5}\mathrm{S} \end{bmatrix}}$$

(c) $$[\mathbf{T}] = \begin{bmatrix} \frac{\mathbf{z}_{11}}{\mathbf{z}_{21}} & \frac{\Delta_z}{\mathbf{z}_{21}} \\ \frac{1}{\mathbf{z}_{21}} & \frac{\mathbf{z}_{22}}{\mathbf{z}_{21}} \end{bmatrix} = \underline{\begin{bmatrix} \frac{5}{4} & \frac{3}{4}\Omega \\ \frac{3}{4}\mathrm{S} & \frac{5}{4} \end{bmatrix}}$$

Chapter 19, Problem 62.

Find the z parameters of the op amp circuit in Fig. 19.109. Obtain the transmission parameters.

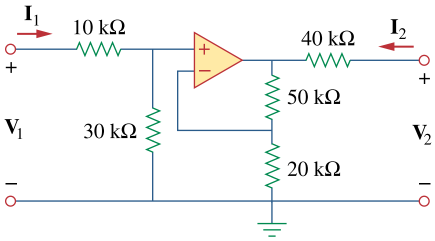
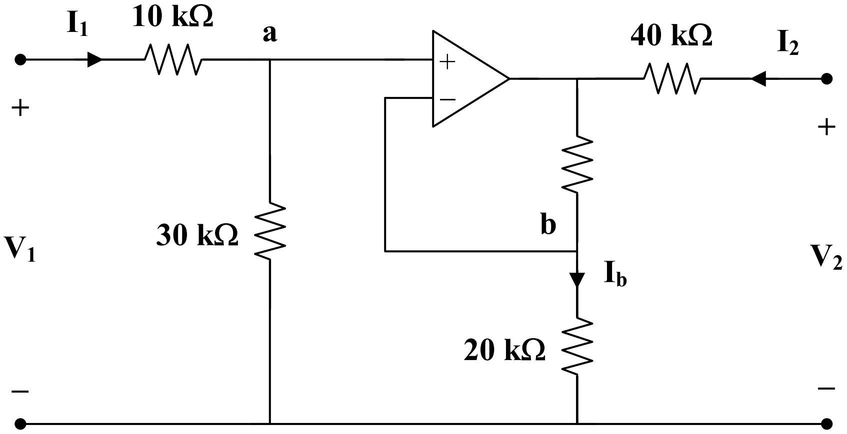


Figure 19.109

For Prob. 19.62.

Chapter 19, Solution 62.

Consider the circuit shown below.



Since no current enters the input terminals of the op amp,

$$\mathbf{V}_1 = (10+30)\times 10^3\,\mathbf{I}_1 \qquad (1)$$

But
$$\mathbf{V}_a = \mathbf{V}_b = \frac{30}{40}\mathbf{V}_1 = \frac{3}{4}\mathbf{V}_1$$

$$\mathbf{I}_b = \frac{\mathbf{V}_b}{20\times 10^3} = \frac{3}{80\times 10^3}\mathbf{V}_1$$

which is the same current that flows through the 50-kΩ resistor.

Thus,
$$\mathbf{V}_2 = 40\times 10^3\,\mathbf{I}_2 + (50+20)\times 10^3\,\mathbf{I}_b$$

$$\mathbf{V}_2 = 40\times 10^3\,\mathbf{I}_2 + 70\times 10^3\cdot\frac{3}{80\times 10^3}\mathbf{V}_1$$

$$\mathbf{V}_2 = \frac{21}{8}\mathbf{V}_1 + 40\times 10^3\,\mathbf{I}_2$$

$$\mathbf{V}_2 = 105\times 10^3\,\mathbf{I}_1 + 40\times 10^3\,\mathbf{I}_2 \qquad (2)$$

From (1) and (2),

$$\underline{[\mathbf{z}] = \begin{bmatrix} \mathbf{40} & \mathbf{0} \\ \mathbf{105} & \mathbf{40} \end{bmatrix}\mathbf{k\Omega}}$$

$$\Delta_z = \mathbf{z}_{11}\mathbf{z}_{22} - \mathbf{z}_{12}\mathbf{z}_{21} = 16\times 10^8$$

$$[\mathbf{T}] = \begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{C} & \mathbf{D} \end{bmatrix} = \begin{bmatrix} \dfrac{\mathbf{z}_{11}}{\mathbf{z}_{21}} & \dfrac{\Delta_z}{\mathbf{z}_{21}} \\ \dfrac{1}{\mathbf{z}_{21}} & \dfrac{\mathbf{z}_{22}}{\mathbf{z}_{21}} \end{bmatrix} = \underline{\begin{bmatrix} \mathbf{0.381} & \mathbf{15.24\ k\Omega} \\ \mathbf{9.52\ \mu S} & \mathbf{0.381} \end{bmatrix}}$$

Chapter 19, Problem 63.

Determine the *z* parameters of the two-port in Fig. 19.110.

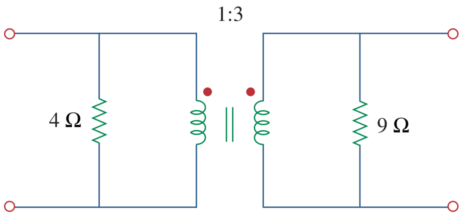
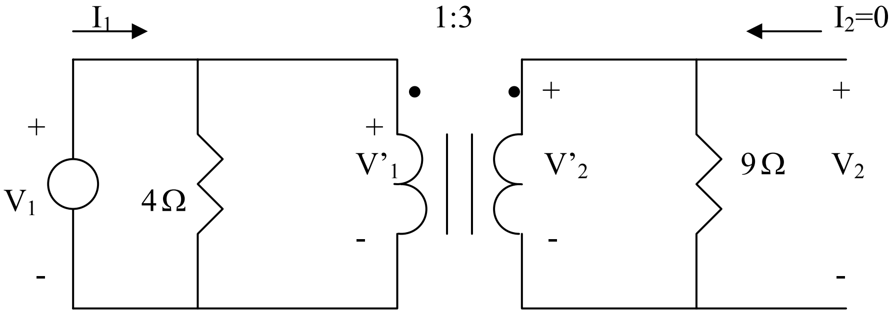


Figure 19.110

For Prob. 19.63.

Chapter 19, Solution 63.

To get z_{11} and z_{21}, consider the circuit below.

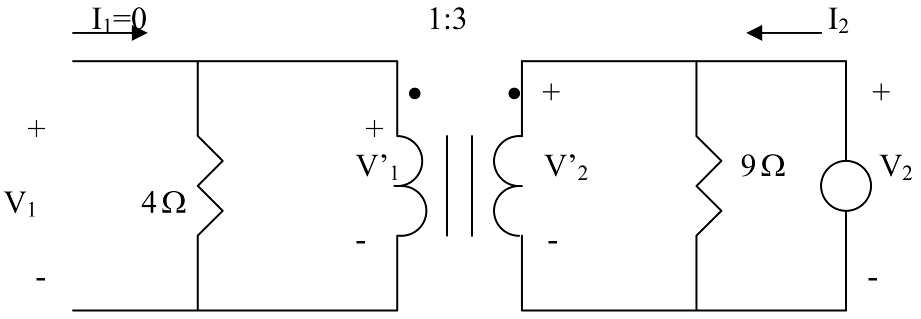


$$Z_R = \frac{9}{n^2} = 1, \qquad n = 3$$

$$V_1 = (4 // Z_R)I_1 = \frac{4}{5}I_1 \quad \longrightarrow \quad z_{11} = \frac{V_1}{I_1} = 0.8$$

$$V_2 = V_2' = nV_1' = nV_1 = 3(4/5)I_1 \quad \longrightarrow \quad z_{21} = \frac{V_2}{I_1} = 2.4$$

To get z_{21} and z_{22}, consider the circuit below.



$$Z_R' = n^2(4) = 36, \qquad n = 3$$

$$V_2 = (9 // Z_R')I_2 = \frac{9x36}{45}I_2 \quad \longrightarrow \quad z_{22} = \frac{V_2}{I_2} = 7.2$$

$$V_1 = \frac{V_2}{n} = \frac{V_2}{3} = 2.4I_2 \quad \longrightarrow \quad z_{21} = \frac{V_1}{I_2} = 2.4$$

Thus,

$$\underline{[z] = \begin{bmatrix} 0.8 & 2.4 \\ 2.4 & 7.2 \end{bmatrix}\Omega}$$

Chapter 19, Problem 64.

Determine the *y* parameters at $\omega = 1{,}000$ rad/s for the op amp circuit in Fig. 19.111. Find the corresponding *h* parameters.

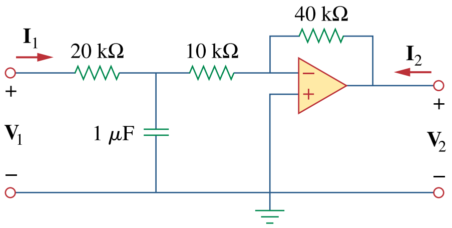


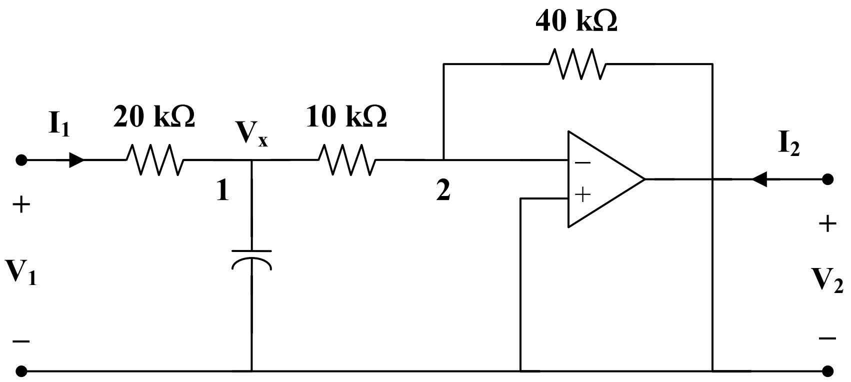
Figure 19.111

For Prob. 19.64.

Chapter 19, Solution 64.

$$1\ \mu\text{F} \longrightarrow \frac{1}{j\omega C} = \frac{-j}{(10^3)(10^{-6})} = -j\ \text{k}\Omega$$

Consider the op amp circuit below.



At node 1,

$$\frac{\mathbf{V}_1 - \mathbf{V}_x}{20} = \frac{\mathbf{V}_x}{-j} + \frac{\mathbf{V}_x - 0}{10}$$

$$\mathbf{V}_1 = (3 + j20)\,\mathbf{V}_x \qquad (1)$$

At node 2,

$$\frac{\mathbf{V}_x - 0}{10} = \frac{0 - \mathbf{V}_2}{40} \longrightarrow \mathbf{V}_x = \frac{-1}{4}\mathbf{V}_2 \qquad (2)$$

But $$\mathbf{I}_1 = \frac{\mathbf{V}_1 - \mathbf{V}_x}{20 \times 10^3} \qquad (3)$$

Substituting (2) into (3) gives

$$\mathbf{I}_1 = \frac{\mathbf{V}_1 + 0.25\,\mathbf{V}_2}{20 \times 10^3} = 50 \times 10^{-6}\,\mathbf{V}_1 + 12.5 \times 10^{-6}\,\mathbf{V}_2 \qquad (4)$$

Substituting (2) into (1) yields

$$\mathbf{V}_1 = \frac{-1}{4}(3 + j20)\,\mathbf{V}_2$$

or $$0 = \mathbf{V}_1 + (0.75 + j5)\,\mathbf{V}_2 \qquad (5)$$

Comparing (4) and (5) with the following equations

$$\mathbf{I}_1 = \mathbf{y}_{11}\,\mathbf{V}_1 + \mathbf{y}_{12}\,\mathbf{V}_2$$
$$\mathbf{I}_2 = \mathbf{y}_{21}\,\mathbf{V}_1 + \mathbf{y}_{22}\,\mathbf{V}_2$$

indicates that $\mathbf{I}_2 = 0$ and that

$$\underline{[\mathbf{y}] = \begin{bmatrix} \mathbf{50 \times 10^{-6}} & \mathbf{12.5 \times 10^{-6}} \\ \mathbf{1} & \mathbf{0.75 + j5} \end{bmatrix} \mathbf{S}}$$

$$\Delta_y = (77.5 + j25. - 12.5) \times 10^{-6} = (65 + j250) \times 10^{-6}$$

$$[\mathbf{h}] = \begin{bmatrix} \dfrac{1}{\mathbf{y}_{11}} & \dfrac{-\mathbf{y}_{12}}{\mathbf{y}_{11}} \\ \dfrac{\mathbf{y}_{21}}{\mathbf{y}_{11}} & \dfrac{\Delta_y}{\mathbf{y}_{11}} \end{bmatrix} = \underline{\begin{bmatrix} \mathbf{2 \times 10^4\ \Omega} & \mathbf{-0.25} \\ \mathbf{2 \times 10^4} & \mathbf{1.3 + j5\ S} \end{bmatrix}}$$

Chapter 19, Problem 65.

What is the *y* parameter presentation of the circuit in Fig. 19.112?

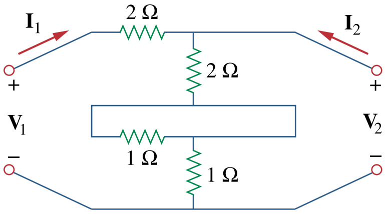


Figure 19.112

For Prob. 19.65.

Chapter 19, Solution 65.

The network consists of two two-ports in series. It is better to work with z parameters and then convert to y parameters.

For N_a, $$[\mathbf{z}_a] = \begin{bmatrix} 4 & 2 \\ 2 & 2 \end{bmatrix}$$

For N_b, $$[\mathbf{z}_b] = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$$

$$[\mathbf{z}] = [\mathbf{z}_a] + [\mathbf{z}_b] = \begin{bmatrix} 6 & 3 \\ 3 & 3 \end{bmatrix}$$

$$\Delta_z = 18 - 9 = 9$$

$$[\mathbf{y}] = \begin{bmatrix} \frac{\mathbf{z}_{22}}{\Delta_z} & \frac{-\mathbf{z}_{12}}{\Delta_z} \\ \frac{-\mathbf{z}_{21}}{\Delta_z} & \frac{\mathbf{z}_{11}}{\Delta_z} \end{bmatrix} = \underline{\begin{bmatrix} \mathbf{\frac{1}{3}} & \mathbf{\frac{-1}{3}} \\ \mathbf{\frac{-1}{3}} & \mathbf{\frac{2}{3}} \end{bmatrix} \mathbf{S}}$$

Chapter 19, Problem 66.

In the two-port of Fig. 19.113, let $\mathbf{y}_{12} = \mathbf{y}_{21} = 0$, $\mathbf{y}_{11} = 2$ mS, and $\mathbf{y}_{22} = 10$ mS. Find $\mathbf{V}_o/\mathbf{V}_s$.

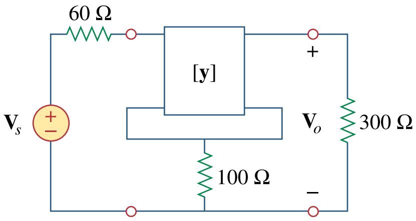


Figure 19.113

For Prob. 19.66.

Chapter 19, Solution 66.

Since we have two two-ports in series, it is better to convert the given y parameters to z parameters.

$$\Delta_y = \mathbf{y}_{11}\mathbf{y}_{22} - \mathbf{y}_{12}\mathbf{y}_{21} = (2\times 10^{-3})(10\times 10^{-3}) - 0 = 20\times 10^{-6}$$

$$[\mathbf{z}_a] = \begin{bmatrix} \dfrac{\mathbf{y}_{22}}{\Delta_y} & \dfrac{-\mathbf{y}_{12}}{\Delta_y} \\ \dfrac{-\mathbf{y}_{21}}{\Delta_y} & \dfrac{\mathbf{y}_{11}}{\Delta_y} \end{bmatrix} = \begin{bmatrix} 500\,\Omega & 0 \\ 0 & 100\,\Omega \end{bmatrix}$$

$$[\mathbf{z}] = \begin{bmatrix} 500 & 0 \\ 0 & 100 \end{bmatrix} + \begin{bmatrix} 100 & 100 \\ 100 & 100 \end{bmatrix} = \begin{bmatrix} 600 & 100 \\ 100 & 200 \end{bmatrix}$$

i.e.

$$\mathbf{V}_1 = \mathbf{z}_{11}\mathbf{I}_1 + \mathbf{z}_{12}\mathbf{I}_2$$
$$\mathbf{V}_2 = \mathbf{z}_{21}\mathbf{I}_1 + \mathbf{z}_{22}\mathbf{I}_2$$

or

$$\mathbf{V}_1 = 600\mathbf{I}_1 + 100\mathbf{I}_2 \qquad (1)$$
$$\mathbf{V}_2 = 100\mathbf{I}_1 + 200\mathbf{I}_2 \qquad (2)$$

But, at the input port,

$$\mathbf{V}_s = \mathbf{V}_1 + 60\mathbf{I}_1 \qquad (3)$$

and at the output port,

$$\mathbf{V}_2 = \mathbf{V}_o = \text{-}300\mathbf{I}_2 \qquad (4)$$

From (2) and (4),

$$100\mathbf{I}_1 + 200\mathbf{I}_2 = \text{-}300\mathbf{I}_2$$
$$\mathbf{I}_1 = \text{-}5\mathbf{I}_2 \qquad (5)$$

Substituting (1) and (5) into (3),

$$\mathbf{V}_s = 600\mathbf{I}_1 + 100\mathbf{I}_2 + 60\mathbf{I}_1$$
$$= (660)(\text{-}5)\mathbf{I}_2 + 100\mathbf{I}_2$$
$$= \text{-}3200\mathbf{I}_2 \qquad (6)$$

From (4) and (6),

$$\frac{\mathbf{V}_o}{\mathbf{V}_2} = \frac{\text{-}300\mathbf{I}_2}{\text{-}3200\mathbf{I}_2} = \underline{\mathbf{0.09375}}$$

Chapter 19, Problem 67.

If three copies of the circuit in Fig. 19.114 are connected in parallel, find the overall transmission parameters.

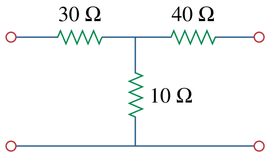
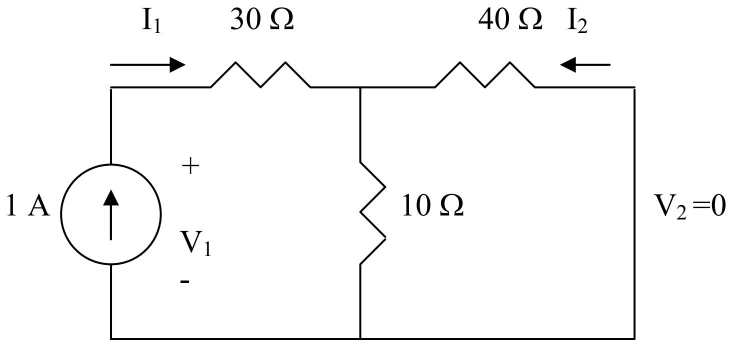


Figure 19.114

For Prob. 19.67.

Chapter 19, Solution 67.

We first the y parameters, to find y_{11} and y_{21} consider the circuit below.

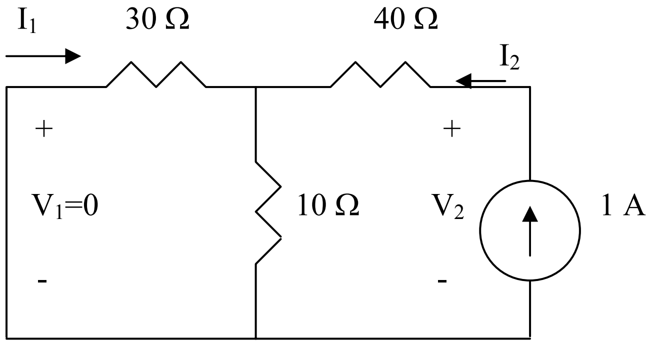


$$V_1 = I_1(30 + 10 // 40) = 38I_1 \longrightarrow y_{11} = \frac{I_1}{V_1} = \frac{1}{38}$$

By current division,

$$I_2 = \frac{-10}{50} I_1 = -0.2I_1 \longrightarrow y_{21} = \frac{I_2}{V_1} = \frac{-0.2I_1}{38I_1} = \frac{-1}{190}$$

To find y_{22} and y_{12} consider the circuit below.



$$V_2 = (40+10//30)I_2 = 47.5I_2 \quad \longrightarrow \quad y_{22} = \frac{I_2}{V_2} = \frac{2}{93} \; y_{22} = 2/95$$

By current division,

$$I_1 = -\frac{10}{30+10}I_2 = -\frac{I_2}{4} \quad \longrightarrow \quad y_{12} = \frac{I_1}{V_2} = \frac{-\frac{1}{4}I_2}{47.5I_2} = -\frac{1}{190}$$

$$[y] = \begin{bmatrix} 1/38 & -1/190 \\ -1/190 & 2/95 \end{bmatrix}$$

For three copies cascaded in parallel, we can use MATLAB.

```
>> Y=[1/38,-1/190;-1/190,2/95]
Y =
   0.0263  -0.0053
  -0.0053   0.0211
>> Y3=3*Y
Y3 =
   0.0789  -0.0158
  -0.0158   0.0632
>> DY=0.0789*0.0632-0.0158*0.158
DY =
   0.0025
>> T=[0.0632/0.0158,1/0.0158;DY/0.0158,0.0789/0.0158]
T =
   4.0000  63.2911
   0.1576   4.9937
```

$$\underline{T = \begin{bmatrix} 4 & 63.29 \\ 0.1576 & 4.994 \end{bmatrix}}$$

Chapter 19, Problem 68.

Obtain the *h* parameters for the network in Fig. 19.115.

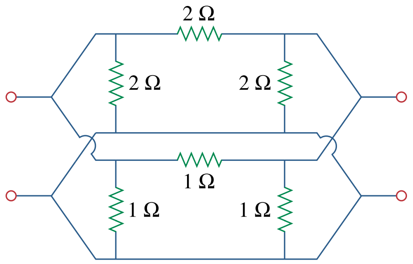


Figure 19.115
For Prob. 19.68.

Chapter 19, Solution 68.

For the upper network $\mathrm{N_a}$, $[\mathbf{y}_a] = \begin{bmatrix} 4 & -2 \\ -2 & 4 \end{bmatrix}$

and for the lower network $\mathrm{N_b}$, $[\mathbf{y}_b] = \begin{bmatrix} 2 & -1 \\ 1 & 2 \end{bmatrix}$

For the overall network,

$$[\mathbf{y}] = [\mathbf{y}_a] + [\mathbf{y}_b] = \begin{bmatrix} 6 & -3 \\ -3 & 6 \end{bmatrix}$$

$$\Delta_y = 36 - 9 = 27$$

$$[\mathbf{h}] = \begin{bmatrix} \dfrac{1}{\mathbf{y}_{11}} & \dfrac{-\mathbf{y}_{12}}{\mathbf{y}_{11}} \\ \dfrac{\mathbf{y}_{21}}{\mathbf{y}_{11}} & \dfrac{\Delta_y}{\mathbf{y}_{11}} \end{bmatrix} = \underline{\begin{bmatrix} \dfrac{1}{6}\Omega & \dfrac{1}{2} \\ \dfrac{1}{2} & \dfrac{9}{2}\mathrm{S} \end{bmatrix}}$$

Chapter 19, Problem 69.

* The circuit in Fig. 19.116 may be regarded as two two-ports connected in parallel. Obtain the *y* parameters as functions of *s*.

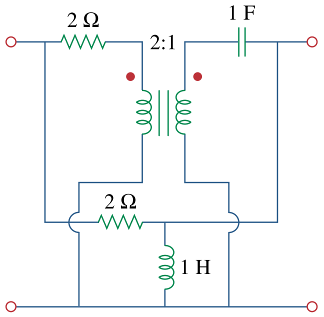


Figure 19.116

For Prob. 19.69.

* An asterisk indicates a challenging problem.

Chapter 19, Solution 69.

We first determine the y parameters for the upper network N_a.

To get $\mathbf{y}_{11}$ and $\mathbf{y}_{21}$, consider the circuit in Fig. (a).

$$n = \frac{1}{2}, \qquad \mathbf{Z}_R = \frac{1/s}{n^2} = \frac{4}{s}$$

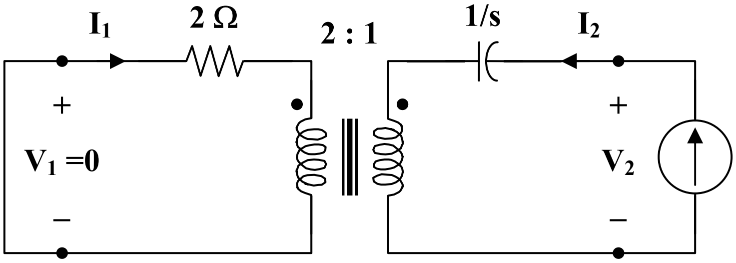
$$\mathbf{V}_1 = (2 + \mathbf{Z}_R)\mathbf{I}_1 = \left(2 + \frac{4}{s}\right)\mathbf{I}_1 = \left(\frac{2s+4}{s}\right)\mathbf{I}_1$$

$$\mathbf{y}_{11} = \frac{\mathbf{I}_1}{\mathbf{V}_1} = \frac{s}{2(s+2)}$$

$$\mathbf{I}_2 = \frac{-\mathbf{I}_1}{n} = -2\mathbf{I}_1 = \frac{-s\mathbf{V}_1}{s+2}$$

$$\mathbf{y}_{21} = \frac{\mathbf{I}_2}{\mathbf{V}_1} = \frac{-s}{s+2}$$

To get $\mathbf{y}_{22}$ and $\mathbf{y}_{12}$, consider the circuit in Fig. (b).



(b)

$$\mathbf{Z}_R{}' = (n^2)(2) = \left(\frac{1}{4}\right)(2) = \frac{1}{2}$$

$$\mathbf{V}_2 = \left(\frac{1}{s} + \mathbf{Z}_R{}'\right)\mathbf{I}_2 = \left(\frac{1}{s} + \frac{1}{2}\right)\mathbf{I}_2 = \left(\frac{s+2}{2s}\right)\mathbf{I}_2$$

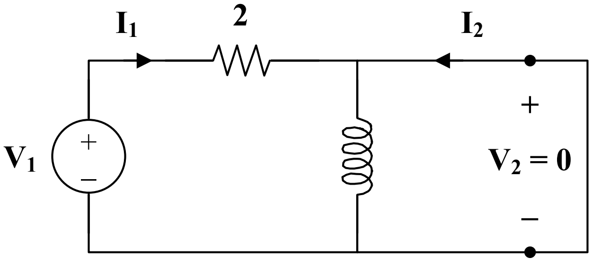
$$\mathbf{y}_{22} = \frac{\mathbf{I}_2}{\mathbf{V}_2} = \frac{2s}{s+2}$$

$$\mathbf{I}_1 = -n\,\mathbf{I}_2 = \left(\frac{-1}{2}\right)\left(\frac{2s}{s+2}\right)\mathbf{V}_2 = \left(\frac{-s}{s+2}\right)\mathbf{V}_2$$

$$\mathbf{y}_{12} = \frac{\mathbf{I}_1}{\mathbf{V}_2} = \frac{-s}{s+2}$$

$$[\mathbf{y}_a] = \begin{bmatrix} \dfrac{s}{2(s+2)} & \dfrac{-s}{s+2} \\ \dfrac{-s}{s+2} & \dfrac{2s}{s+2} \end{bmatrix}$$

For the lower network N_b, we obtain $\mathbf{y}_{11}$ and $\mathbf{y}_{21}$ by referring to the network in Fig. (c).

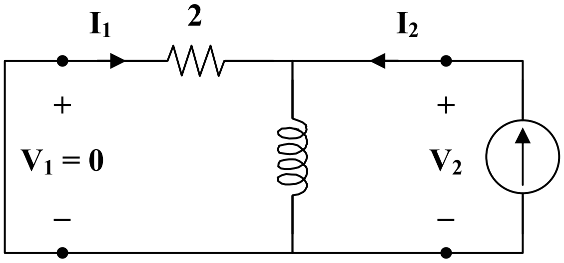


(c)

$$\mathbf{V}_1 = 2\mathbf{I}_1 \longrightarrow \mathbf{y}_{11} = \frac{\mathbf{I}_1}{\mathbf{V}_1} = \frac{1}{2}$$

$$\mathbf{I}_2 = -\mathbf{I}_1 = \frac{-\mathbf{V}_1}{2} \longrightarrow \mathbf{y}_{21} = \frac{\mathbf{I}_2}{\mathbf{V}_1} = \frac{-1}{2}$$

To get $\mathbf{y}_{22}$ and $\mathbf{y}_{12}$, refer to the circuit in Fig. (d).



(d)

$$\mathbf{V}_2 = (\mathrm{s}\,||\,2)\mathbf{I}_2 = \frac{2\mathrm{s}}{\mathrm{s}+2}\mathbf{I}_2 \longrightarrow \mathbf{y}_{22} = \frac{\mathbf{I}_2}{\mathbf{V}_2} = \frac{\mathrm{s}+2}{2\mathrm{s}}$$

$$\mathbf{I}_1 = -\mathbf{I}_2 \cdot \frac{-\mathrm{s}}{\mathrm{s}+2} = \left(\frac{-\mathrm{s}}{\mathrm{s}+2}\right)\left(\frac{\mathrm{s}+2}{2\mathrm{s}}\right)\mathbf{V}_2 = \frac{-\mathbf{V}_2}{2}$$

$$\mathbf{y}_{12} = \frac{\mathbf{I}_1}{\mathbf{V}_2} = \frac{-1}{2}$$

$$[\mathbf{y}_b] = \begin{bmatrix} 1/2 & -1/2 \\ -1/2 & (\mathrm{s}+2)/2\mathrm{s} \end{bmatrix}$$

$$[\mathbf{y}] = [\mathbf{y}_a] + [\mathbf{y}_b] = \underline{\begin{bmatrix} \dfrac{\mathbf{s}+1}{\mathbf{s}+2} & \dfrac{-(3\mathbf{s}+2)}{2(\mathbf{s}+2)} \\ \dfrac{-(3\mathbf{s}+2)}{2(\mathbf{s}+2)} & \dfrac{5\mathbf{s}^2+4\mathbf{s}+4}{2\mathbf{s}(\mathbf{s}+2)} \end{bmatrix}}$$

Chapter 19, Problem 70.

* For the parallel-series connection of the two two-ports in Fig. 19.117, find the *g* parameters.

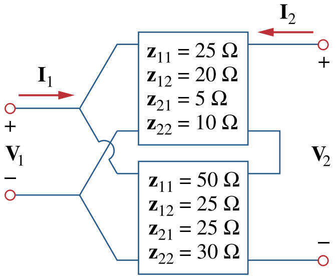


Figure 19.117

For Prob. 19.70.

* An asterisk indicates a challenging problem.

Chapter 19, Solution 70.

We may obtain the g parameters from the given z parameters.

$$[\mathbf{z}_a] = \begin{bmatrix} 25 & 20 \\ 5 & 10 \end{bmatrix}, \qquad \Delta_{z_a} = 250 - 100 = 150$$

$$[\mathbf{z}_b] = \begin{bmatrix} 50 & 25 \\ 25 & 30 \end{bmatrix}, \qquad \Delta_{z_b} = 1500 - 625 = 875$$

$$[\mathbf{g}] = \begin{bmatrix} \dfrac{1}{z_{11}} & \dfrac{-z_{12}}{z_{11}} \\ \dfrac{z_{21}}{z_{11}} & \dfrac{\Delta_z}{z_{11}} \end{bmatrix}$$

$$[\mathbf{g}_a] = \begin{bmatrix} 0.04 & -0.8 \\ 0.2 & 6 \end{bmatrix}, \qquad [\mathbf{g}_b] = \begin{bmatrix} 0.02 & -0.5 \\ 0.5 & 17.5 \end{bmatrix}$$

$$[\mathbf{g}] = [\mathbf{g}_a] + [\mathbf{g}_b] = \underline{\begin{bmatrix} \mathbf{0.06\ S} & \mathbf{-1.3} \\ \mathbf{0.7} & \mathbf{23.5\ \Omega} \end{bmatrix}}$$

Chapter 19, Problem 71.

* Determine the *z* parameters for the network in Fig. 19.118.

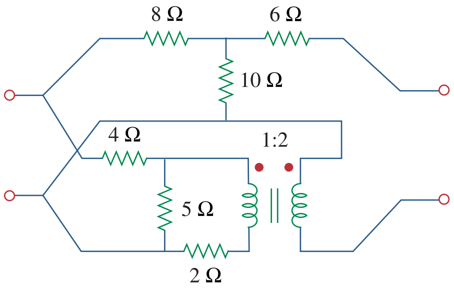


Figure 19.118

For Prob. 19.71.

* An asterisk indicates a challenging problem.

Chapter 19, Solution 71.

This is a parallel-series connection of two two-ports. We need to add their g parameters together and obtain z parameters from there.

For the transformer,

$$V_1 = \frac{1}{2}V_2, \quad I_1 = -2I_2$$

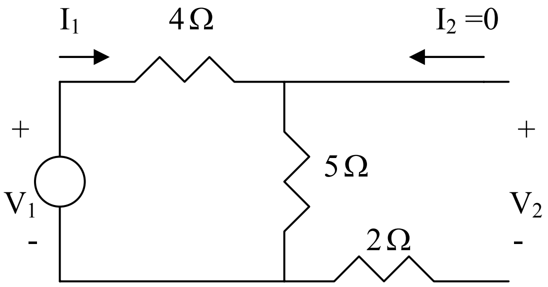
Comparing this with

$$V_1 = AV_2 - BI_2, \qquad I_1 = CV_2 - DI_2$$

shows that

$$[T_{b1}] = \begin{bmatrix} 0.5 & 0 \\ 0 & 2 \end{bmatrix}$$

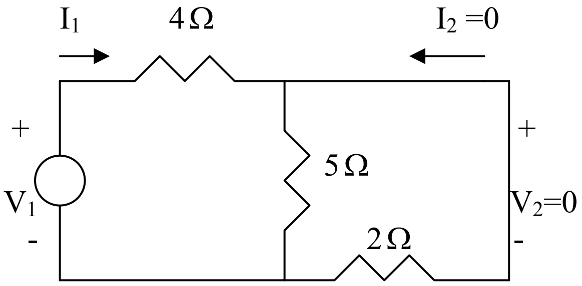
To get A and C for T_{b2} , consider the circuit below.



$$V_1 = 9I_1, \qquad V_2 = 5I_1$$

$$A = \frac{V_1}{V_2} = 9/5 = 1.8, \qquad C = \frac{I_1}{V_2} = 1/5 = 0.2$$

We obtain B and D by looking at the circuit below.



$$I_2 = -\frac{5}{7}I_1 \qquad \longrightarrow \qquad D = -\frac{I_1}{I_2} = 7/5 = 1.4$$

$$V_1 = 4I_1 - 2I_2 = 4(-\frac{7}{5}I_2) - 2I_2 = -\frac{38}{5}I_2 \qquad \longrightarrow \qquad B = -\frac{V_1}{I_2} = 7.6$$

$$[T_{b2}] = \begin{bmatrix} 1.8 & 7.6 \\ 0.2 & 1.4 \end{bmatrix}$$

$$[T] = [T_{b1}][T_{b2}] = \begin{bmatrix} 0.9 & 3.8 \\ 0.4 & 2.8 \end{bmatrix}, \qquad \Delta_T = 1$$

$$[g_b] = \begin{bmatrix} C/A & -\Delta_T/A \\ 1/A & B/A \end{bmatrix} = \begin{bmatrix} 0.4444 & -1.1111 \\ 1.1111 & 4.2222 \end{bmatrix}$$

From Prob. 19.52,

$$[T_a] = \begin{bmatrix} 1.8 & 18.8 \\ 0.1 & 1.6 \end{bmatrix}$$

$$[g_a] = \begin{bmatrix} C/A & -\Delta_T/A \\ 1/A & B/A \end{bmatrix} = \begin{bmatrix} 0.05555 & -0.5555 \\ 0.5555 & 10.4444 \end{bmatrix}$$

$$[g] = [g_a] + [g_b] = \begin{bmatrix} 0.4999 & -1.6667 \\ 1.6667 & 14.667 \end{bmatrix}$$

Thus,

$$[z] = \begin{bmatrix} 1/g_{11} & -g_{21}/g_{11} \\ g_{21}/g_{11} & \Delta_g/g_{11} \end{bmatrix} = \underline{\begin{bmatrix} 2 & -3.334 \\ 3.334 & 20.22 \end{bmatrix}\Omega}$$

Chapter 19, Problem 72.

* A series-parallel connection of two two-ports is shown in Fig. 19.119. Determine the *z* parameter representation of the network.

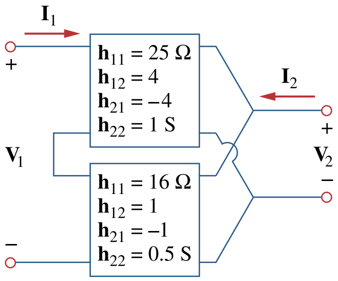


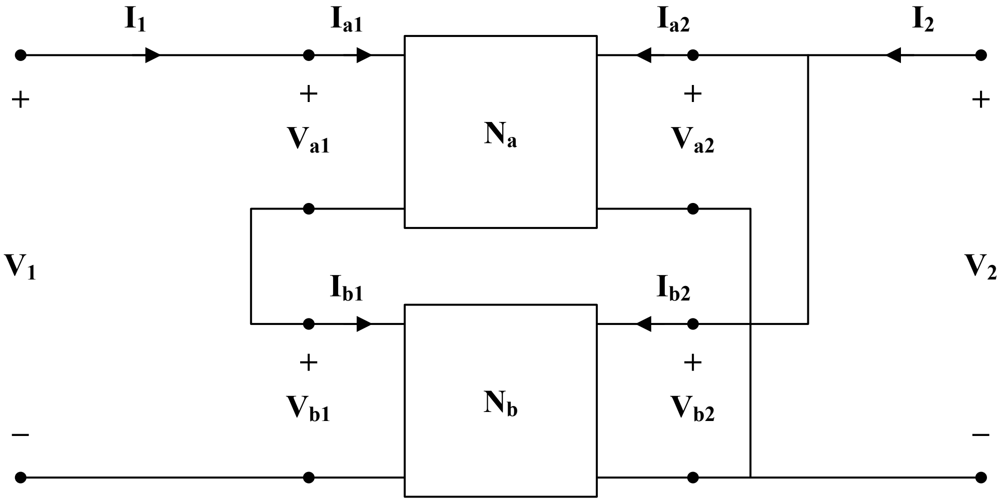
Figure 19.119

For Prob. 19.72.

* An asterisk indicates a challenging problem.

Chapter 19, Solution 72.

Consider the network shown below.



$$\mathbf{V}_{a1} = 25\mathbf{I}_{a1} + 4\mathbf{V}_{a2} \quad (1)$$
$$\mathbf{I}_{a2} = -4\mathbf{I}_{a1} + \mathbf{V}_{a2} \quad (2)$$
$$\mathbf{V}_{b1} = 16\mathbf{I}_{b1} + \mathbf{V}_{b2} \quad (3)$$
$$\mathbf{I}_{b2} = -\mathbf{I}_{b1} + 0.5\mathbf{V}_{b2} \quad (4)$$

$$\mathbf{V}_1 = \mathbf{V}_{a1} + \mathbf{V}_{b1}$$
$$\mathbf{V}_2 = \mathbf{V}_{a2} = \mathbf{V}_{b2}$$
$$\mathbf{I}_2 = \mathbf{I}_{a2} + \mathbf{I}_{b2}$$
$$\mathbf{I}_1 = \mathbf{I}_{a1}$$

Now, rewrite (1) to (4) in terms of $\mathbf{I}_1$ and $\mathbf{V}_2$

$$\mathbf{V}_{a1} = 25\mathbf{I}_1 + 4\mathbf{V}_2 \quad (5)$$
$$\mathbf{I}_{a2} = -4\mathbf{I}_1 + \mathbf{V}_2 \quad (6)$$
$$\mathbf{V}_{b1} = 16\mathbf{I}_{b1} + \mathbf{V}_2 \quad (7)$$
$$\mathbf{I}_{b2} = -\mathbf{I}_{b1} + 0.5\mathbf{V}_2 \quad (8)$$

Adding (5) and (7),

$$\mathbf{V}_1 = 25\mathbf{I}_1 + 16\mathbf{I}_{b1} + 5\mathbf{V}_2 \quad (9)$$

Adding (6) and (8),

$$\mathbf{I}_2 = -4\mathbf{I}_1 - \mathbf{I}_{b1} + 1.5\mathbf{V}_2 \quad (10)$$
$$\mathbf{I}_{b1} = \mathbf{I}_{a1} = \mathbf{I}_1 \quad (11)$$

Because the two networks N_a and N_b are independent,

$$\mathbf{I}_2 = -5\mathbf{I}_1 + 1.5\mathbf{V}_2$$

or

$$\mathbf{V}_2 = 3.333\mathbf{I}_1 + 0.6667\mathbf{I}_2 \qquad (12)$$

Substituting (11) and (12) into (9),

$$\mathbf{V}_1 = 41\mathbf{I}_1 + \frac{25}{1.5}\mathbf{I}_1 + \frac{5}{1.5}\mathbf{I}_2$$

$$\mathbf{V}_1 = 57.67\mathbf{I}_1 + 3.333\mathbf{I}_2 \qquad (13)$$

Comparing (12) and (13) with the following equations

$$\mathbf{V}_1 = \mathbf{z}_{11}\,\mathbf{I}_1 + \mathbf{z}_{12}\,\mathbf{I}_2$$

$$\mathbf{V}_2 = \mathbf{z}_{21}\,\mathbf{I}_1 + \mathbf{z}_{22}\,\mathbf{I}_2$$

indicates that

$$\underline{[\mathbf{z}] = \begin{bmatrix} \mathbf{57.67} & \mathbf{3.333} \\ \mathbf{3.333} & \mathbf{0.6667} \end{bmatrix}\Omega}$$

Alternatively,

$$[\mathbf{h}_a] = \begin{bmatrix} 25 & 4 \\ -4 & 1 \end{bmatrix}, \qquad [\mathbf{h}_b] = \begin{bmatrix} 16 & 1 \\ -1 & 0.5 \end{bmatrix}$$

$$[\mathbf{h}] = [\mathbf{h}_a] + [\mathbf{h}_b] = \begin{bmatrix} 41 & 5 \\ -5 & 1.5 \end{bmatrix} \qquad \Delta_h = 61.5 + 25 = 86.5$$

$$[\mathbf{z}] = \begin{bmatrix} \dfrac{\Delta_h}{\mathbf{h}_{22}} & \dfrac{\mathbf{h}_{12}}{\mathbf{h}_{22}} \\ \dfrac{-\mathbf{h}_{21}}{\mathbf{h}_{22}} & \dfrac{1}{\mathbf{h}_{22}} \end{bmatrix} = \underline{\begin{bmatrix} \mathbf{57.67} & \mathbf{3.333} \\ \mathbf{3.333} & \mathbf{0.6667} \end{bmatrix}\Omega}$$

as obtained previously.

Chapter 19, Problem 73.

Three copies of the circuit shown in Fig. 19.70 are connected in cascade. Determine the *z* parameters.

Chapter 19, Solution 73.

From Problem 19.6,

$$[z] = \begin{bmatrix} 25 & 20 \\ 24 & 30 \end{bmatrix}, \qquad \Delta z = 25x30 - 20x24 = 270$$

$$A = \frac{z_{11}}{z_{21}} = \frac{25}{24}, \qquad B = \frac{\Delta z}{z_{21}} = \frac{270}{24}$$

$$C = \frac{1}{z_{21}} = \frac{1}{24}, \; D = \frac{z_{22}}{z_{21}} = \frac{30}{24}$$

The overall ABCD parameters can be found using MATLAB.

```
 >> T=[25/24,270/24;1/24,30/24]
T =
   1.0417   11.2500
   0.0417    1.2500
>> T3=T*T*T
T3 =
   2.6928   49.7070
   0.1841    3.6133
>> Z=[2.693/0.1841,(2.693*3.613-0.1841*49.71)/0.1841;1/0.1841,3.613/0.1841]
Z =
  14.6279    3.1407
   5.4318   19.6252
```

$$\underline{Z = \begin{bmatrix} 14.628 & 3.141 \\ 5.432 & 19.625 \end{bmatrix}}$$

Chapter 19, Problem 74.

ML

* Determine the **ABCD** parameters of the circuit in Fig. 19.120 as functions of *s*. (*Hint:* Partition the circuit into subcircuits and cascade them using the results of Prob. 19.43.)

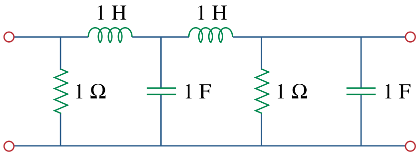


Figure 19.120

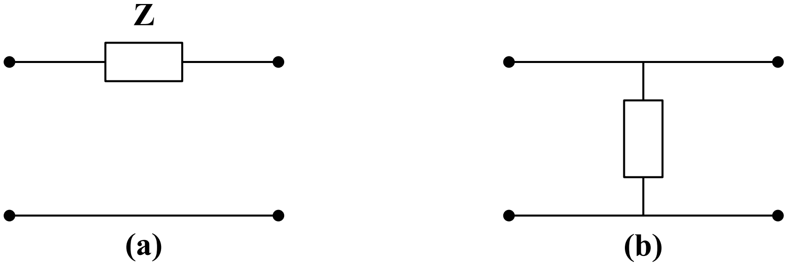
For Prob. 19.74.

* An asterisk indicates a challenging problem.

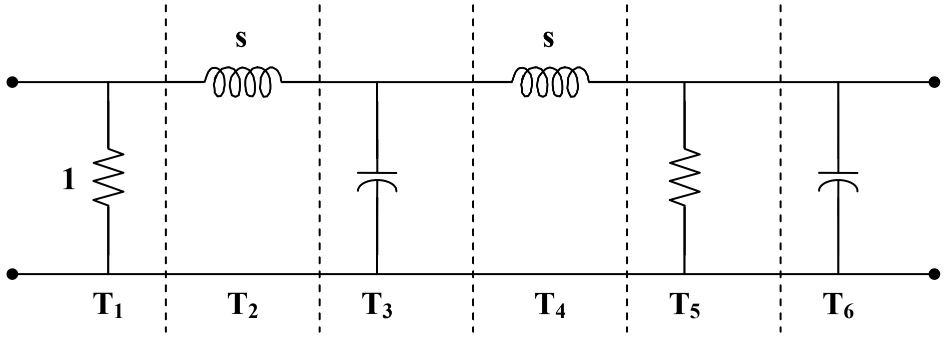
Chapter 19, Solution 74.

From Prob. 18.35, the transmission parameters for the circuit in Figs. (a) and (b) are

$$[\mathbf{T}_a] = \begin{bmatrix} 1 & \mathbf{Z} \\ 0 & 1 \end{bmatrix}, \qquad [\mathbf{T}_b] = \begin{bmatrix} 1 & 0 \\ 1/\mathbf{Z} & 1 \end{bmatrix}$$



We partition the given circuit into six subcircuits similar to those in Figs. (a) and (b) as shown in Fig. (c) and obtain $[\mathbf{T}]$ for each.



$$[\mathbf{T}_1] = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}, \qquad [\mathbf{T}_2] = \begin{bmatrix} 1 & s \\ 0 & 1 \end{bmatrix}, \qquad [\mathbf{T}_3] = \begin{bmatrix} 1 & 0 \\ s & 1 \end{bmatrix}$$

$$[\mathbf{T}_4] = [\mathbf{T}_2], \qquad [\mathbf{T}_5] = [\mathbf{T}_1], \qquad [\mathbf{T}_6] = [\mathbf{T}_3]$$

$$[\mathbf{T}] = [\mathbf{T}_1][\mathbf{T}_2][\mathbf{T}_3][\mathbf{T}_4][\mathbf{T}_5][\mathbf{T}_6] = [\mathbf{T}_1][\mathbf{T}_2][\mathbf{T}_3][\mathbf{T}_4]\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ s & 1 \end{bmatrix}$$

$$= [\mathbf{T}_1][\mathbf{T}_2][\mathbf{T}_3][\mathbf{T}_4]\begin{bmatrix} 1 & 0 \\ s+1 & 1 \end{bmatrix} = [\mathbf{T}_1][\mathbf{T}_2][\mathbf{T}_3]\begin{bmatrix} 1 & s \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ s+1 & 1 \end{bmatrix}$$

$$= [\mathbf{T}_1][\mathbf{T}_2]\begin{bmatrix} 1 & 0 \\ s & 1 \end{bmatrix}\begin{bmatrix} s^2+s+1 & s \\ s+1 & 1 \end{bmatrix}$$

$$= [\mathbf{T}_1]\begin{bmatrix} 1 & s \\ 0 & 1 \end{bmatrix}\begin{bmatrix} s^2+s+1 & s \\ s^3+s^2+2s+1 & s^2+1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} s^4+s^3+3s^2+2s+1 & s^3+2s \\ s^3+s^2+2s+1 & s^2+1 \end{bmatrix}$$

$$[\mathbf{T}] = \underline{\begin{bmatrix} \mathbf{s^4+s^3+3s^2+2s+1} & \mathbf{s^3+2s} \\ \mathbf{s^4+2s^3+4s^2+4s+2} & \mathbf{s^3+s^2+2s+1} \end{bmatrix}}$$

Note that $\mathbf{AB} - \mathbf{CD} = 1$ as expected.

Chapter 19, Problem 75.

* For the individual two-ports shown in Fig. 19.121 where,

$$[\mathbf{z}_a] = \begin{bmatrix} 8 & 6 \\ 4 & 5 \end{bmatrix}\Omega \qquad [\mathbf{y}_b] = \begin{bmatrix} 8 & -4 \\ 2 & 10 \end{bmatrix}\text{S}$$

(a) Determine the *y* parameters of the overall two-port.
(b) Find the voltage ratio $\mathbf{V}_o/\mathbf{V}_i$ when $\mathbf{Z}_L = 2\ \Omega$.

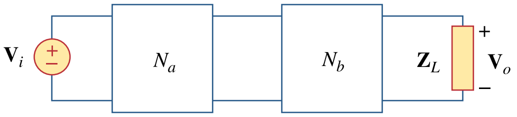


Figure 19.110

For Prob. 19.63.

* An asterisk indicates a challenging problem.

Chapter 19, Solution 75.

(a) We convert [z$_a$] and [z$_b$] to T-parameters. For N$_a$, $\Delta_z = 40 - 24 = 16$.

$$[T_a] = \begin{bmatrix} z_{11}/z_{21} & \Delta_z/z_{21} \\ 1/z_{21} & z_{22}/z_{21} \end{bmatrix} = \begin{bmatrix} 2 & 4 \\ 0.25 & 1.25 \end{bmatrix}$$

For N$_b$, $\Delta_y = 80 + 8 = 88$.

$$[T_b] = \begin{bmatrix} -y_{22}/y_{21} & -1/y_{21} \\ -\Delta_y/y_{21} & -y_{11}/y_{21} \end{bmatrix} = \begin{bmatrix} -5 & -0.5 \\ -44 & -4 \end{bmatrix}$$

$$[T] = [T_a][T_b] = \begin{bmatrix} -186 & -17 \\ -56.25 & -5.125 \end{bmatrix}$$

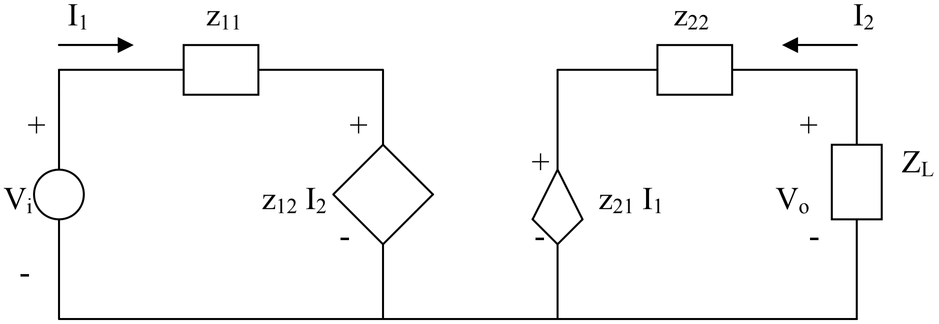
We convert this to y-parameters. $\Delta_T = AD - BC = -3$.

$$[y] = \begin{bmatrix} D/B & -\Delta_T/B \\ -1/B & A/B \end{bmatrix} = \underline{\begin{bmatrix} 0.3015 & -0.1765 \\ 0.0588 & 10.94 \end{bmatrix}}$$

(b) The equivalent z-parameters are

$$[z] = \begin{bmatrix} A/C & \Delta_T/C \\ 1/C & D/C \end{bmatrix} = \begin{bmatrix} 3.3067 & 0.0533 \\ -0.0178 & 0.0911 \end{bmatrix}$$

Consider the equivalent circuit below.



$$V_i = z_{11}I_1 + z_{12}I_2 \qquad (1)$$

$$V_o = z_{21}I_1 + z_{22}I_2 \qquad (2)$$

But $V_o = -I_2Z_L \longrightarrow I_2 = -V_o/Z_L$ (3)

From (2) and (3) ,

$$V_o = z_{21}I_1 - z_{22}\frac{V_o}{Z_L} \longrightarrow I_1 = V_o\left(\frac{1}{z_{21}} + \frac{z_{22}}{Z_L z_{21}}\right) \qquad (4)$$

Substituting (3) and (4) into (1) gives

$$\frac{V_i}{V_o} = \left(\frac{z_{11}}{z_{21}} + \frac{z_{11}z_{22}}{z_{21}Z_L}\right) - \frac{z_{12}}{Z_L} = -194.3 \longrightarrow \underline{\frac{V_{o.}}{V_i} = -0.0051}$$

Chapter 19, Problem 76.

Use *PSpice* to obtain the *z* parameters of the network in Fig. 19.122.

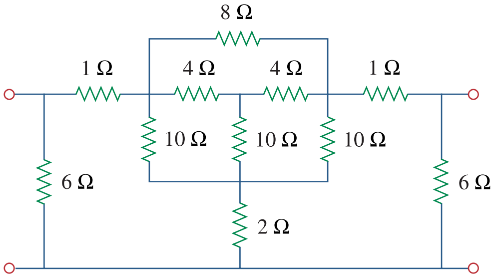


Figure 19.122

For Prob. 19.76.

Chapter 19, Solution 76.

To get z_{11} and z_{21}, we open circuit the output port and let $I_1 = 1A$ so that

$$z_{11} = \frac{V_1}{I_1} = V_1, \qquad z_{21} = \frac{V_2}{I_1} = V_2$$

The schematic is shown below. After it is saved and run, we obtain

$$z_{11} = V_1 = 3.849, \qquad z_{21} = V_2 = 1.122$$

Similarly, to get z_{22} and z_{12}, we open circuit the input port and let $I_2 = 1A$ so that

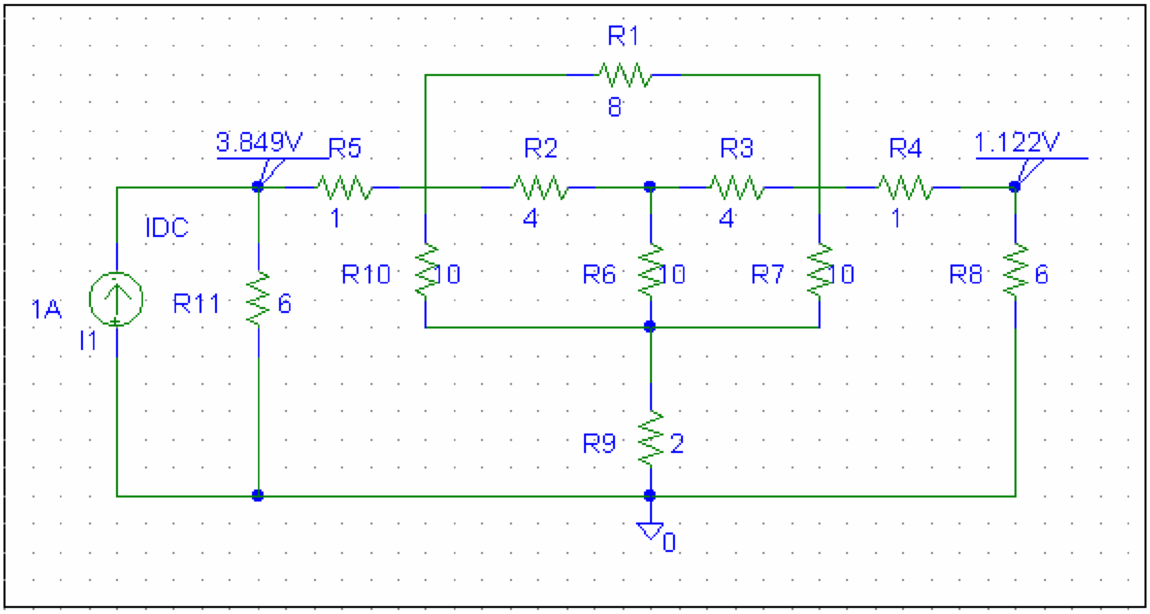
$$z_{12} = \frac{V_1}{I_2} = V_1, \qquad z_{22} = \frac{V_2}{I_2} = V_2$$

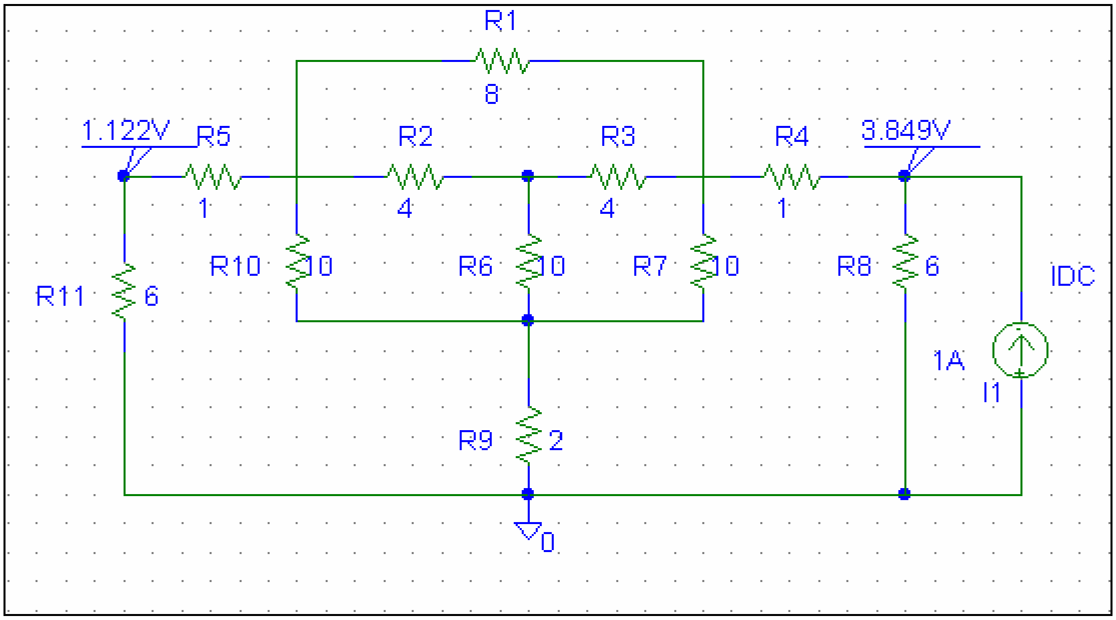
The schematic is shown below. After it is saved and run, we obtain

$$z_{12} = V_1 = 1.122, \qquad z_{22} = V_2 = 3.849$$

Thus,

$$\underline{[z] = \begin{bmatrix} 3.949 & 1.122 \\ 1.122 & 3.849 \end{bmatrix} \Omega}$$


R1
8
3.849V
R5
R2
R3
R4
1.122V
1
4
4
1
IDC
R10
10
R6
10
R7
10
R8
6
1A
R11
6
I1
R9
2
0


R1
8
1.122V
R5
R2
R3
R4
3.849V
1
4
4
1
R10
10
R6
10
R7
10
R8
6
IDC
R11
6
1A
I1
R9
2
0

Chapter 19, Problem 77.

Using *PSpice*, find the *h* parameters of the network in Fig. 19.123. Take $\omega = 1$ rad/s

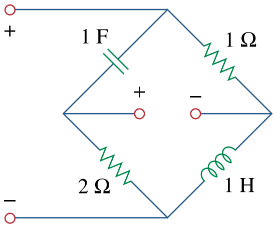


Figure 19.123

For Prob. 19.77.

Chapter 19, Solution 77.

We follow Example 19.15 except that this is an AC circuit.

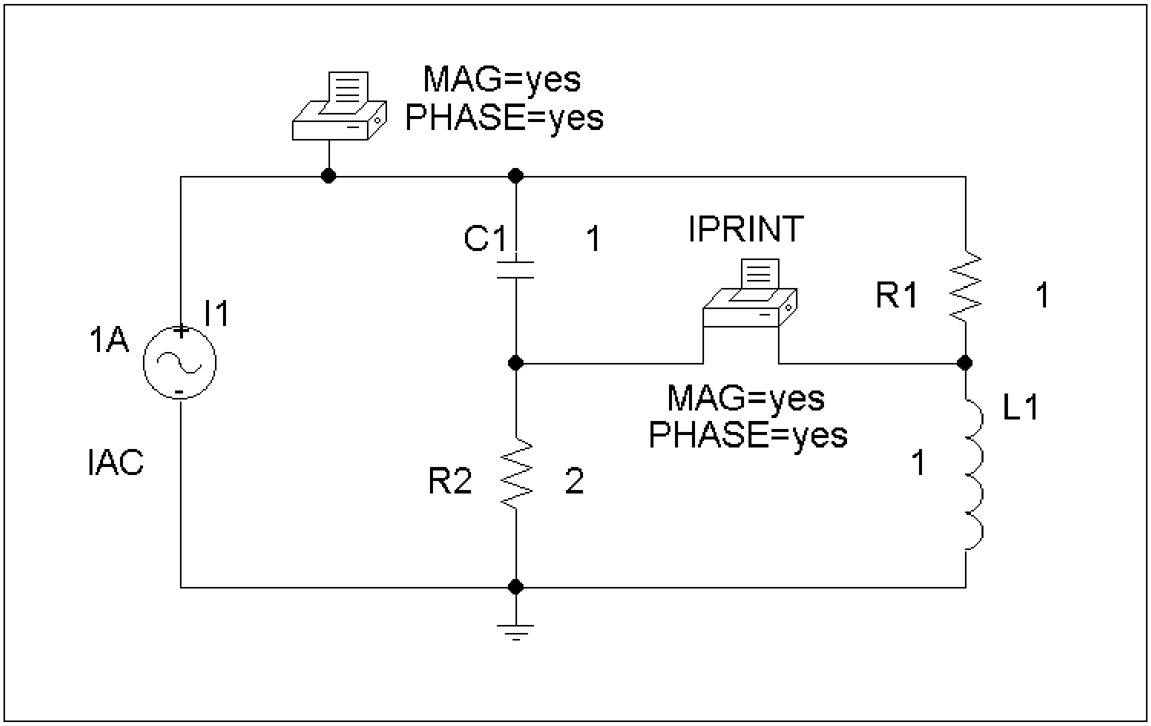
(a) We set $V_2 = 0$ and $I_1 = 1$ A. The schematic is shown below. In the AC Sweep Box, set Total Pts = 1, Start Freq = 0.1592, and End Freq = 0.1592. After simulation, the output file includes

```
FREQ          IM(V_PRINT2)        IP(V_PRINT2)

1.592 E–01    3.163 E–.01         –1.616 E+02


FREQ          VM($N_0001)         VP($N_0001)

1.592 E–01    9.488 E–01          –1.616 E+02
```

From this we obtain

$$h_{11} = V_1/1 = 0.9488\angle{-161.6^\circ}$$

$$h_{21} = I_2/1 = 0.3163\angle{-161.6^\circ}.$$



(b) In this case, we set $I_1 = 0$ and $V_2 = 1$V. The schematic is shown below. In the AC Sweep box, we set Total Pts = 1, Start Freq = 0.1592, and End Freq = 0.1592. After simulation, we obtain an output file which includes

```
FREQ          VM($N_0001)          VP($N_0001)

1.592 E–01    3.163 E–.01          1.842 E+01


FREQ          IM(V_PRINT2)         IP(V_PRINT2)

1.592 E–01    9.488 E–01           –1.616 E+02
```

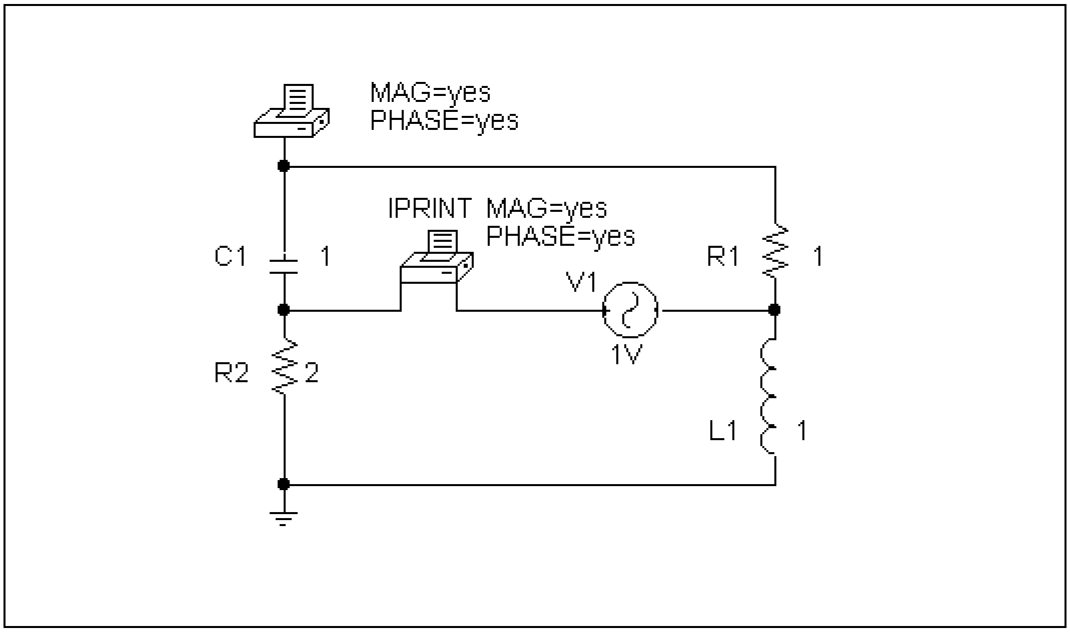
From this,

$$h_{12} = V_1/1 = 0.3163\angle 18.42^\circ$$

$$h_{21} = I_2/1 = 0.9488\angle -161.6^\circ.$$

Thus,

$$\underline{[h] = \begin{bmatrix} \mathbf{0.9488\angle -161.6^\circ} & \mathbf{0.3163\angle 18.42^\circ} \\ \mathbf{0.3163\angle -161.6^\circ} & \mathbf{0.9488\angle -161.6^\circ} \end{bmatrix}}$$



Chapter 19, Problem 78.

Obtain the h parameters at $\omega = 4$ rad/s for the circuit in Fig. 19.124 using *PSpice*.

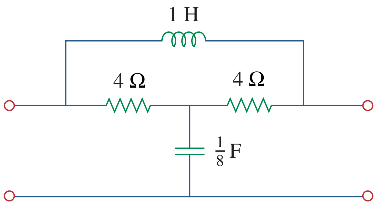


Figure 19.124

For Prob. 19.78.

Chapter 19, Solution 78

For h_{11} and h_{21}, short-circuit the output port and let $I_1 = 1A$. $f = \omega / 2\pi = 0.6366$. The schematic is shown below. When it is saved and run, the output file contains the following:

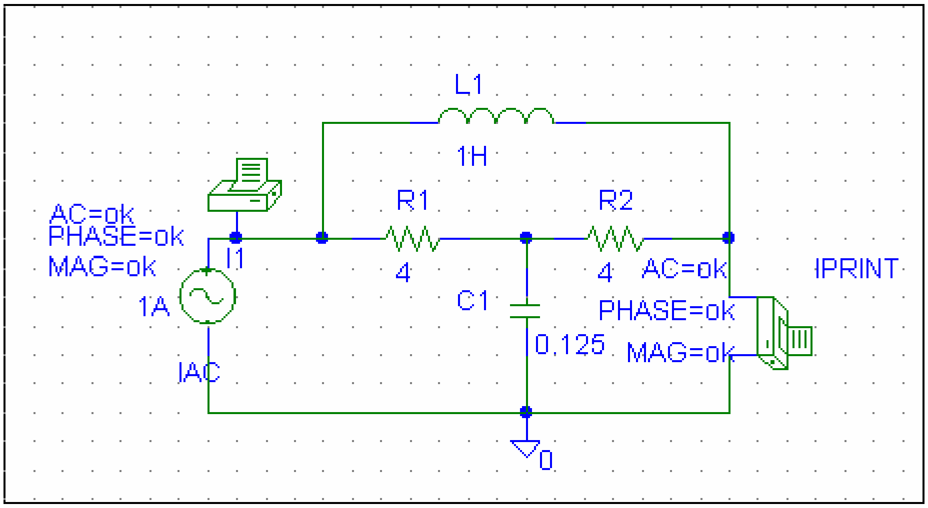
```
FREQ       IM(V_PRINT1)IP(V_PRINT1)


  6.366E-01   1.202E+00   1.463E+02


 FREQ       VM($N_0003) VP($N_0003)


  6.366E-01   3.771E+00  -1.350E+02
```

From the output file, we obtain

$$I_2 = 1.202\angle 146.3^{o}, \qquad V_1 = 3.771\angle -135^{o}$$

so that

$$h_{11} = \frac{V_1}{1} = 3.771\angle -135^{o}, \qquad h_{21} = \frac{I_2}{1} = 1.202\angle 146.3^{o}$$



For h_{12} and h_{22}, open-circuit the input port and let $V_2 = 1V$. The schematic is shown below. When it is saved and run, the output file includes:

```
 FREQ       VM($N_0003) VP($N_0003)


  6.366E-01   1.202E+00  -3.369E+01

 FREQ       IM(V_PRINT1)IP(V_PRINT1)


  6.366E-01   3.727E-01  -1.534E+02
```

From the output file, we obtain

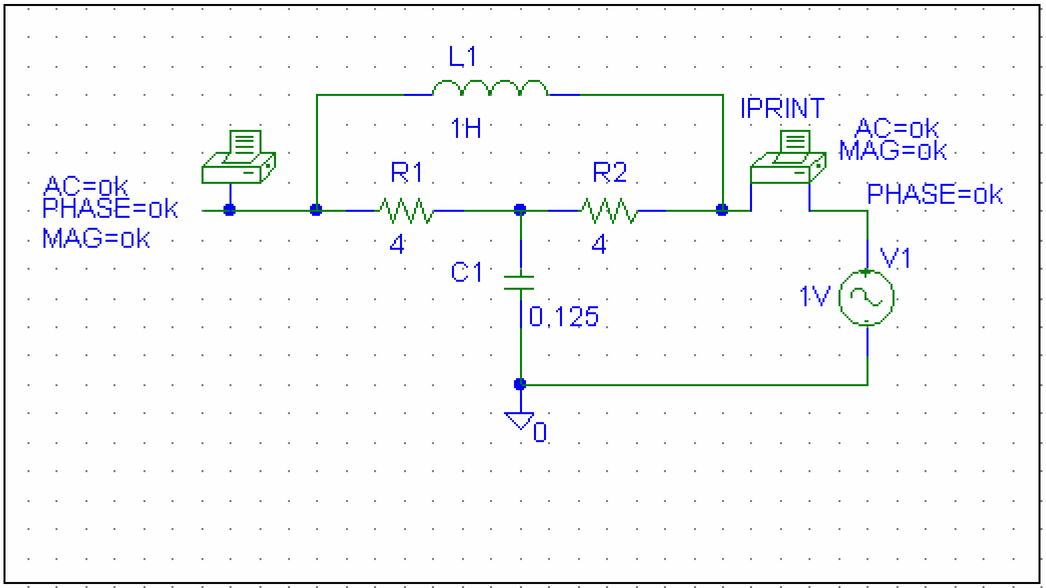
$$I_2 = 0.3727\angle -153.4^o, \qquad V_1 = 1.202\angle -33.69^o$$

so that

$$h_{12} = \frac{V_1}{1} = 1.202\angle -33.69^o, \qquad h_{22} = \frac{I_2}{1} = 0.3727\angle -153.4^o$$

Thus,

$$\underline{[h] = \begin{bmatrix} 3.771\angle -135^o & 1.202\angle -33.69^o \\ 1.202\angle 146.3 & 0.3727\angle -153.4^o \end{bmatrix}}$$



Chapter 19, Problem 79.

Use *PSpice* to determine the *z* parameters of the circuit in Fig. 19.125. Take $\omega = 2$ rad/s.

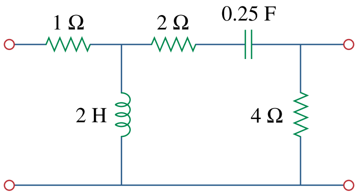


Figure 19.125

For Prob. 19.79.

Chapter 19, Solution 79

We follow Example 19.16.

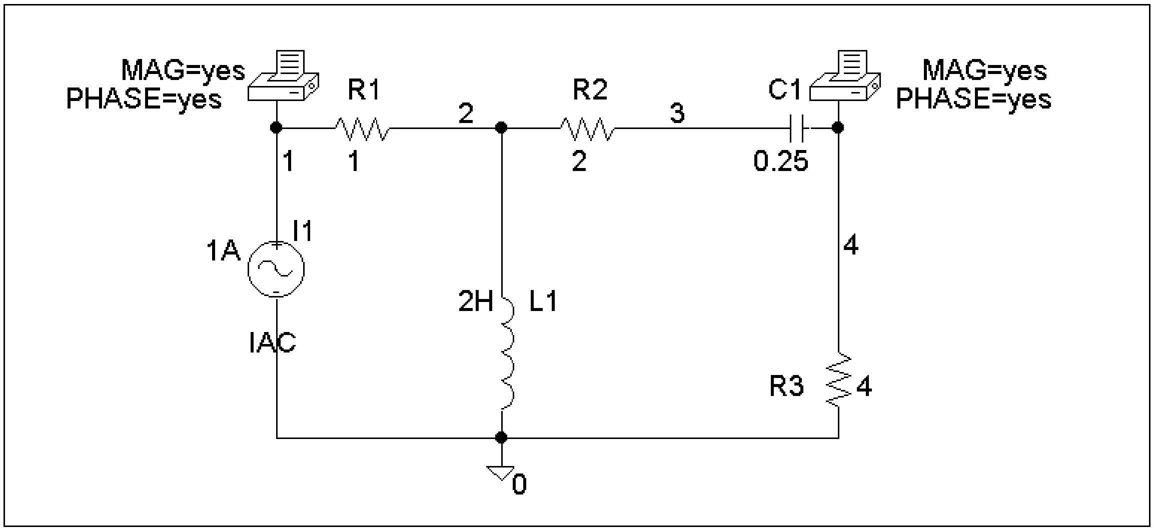
(a) We set $I_1 = 1$ A and open-circuit the output-port so that $I_2 = 0$. The schematic is shown below with two VPRINT1s to measure V_1 and V_2. In the AC Sweep box, we enter Total Pts = 1, Start Freq = 0.3183, and End Freq = 0.3183. After simulation, the output file includes

```
FREQ          VM(1)              VP(1)

3.183 E–01    4.669 E+00         –1.367 E+02


FREQ          VM(4)              VP(4)

3.183 E–01    2.530 E+00         –1.084 E+02
```

From this,

$$z_{11} = V_1/I_1 = 4.669\angle{-136.7^\circ}/1 = 4.669\angle{-136.7^\circ}$$

$$z_{21} = V_2/I_1 = 2.53\angle{-108.4^\circ}/1 = 2.53\angle{-108.4^\circ}.$$



(b) In this case, we let $I_2 = 1$ A and open-circuit the input port. The schematic is shown below. In the AC Sweep box, we type Total Pts = 1, Start Freq = 0.3183, and End Freq = 0.3183. After simulation, the output file includes

```
FREQ          VM(1)           VP(1)

3.183 E–01    2.530 E+00      –1.084 E+02


FREQ          VM(2)           VP(2)

3.183 E–01    1.789 E+00      –1.534 E+02
```

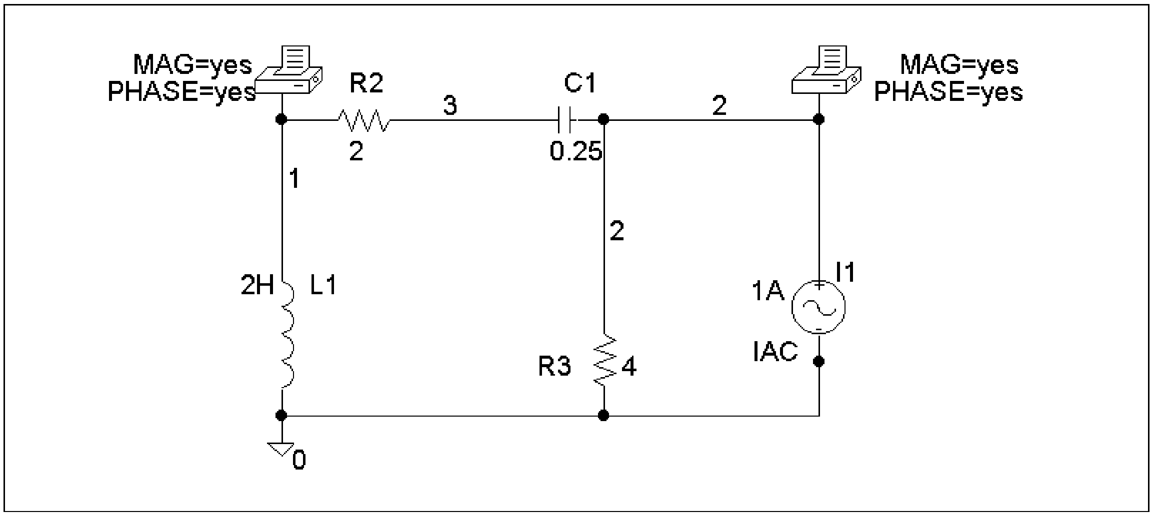
From this,

$$z_{12} = V_1/I_2 = 2.53\angle{-108.4°}/1 = 2.53\angle{-108..4°}$$

$$z_{22} = V_2/I_2 = 1.789\angle{-153.4°}/1 = 1.789\angle{-153.4°}.$$

Thus,

$$[z] = \underline{\begin{bmatrix} \mathbf{4.669\angle-136.7°} & \mathbf{2.53\angle-108.4°} \\ \mathbf{2.53\angle-108.4°} & \mathbf{1.789\angle-153.4°} \end{bmatrix}\boldsymbol{\Omega}}$$



Chapter 19, Problem 80.

Use *PSpice* to find the *z* parameters of the circuit in Fig. 19.71.

Chapter 19, Solution 80

To get z_{11} and z_{21}, we open circuit the output port and let $I_1 = 1A$ so that

$$z_{11} = \frac{V_1}{I_1} = V_1, \qquad z_{21} = \frac{V_2}{I_1} = V_2$$

The schematic is shown below. After it is saved and run, we obtain

$$z_{11} = V_1 = 29.88, \qquad z_{21} = V_2 = -70.37$$

Similarly, to get z_{22} and z_{12}, we open circuit the input port and let $I_2 = 1A$ so that

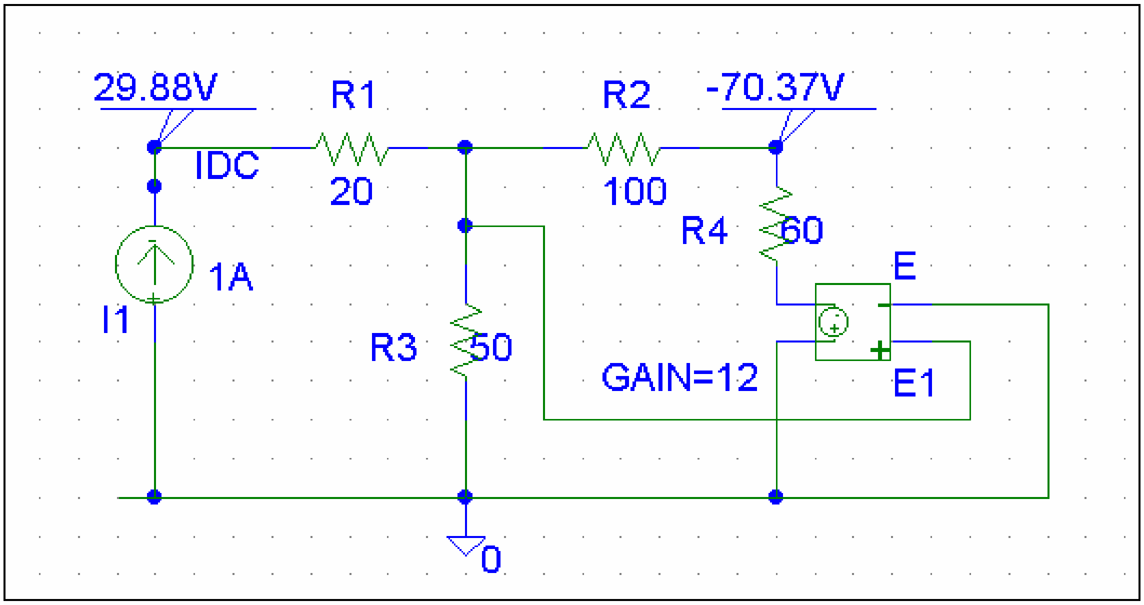
$$z_{12} = \frac{V_1}{I_2} = V_1, \qquad z_{22} = \frac{V_2}{I_2} = V_2$$

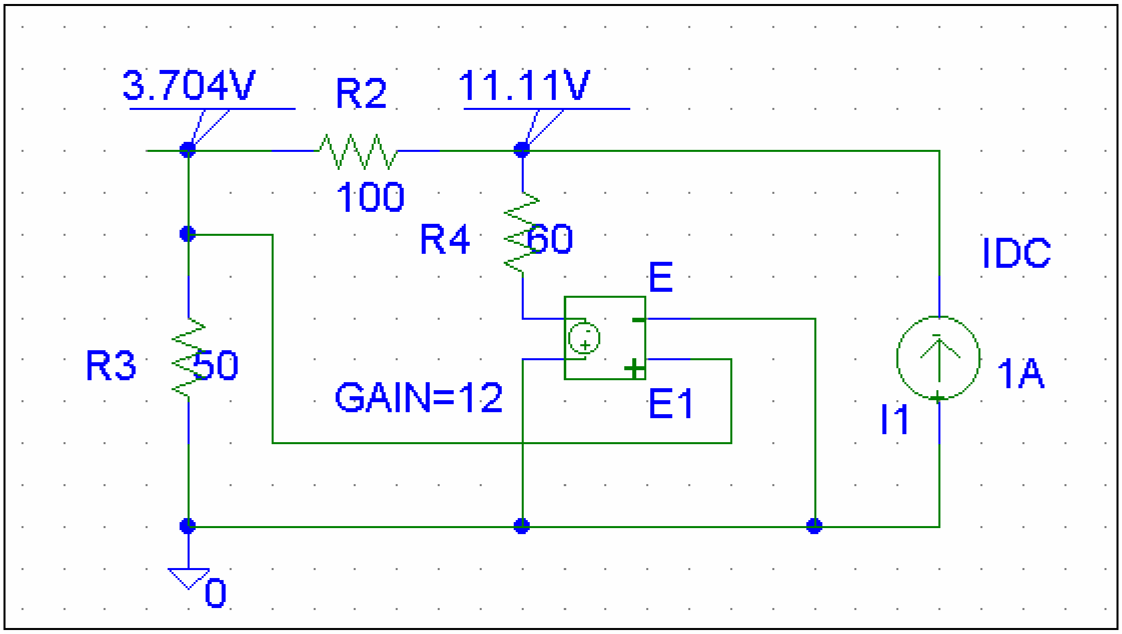
The schematic is shown below. After it is saved and run, we obtain

$$z_{12} = V_1 = 3.704, \qquad z_{22} = V_2 = 11.11$$

Thus,

$$[z] = \underline{\begin{bmatrix} 29.88 & 3.704 \\ -70.37 & 11.11 \end{bmatrix} \Omega}$$


29.88V
R1
R2
-70.37V
IDC
20
100
R4
60
E
1A
I1
R3
50
GAIN=12
E1
0


3.704V
R2
11.11V
100
R4
60
E
IDC
R3
50
GAIN=12
E1
I1
1A
0

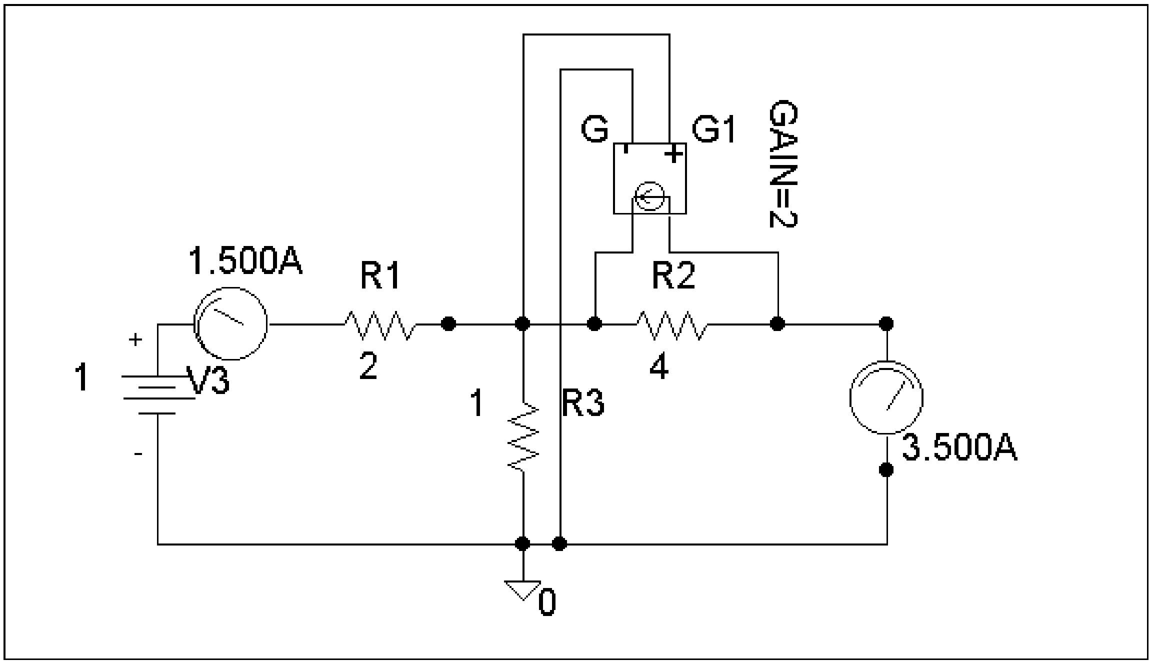
Chapter 19, Problem 81.

Repeat Prob. 19.26 using *PSpice*.

Chapter 19, Solution 81

(a) We set $V_1 = 1$ and short circuit the output port. The schematic is shown below. After simulation we obtain

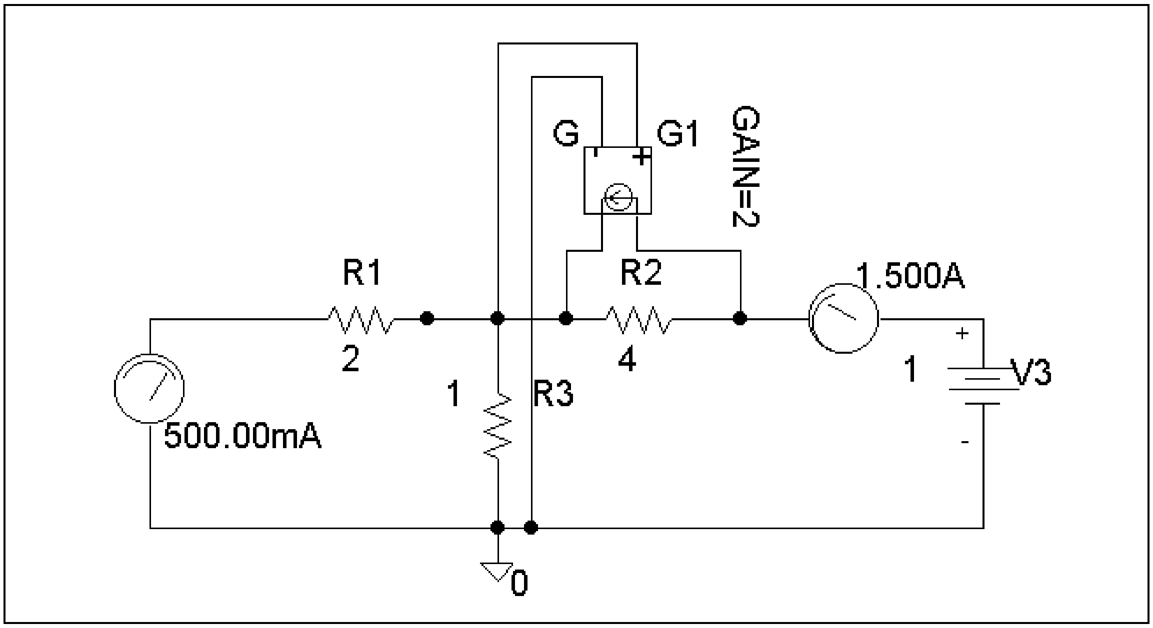
$$y_{11} = I_1 = 1.5, \ y_{21} = I_2 = 3.5$$



(b) We set $V_2 = 1$ and short-circuit the input port. The schematic is shown below. Upon simulating the circuit, we obtain

$$y_{12} = I_1 = -0.5, \; y_{22} = I_2 = 1.5$$

$$\underline{[Y] = \begin{bmatrix} \mathbf{1.5} & \mathbf{-0.5} \\ \mathbf{3.5} & \mathbf{1.5} \end{bmatrix} \mathbf{S}}$$



Chapter 19, Problem 82.

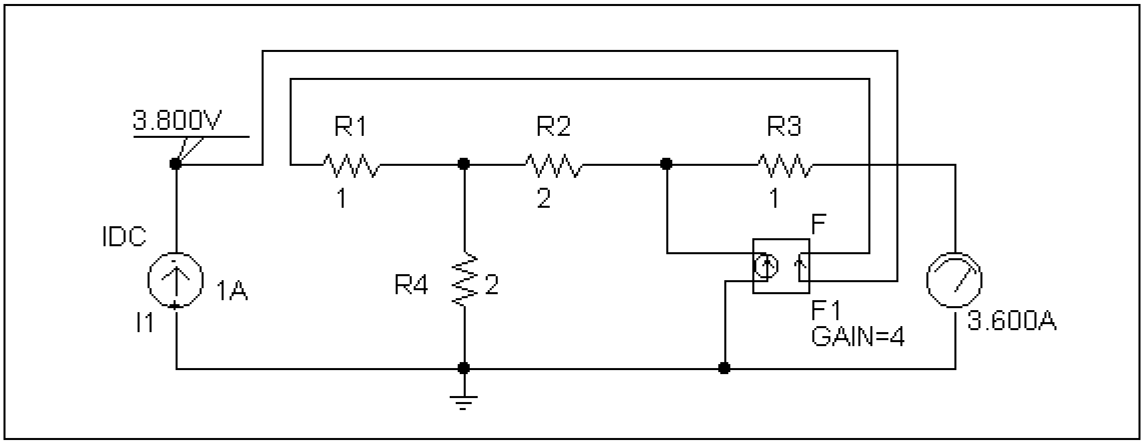
Use *PSpice* to rework Prob. 19.31.

Chapter 19, Solution 82

We follow Example 19.15.

(a) Set $V_2 = 0$ and $I_1 = 1A$. The schematic is shown below. After simulation, we obtain

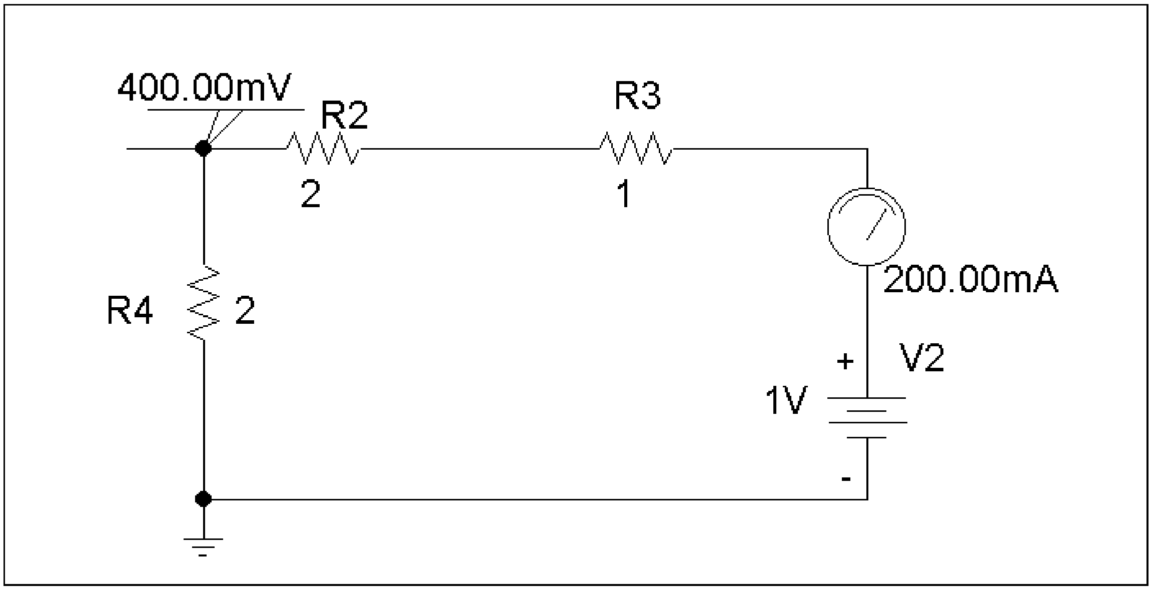
$$h_{11} = V_1/1 = 3.8, \ h_{21} = I_2/1 = 3.6$$



(b) Set $V_1 = 1$ V and $I_1 = 0$. The schematic is shown below. After simulation, we obtain

$$h_{12} = V_1/1 = 0.4, \; h_{22} = I_2/1 = 0.25$$

Hence, $$\underline{[h] = \begin{bmatrix} \mathbf{3.8} & \mathbf{0.4} \\ \mathbf{3.6} & \mathbf{0.25} \end{bmatrix}}$$



Chapter 19, Problem 83.

Rework Prob. 19.47 using *PSpice*.

Chapter 19, Solution 83

To get A and C, we open-circuit the output and let $I_1 = 1A$. The schematic is shown below. When the circuit is saved and simulated, we obtain $V_1 = 11$ and $V_2 = 34$.

$$A = \frac{V_1}{V_2} = 0.3235, \qquad C = \frac{I_1}{V_2} = \frac{1}{34} = 0.02941$$

Similarly, to get B and D, we open-circuit the output and let $I_1 = 1A$. The schematic is shown below. When the circuit is saved and simulated, we obtain $V_1 = 2.5$ and $I_2 = -2.125$.

$$B = -\frac{V_1}{I_2} = \frac{2.5}{2.125} = 1.1765, \qquad D = -\frac{I_1}{I_2} = \frac{1}{2.125} = 0.4706$$

Thus,

$$\underline{[T] = \begin{bmatrix} 0.3235 & 1.1765 \\ 0.02941 & 0.4706 \end{bmatrix}}$$

Chapter 19, Problem 84.

Using *PSpice*, find the transmission parameters for the network in Fig. 19.126.

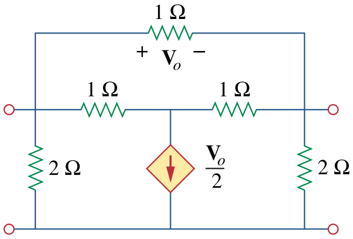


Figure 19.126

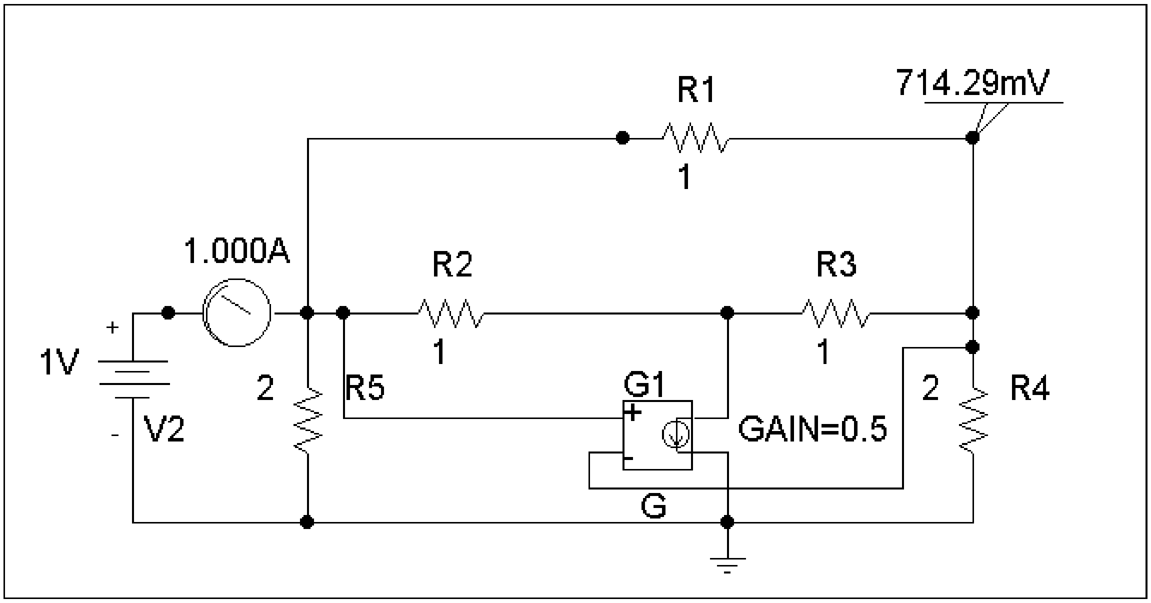
For Prob. 19.84.

Chapter 19, Solution 84

(a) Since $A = \left.\frac{V_1}{V_2}\right|_{I_2=0}$ and $C = \left.\frac{I_1}{V_2}\right|_{I_2=0}$, we open-circuit the output port and let V_1 = 1 V. The schematic is as shown below. After simulation, we obtain

$$A = 1/V_2 = 1/0.7143 = 1.4$$

$$C = I_2/V_2 = 1.0/0.7143 = 1.4$$

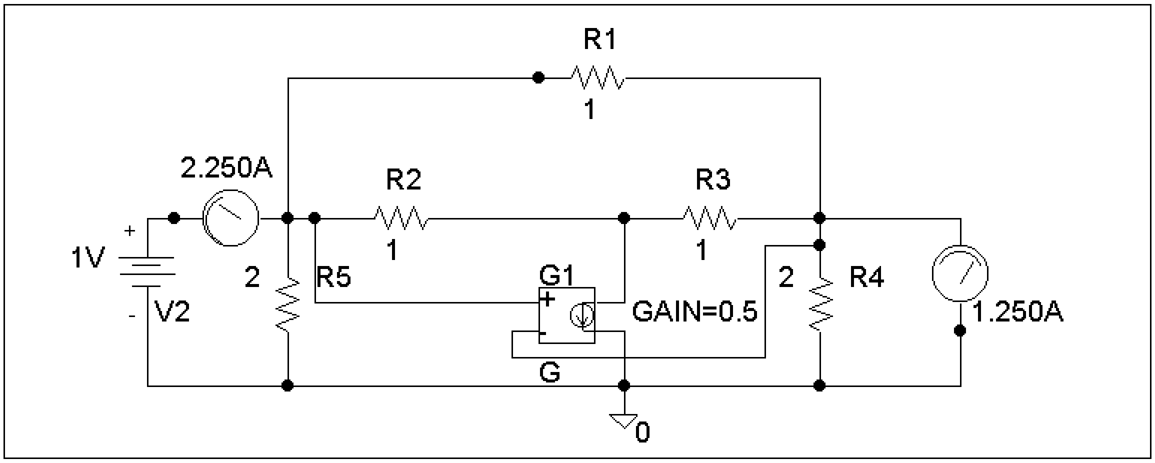


(b) To get B and D, we short-circuit the output port and let $V_1 = 1$. The schematic is shown below. After simulating the circuit, we obtain

$$B = -V_1/I_2 = -1/1.25 = -0.8$$

$$D = -I_1/I_2 = -2.25/1.25 = -1.8$$

Thus
$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \underline{\begin{bmatrix} \mathbf{1.4} & \mathbf{-0.8} \\ \mathbf{1.4} & \mathbf{-1.8} \end{bmatrix}}$$



Chapter 19, Problem 85.

At ω = 1 rad/s find the transmission parameters of the network in Fig. 19.127 using *PSpice*.

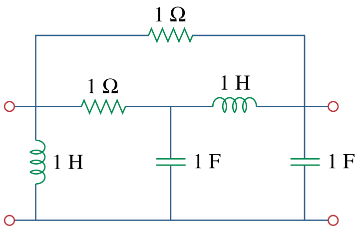


Figure 19.127

For Prob. 19.85.

Chapter 19, Solution 85

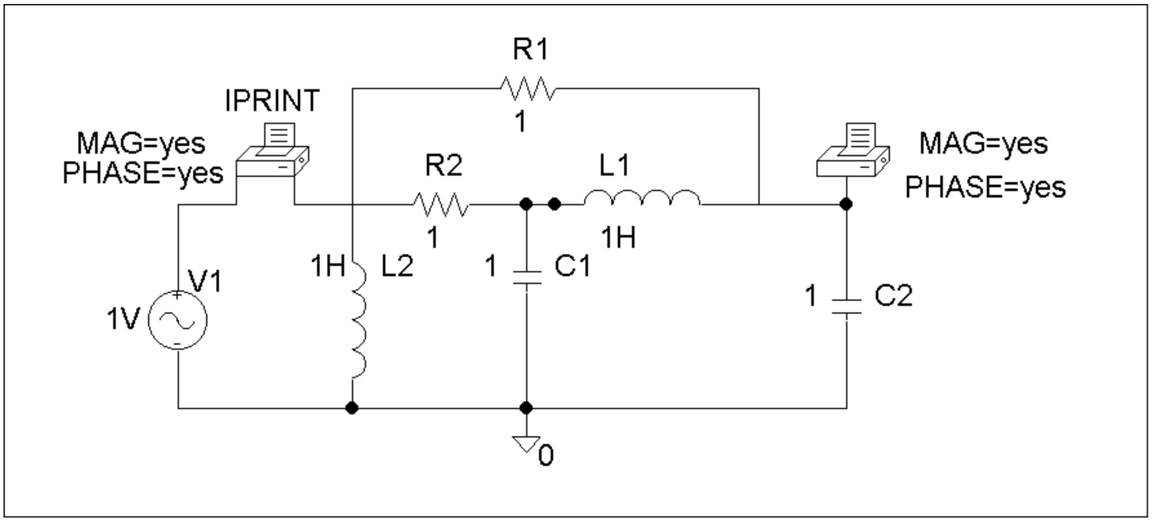
(a) Since $A = \left.\frac{V_1}{V_2}\right|_{I_2=0}$ and $C = \left.\frac{I_1}{V_2}\right|_{I_2=0}$, we let $V_1 = 1$ V and open-circuit the output port. The schematic is shown below. In the AC Sweep box, we set Total Pts = 1, Start Freq = 0.1592, and End Freq = 0.1592. After simulation, we obtain an output file which includes

```
FREQ          IM(V_PRINT1)        IP(V_PRINT1)
1.592 E–01    6.325 E–01          1.843 E+01

FREQ          VM($N_0002)         VP($N_0002)
1.592 E–01    6.325 E–01          –7.159 E+01
```

From this, we obtain

$$A = \frac{1}{V_2} = \frac{1}{0.6325\angle -71.59°} = 1.581\angle 71.59°$$

$$C = \frac{I_1}{V_2} = \frac{0.6325\angle 18.43°}{0.6325\angle -71.59°} = 1\angle 90° \;=\; j$$



(b) Similarly, since $B = \left.\frac{V_1}{I_2}\right|_{V_2=0}$ and $D = -\left.\frac{I_1}{I_2}\right|_{V_2=0}$, we let $V_1 = 1$ V and short-circuit the output port. The schematic is shown below. Again, we set Total Pts = 1, Start Freq = 0.1592, and End Freq = 0.1592 in the AC Sweep box. After simulation, we get an output file which includes the following results:

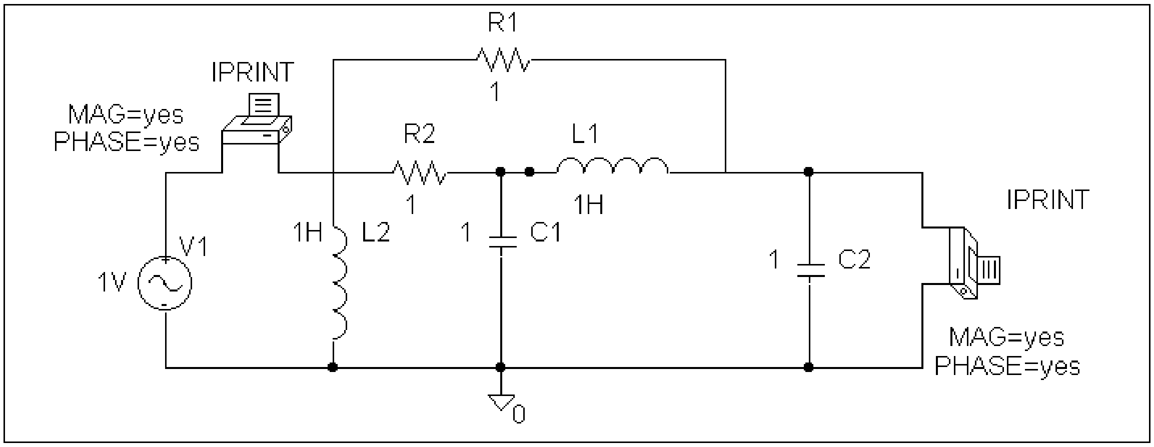
```
FREQ          IM(V_PRINT1)        IP(V_PRINT1)
1.592 E–01    5.661 E–04          8.997 E+01

FREQ          IM(V_PRINT3)        IP(V_PRINT3)
1.592 E–01    9.997 E–01          –9.003 E+01
```

From this,

$$B = -\frac{1}{I_2} = -\frac{1}{0.9997\angle -90^\circ} = -1\angle 90^\circ = -j$$

$$D = -\frac{I_1}{I_2} = -\frac{5.661\text{x}10^{-4}\angle 89.97^\circ}{0.9997\angle -90^\circ} = 5.561\text{x}10^{-4}$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \underline{\begin{bmatrix} \mathbf{1.581\angle 71.59^\circ} & \mathbf{-j} \\ \mathbf{j} & \mathbf{5.661x10^{-4}} \end{bmatrix}}$$



Chapter 19, Problem 86.

Obtain the *g* parameters for the network in Fig. 19.128 using *PSpice*.

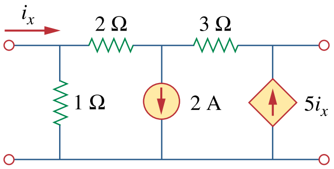


Figure 19.128
For Prob. 19.86.

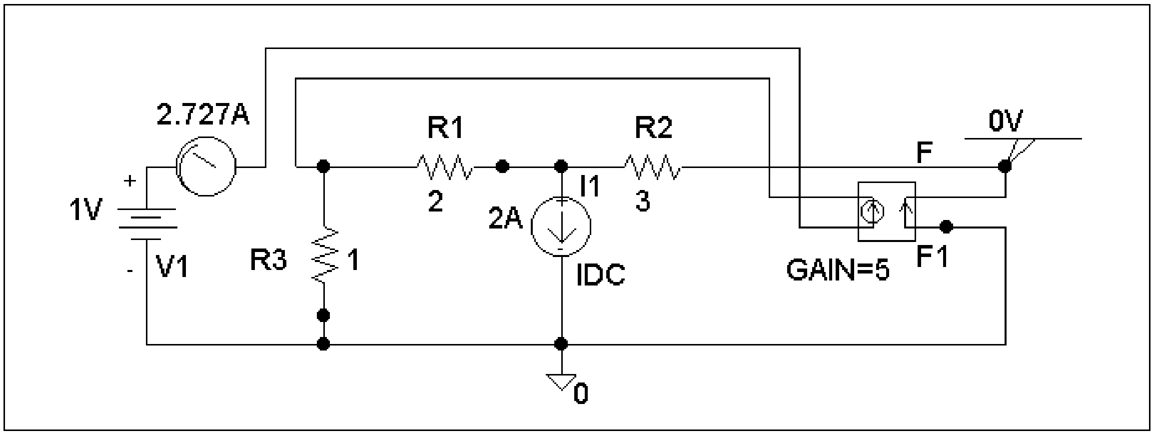
Chapter 19, Solution 86

(a) By definition, $g_{11} = \left.\frac{I_1}{V_1}\right|_{I_2=0}$, $g_{21} = \left.\frac{V_1}{V_2}\right|_{I_2=0}$.

We let $V_1 = 1$ V and open-circuit the output port. The schematic is shown below. After simulation, we obtain

$$g_{11} = I_1 = 2.7$$

$$g_{21} = V_2 = 0.0$$



(b) Similarly,

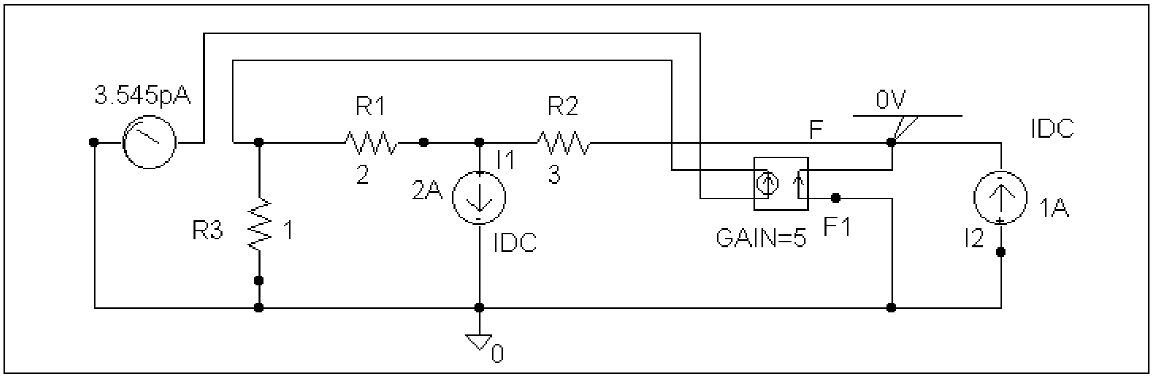
$$g_{12} = \left.\frac{I_1}{I_2}\right|_{V_1=0}, \quad g_{22} = \left.\frac{V_2}{I_2}\right|_{V_1=0}$$

We let $I_2 = 1$ A and short-circuit the input port. The schematic is shown below. After simulation,

$$g_{12} = I_1 = 0$$

$$g_{22} = V_2 = 0$$

Thus
$$[g] = \underline{\begin{bmatrix} \mathbf{2.727S} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix}}$$



Chapter 19, Problem 87.

For the circuit shown in Fig. 19.129, use *PSpice* to obtain the *t* parameters. Assume ω = 1 rad/s.

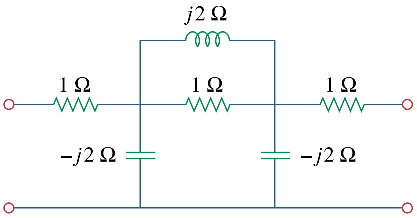


Figure 19.129

For Prob. 19.87.

Chapter 19, Solution 87

(a) Since $a = \left.\frac{V_2}{V_1}\right|_{I_1=0}$ and $c = \left.\frac{I_2}{V_1}\right|_{I_1=0}$,

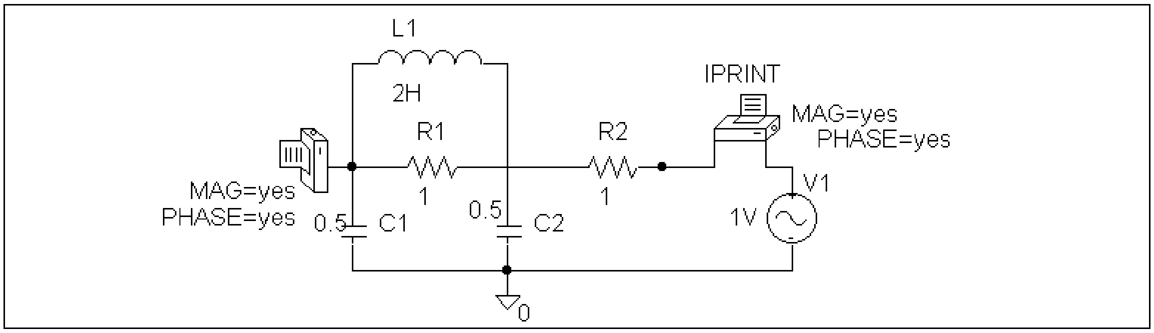
we open-circuit the input port and let V_2 = 1 V. The schematic is shown below. In the AC Sweep box, set Total Pts = 1, Start Freq = 0.1592, and End Freq = 0.1592. After simulation, we obtain an output file which includes

```
FREQ          IM(V_PRINT2)        IP(V_PRINT2)
1.592 E–01    5.000 E–01          1.800 E+02

FREQ          VM($N_0001)         VP($N_0001)
1.592 E–01    5.664 E–04          8.997 E+01
```

From this,

$$a = \frac{1}{5.664x10^{-4}\angle 89.97^\circ} = 1765\angle -89.97^\circ$$

$$c = \frac{0.5\angle 180^\circ}{5.664x10^{-4}\angle 89.97^\circ} = -882.28\angle -89.97^\circ$$



(b) Similarly,

$$b = -\left.\frac{V_2}{I_1}\right|_{V_1=0} \quad \text{and} \quad d = -\left.\frac{I_2}{I_1}\right|_{V_1=0}$$

We short-circuit the input port and let $V_2 = 1$ V. The schematic is shown below. After simulation, we obtain an output file which includes

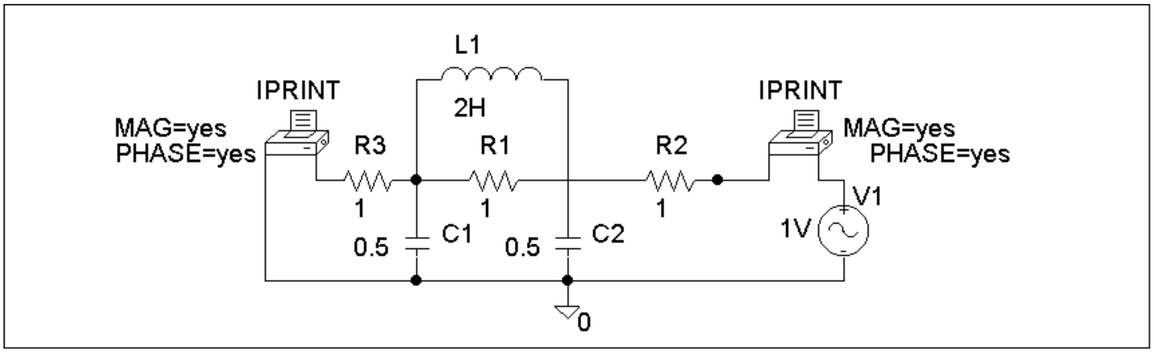
```
FREQ          IM(V_PRINT2)        IP(V_PRINT2)
1.592 E–01    5.000 E–01          1.800 E+02

FREQ          IM(V_PRINT3)        IP(V_PRINT3)
1.592 E–01    5.664 E–04          –9.010 E+01
```

From this, we get

$$b = -\frac{1}{5.664\text{x}10^{-4}\angle -90.1^\circ} = -j1765$$

$$d = -\frac{0.5\angle 180^\circ}{5.664\text{x}10^{-4}\angle -90.1^\circ} = j888.28$$

Thus $$[t] = \underline{\begin{bmatrix} \mathbf{-j1765} & \mathbf{-j1765} \\ \mathbf{j888.2} & \mathbf{j888.2} \end{bmatrix}}$$

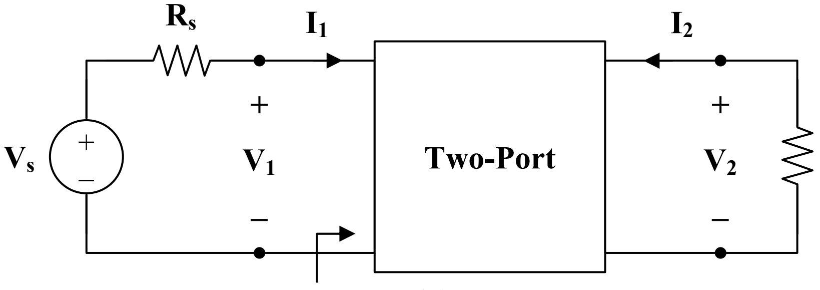


Chapter 19, Problem 88.

Using the *y* parameters, derive formulas for Z_{in}, Z_{out}, A_i, and A_v for the common-emitter transistor circuit.

Chapter 19, Solution 88

To get Z_{in}, consider the network in Fig. (a).



(a)

$$\mathbf{I}_1 = \mathbf{y}_{11}\mathbf{V}_1 + \mathbf{y}_{12}\mathbf{V}_2 \qquad (1)$$

$$\mathbf{I}_2 = \mathbf{y}_{21}\mathbf{V}_1 + \mathbf{y}_{22}\mathbf{V}_2 \qquad (2)$$

But $$\mathbf{I}_2 = \frac{-\mathbf{V}_2}{R_L} = \mathbf{y}_{21}\mathbf{V}_1 + \mathbf{y}_{22}\mathbf{V}_2$$

$$\mathbf{V}_2 = \frac{-\mathbf{y}_{21}\mathbf{V}_1}{\mathbf{y}_{22} + 1/R_L} \qquad (3)$$

Substituting (3) into (1) yields

$$\mathbf{I}_1 = \mathbf{y}_{11}\mathbf{V}_1 + \mathbf{y}_{12}\cdot\left(\frac{-\mathbf{y}_{21}\mathbf{V}_1}{\mathbf{y}_{22} + 1/R_L}\right), \qquad \mathbf{Y}_L = \frac{1}{R_L}$$

$$\mathbf{I}_1 = \left(\frac{\Delta_y + \mathbf{y}_{11}\mathbf{Y}_L}{\mathbf{y}_{22} + \mathbf{Y}_L}\right)\mathbf{V}_1, \qquad \Delta_y = \mathbf{y}_{11}\mathbf{y}_{22} - \mathbf{y}_{12}\mathbf{y}_{21}$$

or $$Z_{\text{in}} = \frac{\mathbf{V}_1}{\mathbf{I}_1} = \underline{\frac{\mathbf{y}_{22} + \mathbf{Y}_L}{\Delta_y + \mathbf{y}_{11}\mathbf{Y}_L}}$$

$$A_i = \frac{\mathbf{I}_2}{\mathbf{I}_1} = \frac{\mathbf{y}_{21}\mathbf{V}_1 + \mathbf{y}_{22}\mathbf{V}_2}{\mathbf{I}_1} = \mathbf{y}_{21}Z_{\text{in}} + \left(\frac{\mathbf{y}_{22}}{\mathbf{I}_1}\right)\left(\frac{-\mathbf{y}_{21}\mathbf{V}_1}{\mathbf{y}_{22} + \mathbf{Y}_L}\right)$$

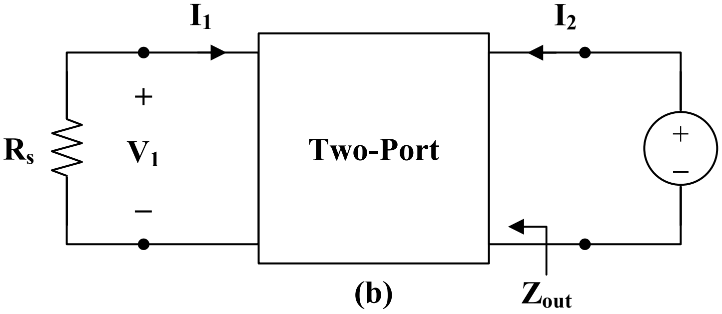
$$= \mathbf{y}_{21}Z_{\text{in}} - \frac{\mathbf{y}_{22}\mathbf{y}_{21}Z_{\text{in}}}{\mathbf{y}_{22} + \mathbf{Y}_L} = \left(\frac{\mathbf{y}_{22} + \mathbf{Y}_L}{\Delta_y + \mathbf{y}_{11}\mathbf{Y}_L}\right)\left(\mathbf{y}_{21} - \frac{\mathbf{y}_{22}\mathbf{y}_{21}}{\mathbf{y}_{22} + \mathbf{Y}_L}\right)$$

$$A_i = \underline{\frac{\mathbf{y}_{21}\mathbf{Y}_L}{\Delta_y + \mathbf{y}_{11}\mathbf{Y}_L}}$$

From (3),

$$A_v = \frac{\mathbf{V}_2}{\mathbf{V}_1} = \underline{\frac{\mathbf{-y_{21}}}{\mathbf{y_{22} + Y_L}}}$$

To get Z_{out}, consider the circuit in Fig. (b).



(b)

$$Z_{out} = \frac{\mathbf{V}_2}{\mathbf{I}_2} = \frac{\mathbf{V}_2}{\mathbf{y}_{21}\,\mathbf{V}_1 + \mathbf{y}_{22}\,\mathbf{V}_2} \qquad (4)$$

But $\mathbf{V}_1 = \text{-}R_s\,\mathbf{I}_1$

Substituting this into (1) yields

$$\mathbf{I}_1 = \text{-}\mathbf{y}_{11}\,R_s\,\mathbf{I}_1 + \mathbf{y}_{12}\,\mathbf{V}_2$$

$$(1 + \mathbf{y}_{11}\,R_s)\mathbf{I}_1 = \mathbf{y}_{12}\,\mathbf{V}_2$$

$$\mathbf{I}_1 = \frac{\mathbf{y}_{12}\,\mathbf{V}_2}{1 + \mathbf{y}_{11}\,R_s} = \frac{\text{-}\mathbf{V}_1}{R_s}$$

or

$$\frac{\mathbf{V}_1}{\mathbf{V}_2} = \frac{\text{-}\mathbf{y}_{12}\,R_s}{1 + \mathbf{y}_{11}\,R_s}$$

Substituting this into (4) gives

$$Z_{out} = \frac{1}{\mathbf{y}_{22} - \dfrac{\mathbf{y}_{12}\,\mathbf{y}_{21}\,R_s}{1 + \mathbf{y}_{11}\,R_s}}$$

$$= \frac{1 + \mathbf{y}_{11}\,R_s}{\mathbf{y}_{22} + \mathbf{y}_{11}\,\mathbf{y}_{22}\,R_s - \mathbf{y}_{21}\,\mathbf{y}_{22}\,R_s}$$

$$Z_{out} = \underline{\frac{\mathbf{y_{11} + Y_s}}{\mathbf{\Delta_y + y_{22}\,Y_s}}}$$

Chapter 19, Problem 89.

A transistor has the following parameters in a common-emitter circuit:

$h_{ie} = 2{,}640\,\Omega$, $h_{re} = 2.6 \times 10^{-4}$

$h_{fe} = 72$, $h_{oe} = 16\,\mu\text{S}$, $R_L = 100\text{ k}\Omega$

What is the voltage amplification of the transistor? How many decibels gain is this?

Chapter 19, Solution 89

$$A_v = \frac{-h_{fe} R_L}{h_{ie} + (h_{ie} h_{oe} - h_{re} h_{fe}) R_L}$$

$$A_v = \frac{-72 \cdot 10^5}{2640 + (2640 \times 16 \times 10^{-6} - 2.6 \times 10^{-4} \times 72) \cdot 10^5}$$

$$A_v = \frac{-72 \cdot 10^5}{2640 + 1824} = \underline{\mathbf{-1613}}$$

$$\text{dc gain} = 20 \log \left| A_v \right| = 20 \log(1613) = \underline{\mathbf{64.15}}$$

Chapter 19, Problem 90.

A transistor with

$h_{fe} = 120$, $h_{ie} = 2\text{k}\Omega$

$h_{re} = 10^{-4}$, $h_{oe} = 20\,\mu\text{S}$

is used for a CE amplifier to provide an input resistance of $1.5\text{ k}\Omega$.

(a) Determine the necessary load resistance R_L.
(b) Calculate A_v, A_i, and Z_{out} if the amplifier is driven by a 4-mV source having an internal resistance of $600\,\Omega$.
(c) Find the voltage across the load.

Chapter 19, Solution 90

(a) $$Z_{in} = h_{ie} - \frac{h_{re} h_{fe} R_L}{1 + h_{oe} R_L}$$

$$1500 = 2000 - \frac{10^{-4} \times 120 R_L}{1 + 20 \times 10^{-6} R_L}$$

$$500 = \frac{12 \times 10^{-3}}{1 + 2 \times 10^{-5} R_L}$$

$$500 + 10^{-2} R_L = 12 \times 10^{-3} R_L$$

$$500 \times 10^2 = 0.2 R_L$$

$$R_L = \underline{\mathbf{250\ k\Omega}}$$

(b) $$A_v = \frac{-h_{fe} R_L}{h_{ie} + (h_{ie} h_{oe} - h_{re} h_{fe}) R_L}$$

$$A_v = \frac{-120 \times 250 \times 10^3}{2000 + (2000 \times 20 \times 10^{-6} - 120 \times 10^{-4}) \times 250 \times 10^3}$$

$$A_v = \frac{-30 \times 10^6}{2 \times 10^3 + 7 \times 10^3} = \underline{-3333}$$

$$A_i = \frac{h_{fe}}{1 + h_{oe} R_L} = \frac{120}{1 + 20 \times 10^{-6} \times 250 \times 10^3} = \underline{\mathbf{20}}$$

$$Z_{out} = \frac{R_s + h_{ie}}{(R_s + h_{ie}) h_{oe} - h_{re} h_{fe}} = \frac{600 + 2000}{(600 + 2000) \times 20 \times 10^{-6} - 10^{-4} \times 120}$$

$$Z_{out} = \frac{2600}{40}\ k\Omega = \underline{\mathbf{65\ k\Omega}}$$

(c) $$A_v = \frac{\mathbf{V}_c}{\mathbf{V}_b} = \frac{\mathbf{V}_c}{\mathbf{V}_s} \longrightarrow \mathbf{V}_c = A_v \mathbf{V}_s = -3333 \times 4 \times 10^{-3} = \underline{\mathbf{-13.33\ V}}$$

Chapter 19, Problem 91.

For the transistor network of Fig. 19.130,

$h_{fe} = 80,$ $h_{ie} = 1.2\text{k}\Omega$

$h_{re} = 1.5 \times 10^{-4},$ $h_{oe} = 20\,\mu\text{S}$

Determine the following:

(a) voltage gain $A_v = V_o/V_s$,

(b) current gain $A_i = I_0/I_i$,

(c) input impedance Z_{in},

(d) output impedance Z_{out}.

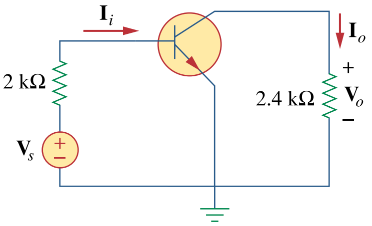


Figure 19.130

For Prob. 19.91.

Chapter 19, Solution 91

$$R_s = 1.2\ \text{k}\Omega, \qquad R_L = 4\ \text{k}\Omega$$

(a) $$A_v = \frac{-h_{fe} R_L}{h_{ie} + (h_{ie} h_{oe} - h_{re} h_{fe}) R_L}$$

$$A_v = \frac{-80 \times 4 \times 10^3}{1200 + (1200 \times 20 \times 10^{-6} - 1.5 \times 10^{-4} \times 80) \times 4 \times 10^3}$$

$A_V = \dfrac{-32000}{1248} = \underline{\mathbf{-25.64}}$ for the transistor. However, the problem asks for V_o/V_s.

Thus,

$V_b = Vo/ATransV = –V_o/25.64$

$I_b = V_s/(2000 + 1200) = V_s/3200$ (Note, we used Z_{in} from (c) below.)

$V_b = 1200xI_b = (1200/3200)V_s = 0.375V_s = –V_o/25.64$

A_V for the circuit = V_o/V_s = $\underline{\mathbf{–9.615}}$

(b) $$A_i = \frac{h_{fe}}{1 + h_{oe} R_L} = \frac{80}{1 + 20 \times 10^{-6} \times 4 \times 10^3} = \underline{74.07}$$

(c) $$Z_{in} = h_{ie} - h_{re} A_i$$

$$Z_{in} = 1200 - 1.5 \times 10^{-4} \times 74.074 \cong \underline{\mathbf{1.2\ k\Omega}}$$

(d) $$Z_{out} = \frac{R_s + h_{ie}}{(R_s + h_{ie}) h_{oe} - h_{re} h_{fe}}$$

$$Z_{out} = \frac{1200 + 1200}{2400 \times 20 \times 10^{-6} - 1.5 \times 10^{-4} \times 80} = \frac{2400}{0.0468} = \underline{51.28\ \text{k}\Omega}$$

Chapter 19, Problem 92.

* Determine A_v, A_i, Z_{in}, and Z_{out} for the amplifier shown in Fig. 19.131. Assume that

$h_{ie} = 4\,\text{k}\Omega$, $h_{re} = 10^{-4}$

$h_{fe} = 100$, $h_{oe} = 30\,\mu\text{S}$

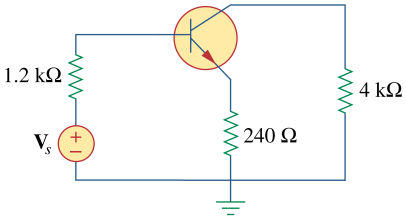
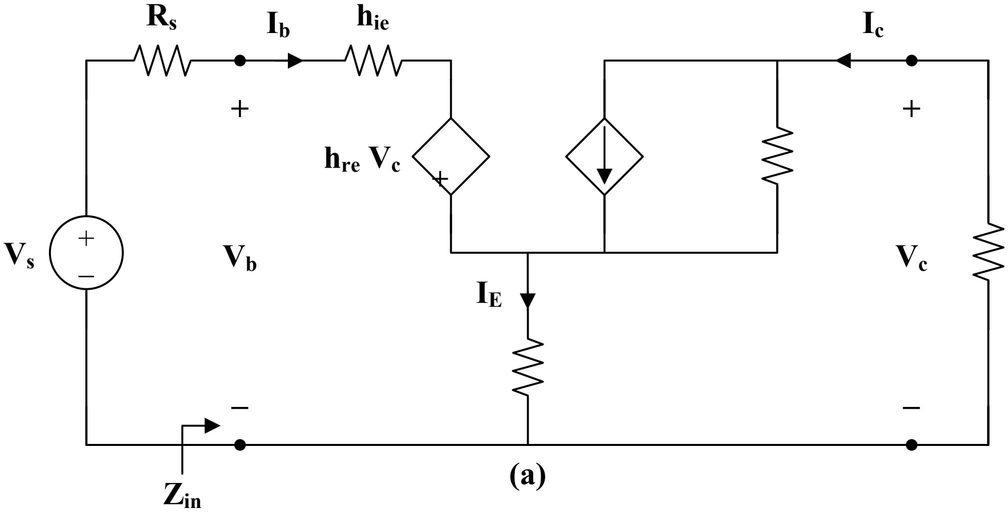


Figure 19.131
For Prob. 19.92.

* An asterisk indicates a challenging problem.

Chapter 19, Solution 92

Due to the resistor $R_E = 240\,\Omega$, we cannot use the formulas in section 18.9.1. We will need to derive our own. Consider the circuit in Fig. (a).



$$\mathbf{I}_E = \mathbf{I}_b + \mathbf{I}_c \tag{1}$$

$$\mathbf{V}_b = h_{ie}\mathbf{I}_b + h_{re}\mathbf{V}_c + (\mathbf{I}_b + \mathbf{I}_c)R_E \tag{2}$$

$$\mathbf{I}_c = h_{fe}\mathbf{I}_b + \frac{\mathbf{V}_c}{R_E + 1/h_{oe}} \tag{3}$$

But
$$\mathbf{V}_c = -\mathbf{I}_c R_L \tag{4}$$

Substituting (4) into (3),

$$\mathbf{I}_c = h_{fe}\mathbf{I}_b - \frac{R_L}{R_E + 1/h_{oe}}\mathbf{I}_c$$

or
$$A_i = \frac{\mathbf{I}_c}{\mathbf{I}_b} = \frac{h_{fe}(1 + R_E h_{oe})}{1 + h_{oe}(R_L} \tag{5}$$

$$A_i = \frac{100(1 + 240x30x10^{-6})}{1 + 30 \times 10^{-6}(4{,}000 + 240)}$$

$$A_i = \underline{\mathbf{79.18}}$$

From (3) and (5),

$$\mathbf{I}_c = \frac{h_{fe}(1 + R_E)h_{oe}}{1 + h_{oe}(R_L + R_E)}\mathbf{I}_b = h_{fe}\mathbf{I}_b + \frac{\mathbf{V}_c}{R_E + 1/h_{oe}} \tag{6}$$

Substituting (4) and (6) into (2),

$$\mathbf{V}_b = (h_{ie} + R_E)\mathbf{I}_b + h_{re}\mathbf{V}_c + \mathbf{I}_c R_E$$

$$\mathbf{V}_b = \frac{\mathbf{V}_c(h_{ie} + R_E)}{\left(R_E + \frac{1}{h_{oe}}\right)\left[\frac{h_{fe}(1 + R_E h_{oe})}{1 + h_{oe}(R_L + R_E)} - h_{fe}\right]} + h_{re}\mathbf{V}_c - \frac{\mathbf{V}_c}{R_L}R_E$$

$$\frac{1}{A_v} = \frac{\mathbf{V}_b}{\mathbf{V}_c} = \frac{(h_{ie} + R_E)}{\left(R_E + \frac{1}{h_{oe}}\right)\left[\frac{h_{fe}(1 + R_E h_{oe})}{1 + h_{oe}(R_L + R_E)} - h_{fe}\right]} + h_{re} - \frac{R_E}{R_L} \tag{7}$$

$$\frac{1}{A_v} = \frac{(4000 + 240)}{\left(240 + \frac{1}{30x10^{-6}}\right)\left[\frac{100(1 + 240x30x10^{-6})}{1 + 30 \times 10^{-6} \times 4240} - 100\right]} + 10^{-4} - \frac{240}{4000}$$

$$\frac{1}{A_v} = -6.06x10^{-3} + 10^{-4} - 0.06 = -0.066$$

$$A_v = \underline{\mathbf{-15.15}}$$

From (5),

$$\mathbf{I}_c = \frac{h_{fe}}{1 + h_{oe} R_L} \mathbf{I}_b$$

We substitute this with (4) into (2) to get

$$\mathbf{V}_b = (h_{ie} + R_E)\mathbf{I}_b + (R_E - h_{re} R_L)\mathbf{I}_c$$

$$\mathbf{V}_b = (h_{ie} + R_E)\mathbf{I}_b + (R_E - h_{re} R_L)\left(\frac{h_{fe}(1 + R_E h_{oe})}{1 + h_{oe}(R_L + R_E)}\mathbf{I}_b\right)$$

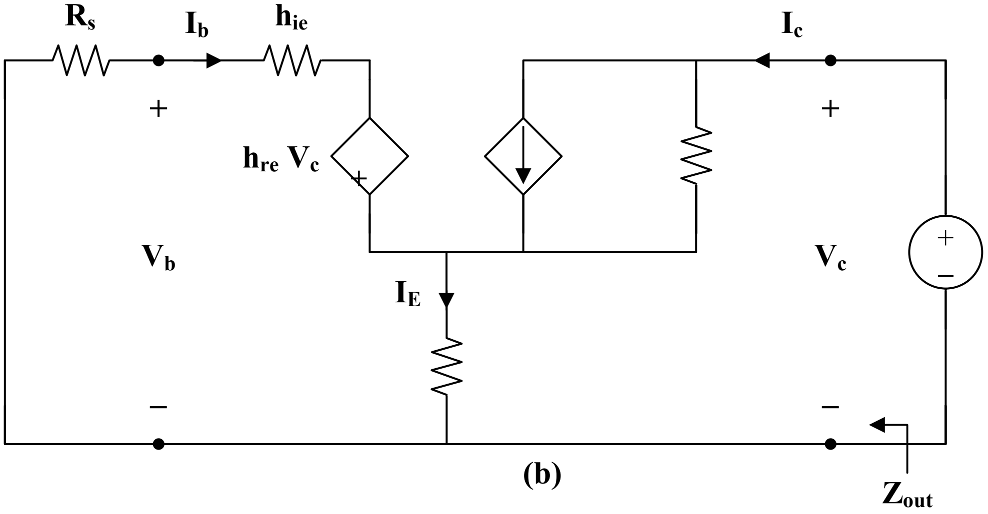
$$Z_{in} = \frac{\mathbf{V}_b}{\mathbf{I}_b} = h_{ie} + R_E + \frac{h_{fe}(R_E - h_{re} R_L)(1 + R_E h_{oe})}{1 + h_{oe}(R_L + R_E)}$$

(8)

$$Z_{in} = 4000 + 240 + \frac{(100)(240 \times 10^{-4} \times 4 \times 10^3)(1 + 240\text{x}30\text{x}10^{-6})}{1 + 30 \times 10^{-6} \times 4240}$$

Z_{in} = **12.818 kΩ**

To obtain Z_{out}, which is the same as the Thevenin impedance at the output, we introduce a 1-V source as shown in Fig. (b).



(b)

From the input loop,

$$\mathbf{I}_b(R_s + h_{ie}) + h_{re}\mathbf{V}_c + R_E(\mathbf{I}_b + \mathbf{I}_c) = 0$$

But $\mathbf{V}_c = 1$

So,

$$\mathbf{I}_b (R_s + h_{ie} + R_E) + h_{re} + R_E\,\mathbf{I}_c = 0 \tag{9}$$

From the output loop,

$$\mathbf{I}_c = \frac{\mathbf{V}_c}{R_E + \dfrac{1}{h_{oe}}} + h_{fe}\,\mathbf{I}_b = \frac{h_{oe}}{R_E h_{oe} + 1} + h_{fe}\,\mathbf{I}_b$$

or

$$\mathbf{I}_b = \frac{\mathbf{I}_c}{h_{fe}} - \frac{h_{oe}\big/ h_{fe}}{1 + R_E h_{oe}} \tag{10}$$

Substituting (10) into (9) gives

$$(R_s + R_E + h_{ie})\left(\frac{\mathbf{I}_c}{h_{fe}}\right) + h_{re} + R_E\,\mathbf{I}_c - \frac{(R_s + R_E + h_{ie})\left(h_{oe}\big/ h_{fe}\right)}{1 + R_E h_{oe}} = 0$$

$$\frac{R_s + R_E + h_{ie}}{h_{fe}}\mathbf{I}_c + R_E\,\mathbf{I}_c = \frac{R_s + R_E + h_{ie}}{1 + R_E h_{oe}}\left(\frac{h_{oe}}{h_{fe}}\right) - h_{re}$$

$$\mathbf{I}_c = \frac{(h_{oe}/h_{fe})\left[\dfrac{R_s + R_E + h_{ie}}{1 + R_E h_{oe}}\right] - h_{re}}{R_E + (R_s + R_E + h_{ie})/h_{fe}}$$

$$Z_{out} = \frac{1}{\mathbf{I}_c} = \frac{R_E\,h_{fe} + R_s + R_E + h_{ie}}{\left[\dfrac{R_s + R_E + h_{ie}}{1 + R_E h_{oe}}\right] h_{oe} - h_{re}\,h_{fe}}$$

$$Z_{out} = \frac{240 \times 100 + (1200 + 240 + 4000)}{\left[\dfrac{1200 + 240 + 4000}{1 + 240\text{x}30\text{x}10^{-6}}\right] \times 30 \times 10^{-6} - 10^{-4} \times 100}$$

$$Z_{out} = \frac{24000 + 5440}{0.152} = \underline{\mathbf{193.7\ k\Omega}}$$

***Chapter 19, Problem 93.**

Calculate A_v, A_i, Z_{in}, and Z_{out}, for the transistor network in Fig. 19.132. Assume that

$h_{ie} = 2\text{ k}\Omega$, $h_{re} = 2.5 \times 10^{-4}$

$h_{fe} = 150$, $h_{oe} = 10\,\mu\text{S}$

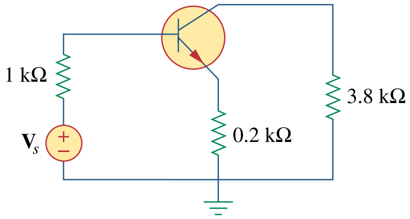


Figure 19.110

For Prob. 19.63.

*An asterisk indicates a challenging problem.

Chapter 19, Solution 93

We apply the same formulas derived in the previous problem.

$$\frac{1}{A_v} = \frac{(h_{ie} + R_E)}{\left(R_E + \dfrac{1}{h_{oe}}\right)\left[\dfrac{h_{fe}(1 + R_E h_{oe})}{1 + h_{oe}(R_L + R_E)} - h_{fe}\right]} + h_{re} - \frac{R_E}{R_L}$$

$$\frac{1}{A_v} = \frac{(2000 + 200)}{(200 + 10^5)\left[\dfrac{150(1 + 0.002)}{1 + 0.04} - 150\right]} + 2.5 \times 10^{-4} - \frac{200}{3800}$$

$$\frac{1}{A_v} = -0.004 + 2.5 \times 10^{-4} - 0.05263 = \text{-}0.05638$$

$$A_v = \underline{\mathbf{-17.74}}$$

$$A_i = \frac{h_{fe}(1 + R_E h_{oe})}{1 + h_{oe}(R_L + R_E)} = \frac{150(1 + 200\text{x}10^{-5})}{1 + 10^{-5} \times (200 + 3800)} = \underline{\mathbf{144.5}}$$

$$Z_{in} = h_{ie} + R_E + \frac{h_{fe}(R_E - h_{re} R_L)(1 + R_E h_{oe})}{1 + h_{oe}(R_L + R_E)}$$

$$Z_{in} = 2000 + 200 + \frac{(150)(200 - 2.5 \times 10^{-4} \times 3.8 \times 10^3)(1.002)}{1.04}$$

$$Z_{in} = 2200 + 28966$$

$$Z_{in} = \underline{\mathbf{31.17\ k\Omega}}$$

$$Z_{out} = \frac{R_E h_{fe} + R_s + R_E + h_{ie}}{\left[\dfrac{R_s + R_E + h_{ie}}{1 + R_E h_{oe}}\right] h_{oe} - h_{re} h_{fe}}$$

$$Z_{out} = \frac{200 \times 150 + 1000 + 200 + 2000}{\left[\dfrac{3200 \times 10^{-5}}{1.002}\right] - 2.5 \times 10^{-4} \times 150} = \frac{33200}{\text{-}\,0.0055}$$

$$Z_{out} = \underline{\mathbf{-6.148\ M\Omega}}$$

Chapter 19, Problem 94.

A transistor in its common-emitter mode is specified by

$$[\mathbf{h}] = \begin{bmatrix} 200\Omega & 0 \\ 100 & 10^{-6}\mathrm{S} \end{bmatrix}$$

Two such identical transistors are connected in cascade to form a two-stage amplifier used at audio frequencies. If the amplifier is terminated by a 4-kΩ resistor, calculate the overall A_v and Z_{in}.

Chapter 19, Solution 94

We first obtain the **ABCD** parameters.

Given $[\mathbf{h}] = \begin{bmatrix} 200 & 0 \\ 100 & 10^{-6} \end{bmatrix}$, $\Delta_h = \mathbf{h}_{11}\mathbf{h}_{22} - \mathbf{h}_{12}\mathbf{h}_{21} = 2\times10^{-4}$

$$[\mathbf{T}] = \begin{bmatrix} \dfrac{\Delta_h}{\mathbf{h}_{21}} & \dfrac{-\mathbf{h}_{11}}{\mathbf{h}_{21}} \\ \dfrac{-\mathbf{h}_{22}}{\mathbf{h}_{21}} & \dfrac{-1}{\mathbf{h}_{21}} \end{bmatrix} = \begin{bmatrix} -2\times10^{-6} & -2 \\ -10^{-8} & -10^{-2} \end{bmatrix}$$

The overall **ABCD** parameters for the amplifier are

$$[\mathbf{T}] = \begin{bmatrix} -2\times10^{-6} & -2 \\ -10^{-8} & -10^{-2} \end{bmatrix}\begin{bmatrix} -2\times10^{-6} & -2 \\ -10^{-8} & -10^{-2} \end{bmatrix} \cong \begin{bmatrix} 2\times10^{-8} & 2\times10^{-2} \\ 10^{-10} & 10^{-4} \end{bmatrix}$$

$$\Delta_T = 2\times10^{-12} - 2\times10^{-12} = 0$$

$$[\mathbf{h}] = \begin{bmatrix} \dfrac{\mathbf{B}}{\mathbf{D}} & \dfrac{\Delta_T}{\mathbf{D}} \\ \dfrac{-1}{\mathbf{D}} & \dfrac{\mathbf{C}}{\mathbf{D}} \end{bmatrix} = \begin{bmatrix} 200 & 0 \\ -10^4 & 10^{-6} \end{bmatrix}$$

Thus, $h_{ie} = 200$, $h_{re} = 0$, $h_{fe} = -10^4$, $h_{oe} = 10^{-6}$

$$A_v = \frac{(10^4)(4\times10^3)}{200 + (2\times10^{-4} - 0)\times4\times10^3} = \underline{\mathbf{2\times10^5}}$$

$$Z_{in} = h_{ie} - \frac{h_{re}h_{fe}R_L}{1 + h_{oe}R_L} = 200 - 0 = \underline{\mathbf{200\,\Omega}}$$

Chapter 19, Problem 95.

Realize an *LC* ladder network such that

$$y_{22} = \frac{s^3 + 5s}{s^4 + 10s^2 + 8}$$

Chapter 19, Solution 95

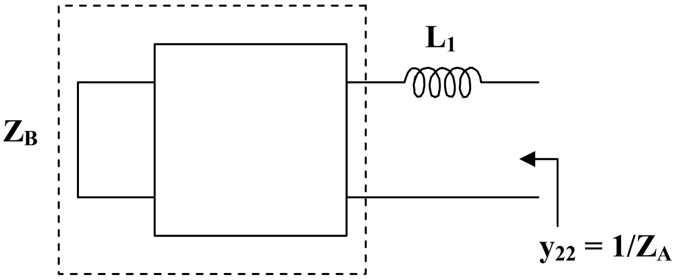
Let $\mathbf{Z}_A = \frac{1}{\mathbf{y}_{22}} = \frac{s^4 + 10s^2 + 8}{s^3 + 5s}$

Using long division,

$$\mathbf{Z}_A = s + \frac{5s^2 + 8}{s^3 + 5s} = s L_1 + \mathbf{Z}_B$$

i.e. $L_1 = 1\,H$ and $\mathbf{Z}_B = \frac{5s^2 + 8}{s^3 + 5s}$

as shown in Fig (a).



(a)

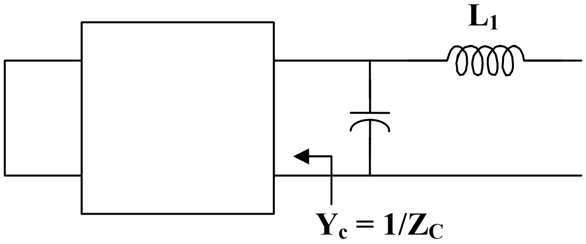
$$\mathbf{Y}_B = \frac{1}{\mathbf{Z}_B} = \frac{s^3 + 5s}{5s^2 + 8}$$

Using long division,

$$\mathbf{Y}_B = 0.2s + \frac{3.4s}{5s^2 + 8} = sC_2 + \mathbf{Y}_C$$

where $C_2 = 0.2\text{ F}$ and $\mathbf{Y}_C = \dfrac{3.4s}{5s^2 + 8}$

as shown in Fig. (b).



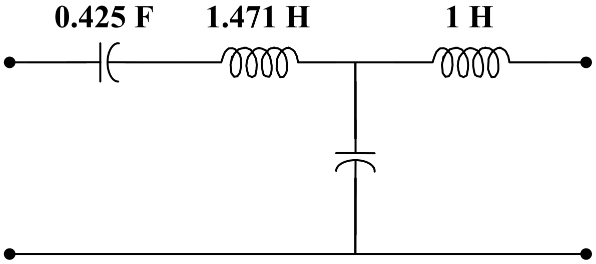
(b)

$$\mathbf{Z}_C = \frac{1}{\mathbf{Y}_C} = \frac{5s^2 + 8}{3.4s} = \frac{5s}{3.4} + \frac{8}{3.4s} = sL_3 + \frac{1}{sC_4}$$

i.e. an inductor in series with a capacitor

$$L_3 = \frac{5}{3.4} = 1.471\text{ H} \quad \text{and} \quad C_4 = \frac{3.4}{8} = 0.425\text{ F}$$

Thus, **<u>the LC network is shown in Fig. (c)</u>**.



(c)

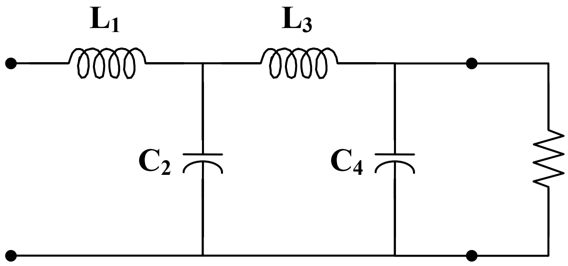
Chapter 19, Problem 96.

Design an *LC* ladder network to realize a lowpass filter with transfer function

$$H(s)=\frac{1}{s^4+2.613s^2+3.414s^2+2.613s+1}$$

Chapter 19, Solution 96

This is a fourth order network which can be realized with the network shown in Fig. (a).



(a)

$$\Delta(s) = (s^4+3.414s^2+1)+(2.613s^3+2.613s)$$

$$H(s)=\frac{\dfrac{1}{2.613s^3+2.613s}}{1+\dfrac{s^4+3.414s^2+1}{2.613s^3+2.613s}}$$

which indicates that

$$\mathbf{y}_{21}=\frac{-1}{2.613s^3+2.613s}$$

$$\mathbf{y}_{22}=\frac{s^4+3.414s+1}{2.613s^3+2.613s}$$

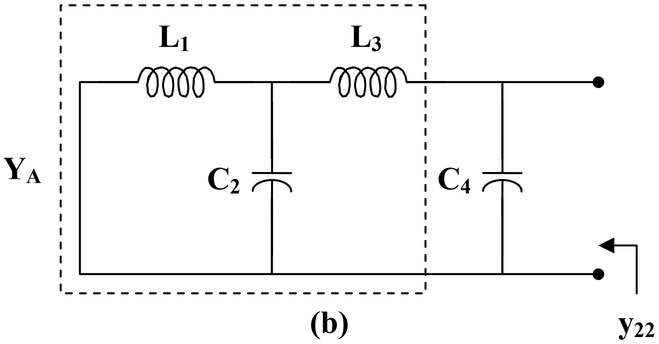
We seek to realize $\mathbf{y}_{22}$.

By long division,

$$\mathbf{y}_{22} = 0.383s + \frac{2.414s^2 + 1}{2.613s^3 + 2.613s} = s\,C_4 + \mathbf{Y}_A$$

i.e. $C_4 = 0.383\ \text{F}$ and $\mathbf{Y}_A = \dfrac{2.414s^2 + 1}{2.613s^3 + 2.613s}$

as shown in Fig. (b).



(b)

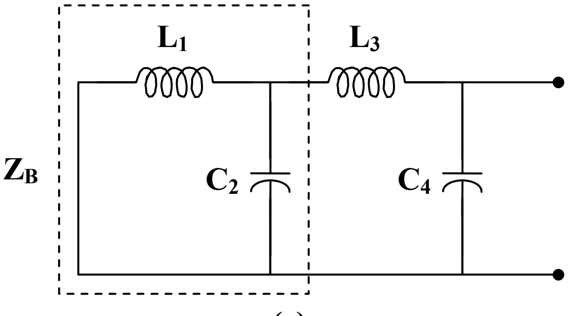
$$\mathbf{Z}_A = \frac{1}{\mathbf{Y}_A} = \frac{2.613s^3 + 2.613s}{2.414s^2 + 1}$$

By long division,

$$\mathbf{Z}_A = 1.082s + \frac{1.531s}{2.414s^2 + 1} = s\,L_3 + \mathbf{Z}_B$$

i.e. $L_3 = 1.082\ \text{H}$ and $\mathbf{Z}_B = \dfrac{1.531s}{2.414s^2 + 1}$

as shown in Fig.(c).

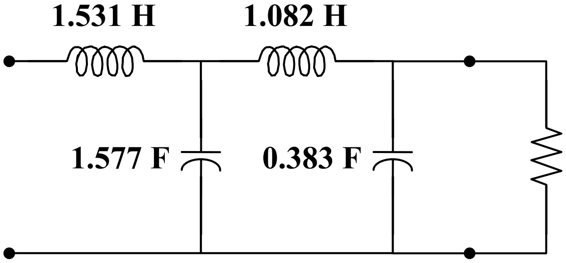


(c)

$$\mathbf{Y}_B = \frac{1}{\mathbf{Z}_B} = 1.577s + \frac{1}{1.531s} = sC_2 + \frac{1}{sL_1}$$

i.e. $C_2 = 1.577\text{ F}$ and $L_1 = 1.531\text{ H}$

Thus, **the network is shown in Fig. (d)**.



(d)

Chapter 19, Problem 97.

Synthesize the transfer function

$$H(s)=\frac{V_o}{V_s}=\frac{s^3}{s^3+6s+12s+24}$$

using the *LC* ladder network in Fig. 19.133.

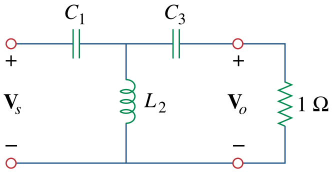


Figure 19.133

For Prob. 19.97.

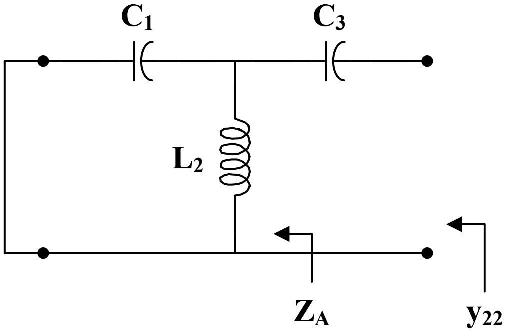
Chapter 19, Solution 97

$$H(s)=\frac{s^3}{(s^3+12s)+(6s^2+24)}=\frac{\dfrac{s^3}{s^3+12s}}{1+\dfrac{6s^2+24}{s^3+12s}}$$

Hence,

$$\mathbf{y}_{22}=\frac{6s^2+24}{s^3+12s}=\frac{1}{sC_3}+\mathbf{Z}_A \qquad (1)$$

where $\mathbf{Z}_A$ is shown in the figure below.



We now obtain C_3 and $\mathbf{Z}_A$ using partial fraction expansion.

Let
$$\frac{6s^2+24}{s(s^2+12)} = \frac{A}{s} + \frac{Bs+C}{s^2+12}$$
$$6s^2+24 = A(s^2+12) + Bs^2 + Cs$$

Equating coefficients :

$$s^0: \quad 24 = 12A \longrightarrow A = 2$$
$$s^1: \quad 0 = C$$
$$s^2: \quad 6 = A + B \longrightarrow B = 4$$

Thus,
$$\frac{6s^2+24}{s(s^2+12)} = \frac{2}{s} + \frac{4s}{s^2+12} \tag{2}$$

Comparing (1) and (2),
$$C_3 = \frac{1}{A} = \frac{1}{2}\text{ F}$$

$$\frac{1}{\mathbf{Z}_A} = \frac{s^2+12}{4s} = \frac{1}{4}s + \frac{3}{s} \tag{3}$$

But
$$\frac{1}{\mathbf{Z}_A} = sC_1 + \frac{1}{sL_2} \tag{4}$$

Comparing (3) and (4),
$$C_1 = \frac{1}{4}\text{ F} \qquad \text{and} \qquad L_2 = \frac{1}{3}\text{ H}$$

Therefore,
$$C_1 = \underline{\mathbf{0.25\ F}}, \qquad L_2 = \underline{\mathbf{0.3333\ H}}, \qquad C_3 = \underline{\mathbf{0.5\ F}}$$

Chapter 19, Problem 98.

A two-stage amplifier in Fig. 19.134 contains two identical stages with

$$[\mathbf{h}] = \begin{bmatrix} 2\,k\Omega & 0.004 \\ 200 & 500\,\mu\text{S} \end{bmatrix}$$

If $\mathbf{Z}_L = 20\,\text{k}\Omega$, find the required value of $\mathbf{V}_s$ to produce $\mathbf{V}_o$ = 16 V.

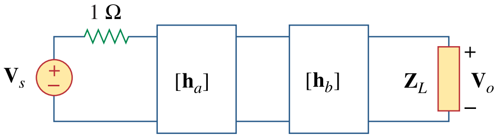


Figure 19.134
For Prob. 19.98.

Chapter 19, Solution 98

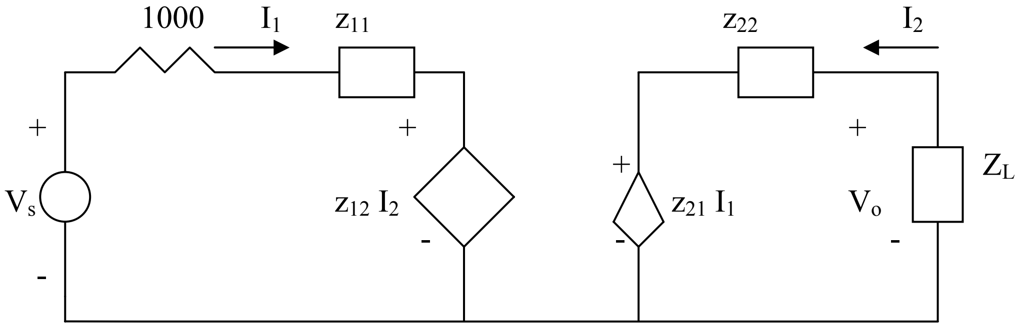
$$\Delta_h = 1 - 0.8 = 0.2$$

$$[T_a] = [T_b] = \begin{bmatrix} -\Delta_h / h_{21} & -h_{11}/h_{21} \\ -h_{22}/h_{21} & -1/h_{21} \end{bmatrix} = \begin{bmatrix} -0.001 & -10 \\ -2.5x10^{-6} & -0.005 \end{bmatrix}$$

$$[T] = [T_a][T_b] = \begin{bmatrix} 2.6x10^{-5} & 0.06 \\ 1.5x10^{-8} & 5x10^{-5} \end{bmatrix}$$

We now convert this to z-parameters

$$[z] = \begin{bmatrix} A/C & \Delta_T/C \\ 1/C & D/C \end{bmatrix} = \begin{bmatrix} 1.733x10^3 & 0.0267 \\ 6.667x10^7 & 3.33x10^3 \end{bmatrix}$$



$$V_s = (1000 + z_{11})I_1 + z_{12}I_2 \qquad (1)$$

$$V_o = z_{22}I_2 + z_{21}I_1 \qquad (2)$$

But $V_o = -I_2Z_L \quad \longrightarrow \quad I_2 = -V_o / Z_L \qquad (3)$

Substituting (3) into (2) gives

$$I_1 = V_o\left(\frac{1}{z_{21}} + \frac{z_{22}}{z_{21}Z_L}\right) \qquad (4)$$

We substitute (3) and (4) into (1)

$$\begin{aligned} V_s &= (1000 + z_{11})\left(\frac{1}{z_{11}} + \frac{z_{22}}{z_{21}Z_L}\right)V_o - \frac{z_{12}}{Z_L}V_o \\ &= 7.653\text{x}10^{-4} - 2.136\text{x}10^{-5} = \underline{744\mu V} \end{aligned}$$

Chapter 19, Problem 99.

Assume that the two circuits in Fig. 19.135 are equivalent. The parameters of the two circuits must be equal. Using this factor and the *z* parameters, derive Eqs. (9.67) and (9.68).

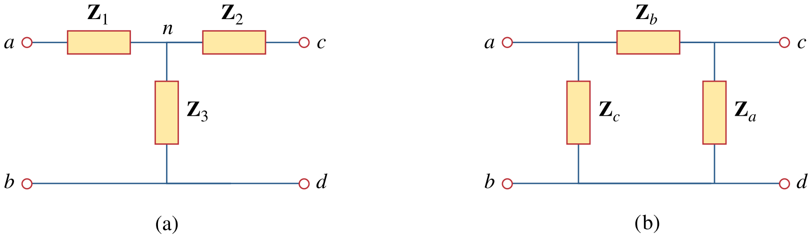


Figure 19.135
For Prob. 19.99.

Chapter 19, Solution 99

$$\mathbf{Z}_{ab} = \mathbf{Z}_1 + \mathbf{Z}_3 = \mathbf{Z}_c \,||\, (\mathbf{Z}_b + \mathbf{Z}_a)$$

$$\mathbf{Z}_1 + \mathbf{Z}_3 = \frac{\mathbf{Z}_c(\mathbf{Z}_a + \mathbf{Z}_b)}{\mathbf{Z}_a + \mathbf{Z}_b + \mathbf{Z}_c} \tag{1}$$

$$\mathbf{Z}_{cd} = \mathbf{Z}_2 + \mathbf{Z}_3 = \mathbf{Z}_a \,||\, (\mathbf{Z}_b + \mathbf{Z}_c)$$

$$\mathbf{Z}_2 + \mathbf{Z}_3 = \frac{\mathbf{Z}_a(\mathbf{Z}_b + \mathbf{Z}_c)}{\mathbf{Z}_a + \mathbf{Z}_b + \mathbf{Z}_c} \tag{2}$$

$$\mathbf{Z}_{ac} = \mathbf{Z}_1 + \mathbf{Z}_2 = \mathbf{Z}_b \,||\, (\mathbf{Z}_a + \mathbf{Z}_c)$$

$$\mathbf{Z}_1 + \mathbf{Z}_2 = \frac{\mathbf{Z}_b(\mathbf{Z}_a + \mathbf{Z}_c)}{\mathbf{Z}_a + \mathbf{Z}_b + \mathbf{Z}_c} \tag{3}$$

Subtracting (2) from (1),

$$\mathbf{Z}_1 - \mathbf{Z}_2 = \frac{\mathbf{Z}_b(\mathbf{Z}_c - \mathbf{Z}_a)}{\mathbf{Z}_a + \mathbf{Z}_b + \mathbf{Z}_c} \tag{4}$$

Adding (3) and (4),

$$\mathbf{Z}_1 = \underline{\frac{\mathbf{Z}_b\mathbf{Z}_c}{\mathbf{Z}_a + \mathbf{Z}_b + \mathbf{Z}_c}} \tag{5}$$

Subtracting (5) from (3),

$$\mathbf{Z}_2 = \underline{\frac{\mathbf{Z_a Z_b}}{\mathbf{Z_a + Z_b + Z_c}}} \qquad (6)$$

Subtracting (5) from (1),

$$\mathbf{Z}_3 = \underline{\frac{\mathbf{Z_c Z_a}}{\mathbf{Z_a + Z_b + Z_c}}} \qquad (7)$$

Using (5) to (7)

$$\mathbf{Z}_1\mathbf{Z}_2 + \mathbf{Z}_2\mathbf{Z}_3 + \mathbf{Z}_3\mathbf{Z}_1 = \frac{\mathbf{Z}_a\mathbf{Z}_b\mathbf{Z}_c\,(\mathbf{Z}_a + \mathbf{Z}_b + \mathbf{Z}_c)}{(\mathbf{Z}_a + \mathbf{Z}_b + \mathbf{Z}_c)^2}$$

$$\mathbf{Z}_1\mathbf{Z}_2 + \mathbf{Z}_2\mathbf{Z}_3 + \mathbf{Z}_3\mathbf{Z}_1 = \frac{\mathbf{Z}_a\mathbf{Z}_b\mathbf{Z}_c}{\mathbf{Z}_a + \mathbf{Z}_b + \mathbf{Z}_c} \qquad (8)$$

Dividing (8) by each of (5), (6), and (7),

$$\mathbf{Z}_a = \underline{\frac{\mathbf{Z_1 Z_2 + Z_2 Z_3 + Z_3 Z_1}}{\mathbf{Z_1}}}$$

$$\mathbf{Z}_b = \underline{\frac{\mathbf{Z_1 Z_2 + Z_2 Z_3 + Z_3 Z_1}}{\mathbf{Z_3}}}$$

$$\mathbf{Z}_c = \underline{\frac{\mathbf{Z_1 Z_2 + Z_2 Z_3 + Z_3 Z_1}}{\mathbf{Z_2}}}$$

as required. Note that the formulas above are not exactly the same as those in Chapter 9 because the locations of $\mathbf{Z}_b$ and $\mathbf{Z}_c$ are interchanged in Fig. 18.122.

Attia, John Okyere. “Two-Port Networks.”
Electronics and Circuit Analysis using MATLAB.
Ed. John Okyere Attia
Boca Raton: CRC Press LLC, 1999

CHAPTER SEVEN

TWO-PORT NETWORKS

This chapter discusses the application of MATLAB for analysis of two-port networks. The describing equations for the various two-port network representations are given. The use of MATLAB for solving problems involving parallel, series and cascaded two-port networks is shown. Example problems involving both passive and active circuits will be solved using MATLAB.

7.1 TWO-PORT NETWORK REPRESENTATIONS

A general two-port network is shown in Figure 7.1.

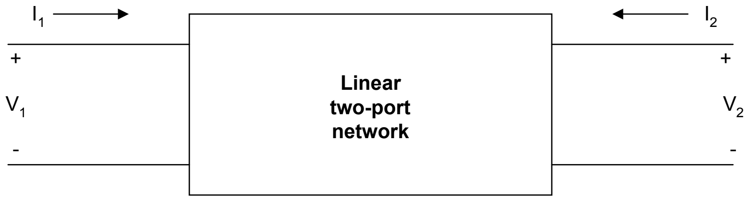


Figure 7.1 General Two-Port Network

I_1 and V_1 are input current and voltage, respectively. Also, I_2 and V_2 are output current and voltage, respectively. It is assumed that the linear two-port circuit contains no independent sources of energy and that the circuit is initially at rest (no stored energy). Furthermore, any controlled sources within the linear two-port circuit cannot depend on variables that are outside the circuit.

7.1.1 z-parameters

A two-port network can be described by z-parameters as

$$V_1 = z_{11} I_1 + z_{12} I_2 \tag{7.1}$$

$$V_2 = z_{21} I_1 + z_{22} I_2 \tag{7.2}$$

In matrix form, the above equation can be rewritten as

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \tag{7.3}$$

The *z*-parameter can be found as follows

$$z_{11} = \frac{V_1}{I_1}\Big|_{I_2=0} \tag{7.4}$$

$$z_{12} = \frac{V_1}{I_2}\Big|_{I_1=0} \tag{7.5}$$

$$z_{21} = \frac{V_2}{I_1}\Big|_{I_2=0} \tag{7.6}$$

$$z_{22} = \frac{V_2}{I_2}\Big|_{I_1=0} \tag{7.7}$$

The z-parameters are also called open-circuit impedance parameters since they are obtained as a ratio of voltage and current and the parameters are obtained by open-circuiting port 2 ($I_2 = 0$) or port1 ($I_1 = 0$). The following example shows a technique for finding the z-parameters of a simple circuit.

Example 7.1

For the T-network shown in Figure 7.2, find the z-parameters.

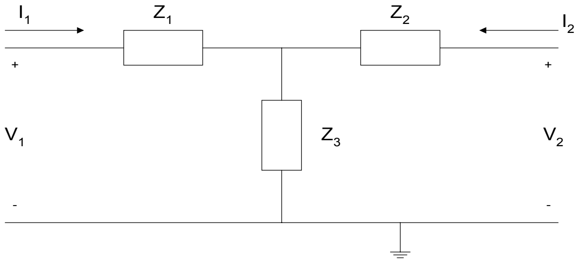


Figure 7.2 T-Network

Solution

Using KVL

$$V_1 = Z_1 I_1 + Z_3(I_1 + I_2) = (Z_1 + Z_3)I_1 + Z_3 I_2 \tag{7.8}$$

$$V_2 = Z_2 I_2 + Z_3(I_1 + I_2) = (Z_3)I_1 + (Z_2 + Z_3)I_2 \tag{7.9}$$

thus

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_1 + Z_3 & Z_3 \\ Z_3 & Z_2 + Z_3 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \tag{7.10}$$

and the z-parameters are

$$[Z] = \begin{bmatrix} Z_1 + Z_3 & Z_3 \\ Z_3 & Z_2 + Z_3 \end{bmatrix} \tag{7.11}$$

7.1.2 y-parameters

A two-port network can also be represented using y-parameters. The describing equations are

$$I_1 = y_{11} V_1 + y_{12} V_2 \tag{7.12}$$

$$I_2 = y_{21} V_1 + y_{22} V_2 \tag{7.13}$$

where

V_1 and V_2 are independent variables and
I_1 and I_2 are dependent variables.

In matrix form, the above equations can be rewritten as

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \tag{7.14}$$

The y-parameters can be found as follows:

$$y_{11} = \frac{I_1}{V_1}\bigg|_{V_2=0} \tag{7.15}$$

$$y_{12} = \frac{I_1}{V_2}\bigg|_{V_1=0} \tag{7.16}$$

$$y_{21} = \frac{I_2}{V_1}\bigg|_{V_2=0} \tag{7.17}$$

$$y_{22} = \frac{I_2}{V_2}\bigg|_{V_1=0} \tag{7.18}$$

The y-parameters are also called short-circuit admittance parameters. They are obtained as a ratio of current and voltage and the parameters are found by short-circuiting port 2 ($V_2 = 0$) or port 1 ($V_1 = 0$). The following two examples show how to obtain the y-parameters of simple circuits.

Example 7.2

Find the y-parameters of the pi (π) network shown in Figure 7.3.

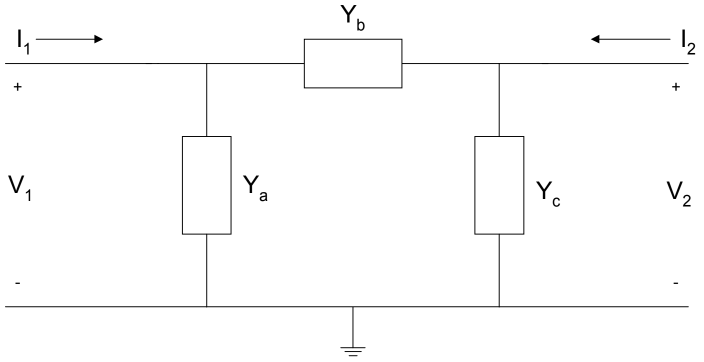


Figure 7.3 Pi-Network

Solution

Using KCL, we have

$$I_1 = V_1 Y_a + (V_1 - V_2)Y_b = V_1(Y_a + Y_b) - V_2 Y_b \tag{7.19}$$

$$I_2 = V_2Y_c + (V_2 - V_1)Y_b = -V_1Y_b + V_2(Y_b + Y_c) \qquad (7.20)$$

Comparing Equations (7.19) and (7.20) to Equations (7.12) and (7.13), the y-parameters are

$$[Y] = \begin{bmatrix} Y_a + Y_b & -Y_b \\ -Y_b & Y_b + Y_c \end{bmatrix} \qquad (7.21)$$

Example 7.3

Figure 7.4 shows the simplified model of a field effect transistor. Find its y-parameters.

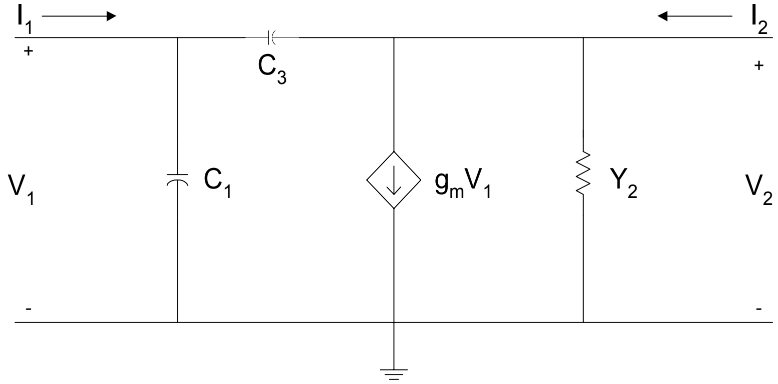


Figure 7.4 Simplified Model of a Field Effect Transistor

Using KCL,

$$I_1 = V_1sC_1 + (V_1 - V_2)sC_3 = V_1(sC_1 + sC_3) + V_2(-sC_3) \qquad (7.22)$$

$$I_2 = V_2Y_2 + g_mV_1 + (V_2 - V_1)sC_3 = V_1(g_m - sC_3) + V_2(Y_2 + sC_3) \qquad (7.23)$$

Comparing the above two equations to Equations (7.12) and (7.13), the y-parameters are

$$[Y] = \begin{bmatrix} sC_1 + sC_3 & -sC_3 \\ g_m - sC_3 & Y_2 + sC_3 \end{bmatrix} \tag{7.24}$$

7.1.3 h-parameters

A two-port network can be represented using the h-parameters. The describing equations for the h-parameters are

$$V_1 = h_{11} I_1 + h_{12} V_2 \tag{7.25}$$

$$I_2 = h_{21} I_1 + h_{22} V_2 \tag{7.26}$$

where

I_1 and V_2 are independent variables and
V_1 and I_2 are dependent variables.

In matrix form, the above two equations become

$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix} \tag{7.27}$$

The h-parameters can be found as follows:

$$h_{11} = \left. \frac{V_1}{I_1} \right|_{V_2 = 0} \tag{7.28}$$

$$h_{12} = \left. \frac{V_1}{V_2} \right|_{I_1 = 0} \tag{7.29}$$

$$h_{21} = \left. \frac{I_2}{I_1} \right|_{V_2 = 0} \tag{7.30}$$

$$h_{22} = \left. \frac{I_2}{V_2} \right|_{I_1 = 0} \tag{7.31}$$

The h-parameters are also called hybrid parameters since they contain both open-circuit parameters ($I_1 = 0$) and short-circuit parameters ($V_2 = 0$). The h-parameters of a bipolar junction transistor are determined in the following example.

Example 7.4

A simplified equivalent circuit of a bipolar junction transistor is shown in Figure 7.5, find its h-parameters.

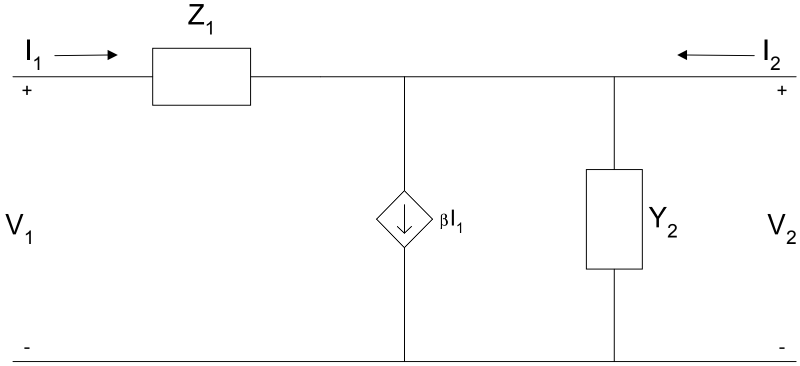


Figure 7.5 Simplified Equivalent Circuit of a Bipolar Junction Transistor

Solution

Using KCL for port 1,

$$V_1 = I_1 Z_1 \tag{7.32}$$

Using KCL at port 2, we get

$$I_2 = \beta I_1 + Y_2 V_2 \tag{7.33}$$

Comparing the above two equations to Equations (7.25) and (7.26) we get the h-parameters.

$$[h] = \begin{bmatrix} Z_1 & 0 \\ \beta & Y_2 \end{bmatrix} \quad ` \tag{7.34}$$

7.1.4 Transmission parameters

A two-port network can be described by transmission parameters. The describing equations are

$$V_1 = a_{11}V_2 - a_{12}I_2 \tag{7.35}$$

$$I_1 = a_{21}V_2 - a_{22}I_2 \tag{7.36}$$

where

V_2 and I_2 are independent variables and
V_1 and I_1 are dependent variables.

In matrix form, the above two equations can be rewritten as

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix} \tag{7.37}$$

The transmission parameters can be found as

$$a_{11} = \frac{V_1}{V_2}\bigg|_{I_2=0} \tag{7.38}$$

$$a_{12} = -\frac{V_1}{I_2}\bigg|_{V_2=0} \tag{7.39}$$

$$a_{21} = \frac{I_1}{V_2}\bigg|_{I_2=0} \tag{7.40}$$

$$a_{22} = -\frac{I_1}{I_2}\bigg|_{V_2=0} \tag{7.41}$$

The transmission parameters express the primary (sending end) variables V_1 and I_1 in terms of the secondary (receiving end) variables V_2 and $-I_2$. The negative of I_2 is used to allow the current to enter the load at the receiving end. Examples 7.5 and 7.6 show some techniques for obtaining the transmission parameters of impedance and admittance networks.

Example 7.5

Find the transmission parameters of Figure 7.6.

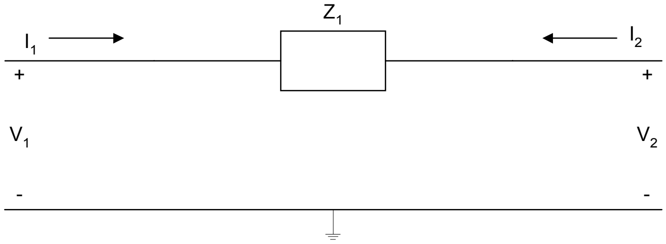


Figure 7.6 Simple Impedance Network

Solution

By inspection,

$$I_1 = -I_2 \tag{7.42}$$

Using KVL,

$$V_1 = V_2 + Z_1 I_1 \tag{7.43}$$

Since $I_1 = -I_2$, Equation (7.43) becomes

$$V_1 = V_2 - Z_1 I_2 \tag{7.44}$$

Comparing Equations (7.42) and (7.44) to Equations (7.35) and (7.36), we have

$$\begin{aligned} a_{11} &= 1 \qquad & a_{12} &= Z_1 \\ a_{21} &= 0 \qquad & a_{22} &= 1 \end{aligned} \tag{7.45}$$

Example 7.6

Find the transmission parameters for the network shown in Figure 7.7.

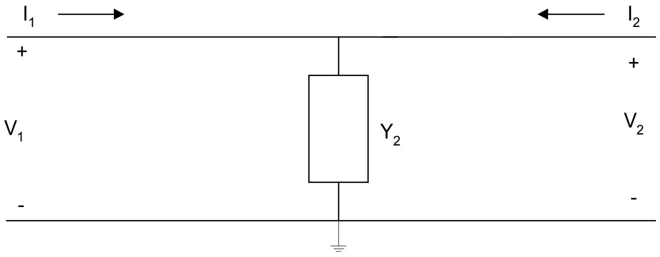


Figure 7.7 Simple Admittance Network

Solution

By inspection,

$$V_1 = V_2 \tag{7.46}$$

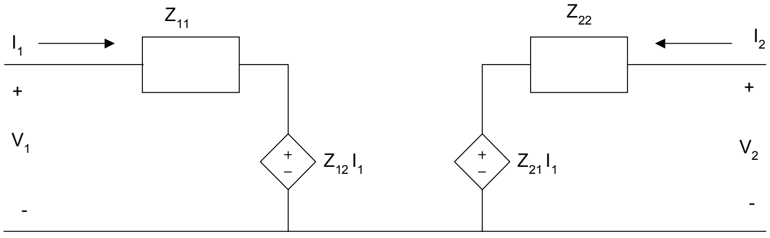
Using KCL, we have

$$I_1 = V_2 Y_2 - I_2 \tag{7.47}$$

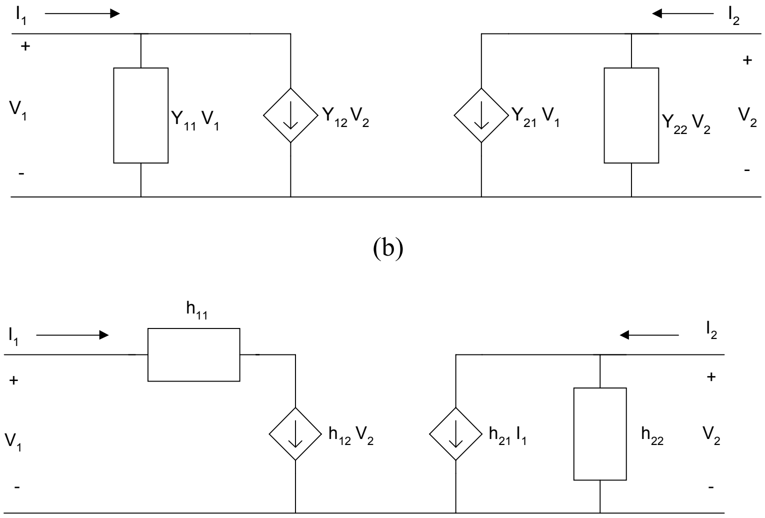
Comparing Equations (7.46) and 7.47) to equations (7.35) and (7.36) we have

$$\begin{aligned} a_{11} &= 1 \qquad & a_{12} &= 0 \\ a_{21} &= Y_2 \qquad & a_{22} &= 1 \end{aligned} \tag{7.48}$$

Using the describing equations, the equivalent circuits of the various two-port network representations can be drawn. These are shown in Figure 7.8.



(a)



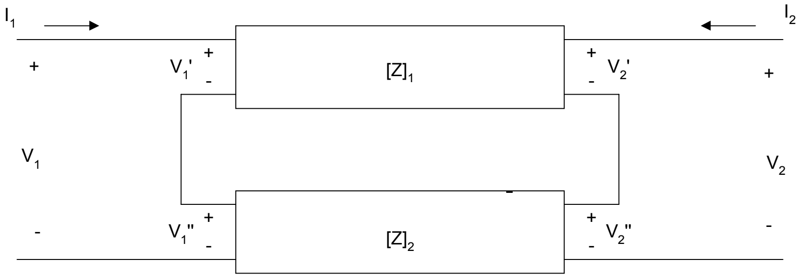
(b)

(c)

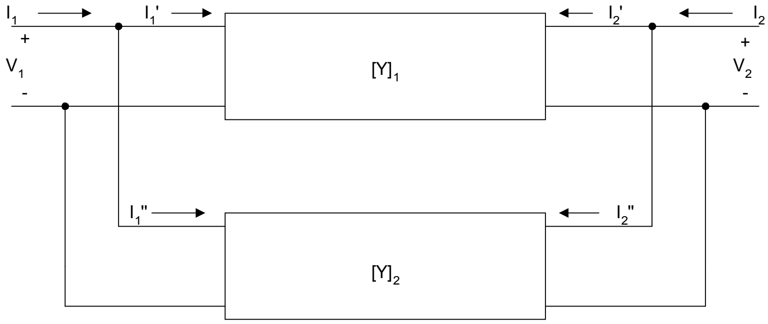
Figure 7.8 Equivalent Circuit of Two-port Networks (a) z-parameters, (b) y-parameters and (c) h-parameters

7.2 INTERCONNECTION OF TWO-PORT NETWORKS

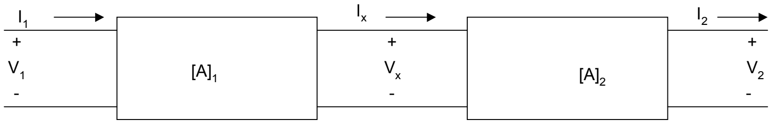
Two-port networks can be connected in series, parallel or cascade. Figure 7.9 shows the various two-port interconnections.



(a) Series-connected Two-port Network



(b) Parallel-connected Two-port Network



(c) Cascade Connection of Two-port Network

Figure 7.9 Interconnection of Two-port Networks (a) Series (b) Parallel (c) Cascade

It can be shown that if two-port networks with z-parameters $[Z]_1, [Z]_2, [Z]_3, \ldots, [Z]_n$ are connected in series, then the equivalent two-port z-parameters are given as

$$[Z]_{eq} = [Z]_1 + [Z]_2 + [Z]_3 + \ldots + [Z]_n \tag{7.49}$$

If two-port networks with y-parameters $[Y]_1, [Y]_2, [Y]_3, \ldots, [Y]_n$ are connected in parallel, then the equivalent two-port y-parameters are given as

$$[Y]_{eq} = [Y]_1 + [Y]_2 + [Y]_3 + \ldots + [Y]_n \tag{7.50}$$

When several two-port networks are connected in cascade, and the individual networks have transmission parameters $[A]_1, [A]_2, [A]_3, \ldots, [A]_n$, then the equivalent two-port parameter will have a transmission parameter given as

$$[A]_{eq} = [A]_1 * [A]_2 * [A]_3 * \ldots * [A]_n \tag{7.51}$$

The following three examples illustrate the use of MATLAB for determining the equivalent parameters of interconnected two-port networks.

Example 7.7

Find the equivalent y-parameters for the bridge T-network shown in Figure 7.10.

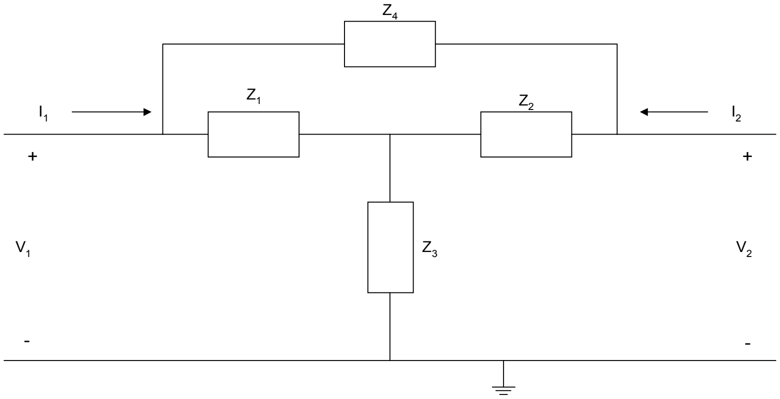


Figure 7.10 Bridge-T Network

Solution

The bridge-T network can be redrawn as

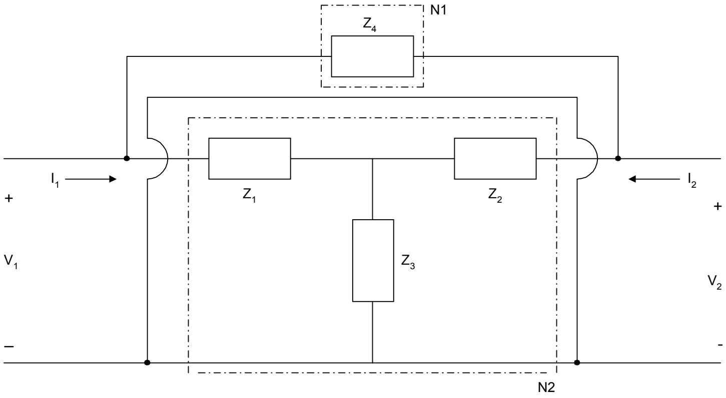


Figure 7.11 An Alternative Representation of Bridge-T Network

From Example 7.1, the z-parameters of network N2 are

$$[Z] = \begin{bmatrix} Z_1 + Z_3 & Z_3 \\ Z_3 & Z_2 + Z_3 \end{bmatrix}$$

We can convert the z-parameters to y-parameters [refs. 4 and 6] and we get

$$\begin{aligned} y_{11} &= \frac{Z_2 + Z_3}{Z_1 Z_2 + Z_1 Z_3 + Z_2 Z_3} \\ y_{12} &= \frac{-Z_3}{Z_1 Z_2 + Z_1 Z_3 + Z_2 Z_3} \\ y_{21} &= \frac{-Z_3}{Z_1 Z_2 + Z_1 Z_3 + Z_2 Z_3} \\ y_{22} &= -\frac{Z_1 + Z_3}{Z_1 Z_2 + Z_1 Z_3 + Z_2 Z_3} \end{aligned} \tag{7.52}$$

From Example 7.5, the transmission parameters of network N1 are

$$\begin{aligned} a_{11} &= 1 & a_{12} &= Z_4 \\ a_{21} &= 0 & a_{22} &= 1 \end{aligned}$$

We convert the transmission parameters to y-parameters[refs. 4 and 6] and we get

$$\begin{aligned} y_{11} &= \frac{1}{Z_4} \\ y_{12} &= -\frac{1}{Z_4} \\ y_{21} &= -\frac{1}{Z_4} \\ y_{22} &= \frac{1}{Z_4} \end{aligned} \tag{7.53}$$

Using Equation (7.50), the equivalent y-parameters of the bridge-T network are

$$\begin{aligned} y_{11eq} &= \frac{1}{Z_4} + \frac{Z_2 + Z_3}{Z_1Z_2 + Z_1Z_3 + Z_2Z_3} \\ y_{12eq} &= -\frac{1}{Z_4} - \frac{Z_3}{Z_1Z_2 + Z_1Z_3 + Z_2Z_3} \\ y_{21eq} &= -\frac{1}{Z_4} - \frac{Z_3}{Z_1Z_2 + Z_1Z_3 + Z_2Z_3} \\ y_{22eq} &= \frac{1}{Z_4} + \frac{Z_1 + Z_3}{Z_1Z_2 + Z_1Z_3 + Z_2Z_3} \end{aligned} \tag{7.54}$$

Example 7.8

Find the transmission parameters of Figure 7.12.

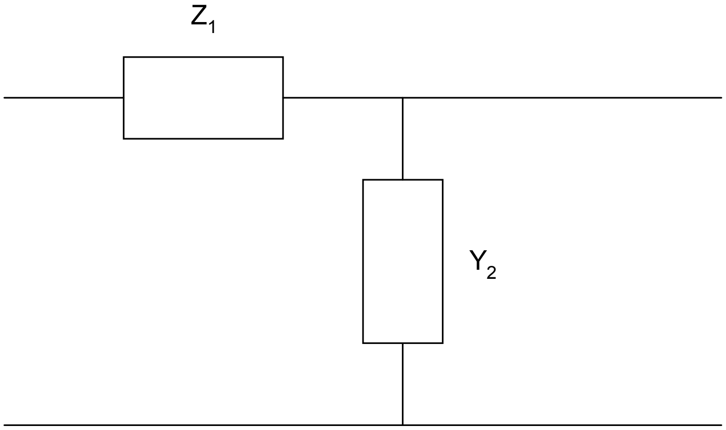


Figure 7.12 Simple Cascaded Network

Solution

Figure 7.12 can be redrawn as

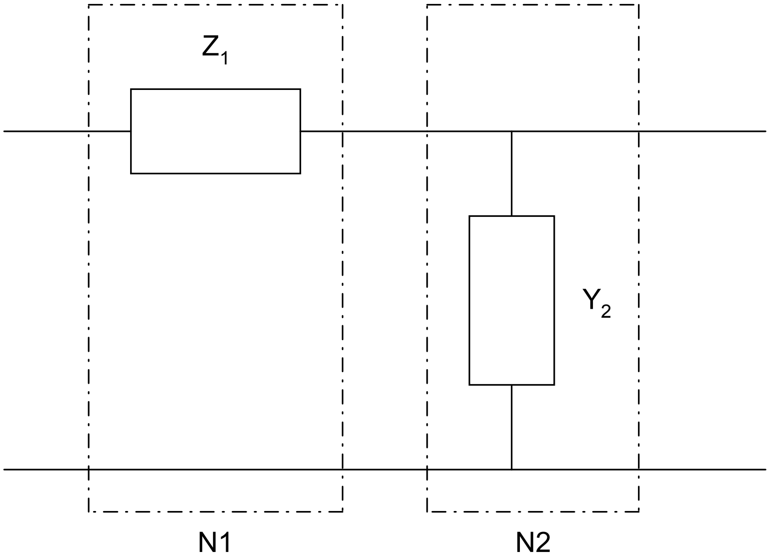


Figure 7.13 Cascade of Two Networks N1 and N2

From Example 7.5, the transmission parameters of network N1 are

$$a_{11} = 1 \qquad a_{12} = Z_1$$
$$a_{21} = 0 \qquad a_{22} = 1$$

From Example 7.6, the transmission parameters of network N2 are

$$a_{11} = 1 \qquad a_{12} = 0$$
$$a_{21} = Y_2 \qquad a_{22} = 1$$

From Equation (7.51), the transmission parameters of Figure 7.13 are

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}_{eq} = \begin{bmatrix} 1 & Z_1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ Y_2 & 1 \end{bmatrix} = \begin{bmatrix} 1 + Z_1 Y_2 & Z_1 \\ Y_2 & 1 \end{bmatrix} \qquad (7.55)$$

Example 7.9

Find the transmission parameters for the cascaded system shown in Figure 7.14. The resistance values are in Ohms.

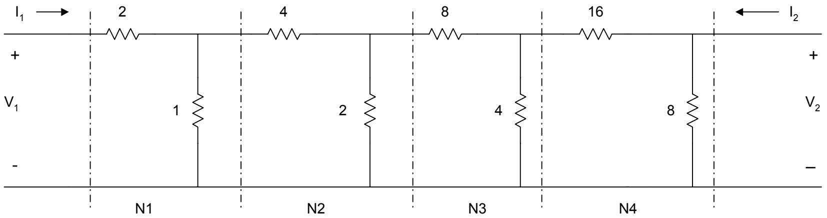


Figure 7.14 Cascaded Resistive Network

Solution

Figure 7.14 can be considered as four networks, N1, N2, N3, and N4 connected in cascade. From Example 7.8, the transmission parameters of Figure 7.12 are

$$[a]_{N1} = \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}$$

$$[a]_{N2} = \begin{bmatrix} 3 & 4 \\ 0.5 & 1 \end{bmatrix}$$

$$[a]_{N3} = \begin{bmatrix} 3 & 8 \\ 0.25 & 1 \end{bmatrix}$$

$$[a]_{N4} = \begin{bmatrix} 3 & 16 \\ 0.125 & 1 \end{bmatrix}$$

The transmission parameters of Figure 7.14 can be obtained using the following MATLAB program.

MATLAB Script

```
diary ex7_9.dat
% Transmission parameters of cascaded network

a1 = [3 2; 1 1];
a2 = [3 4; 0.5 1];
a3 = [3 8; 0.25 1];
a4 = [3 16; 0.125 1];

% equivalent transmission parameters
a = a1*(a2*(a3*a4))
diary
```

The value of matrix a is

```
a =
     112.2500    630.0000
       39.3750    221.0000
```

7.3 TERMINATED TWO-PORT NETWORKS

In normal applications, two-port networks are usually terminated. A terminated two-port network is shown in Figure 7.4.

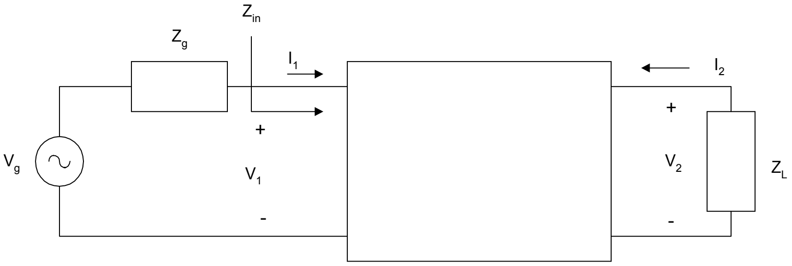


Figure 7.15 Terminated Two-Port Network

In the Figure 7.15, V_g and Z_g are the source generator voltage and impedance, respectively. Z_L is the load impedance. If we use z-parameter representation for the two-port network, the voltage transfer function can be shown to be

$$\frac{V_2}{V_g} = \frac{z_{21} Z_L}{(z_{11} + Z_g)(z_{22} + Z_L) - z_{12} z_{21}} \tag{7.56}$$

and the input impedance,

$$Z_{in} = z_{11} - \frac{z_{12} z_{21}}{z_{22} + Z_L} \tag{7.57}$$

and the current transfer function,

$$\frac{I_2}{I_1} = -\frac{z_{21}}{z_{22} + Z_L} \tag{7.58}$$

A terminated two-port network, represented using the y-parameters, is shown in Figure 7.16.

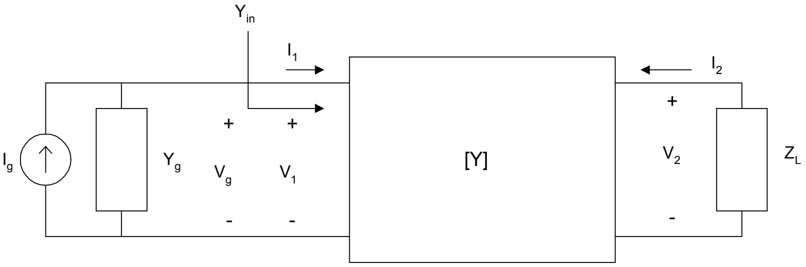


Figure 7.16 A Terminated Two-Port Network with y-parameters Representation

It can be shown that the input admittance, Y_{in}, is

$$Y_{in} = y_{11} - \frac{y_{12} y_{21}}{y_{22} + Y_L} \tag{7.59}$$

and the current transfer function is given as

$$\frac{I_2}{I_g} = \frac{y_{21} Y_L}{(y_{11} + Y_g)(y_{22} + Y_L) - y_{12} y_{21}} \tag{7.60}$$

and the voltage transfer function

$$\frac{V_2}{V_g} = -\frac{y_{21}}{y_{22} + Y_L} \tag{7.61}$$

A doubly terminated two-port network, represented by transmission parameters, is shown in Figure 7.17.

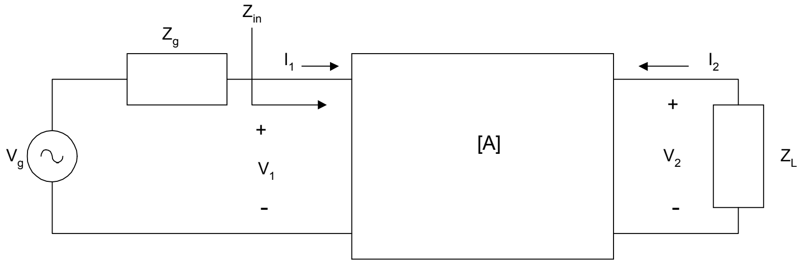


Figure 7.17 A Terminated Two-Port Network with Transmission Parameters Representation

The voltage transfer function and the input impedance of the transmission parameters can be obtained as follows. From the transmission parameters, we have

$$V_1 = a_{11}V_2 - a_{12}I_2 \tag{7.62}$$

$$I_1 = a_{21}V_2 - a_{22}I_2 \tag{7.63}$$

From Figure 7.6,

$$V_2 = -I_2 Z_L \tag{7.64}$$

Substituting Equation (7.64) into Equations (7.62) and (7.63), we get the input impedance,

$$Z_{in} = \frac{a_{11}Z_L + a_{12}}{a_{21}Z_L + a_{22}} \tag{7.65}$$

From Figure 7.17, we have

$$V_1 = V_g - I_1 Z_g \tag{7.66}$$

Substituting Equations (7.64) and (7.66) into Equations (7.62) and (7.63), we have

$$V_g - I_1 Z_g = V_2 [a_{11} + \frac{a_{12}}{Z_L}] \tag{7.67}$$

$$I_1 = V_2 [a_{21} + \frac{a_{22}}{Z_L}] \tag{7.68}$$

Substituting Equation (7.68) into Equation (7.67), we get

$$V_g - V_2 Z_g [a_{21} + \frac{a_{22}}{Z_L}] = V_2 [a_{11} + \frac{a_{12}}{Z_L}] \tag{7.69}$$

Simplifying Equation (7.69), we get the voltage transfer function

$$\frac{V_2}{V_g} = \frac{Z_L}{(a_{11} + a_{21} Z_g) Z_L + a_{12} + a_{22} Z_g} \tag{7.70}$$

The following examples illustrate the use of MATLAB for solving terminated two-port network problems.

Example 7.10

Assuming that the operational amplifier of Figure 7.18 is ideal,

(a) Find the z-parameters of Figure 7.18.

(b) If the network is connected by a voltage source with source resistance of 50Ω and a load resistance of 1 KΩ, find the voltage gain.

(c) Use MATLAB to plot the magnitude response.

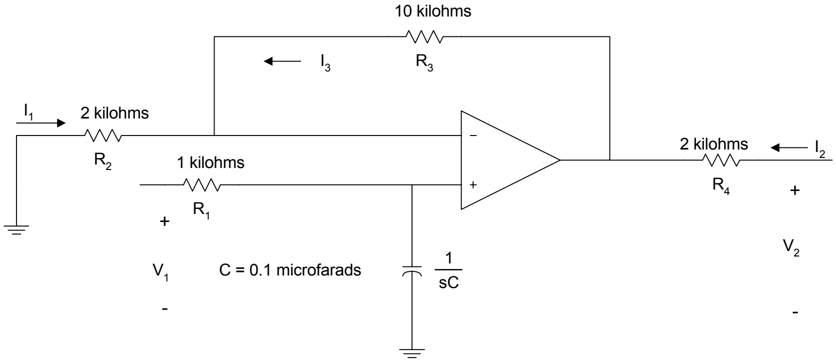


Figure 7.18 An Active Lowpass Filter

Solution

Using KVL,

$$V_1 = R_1 I_1 + \frac{I_1}{sC} \tag{7.71}$$

$$V_2 = R_4 I_2 + R_3 I_3 + R_2 I_3 \tag{7.72}$$

From the concept of virtual circuit discussed in Chapter 11,

$$R_2 I_3 = \frac{I_1}{sC} \tag{7.73}$$

Substituting Equation (7.73) into Equation (7.72), we get

$$V_2 = \frac{(R_2 + R_3) I_1}{sCR_2} + R_4 I_2 \tag{7.74}$$

Comparing Equations (7.71) and (7.74) to Equations (7.1) and (7.2), we have

$$
\begin{aligned}
z_{11} &= R_1 + \frac{1}{sC} \\
z_{12} &= 0 \\
z_{21} &= \left(1 + \frac{R_3}{R_2}\right)\left(\frac{1}{sC}\right) \\
z_{22} &= R_4
\end{aligned}
\tag{7.75}
$$

From Equation (7.56), we get the voltage gain for a terminated two-port network. It is repeated here.

$$\frac{V_2}{V_g} = \frac{z_{21} Z_L}{(z_{11} + Z_g)(z_{22} + Z_L) - z_{12} z_{21}}$$

Substituting Equation (7.75) into Equation (7.56), we have

$$\frac{V_2}{V_g} = \frac{(1 + \frac{R_3}{R_2}) Z_L}{(R_4 + Z_L)[1 + sC(R_1 + Z_g)]} \tag{7.76}$$

For $Z_g = 50\ \Omega$, $Z_L = 1\ K\Omega,\ R_3 = 10\, K\Omega,\ R_2 = 1 K\Omega, R_4 = 2\ K\Omega$ and $C = 0.1\ \mu F$, Equation (7.76) becomes

$$\frac{V_2}{V_g} = \frac{2}{[1 + 1.05 * 10^{-4} s]} \tag{7.77}$$

The MATLAB script is

```
%
num = [2];
den = [1.05e-4 1];
w = logspace(1,5);
h = freqs(num,den,w);
f = w/(2*pi);
mag = 20*log10(abs(h));  % magnitude in dB
semilogx(f,mag)
title('Lowpass Filter Response')
xlabel('Frequency, Hz')
```

```
ylabel('Gain in dB')
```

The frequency response is shown in Figure 7.19.

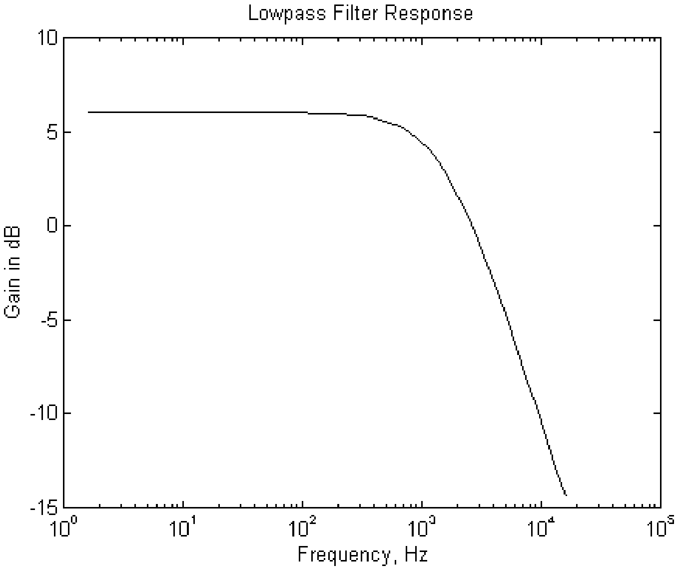


Figure 7.19 Magnitude Response of an Active Lowpass Filter

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4. Nilsson, J.W., Electric Circuits, 3rd Edition, Addison-Wesley Publishing Company, 1990.

5. Meader, D.A., *Laplace Circuit Analysis and Active Filters,* Prentice Hall, 1991.

6. Johnson, D. E. Johnson, J.R., and Hilburn, J.L. *Electric Circuit Analysis*, 3rd Edition, Prentice Hall, 1997.

7. Vlach, J.O., Network Theory and CAD, IEEE Trans. on Education, Vol. 36, No. 1, Feb. 1993, pp. 23 - 27.

EXERCISES

7.1 (a) Find the transmission parameters of the circuit shown in Figure P7.1a. The resistance values are in ohms.

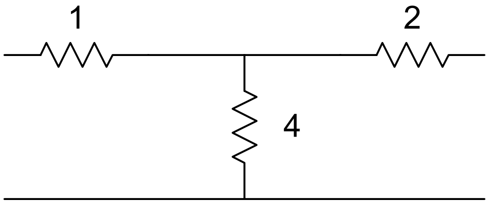


Figure P7.1a Resistive T-Network

(b) From the result of part (a), use MATLAB to find the transmission parameters of Figure P7.2b. The resistance values are in ohms.

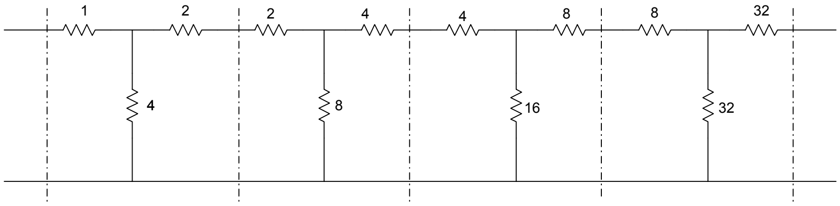


Figure P7.1b Cascaded Resistive Network

7.2 Find the y-parameters of the circuit shown in Figure P7.2 The resistance values are in ohms.

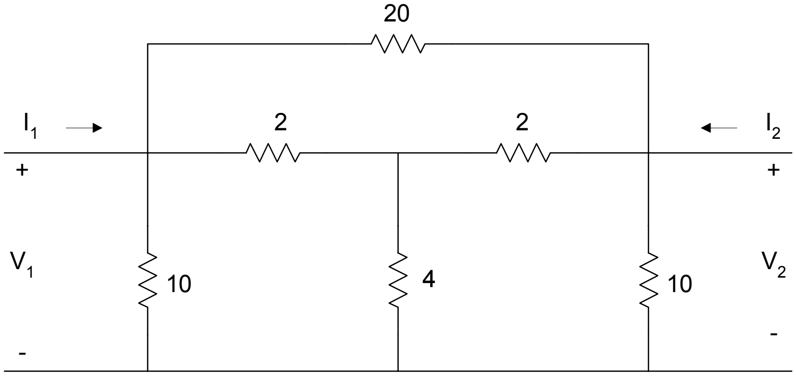


Figure P7.2 A Resistive Network

7.3 (a) Show that for the symmetrical lattice structure shown in Figure P7.3,

$$z_{11} = z_{22} = 0.5(Z_c + Z_d)$$
$$z_{12} = z_{21} = 0.5(Z_c - Z_d)$$

(b) If $Z_c = 10\ \Omega, \quad Z_d = 4\ \Omega,$ find the equivalent y-parameters.

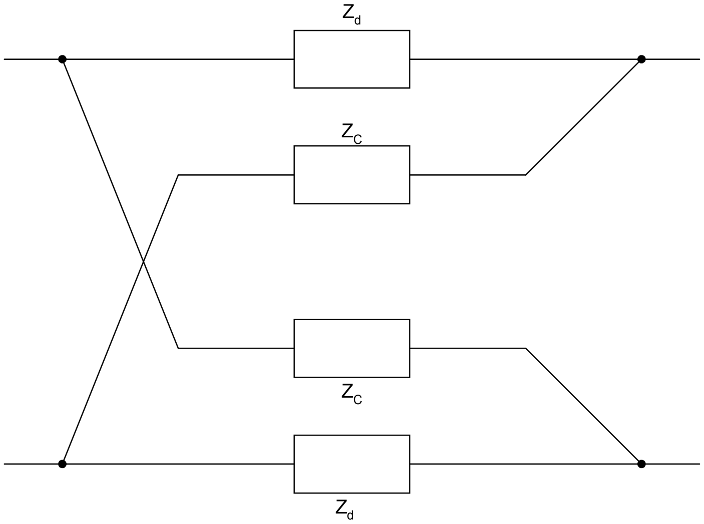


Figure P7.3 Symmetrical Lattice Structure

7.4 (a) Find the equivalent z-parameters of Figure P7.4.
(b) If the network is terminated by a load of 20 ohms and connected to a source of V_S with a source resistance of 4 ohms, use MATLAB to plot the frequency response of the circuit.

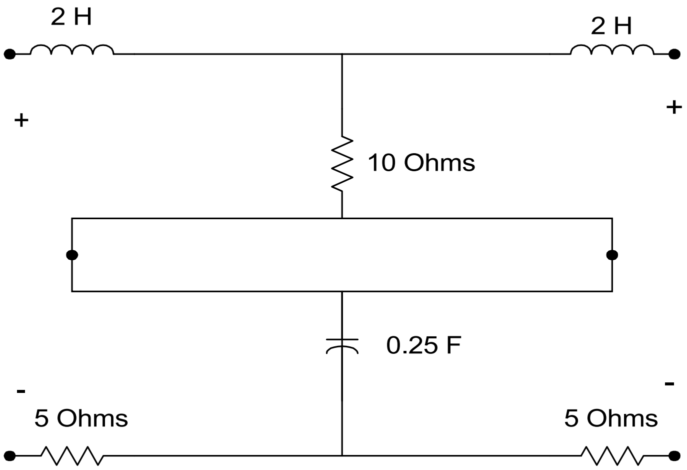


Figure P7.4 Circuit for Problem 7.4

7.5 For Figure P7.5
(a) Find the transmission parameters of the RC ladder network.
(b) Obtain the expression for $\frac{V_2}{V_1}$.
(c) Use MATLAB to plot the phase characteristics of $\frac{V_2}{V_1}$.

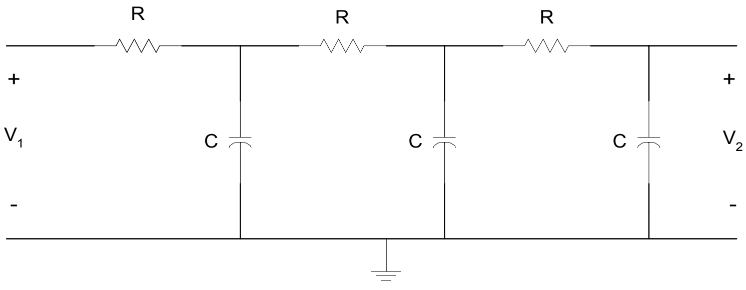


Figure P7.5 RC Ladder Network

7.6 For the circuit shown in Figure P7.6,

(a) Find the y-parameters.

(b) Find the expression for the input admittance.

(c) Use MATLAB to plot the input admittance as a function of frequency.

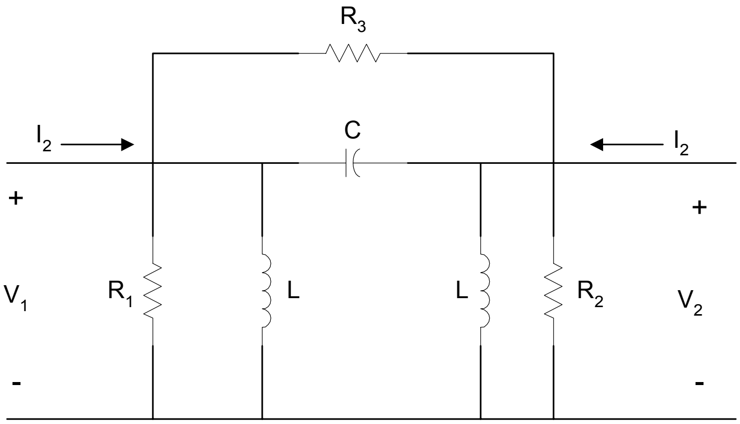


Figure P7.6 Circuit for Problem 7.6

7.7 For the op amp circuit shown in Figure P7.7, find the y-parameters.

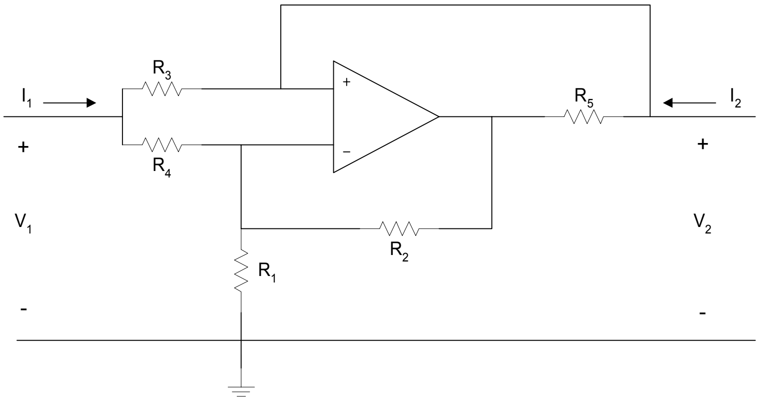


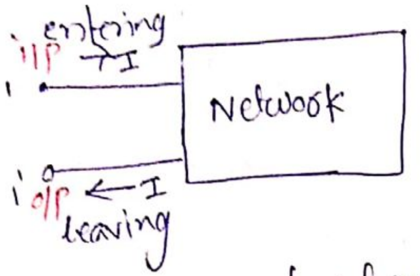
Figure P7.7 Op Amp Circuit

Module-II Two Port Networks

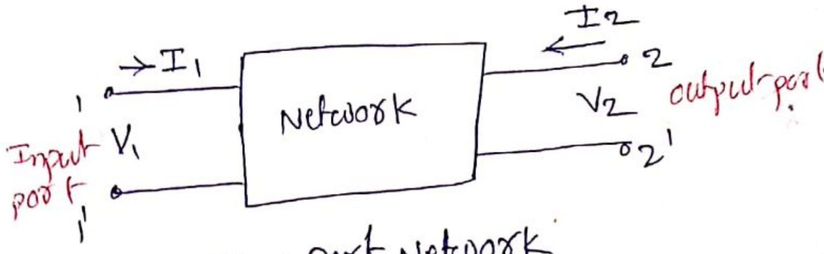
If the current entering one terminal of a path is equal and opposite to the current leaving the other terminal of the path then this type of terminal path is called a "Port".

(or)

A pair of terminals through which a current/signal may enter or leave a network is known as a "port".



(a) one port Network



(b) Two port Network

→ Two ports containing no sources in their branches are called "passive port".
Eg. Power transmission lines and transformers.

→ Two ports containing sources in their branches are called "Active ports".

→ The variables of the two-port network are V_1, V_2, and I_1, I_2. Two of these are dependent variables, the other two are independent variables.

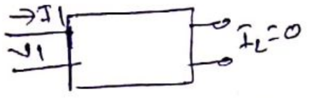
→ The no. of possible combinations generated by the four variables taken two at a time is six. Thus there are six possible sets of equations describing a two port Network.

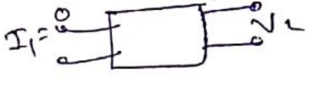
① $V_1 = R_{11}I_1 + R_{12}I_2$, to find R_{11} put $I_2 = 0$ i.e. output port is open circuited.

$V_2 = R_{21}I_1 + R_{22}I_2$

$R_{11} = \frac{V_1}{I_1} = Z$ open circuited impedance parameters

$$\boxed{\begin{aligned} V_1 &= Z_{11}I_1 + Z_{12}I_2 \\ V_2 &= Z_{21}I_1 + Z_{22}I_2 \end{aligned}}$$



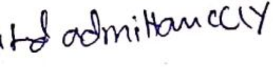


② $I_1 = \frac{V_1}{R_{11}} + \frac{V_2}{R_{12}}$ = $\frac{1}{R} = Y$;

$I_2 = \frac{V_1}{R_{21}} + \frac{V_2}{R_{22}}$

$$\boxed{\begin{aligned} I_1 &= Y_{11}V_1 + Y_{12}V_2 \\ I_2 &= Y_{21}V_1 + Y_{22}V_2 \end{aligned}}$$

To find Y_{11}, put $V_2 = 0$ i.e. output port short



$Y_{11} = \frac{I_1}{V_1}$, short circuited admittance (Y) parameters

③
$$\boxed{\begin{aligned} V_1 &= AV_2 - BI_2 \\ I_1 &= CV_2 - DI_2 \end{aligned}}$$

ABCD parameters

④
$$\boxed{\begin{aligned} V_2 &= A'V_1 - B'I_2 \\ I_2 &= C'V_1 - D'I_2 \end{aligned}}$$

Inverse transmission parameters.

⑤ $V_1 = K_{11} I_1 + K_{12} V_2$

$I_2 = K_{21} I_1 + K_{22} V_2$

$K_{11} = \frac{V_1}{I_1}$ = ~~i/p impedance~~ short circuit i/p impe...

$K_{12} = \frac{V_1}{V_2}$ open ckt = reverse voltage gain

$K_{21} = \frac{I_2}{I_1}$ short ckt = Forward current gain

$K_{22} = \frac{I_2}{V_2}$ open ckt = output admittance.

All are exist so hybrid

$$\boxed{\begin{aligned} V_1 &= h_{11} I_1 + h_{12} V_2 \\ I_2 &= h_{21} I_1 + h_{22} V_2 \end{aligned}}$$

Hybrid parameters.

⑥ $I_1 = P_{11} V_1 + P_{12} I_2$

$V_2 = P_{21} V_1 + P_{22} I_2$

$P_{11} = \frac{I_1}{V_1}$ = open circuit i/p admittance

$P_{12} = \frac{I_1}{I_2}$ = short ~~open~~ circuit ~~vo~~ reverse current gain

$P_{21} = \frac{V_2}{V_1}$ = open ~~short~~ circuit voltage gain

$P_{22} = \frac{V_2}{I_2}$ = short circuit output impedance.

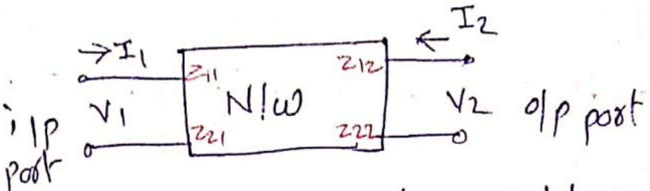
$$\boxed{\begin{aligned} I_1 &= g_{11} V_1 + g_{12} I_2 \\ V_2 &= g_{21} V_1 + g_{22} I_2 \end{aligned}}$$

Inverse hybrid parameters.

In syllabus only ① ② ③ ⑤

① Impedance parameters (open circuit)

$$\boxed{\begin{aligned} V_1 &= Z_{11}I_1 + Z_{12}I_2 \\ V_2 &= Z_{21}I_1 + Z_{22}I_2 \end{aligned}}$$



V_1, V_2 are dependant variables, I_1, I_2 are independent variables

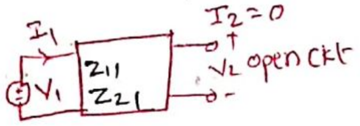
$$(V_1, V_2) = f(I_1, I_2)$$

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$V_1 = Z_{11}I_1 + Z_{12}I_2 \quad -①$$

$$V_2 = Z_{21}I_1 + Z_{22}I_2 \quad -②$$

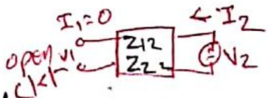
To find Z_{11}, put $I_2 = 0$ i.e. open circuit the o/p port



$Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0}$ open circuit input Impedance

$Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0}$ open circuit forward transfer impedance.

Ily when $I_1 = 0$ i.e. open circuit the input port.



$Z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0}$ open circuit reverse transfer impedance.

$Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0}$ open circuit ~~forward~~ output admittance.

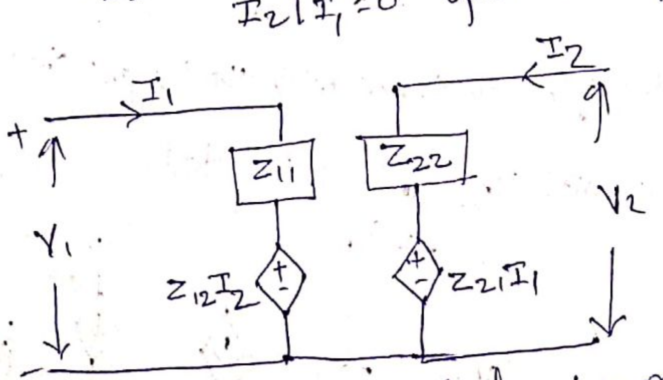


fig. Equivalent circuit of a two port open circuit impedance parameter.

where "$Z_{12}I_2$ and $Z_{21}I_1$ are current controlled voltage sources (CCVS)."

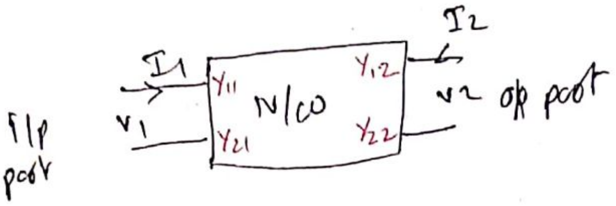
→ If the network under study is reciprocal or bilateral, then with reciprocity principle $\left.\frac{V_2}{I_1}\right|_{I_2=0} = \left.\frac{V_1}{I_2}\right|_{I_1=0}$ $\boxed{Z_{21} = Z_{12}}$

It is observed that all the parameters have the dimensions (or) units of impedance. One of the port being open circuited, hence z parameters called as open circuit impedance parameters.

Y- Parameters (short circuit Admittance Parameters)

$(I_1, I_2) = f(V_1, V_2)$

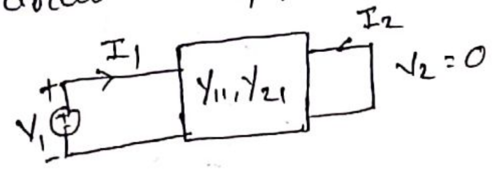
$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$



I_1, I_2 - dependent variables
V_1, V_2 - independent variables.

$$I_1 = Y_{11}V_1 + Y_{12}V_2 \quad -①$$
$$I_2 = Y_{21}V_1 + Y_{22}V_2 \quad -②$$

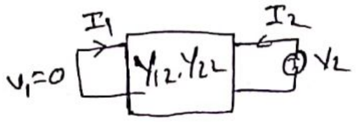
To find Y_{11} & Y_{21}, put $V_2 = 0$ i.e. short circuit the o/p port



From ① & ② $Y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0}$ - short circuit i/p impedance

$Y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0}$ - short circuited forward transfer admittance.

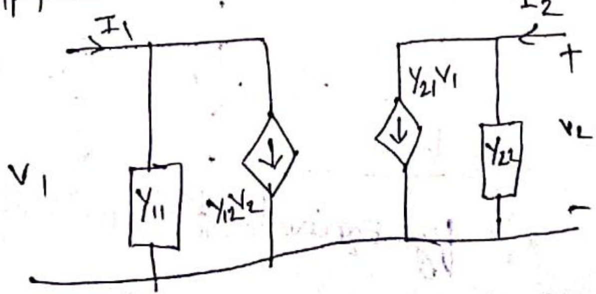
IIly To find Y_{12} & Y_{22}, put $V_1 = 0$ i.e. short circuit the i/p port.



From ① & ② $Y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0}$ Reverse transfer admittance with the i/p port short circuited

$Y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0}$ output admittance with i/p port short circuited.

''$Y_{12}V_2$ & $Y_{21}V_1$ are voltage controlled current sources VCCS''



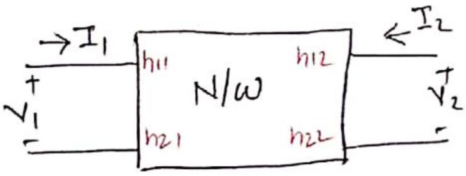
→ All the parameters have the dimensions (or) units of admittance which are obtained by short circuiting either the o/p port or i/p port. Hence Y parameters are called short circuit admittance parameters.

Hybrid Parameters

used in Electronic circuits, especially in constructing models for transistors.

$V_1, I_2 = f(I_1, V_2)$

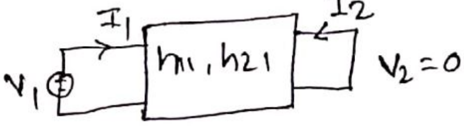
$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$



$V_1 = h_{11} I_1 + h_{12} V_2$ — ①

$I_2 = h_{21} I_1 + h_{22} V_2$ — ②

To find h_{11} & h_{21}, put $V_2 = 0$ i.e. short circuit the o/p port

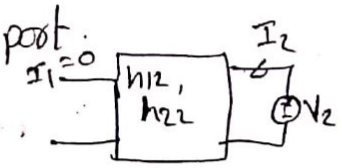


From ① & ②

$h_{11} = \frac{V_1}{I_1}\Big|_{V_2=0}$, i/p impedance with o/p port short circuited

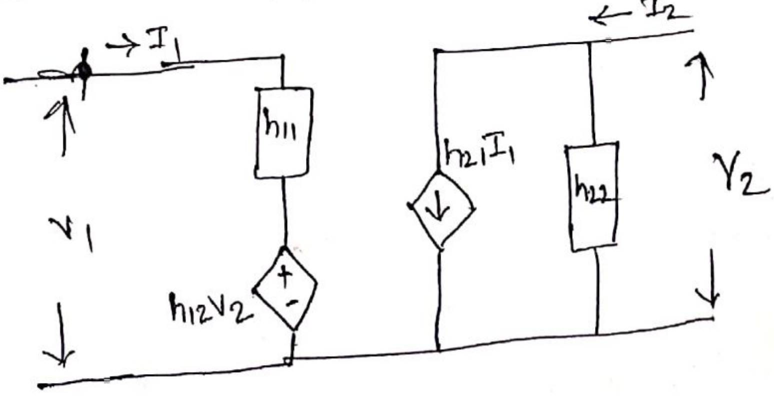
$h_{21} = \frac{I_2}{I_1}\Big|_{V_2=0}$ Forward current gain with o/p port short circuited.

To find h_{12} & h_{22}, put $I_1 = 0$ i.e. open circuit the input port.



$h_{12} = \frac{V_1}{V_2}\Big|_{I_1=0}$, Reverse voltage gain with input port open circuited.

$h_{22} = \frac{I_2}{V_2}\Big|_{I_1=0}$ output admittance with the i/p port open circuited.

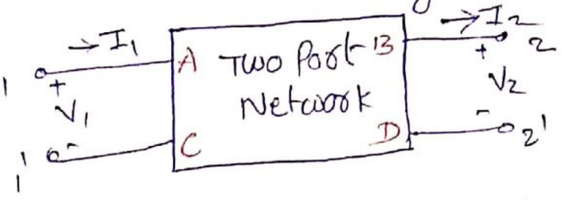


$h_{12} V_2$ - Voltage controlled voltage source (VCVS)

$h_{21} I_1$ - current controlled current source (CCCS)

Transmission (ABCD) Parameters

These are widely used in transmission line theory and cascade Networks.



→ The i/p variables V_1 & I_1 usually called the sending end and are expressed in terms of the output variables V_2 and I_2 called the receiving end.

→ The transmission parameters provide a direct relationship between input and output.

→ ABCD (or) Transmission parameters are also called as general circuit parameters (or) Chain Parameters. $(V_1, I_1) = f(V_2, -I_2)$

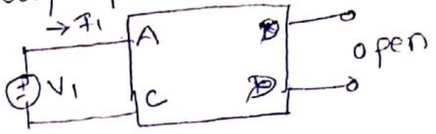
$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

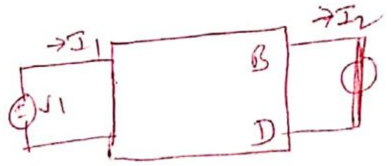
Since, o/p port current is considered outward, therefore negative sign for I_2

$$V_1 = AV_2 - BI_2 \quad ①$$

$$I_1 = CV_2 - DI_2 \quad ②$$

To find A & C, put $I_2=0$, i.e. open circuit the output port





case-1

$$A = \frac{V_1}{V_2}\bigg|_{I_2=0} \; ; \quad \frac{1}{A} = \frac{V_2}{V_1}\bigg|_{I_2=0}$$

A is Reverse voltage gain with the receiving end open circuited

$$C = \frac{I_1}{V_2}\bigg|_{I_2=0}$$, Reverse transfer admittance with the receiving end open

case-2 To find B & D, put $V_2=0$ i.e. short circuit the output port (or) Receiving end

$$B = \frac{V_1}{-I_2}\bigg|_{V_2=0}$$, Reverse transfer impedance with the receiving end short circuited

$$D = \frac{I_1}{-I_2}\bigg|_{V_2=0}$$, Reverse current gain with the receiving end short circuited.

Conversion of one Parameter to other Parameter

(i) Z - Parameters in terms of Y - Parameters.

$V_1 = Z_{11}I_1 + Z_{12}I_2$

$V_2 = Z_{21}I_1 + Z_{22}I_2$

NOTE: A particular sets of parameters sometimes cannot solve a problem (e.g. h-parameters cannot solve all transistor problems). So for this we need to convert one parameter sets to another. This makes a relationship between all parameters.

we know that $[I] = \frac{[V]}{[R]}$

Let $\frac{1}{R} = Y$; $\boxed{[I] = [Y][V]}$ — ①

Ily $[V] = [I][R]$

Let $[R] = [Z]$: $\boxed{[V] = [Z][I]}$ — ②

From ① $\boxed{[V] = [Y]^{-1}[I]}$ — ③

From ② & ③ $[Z] = [Y]^{-1}$ i.e. $\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix}^{-1}$

$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix}^{-1} = \frac{1}{Y_{11}Y_{22} - Y_{12}Y_{21}} \begin{bmatrix} Y_{22} & -Y_{12} \\ -Y_{21} & Y_{11} \end{bmatrix}$$

$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$A^{-1} = \underbrace{\frac{1}{ad-bc}}_{\text{determinant}} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

$Y_{11}Y_{22} - Y_{12}Y_{21} = \Delta Y$ Determinant

$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix}^{-1} = \frac{1}{\Delta Y} \begin{bmatrix} Y_{22} & -Y_{12} \\ -Y_{21} & Y_{11} \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}$$

∴ $Z_{11} = \frac{Y_{22}}{\Delta Y}$; $Z_{12} = \frac{-Y_{12}}{\Delta Y}$

$Z_{21} = -\frac{Y_{21}}{\Delta Y}$; $Z_{22} = \frac{Y_{11}}{\Delta Y}$

(ii) Z - Parameters interms of h-parameters

From h-parameters

$V_1 = h_{11}I_1 + h_{12}V_2$ —①

$I_2 = h_{21}I_1 + h_{22}V_2$ —②

From ②

$$V_2 = \frac{I_2 - h_{21}I_1}{h_{22}}$$

$$\boxed{V_2 = \frac{-h_{21}}{h_{22}}I_1 + \frac{1}{h_{22}}I_2}$$ —③

comparing ③ with z-parameter equation

$$V_2 = Z_{21}I_1 + Z_{22}I_2$$

we get $\boxed{Z_{21} = \frac{-h_{21}}{h_{22}} \; ; \; Z_{22} = \frac{1}{h_{22}}}$ —④

~~From~~ In ①, substitute ③

$$V_1 = h_{11}I_1 + h_{12}\left[\frac{-h_{21}}{h_{22}}I_1 + \frac{1}{h_{22}}I_2\right]$$

$$V_1 = h_{11}I_1 + \frac{h_{12}(-h_{21})I_1}{h_{22}} + \frac{h_{12}}{h_{22}}I_2$$

$$V_1 = \left[h_{11} - \frac{h_{12}h_{21}}{h_{22}}\right]I_1 + \frac{h_{12}}{h_{22}}I_2$$

$$V_1 = \left[\frac{h_{11}h_{22} - h_{12}h_{21}}{h_{22}}\right]I_1 + \frac{h_{12}}{h_{22}}I_2$$

$h_{11}h_{22} - h_{12}h_{21} = \Delta h$ determinant

$$\boxed{V_1 = \frac{\Delta h}{h_{22}}I_1 + \frac{h_{12}}{h_{22}}I_2}$$ —⑤

comparing ⑤ with z-parameter equation

$$V_1 = Z_{11}I_1 + Z_{12}I_2$$

we get $\boxed{Z_{11} = \frac{\Delta h}{h_{22}} \; ; \; Z_{12} = \frac{h_{12}}{h_{22}}}$ —⑥

(iii) Z - Parameters interms of Inverse hybrid (g) parameters

Z. interms of g can determine by taking Inverse of the h-parameters. i.e.

$$Z_{11} = \frac{\Delta h}{h_{22}} \; ; \; Z_{12} = \frac{h_{12}}{h_{22}}$$

$$Z_{21} = \frac{-h_{21}}{h_{22}} \; ; \; Z_{22} = \frac{1}{h_{22}}$$

take Inverse

$$\boxed{\begin{array}{l} Z_{11} = \frac{1}{g_{11}} \; ; \; Z_{12} = \frac{-g_{12}}{g_{11}} \\ Z_{21} = \frac{g_{21}}{g_{11}} \; ; \; Z_{22} = \frac{\Delta g}{g_{11}} \end{array}}$$

where $\Delta g = g_{11}g_{22} - g_{12}g_{21}$

$$\boxed{\begin{array}{l} \therefore Z_{11} = \frac{\Delta h}{h_{22}} , \; Z_{12} = \frac{h_{12}}{h_{22}} \\ Z_{21} = \frac{-h_{21}}{h_{22}} , \; Z_{22} = \frac{1}{h_{22}} \end{array}}$$

(iv) Z- Parameters interms of T-parameters (ABCD parameters)

In T- Parmeters we know that

$$V_1 = AV_2 - BI_2 \quad -①$$

$$I_1 = CV_2 - DI_2 - ②$$

From ② $$\boxed{V_2 = \frac{I_1 + DI_2}{C} = \frac{I_1}{C} + \frac{D}{C} I_2} -③$$

From Z-parameters equation $V_2 = Z_{21}I_1 + Z_{22}I_2$ -④

comparing ③ with ④ $\boxed{Z_{21} = \frac{1}{C} ; \; Z_{22} = \frac{D}{C}}$ -⑤

From ① substitute ③ in ①

$$V_1 = A\left[\frac{1}{C}I_1 + \frac{D}{C}I_2\right] - BI_2$$

$$V_1 = \frac{A}{C}I_1 + \frac{AD}{C}I_2 - BI_2$$

$$= \frac{A}{C}I_1 + \frac{AD}{C}[I_2] - BI_2$$

$$\boxed{V_1 = \frac{A}{C}I_1 + \left[\frac{AD}{C} - B\right]I_2} -⑥$$

From z-parameter equation $\boxed{V_1 = Z_{11}I_1 + Z_{12}I_2}$ -⑦

Comparing ⑥ with ⑦ $\boxed{Z_{11} = \frac{A}{C} ; \; Z_{12} = \frac{AD}{C} - B}$ -8

Therefore Z- Parameters interms of ABCD parameters are

$$\boxed{\begin{array}{ll} Z_{11} = \frac{A}{C} & Z_{12} = \frac{AD}{C} - B \\ Z_{21} = \frac{1}{C} & Z_{22} = \frac{D}{C} \end{array}}$$

(or) $$Z_{12} = \frac{AD - BC}{C} = \frac{\Delta T}{C}$$

$$\boxed{Z_{11} = \frac{A}{C} \; ; \; Z_{12} = \frac{\Delta T}{C} \; ; \; Z_{21} = \frac{1}{C} \; ; \; Z_{22} = \frac{D}{C}}$$

(v) Z-parameters interms of Inverse transmission parameters (or) (A'B'C'D' parameters)

Z-parameters interms of Transmission (ABCD) Parameters are

$$Z_{11} = \frac{A}{C} \; ; \; Z_{12} = \frac{AD-BC}{C} = \frac{\Delta T}{C} \; ; \; Z_{21} = \frac{1}{C} \; ; \; Z_{22} = \frac{D}{C}$$

A'B'C'D'
$$\boxed{Z_{11} = \frac{D'}{C'} \; ; \; Z_{12} = \frac{1}{C'} \; ; \; Z_{21} = \frac{A'D'-B'C'}{C'} = \frac{\Delta T'}{C'} \; ; \; Z_{22} = \frac{A'}{C'}}$$

(a) Y-Parameters interms of Z-parameters

$$Y = [Z]^{-1} \text{ i.e. } \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}^{-1} = \frac{1}{\Delta Z}\begin{bmatrix} Z_{22} & -Z_{12} \\ -Z_{21} & Z_{11} \end{bmatrix}$$

so
$$\boxed{Y_{11} = \frac{Z_{22}}{\Delta Z} \; ; \; Y_{12} = \frac{-Z_{12}}{\Delta Z} \; ; \; Y_{21} = \frac{-Z_{21}}{\Delta Z} \; ; \; Y_{22} = \frac{Z_{11}}{\Delta Z}}$$

where $\Delta Z = Z_{11}Z_{22} - Z_{12}Z_{21}$

(b) Y-parameters interms of h-parameters

h-parameters equations

$V_1 = h_{11}I_1 + h_{12}V_2$ —①

$I_2 = h_{21}I_1 + h_{22}V_2$ —②

From ① $I_1 = \frac{V_1 - h_{12}V_2}{h_{11}} = \left(\frac{1}{h_{11}}\right)V_1 + \left(\frac{-h_{12}}{h_{11}}\right)V_2$ —③

Y-parameter equation is $I_1 = Y_{11}V_1 + Y_{12}V_2$ —④

compare ③ & ④ $\boxed{Y_{11} = \frac{1}{h_{11}} \; ; \; Y_{12} = \frac{-h_{12}}{h_{11}}}$ —⑤

Substitute ③ in ②

$$I_2 = h_{21}\left[\left(\frac{1}{h_{11}}\right)V_1 + \left(\frac{-h_{12}}{h_{11}}\right)V_2\right] + h_{22}V_2$$

$$I_2 = \frac{h_{21}}{h_{11}} V_1 + \left(\frac{-h_{12}h_{21}}{h_{11}}\right) V_2 + h_{22} V_2$$

$$I_2 = \frac{h_{21}}{h_{11}} V_1 + \left(h_{22} - \frac{h_{12}h_{21}}{h_{11}}\right) V_2$$

$$I_2 = \frac{h_{21}}{h_{11}} V_1 + \left[\frac{h_{11}h_{22} - h_{12}h_{21}}{h_{11}}\right] V_2$$

$$I_2 = \frac{h_{21}}{h_{11}} V_1 + \frac{\Delta h}{h_{11}} V_2 \text{ — ⑥}$$

Y-Parameter equation is $I_2 = Y_{21}V_1 + Y_{22}V_2$ — ⑦

Compare ⑥ with ⑦

$$\boxed{Y_{21} = \frac{h_{21}}{h_{11}} \; ; \; Y_{22} = \frac{\Delta h}{h_{11}}} \text{ - ⑧}$$

Therefore Y-parameters interms of h-parameters are

$$\boxed{Y_{11} = \frac{1}{h_{11}} , \; Y_{12} = \frac{-h_{12}}{h_{11}} \; ; \; Y_{21} = \frac{h_{21}}{h_{11}} \; ; \; Y_{22} = \frac{\Delta h}{h_{11}}}$$

(c) Y-Parameters interms of g-parameters.

$$\boxed{Y_{11} = \frac{\Delta g}{g_{22}} , \; Y_{12} = \frac{g_{12}}{g_{22}} , \; Y_{21} = \frac{-g_{21}}{g_{22}} \; ; \; Y_{22} = \frac{1}{g_{22}}}$$

(d) Y-parameters interms of Transmission (ABCD) parameters

T-parameters equations

$V_1 = AV_2 - BI_2$ —①

$I_1 = CV_2 - DI_2$ —②

From ① $I_2 = \frac{AV_2 - V_1}{B}$

$$I_2 = -\frac{1}{B} V_1 + \frac{A}{B} V_2 \text{ —③}$$

Y-parameter equation

$I_2 = Y_{21}V_1 + Y_{22}V_2$ —④

compare ④ with ③

$$\boxed{Y_{21} = -\frac{1}{B} , \; Y_{22} = \frac{A}{B}} \text{ —⑤}$$

substitute ③ in ②

$$I_1 = CV_2 - D\left[\frac{-1}{B}V_1 + \frac{A}{B}V_2\right]$$

$$I_1 = CV_2 + \frac{D}{B}V_1 - \frac{AD}{B}V_2$$

$$I_1 = \frac{D}{B}V_1 + \left(C - \frac{AD}{B}\right)V_2$$

$$I_1 = \frac{D}{B}V_1 + \left(\frac{CB - AD}{B}\right)V_2$$

$$I_1 = \frac{D}{B}V_1 + \left(\frac{-\Delta T}{B}\right)V_2 \text{ —⑥}$$

Y-parameter equation is

$I_1 = Y_{11}V_1 + Y_{12}V_2$ —⑦

Compare ⑥ with ⑦

$$\boxed{Y_{11} = \frac{D}{B} , \; Y_{12} = \frac{-\Delta T}{B}}$$

Therefore Y-Parameters in T parameter

$$\boxed{Y_{11} = \frac{D}{B} \; ; \; Y_{12} = \frac{-\Delta T}{B} ; \; Y_{21} = -\frac{1}{B} , \; Y_{22} = \frac{A}{B}}$$

(e) Y-parameters interms of Inverse transmission ($A'B'C'D'$)

$$\boxed{Y_{11} = \frac{A'}{B'} , \ Y_{12} = -\frac{1}{B'} ; \ Y_{21} = \frac{-\Delta T'}{B'} ; \ Y_{22} = \frac{D'}{B'}}$$

III h-parameters in terms of ~~other~~ z-parameters

we know that z-parameters,

$$V_1 = Z_{11}I_1 + Z_{12}I_2 \quad ①$$
$$V_2 = Z_{21}I_1 + Z_{22}I_2 \quad ②$$

From ② $I_2 = \frac{V_2 - Z_{21}I_1}{Z_{22}}$

$$I_2 = \frac{V_2}{Z_{22}} - \frac{Z_{21}}{Z_{22}} I_1$$

$$I_2 = \frac{-Z_{21}}{Z_{22}} I_1 + \frac{1}{Z_{22}} V_2 \quad ③$$

Compare ③ with h-parameter eqn.

$I_2 = h_{21}I_1 + h_{22}V_2$, we get

$$\boxed{h_{21} = \frac{-Z_{21}}{Z_{22}} ; \ h_{22} = \frac{1}{Z_{22}}} \quad ④$$

substitute ③ in ①

$$V_1 = Z_{11}I_1 + Z_{12}\left[\frac{-Z_{21}}{Z_{22}} I_1 + \frac{1}{Z_{22}} V_2\right]$$

$$V_1 = Z_{11}I_1 - \frac{Z_{12}Z_{21}}{Z_{22}} I_1 + \frac{Z_{12}}{Z_{22}} V_2$$

$$V_1 = \left[Z_{11} - \frac{Z_{12}Z_{21}}{Z_{22}}\right] I_1 + \frac{Z_{12}}{Z_{22}} V_2$$

$$V_1 = \left[\frac{Z_{11}Z_{22} - Z_{12}Z_{21}}{Z_{22}}\right] I_1 + \frac{Z_{12}}{Z_{22}} V_2 \quad ⑤$$

Compare ⑤ with h-parameter eqn.

$V_1 = h_{11}I_1 + h_{12}V_2$

$$\boxed{h_{11} = \frac{\Delta Z}{Z_{22}} ; \ h_{12} = \frac{Z_{12}}{Z_{22}}} \quad ⑥$$

~~Substi~~

(ii) h-parameters interms of Y-Parameters

From Y-parameters $I_1 = Y_{11}V_1 + Y_{12}V_2 \quad ①$

$$I_2 = Y_{21}V_1 + Y_{22}V_2 \quad ②$$

From ① $V_1 = \frac{I_1 - Y_{12}V_2}{Y_{11}}$

$$V_1 = \frac{I_1}{Y_{11}} - \frac{Y_{12}}{Y_{11}} V_2 \quad ③$$

compare ③ with h-parameter eqn.

$V_1 = h_{11}I_1 + h_{12}V_2$

$$\boxed{h_{11} = \frac{1}{Y_{11}} ; \ h_{12} = \frac{-Y_{12}}{Y_{11}}} \quad ④$$

substitute ③ in ②

$$I_2 = Y_{21}\left[\frac{I_1}{Y_{11}} - \frac{Y_{12}}{Y_{11}} V_2\right] + Y_{22}V_2$$

$$I_2 = \frac{Y_{21}}{Y_{11}} I_1 - \frac{Y_{21}Y_{12}}{Y_{11}} V_2 + Y_{22}V_2$$

$$I_2 = \frac{Y_{21}}{Y_{11}} I_1 + \left[Y_{22} - \frac{Y_{12}Y_{21}}{Y_{11}}\right] V_2$$

$$I_2 = \frac{Y_{21}}{Y_{11}} I_1 + \left[\frac{Y_{11}Y_{22} - Y_{12}Y_{21}}{Y_{11}}\right] V_2$$

$$I_2 = \frac{Y_{21}}{Y_{11}} I_1 + \frac{\Delta Y}{Y_{11}} V_2 \quad ⑤$$

compare ⑤ with h-parameter eqn

$I_2 = h_{21}I_1 + h_{22}V_2$

$$\boxed{h_{21} = \frac{Y_{21}}{Y_{11}} ; \ h_{22} = \frac{\Delta Y}{Y_{11}}} \quad ⑥$$

(iii) h- Parameters interms of T parameters (ABCD parameters)

T- Parameters - $V_1 = AV_2 - BI_2$ —①

$I_1 = CV_2 - DI_2$ —②

From ② $I_2 = \frac{CV_2 - I_1}{D}$

$I_2 = -\frac{1}{D}I_1 + \frac{C}{D}V_2$ —③

compare with h-parameter eqn

$I_2 = h_{21}I_1 + h_{22}V_2$

$$\boxed{h_{21} = -\frac{1}{D}, \; h_{22} = \frac{C}{D}} \text{ —④}$$

Substitute ③ in ①

$V_1 = AV_2 - B\left[-\frac{1}{D}I_1 + \frac{C}{D}V_2\right]$

$= AV_2 + \frac{B}{D}I_1 - \frac{BC}{D}V_2$

$= \frac{B}{D}I_1 + \left(A - \frac{BC}{D}\right)V_2$

$= \frac{B}{D}I_1 + \left(\frac{AD-BC}{D}\right)V_2$

$V_1 = \frac{B}{D}I_1 + \frac{\Delta T}{D}V_2$ —⑤

Compare ⑤ with h-parameter eqn.

$V_1 = h_{11}I_1 + h_{12}V_2$

$$\boxed{h_{11} = \frac{B}{D} \; ; \; h_{12} = \frac{\Delta T}{D}} \text{ —⑥}$$

(iv) h- Parameter in terms of T' parameters (A'B'C'D' Parameters)

$V_2 = A'V_1 - B'I_1$ —①

$I_2 = C'V_1 - D'I_1$ —②

From ① $V_1 = \frac{V_2 - B'I_1}{A'}$

$V_1 = -\frac{B'}{A'}I_1 + \frac{V_2}{A'}$ —③

Compare③ with h-parameter eqn

$V_1 = h_{11}I_1 + h_{12}V_2$, we get

$$\boxed{h_{11} = -\frac{B'}{A'} \; , \; h_{12} = \frac{1}{A'}} \text{ —④}$$

Substitute ③ in ②

$I_2 = C'\left[\frac{-B'}{A'}I_1 + \frac{V_2}{A'}\right] - D'I_1$

$I_2 = -\frac{C'B'}{A'}I_1 + \frac{C'}{A'}V_2 - D'I_1$

$I_2 = \left[-\frac{B'C'}{A'} - D'\right]I_1 + \frac{C'}{A'}V_2$

$I_2 = \left[\frac{-B'C' - A'D'}{A'}\right]I_1 + \frac{C'}{A'}V_2$

$I_2 = \frac{-\Delta T'}{A'}I_1 + \frac{C'}{A'}V_2$ —⑤

Compare⑤ with h-parameter eqn

$I_2 = h_{21}I_1 + h_{22}V_2$

$$\boxed{h_{21} = \frac{-\Delta T'}{A'} \; : \; h_{22} = \frac{C'}{A'}} \text{ —⑥}$$

(v) h-parameters interms of g- parameters

$h = [g]^{-1}$

$$\boxed{h_{11} = \frac{g_{22}}{\Delta g} \; , \; h_{12} = \frac{-g_{12}}{\Delta g} \; , \; h_{21} = \frac{-g_{21}}{\Delta g} \; : \; h_{22} = \frac{g_{11}}{\Delta g}}$$

IV (i) T- Parameters interms of Z- parameters

Z- Parameters equations

$V_1 = Z_{11}I_1 + Z_{12}I_2$ -①

$V_2 = Z_{21}I_1 + Z_{22}I_2$ -②

From ②

$$I_1 = \frac{V_2 - I_2Z_{22}}{Z_{21}}$$

$I_1 = \frac{1}{Z_{21}}V_2 - \frac{Z_{22}}{Z_{21}}I_2$ -③

comparing ③ with T- parameter eqn

$$I_1 = CV_2 - DI_2$$

$\boxed{C = \frac{1}{Z_{21}} \ ; \ D = \frac{Z_{22}}{Z_{21}}}$ -④

substitute ③ in ①

$$V_1 = Z_{11}\left[\frac{1}{Z_{21}}V_2 - \frac{Z_{22}}{Z_{21}}I_2\right] + Z_{12}I_2$$

$$V_1 = \frac{Z_{11}}{Z_{21}}V_2 - \frac{Z_{11}Z_{22}}{Z_{21}}I_2 + Z_{12}I_2$$

$$V_1 = \frac{Z_{11}}{Z_{21}}V_2 + \left[-Z_{12} + \frac{Z_{11}Z_{22}}{Z_{21}}\right]I_2$$

$$V_1 = \frac{Z_{11}}{Z_{21}}V_2 + \left[\frac{-Z_{12}Z_{21} + Z_{11}Z_{22}}{Z_{21}}\right]I_2$$

$V_1 = \frac{Z_{11}}{Z_{21}}V_2 + \frac{\Delta Z}{Z_{21}}I_2$ — ⑤

compare ⑤ with T- Parameter eqn

$$V_1 = AV_2 - BI_2$$

$\boxed{A = \frac{Z_{11}}{Z_{21}} \ ; \ B = \frac{\Delta Z}{Z_{21}}}$ -⑥

ii) T- Parameters interms of Y- parameters

y- parameters

$I_1 = Y_{11}V_1 + Y_{12}V_2$ -①

$I_2 = Y_{21}V_1 + Y_{22}V_2$ -②

From ② $V_1 = \frac{I_2 - Y_{22}V_2}{Y_{21}}$

$$V_1 = \frac{1}{Y_{21}}I_2 - \frac{Y_{22}}{Y_{21}}V_2$$

$V_1 = \frac{-Y_{22}}{Y_{21}}V_2 + \frac{1}{Y_{21}}I_2$ -③

compare ③ with T- parameter

$$V_1 = AV_2 - BI_2$$

$\boxed{A = \frac{-Y_{22}}{Y_{21}} \ ; \ B = \frac{-1}{Y_{21}}}$ -④

substitute ③ in ①

$$I_1 = Y_{11}\left[\frac{-Y_{22}}{Y_{21}}V_2 + \frac{1}{Y_{21}}I_2\right] + Y_{12}V_2$$

$$I_1 = -\frac{Y_{11}Y_{22}V_2}{Y_{21}} + \frac{Y_{11}}{Y_{21}}I_2 + Y_{12}V_2$$

$$I_1 = \left[Y_{12} - \frac{Y_{11}Y_{22}}{Y_{21}}\right]V_2 + \frac{Y_{11}}{Y_{21}}I_2$$

$I_1 = \left[\frac{Y_{12}Y_{21} - Y_{11}Y_{22}}{Y_{21}}\right]V_2 + \frac{Y_{11}}{Y_{21}}I_2$ -⑤

compare ⑤ with T parameter eqn

$$I_1 = CV_2 - DI_2$$

$\boxed{C = -\frac{\Delta Y}{Y_{21}} \ . \ D = \frac{-Y_{11}}{Y_{21}}}$ -⑥

iii) T- Parameters in terms of h-parameters.

h-parameters

$V_1 = h_{11}I_1 + h_{12}V_2$ — ①

$I_2 = h_{21}I_1 + h_{22}V_2$ — ②

From ②

$$I_1 = \frac{I_2 - h_{22}V_2}{h_{21}}$$

$$I_1 = \frac{1}{h_{21}}I_2 - \frac{h_{22}}{h_{21}}V_2$$

$$I_1 = \frac{-h_{22}}{h_{21}}V_2 + \frac{1}{h_{21}}I_2 \quad ③$$

compare ③ with T-parameter eqn

$$I_1 = CV_2 - DI_2$$

$$\boxed{C = -\frac{h_{22}}{h_{21}}, \quad D = \frac{-1}{h_{21}}} \quad ④$$

Substitute ③ in ①

$$V_1 = h_{11}\left[\frac{-h_{22}}{h_{21}}V_2 + \frac{1}{h_{21}}I_2\right] + h_{12}V_2$$

$$V_1 = -\frac{h_{11}h_{22}}{h_{21}}V_2 + \frac{h_{11}}{h_{21}}I_2 + h_{12}V_2$$

$$V_1 = \left[-\frac{h_{11}h_{22}}{h_{21}} + h_{12}\right]V_2 + \frac{h_{11}}{h_{21}}I_2$$

$$V_1 = \left[\frac{h_{12}h_{21} - h_{11}h_{22}}{h_{21}}\right]V_2 + \frac{h_{11}}{h_{21}}I_2$$

$$V_1 = \frac{-\Delta h}{h_{21}}V_2 + \frac{h_{11}}{h_{21}}I_2 \quad ⑤$$

compare ⑤ with T-parameter equation

$$V_1 = AV_2 - BI_2$$

$$\boxed{A = \frac{-\Delta h}{h_{21}}, \quad B = \frac{-h_{11}}{h_{21}}} \quad ⑥$$

(iv) T-parameters in terms of T' parameters

$$T = [T']^{-1}$$

$$\boxed{A = \frac{D'}{\Delta T'}, \quad B = \frac{B'}{\Delta T'} \quad C = \frac{C'}{\Delta T'} \quad D = \frac{A'}{\Delta T'}}$$

(v) T- parameters interms of g-parameters

$$\boxed{A = \frac{1}{g_{21}} \quad B = \frac{g_{22}}{g_{21}} \quad C = \frac{g_{11}}{g_{21}} \quad D = \frac{\Delta g}{g_{21}}}$$

V T' parameters in terms of Z-parameters

(i)

$$\boxed{A' = \frac{Z_{22}}{Z_{12}}, \quad B' = \frac{\Delta Z}{Z_{12}}, \quad C' = \frac{1}{Z_{12}}, \quad D' = \frac{Z_{11}}{Z_{12}}}$$

ii) T'- interms of Y-parameters

$$\boxed{A' = \frac{-Y_{11}}{Y_{12}}, \quad B' = \frac{-1}{Y_{12}}, \quad C' = \frac{-\Delta Y}{Y_{12}} \quad D' = \frac{-Y_{22}}{Y_{12}}}$$

(iii) T'- interms of h-parameters

$$A' = \frac{1}{h_{12}}, \quad B' = \frac{h_{11}}{h_{12}}, \quad C' = \frac{h_{22}}{h_{12}} \ \& \ D' = \frac{\Delta h}{h_{12}}$$

(iv) T'- interms of T-parameters:

$$A' = \frac{D}{\Delta T}, \quad B' = \frac{B}{\Delta T}, \quad C' = \frac{C}{\Delta T} \quad D' = \frac{A}{\Delta T}$$

(v) T'- parameters ~~of~~ interms of g parameters

$$A' = \frac{-\Delta g}{g_{12}} \quad B' = \frac{-g_{22}}{g_{12}}, \quad C' = \frac{-g_{11}}{g_{12}} \quad D' = -\frac{1}{g_{12}}$$

VI g-parameters interms of (a) Z-parameters

$$g_{11} = \frac{1}{Z_{11}}, \quad g_{12} = \frac{-Z_{12}}{Z_{11}}, \quad g_{21} = \frac{Z_{21}}{Z_{11}}, \quad g_{22} = \frac{\Delta Z}{Z_{11}}$$

(b) Y-parameters $g_{11} = \frac{\Delta Y}{Y_{22}}, \quad g_{12} = \frac{Y_{12}}{Y_{22}}, \quad g_{21} = \frac{-Y_{21}}{Y_{22}}, \quad g_{22} = \frac{1}{Y_{22}}$

(c) h-parameters: $g_{11} = \frac{h_{22}}{\Delta h}, \quad g_{12} = \frac{-h_{12}}{\Delta h}, \quad g_{21} = \frac{-h_{21}}{\Delta h}, \quad g_{22} = \frac{h_{11}}{\Delta h}$

(d) T-parameters: $g_{11} = \frac{C}{A}, \quad g_{12} = \frac{-\Delta T}{A}, \quad g_{21} = \frac{1}{A}, \quad g_{22} = \frac{B}{A}$

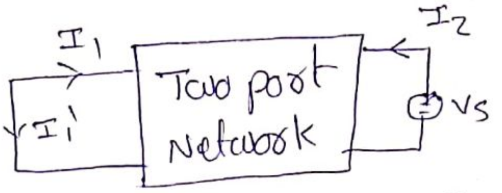
(e) T'-parameters:

$$g_{11} = \frac{C'}{D'}, \quad g_{12} = -\frac{1}{D'}, \quad g_{21} = \frac{\Delta T'}{D'}, \quad g_{22} = \frac{B'}{D'}$$

Condition for Reciprocity in two-port Parameter Representation

A two port network is said to be reciprocal, if the ratio of the response variable to the excitation variable remains identical even if the positions of the response & excitation in the N/w are interchanged.

fig(a) $V_1 = V_s$, $I_1 = I_1$, $V_2 = 0$, $I_2 = -I_2'$



fig(b) $V_2 = V_s$, $I_2 = I_2$, $V_1 = 0$, $I_1 = -I_1'$

$$\frac{V_s}{I_2'} = \frac{V_s}{I_1'} \quad (\text{or}) \quad I_2' = I_1'$$

NOTE: I_2' & I_1' are assumed to be in reverse direction of I_2 & I_1

(a) Reciprocity for Z-parameters

Z-Parameters
$$V_1 = Z_{11}I_1 + Z_{12}I_2 \quad -①$$
$$V_2 = Z_{21}I_1 + Z_{22}I_2 \quad -②$$

From fig (a) $V_1 = V_s$, $I_1 = I_1$, $V_2 = 0$, $I_2 = -I_2'$

substitute above values in ① & ②

$$V_s = Z_{11}I_1 + Z_{12}(-I_2') - ③$$

$$0 = Z_{21}I_1 - Z_{22}I_2' - ④$$

From ④ $I_1 = \frac{Z_{22}I_2'}{Z_{21}} - ⑤$

put ⑤ in ③

$$V_s = Z_{11}\frac{Z_{22}I_2'}{Z_{21}} - Z_{12}I_2'$$

$$V_s = I_2'\left[\frac{Z_{11}Z_{22}}{Z_{21}} - Z_{12}\right]$$

$$V_s = I_2'\left[\frac{Z_{11}Z_{22} - Z_{12}Z_{21}}{Z_{21}}\right]$$

$$\boxed{\therefore I_2' = \frac{V_s Z_{21}}{Z_{11}Z_{22} - Z_{12}Z_{21}}} - ⑥$$

From fig (b) $V_2 = V_s$, $I_2 = I_2$, $V_1 = 0$, $I_1 = -I_1'$

substitute above values in ① & ②

$$0 = -Z_{11}I_1' + Z_{12}I_2 \quad - ⑦$$

$$V_s = -Z_{21}I_1' + Z_{22}I_2 \quad - ⑧$$

From ⑦ $I_2 = \frac{Z_{11}I_1'}{Z_{12}} - ⑨$

Put ⑨ in ⑧

$$V_s = -Z_{21}I_1' + Z_{22}\frac{Z_{11}I_1'}{Z_{12}}$$

$$V_s = I_1'\left[\frac{Z_{22}Z_{11}}{Z_{12}} - Z_{21}\right]$$

$$V_s = I_1'\left[\frac{Z_{11}Z_{22} - Z_{21}Z_{12}}{Z_{12}}\right]$$

$$\boxed{I_1' = \frac{V_s Z_{12}}{Z_{11}Z_{22} - Z_{12}Z_{21}}} - ⑩$$

For Reciprocity $I_2' = I_1'$

Hence condition is

$$\boxed{Z_{21} = Z_{12}}$$

(b) Reciprocity condition for Y-Parameters

Y- Parameters

~~$I_1 = Y$~~ $I_1 = Y_{11} V_1 + Y_{12} Y_2$ — ①

$I_2 = Y_{21} V_1 + Y_{22} V_2$ — ②

From fig(a) $V_1 = V_S,\ I_1 = I_1,\ V_2 = 0,\ I_2 = -I_2'$

using above values for ① or ②

$I_1 = Y_{11} V_S + 0$

$-I_2' = Y_{21} V_S + 0$

$\boxed{I_2' = -Y_{21} V_S}$ — ③

From fig(b)

$V_2 = V_S,\ I_2 = I_2,\ V_1 = 0,\ I_1 = -I_1'$

using above values for ① or ②

$-I_1' = Y_{11}(\times 0) + Y_{12} V_S$

$I_2 = Y_{21} \times 0 + Y_{22} V_S$

$\boxed{I_1' = -Y_{12} V_S}$ — ④

For Reciprocity $I_2' = I_1'$

Hence condition for Reciprocity is $\boxed{Y_{12} = Y_{21}}$

(c) Reciprocity condition for h-parameters

h-Parameters

$V_1 = h_{11} I_1 + h_{12} V_2$ — ①

$I_2 = h_{21} I_1 + h_{22} V_2$ — ②

from fig(a) $V_1 = V_S,\ I_1 = I_1,\ V_2 = 0,\ I_2 = -I_2'$

using above values for ① or ②

$V_S = h_{11} I_1 + 0,\quad I_1 = \frac{V_S}{h_{11}}$ — ③

$-I_2' = h_{21} I_1 + 0,\quad I_2' = -h_{21} I_1$ — ④

put ③ in ④ $I_2' = -\frac{h_{21} V_S}{h_{11}}$

$\boxed{I_2' = -\frac{h_{21} V_S}{h_{11}}}$ — ⑤

From fig(b)

$V_2 = V_S,\ I_2 = I_2,\ V_1 = 0,\ I_1 = -I_1'$

using above values for ① or ②

$0 = -h_{11} I_1' + h_{12} V_S$ — ⑥

$I_2 = -h_{21} I_1' + h_{22} V_S$ — ⑦

From ⑥ $\boxed{I_1' = \frac{h_{12} V_S}{h_{11}}}$ — ⑧

For Reciprocity $I_2' = I_1'$

condition is $\boxed{h_{12} = -h_{21}}$

(d) Reciprocity condition for T- parameters

$V_1 = A V_2 - B I_2$ — ①

$I_1 = C V_2 - D I_2$ — ②

From fig(a) $V_1 = V_S,\ I_1 = I_1,\ V_2 = 0,\ I_2 = -I_2'$

using above values for ① or ②

$V_S = + B I_2'$

$I_1 = D I_2'$

$\boxed{I_2' = \frac{V_S}{B}}$ — ③

From fig(b) $V_2 = V_S,\ I_2 = I_2,\ V_1 = 0,\ I_1 = -I_1'$

using above values for ① or ② $0 = A V_S - B I_2$ — ④

$-I_1' = C V_S - D I_2$ — ⑤

From ④ $I_2 = \frac{A V_S}{B}$ — ⑥, substitute in ⑤

$-I_1' = C V_S - D \frac{A V_S}{B} = -I_1' = \frac{B C V_S - A D V_S}{B}$

$\boxed{I_1' = \left(\frac{AD - BC}{B}\right) V_S}$ — ⑦

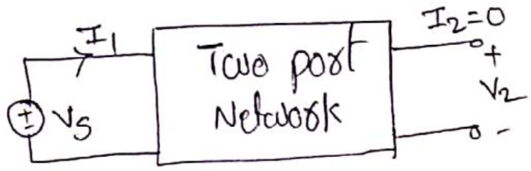
For Reciprocity $I_2' = I_1'$

$\frac{AD - BC}{\cancel{B}} = \frac{1}{\cancel{B}}$ $\boxed{AD - BC = 1 \text{ or } \Delta T = 1 \quad \begin{vmatrix} A & B \\ C & D \end{vmatrix} = 1}$

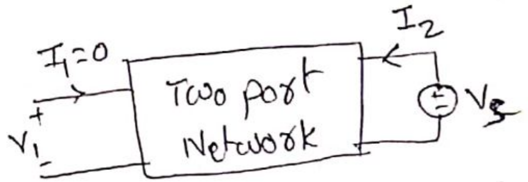
Condition for symmetry

A two port network is said to be symmetrical if the ports can be interchanged without changing the port voltage & currents.

$$\frac{V_s}{I_1}\Big|_{I_2=0} = \frac{V_s}{I_2}\Big|_{I_1=0}$$



Fig(a) - $V_1=V_s, I_1=I_1, I_2=0, V_2=V_2$



fig(b) $V_1=V_1, I_1=0, I_2=I_2, V_2=V_s$

(i) Symmetry condition for z-transform

From fig (a) $V_1=V_s, I_1=I_1, I_2=0, V_2=V_2$ - ①

we know that Z- Parameter equations.

$$V_1 = Z_{11}I_1 + Z_{12}I_2 \quad - ②$$

substitute ① in ②

$$V_s = Z_{11}I_1 + 0$$

$$\boxed{\frac{V_s}{I_1}\Big|_{I_2=0} = Z_{11}} \quad - ③$$

From fig (b) $V_1=V_1, I_1=0, I_2=I_2, V_2=V_s$ ④

we know that Z-Parameter equation

$$V_2 = Z_{12}I_1 + Z_{22}I_2 \quad - ⑤$$

substitute ④ in ⑤

$$V_s = Z_{22}I_2$$

$$\boxed{\frac{V_s}{I_2}\Big|_{I_1=0} = Z_{22}} \longleftarrow ⑥$$

For ~~Reciprocity~~ symmetry condition $\frac{V_s}{I_1}\Big|_{I_2=0} = \frac{V_s}{I_2}\Big|_{I_1=0}$

Hence from ③ & ⑥ ~~Reciprocity~~ Symmetry condition is $\boxed{Z_{11}=Z_{22}}$

(b) Symmetry condition for Y-Parameters

From fig (a) $V_1 = V_s, V_2 = V_2, I_1 = I_1, I_2 = 0$ —①

we know Y-Parameters $I_1 = Y_{11}V_1 + Y_{12}V_2$ —②

$I_2 = Y_{21}V_1 + Y_{22}V_2$ —③

$\therefore I_1 = Y_{11}V_s + Y_{12}V_2$ —④

$0 = Y_{21}V_s + Y_{22}V_2 \Rightarrow V_2 = \frac{-Y_{21}V_s}{Y_{22}}$ —⑤

put ⑤ in ④ $I_1 = Y_{11}V_s + Y_{12}\left(\frac{-Y_{21}V_s}{Y_{22}}\right)$

$$I_1 = Y_{11}V_s + \frac{Y_{12}Y_{21}V_s}{Y_{22}} = \left[Y_{11} - \frac{Y_{12}Y_{21}}{Y_{22}}\right]V_s$$

$$I_1 = \left[\frac{Y_{11}Y_{22}-Y_{12}Y_{21}}{Y_{22}}\right]V_s \quad \therefore \boxed{\frac{V_s}{I_1} = \frac{Y_{22}}{Y_{11}Y_{22}-Y_{12}Y_{21}}} \text{ —⑥}$$

From fig(b) $V_1 = V_1, V_2 = V_s, I_1 = 0, I_2 = I_2$ —⑦

from ② & ③ $0 = Y_{11}V_1 + Y_{12}V_s \Rightarrow \boxed{V_1 = \frac{-Y_{12}V_s}{Y_{11}}}$ —⑧

$I_2 = Y_{21}V_1 + Y_{22}V_s$ —⑨

put ⑧ in ⑨

$$I_2 = Y_{21}\left(\frac{-Y_{12}V_s}{Y_{11}}\right) + Y_{22}V_s$$

$$I_2 = -\frac{Y_{12}Y_{21}V_s}{Y_{11}} + Y_{22}V_s$$

$$= \left[\frac{-Y_{12}Y_{21}}{Y_{11}} + Y_{22}\right]V_s$$

$$I_2 = V_s\left[\frac{Y_{11}Y_{22}-Y_{12}Y_{21}}{Y_{11}}\right]$$

$$\boxed{\frac{V_s}{I_2} = \frac{Y_{11}}{Y_{11}Y_{22}-Y_{12}Y_{21}}} \text{ —⑩}$$

For symmetry $\frac{V_s}{I_1} = \frac{V_s}{I_2} \Rightarrow$ from ⑥ & ⑩

$$\frac{Y_{22}}{\cancel{Y_{11}Y_{22}-Y_{12}Y_{21}}} = \frac{Y_{11}}{\cancel{Y_{11}Y_{22}-Y_{12}Y_{21}}} \qquad \boxed{Y_{11} = Y_{22}}$$

(c) Symmetroy condition for h-Parameters

From fig(a) $V_1 = V_s,\ V_2 = V_2,\ I_1 = I_1,\ I_2 = 0$ —①

we know h-parameters $V_1 = h_{11}I_1 + h_{12}V_2$ —②

$I_2 = h_{21}I_1 + h_{22}V_2$ —③

put ① in ② & ③

$V_s = h_{11}I_1 + h_{12}V_2$ — ④

$0 = h_{21}I_1 + h_{22}V_2 \Rightarrow \boxed{V_2 = \frac{-h_{21}I_1}{h_{22}}}$ —⑤

put ⑤ in ④ $V_s = h_{11}I_1 - \frac{h_{12}h_{21}I_1}{h_{22}}$

$$V_s = \left[h_{11} - \frac{h_{12}h_{21}}{h_{22}}\right]I_1 = \left[\frac{h_{11}h_{22} - h_{12}h_{21}}{h_{22}}\right]I_1$$

$$\boxed{\frac{V_s}{I_1} = \frac{h_{11}h_{22} - h_{12}h_{21}}{h_{22}}} \text{ —⑥}$$

From fig(b) $V_1 = V_1,\ V_2 = V_s,\ I_1 = 0,\ I_2 = I_2$ —⑦

h-parameters $V_1 = h_{11}I_1 + h_{12}V_2$ —⑧

$I_2 = h_{21}I_1 + h_{22}V_2$ —⑨

put ⑦ in ⑧ & ⑨ $V_1 = h_{12}V_s$

$I_2 = h_{22}V_s \Rightarrow \boxed{\frac{V_s}{I_2} = \frac{1}{h_{22}}}$ —⑩

For symmetroy condition $\frac{V_s}{I_1} = \frac{V_s}{I_2}$

Hence from ⑥ & ⑩ $\frac{h_{11}h_{22} - h_{12}h_{21}}{\cancel{h_{22}}} = \frac{1}{\cancel{h_{22}}}$

$$\boxed{h_{11}h_{22} - h_{12}h_{21} = 1} \qquad \boxed{\begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} = 1 \text{ (or) } \Delta h = 1}$$

(d) Symmetry condition for T - Parameters

From fig(a) $V_1 = V_S,\ I_1 = I_1,\ I_2 = 0,\ V_2 = V_2$ — ①

we know that $V_1 = AV_2 - BI_2$ — ②

$I_1 = CV_2 - DI_2$ — ~~④~~③

put ① in ② & ③

$V_S = AV_2$ — ④

$I_1 = CV_2 \Rightarrow V_2 = \frac{I_1}{C}$ — ~~④~~⑤

put ⑤ in ④ $V_S = A\frac{I_1}{C} \Rightarrow \boxed{\frac{V_S}{I_1} = \frac{A}{C}}$ — ⑥

From fig(b) $V_1 = V_1,\ I_1 = 0,\ I_2 = I_2,\ V_2 = V_S$ — ⑦

$V_1 = AV_2 - BI_2$

$I_1 = CV_2 - DI_2$

put ⑦ in above two equations

$V_1 = AV_S - BI_2$

$0 = CV_S - DI_2 \Rightarrow I_2 = \frac{CV_S}{D}$ $\boxed{\frac{V_S}{I_2} = \frac{D}{C}}$ — ⑧

For symmetry $\frac{V_S}{I_1} = \frac{V_S}{I_2}$

From ⑥ & ⑧ $\frac{A}{\cancel{C}} = \frac{D}{\cancel{C}}$; condition for symmetry is $\boxed{A = D}$

(e) Symmetry condition for T' parameters

Condition of symmetry in case of T'-parameters is similar to as in case of T-Parameters i.e. $\boxed{A' = D'}$

(f) Symmetry condition for g parameters

Condition of symmetry in case of g-parameters is similar to as in case of h-parameters i.e.

$$\boxed{g_{11}g_{22} - g_{12}g_{21} = 1 \quad (\text{or}) \quad \begin{vmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{vmatrix} = 1 \quad (\text{or}) \quad \Delta g = 1}$$

Parameter	condition for Reciprocity	condition for symmetry.
$[Z]$	$Z_{12} = Z_{21}$	$Z_{11} = Z_{22}$
$[Y]$	$Y_{12} = Y_{21}$	$Y_{11} = Y_{22}$
$[T]$ (or) $[A\,B\,C\,D]$	$AD - BC = 1$	$A = D$
$[T']$ (or) $[A'B'C'D']$	$A'D' - B'C' = 1$	$A' = D'$
$[h]$	$h_{12} = -h_{21}$	$h_{11}h_{22} - h_{12}h_{21} = 1$
$[g]$	$g_{12} = -g_{21}$	$g_{11}g_{22} - g_{12}g_{21} = 1$

2.12

Interconnection of two port Networks

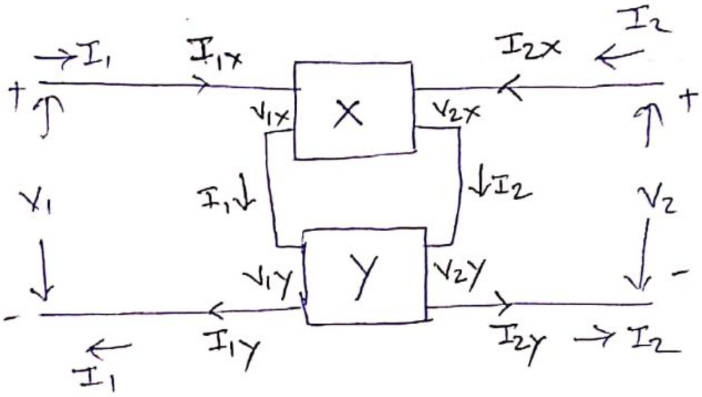
→ When two port networks are connected in cascade, the parameters of the interconnected network can be conveniently expressed with the help of ABCD parameters.

→ The Z-parameters can be used to describe the parameters of series connected two port Networks.

→ The Y-Parameters can be used to describe the parameters of Parallel connected two port networks.

(a) Series connection of two-port Network

If each port has a common reference node for its input and output, and if these references are connected together then the equations of the networks X and Y interms of 'Z' are



$V_{1x} = Z_{11x} I_{1x} + Z_{12x} I_{2x}$ — ①

$V_{2x} = Z_{21x} I_{1x} + Z_{22x} I_{2x}$ — ②

$V_{1y} = Z_{11y} I_{1y} + Z_{12y} I_{2y}$ — ③

$V_{2y} = Z_{21y} I_{1y} + Z_{22y} I_{2y}$ — ④

From the inter-connection of the networks,

$I_1 = I_{1x} = I_{1y}$; $I_2 = I_{2x} = I_{2y}$ — ⑤ in series current is same

$V_1 = V_{1x} + V_{1y}$; $V_2 = V_{2x} + V_{2y}$, then

From ①, ②, ③ & ④

$V_1 = Z_{11x} I_{1x} + Z_{12x} I_{2x} + Z_{11y} I_{1y} + Z_{12y} I_{2y}$

From ⑤.

$V_1 = Z_{11x} I_1 + Z_{12x} I_2 + Z_{11y} I_1 + Z_{12y} I_2$

$V_1 = (Z_{11x} + Z_{11y}) I_1 + (Z_{12x} + Z_{12y}) I_2$ — ⑥

lly $V_2 = (Z_{21x} + Z_{21y}) I_1 + (Z_{22x} + Z_{22y}) I_2$ — ⑦

we know that Z-Parameters

$V_1 = Z_{11} I_1 + Z_{12} I_2$

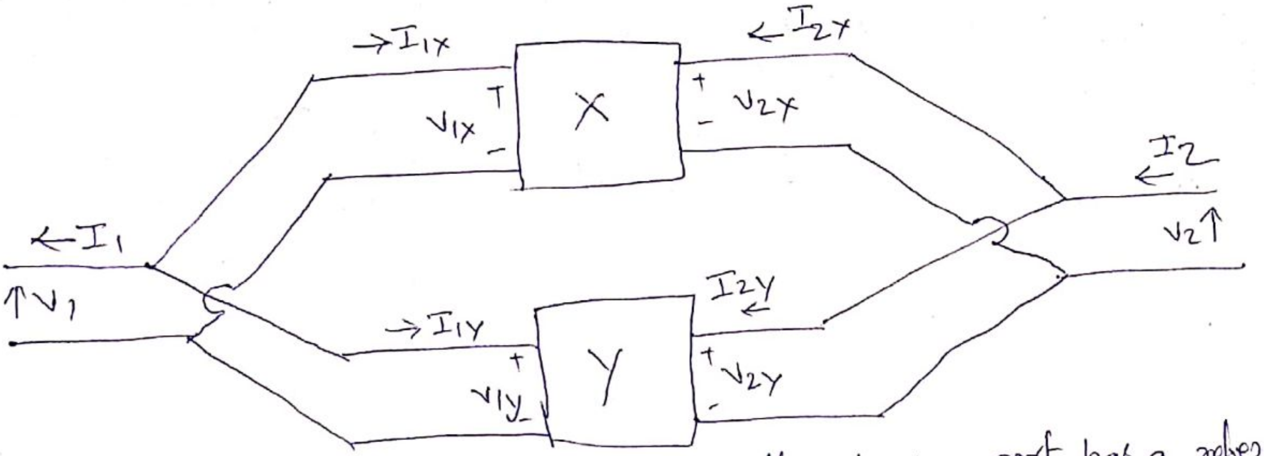
$V_2 = Z_{21} I_1 + Z_{22} I_2$

From ⑥ & ⑦

$$\boxed{\begin{aligned} Z_{11} &= Z_{11x} + Z_{11y} \\ Z_{12} &= Z_{12x} + Z_{12y} \\ Z_{21} &= Z_{21x} + Z_{21y} \\ Z_{22} &= Z_{22x} + Z_{22y} \end{aligned}}$$

Each Z parameter of the series network is given as the sum of the corresponding parameters of the individual networks.

Parallel connection of two, two-port Networks



If each two port has a reference node that is common to its i/p and o/p port, and if the two ports are connected so that they have a common reference node, the the equations of the networks X and Y in terms of Y parameters

$I_{1x} = Y_{11x}V_{1x} + Y_{12x}V_{2x}$ — ①

$I_{2x} = Y_{21x}V_{1x} + Y_{22x}V_{2x}$ — ②

$I_{1y} = Y_{11y}V_{1y} + Y_{12y}V_{2y}$ — ③

$I_{2y} = Y_{21y}V_{1y} + Y_{22y}V_{2y}$ — ④

$V_1 = V_{1x} = V_{1y}$; $V_2 = V_{2x} = V_{2y}$ — ⑤

$I_1 = I_{1x} + I_{1y}$; $I_2 = I_{2x} + I_{2y}$ — ⑥

Substitute ①, ②, ③ & ④ in ⑥

$I_1 = Y_{11x}V_{1x} + Y_{12x}V_{2x} + Y_{11y}V_{1y} + Y_{12y}V_{2y}$

From ⑤

$I_1 = Y_{11x}V_1 + Y_{12x}V_2 + Y_{11y}V_1 + Y_{12y}V_2$

$$\boxed{I_1 = (Y_{11x} + Y_{11y})V_1 + (Y_{12x} + Y_{12y})V_2}$$

Ily $I_2 = Y_{21x}V_{1x} + Y_{22x}V_{2x} + Y_{21y}V_{1y} + Y_{22y}V_{2y}$

From ⑤

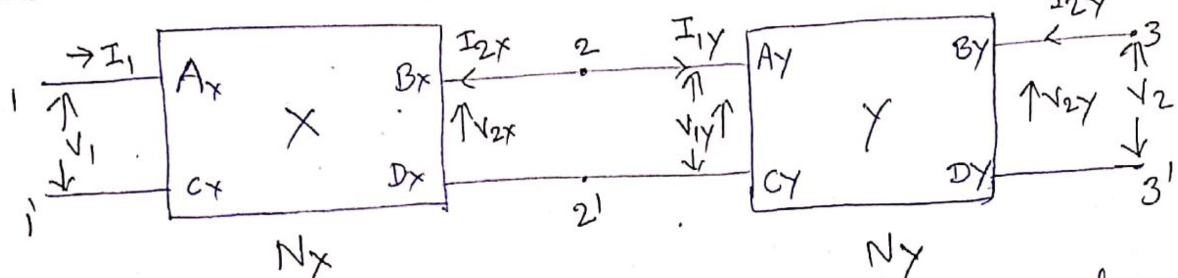
$I_2 = Y_{21x}V_1 + Y_{22x}V_2 + Y_{21y}V_1 + Y_{22y}V_2$

$$\boxed{I_2 = (Y_{21x} + Y_{21y})V_1 + (Y_{22x} + Y_{22y})V_2}$$

$$\boxed{\begin{array}{ll} Y_{11} = Y_{11x} + Y_{11y}; & Y_{12} = Y_{12x} + Y_{12y} \\ Y_{21} = Y_{21x} + Y_{21y}; & V_{22} = Y_{22x} + Y_{22y} \end{array}}$$

Cascade Connection

The main use of the transmission matrix is in dealing with a cascade connection of two-port networks.



Consider 2 two-port networks N_x and N_y connected in cascade with port voltages and currents. The matrix representation of ABCD parameters of the network x, y is

$$X = \begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A_x & B_x \\ C_x & D_x \end{bmatrix} \begin{bmatrix} V_{2x} \\ -I_{2x} \end{bmatrix}$$

$$Y = \begin{bmatrix} V_{1y} \\ I_{1y} \end{bmatrix} = \begin{bmatrix} A_y & B_y \\ C_y & D_y \end{bmatrix} \begin{bmatrix} V_{2y} \\ -I_{2y} \end{bmatrix}$$

At 2-2'; $V_{2x} = V_{1y}$ and $I_{2x} = -I_{1y}$

Combining the results, we have

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A_x & B_x \\ C_x & D_x \end{bmatrix} \begin{bmatrix} A_y & B_y \\ C_y & D_y \end{bmatrix} \begin{bmatrix} V_2 \\ -I_1 \end{bmatrix}$$

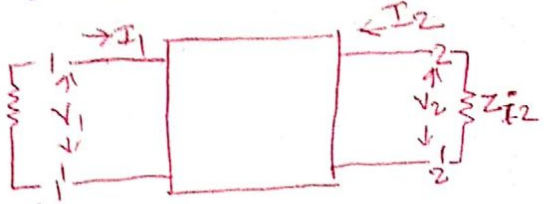
$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

where $\begin{bmatrix} A & B \\ C & D \end{bmatrix}$ is the transmission parameters matrix for the overall network.

Thus the transmission matrix of a cascade of a two-port networks is the product of transmission matrices of the individual two-port networks. This property is used in the design of telephone systems, microwave networks, RADARS etc.

Image Parameters: used to design filters.

The image impedance Z_{I1} and Z_{I2} of a two port network.



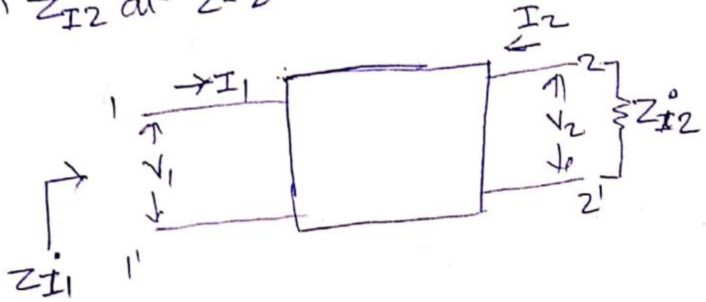
If port 1-1' of the network is terminated in Z_{I1}, the input impedance of port 2-2' is Z_{I2} and if port 2-2' is terminated in Z_{I2}, the i/p impedance at port 1-1' is Z_{I1}. then, Z_{I1} and Z_{I2} are called image impedances of the two-port network.

→ These Z_{I1} and Z_{I2} parameters can be obtained interms of two-port parameters

$V_1 = AV_2 - BI_2$ — ①

$I_1 = CV_2 - DI_2$ — ②

If the network is terminated in Z_{I2} at 2-2'



$V_2 = -I_2 Z_{I2}$ — ③

$$\frac{①}{②}\ \frac{V_1}{I_1} = \frac{AV_2 - BI_2}{CV_2 - DI_2} = Z_{I1} \text{ — ④}$$

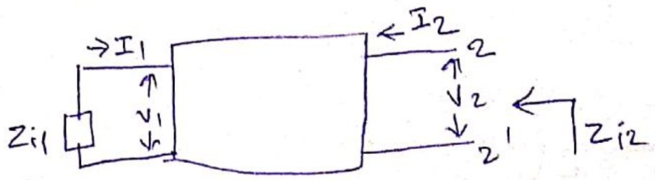
put ③ in ④

$$Z_{I1} = \frac{-AI_2 Z_{I2} - BI_2}{-CI_2 Z_{I2} - DI_2}$$

$$Z_{I1} = \frac{\cancel{I_2}(-AZ_{I2} - B)}{\cancel{I_2}(-CZ_{I2} - D)}$$

$$\boxed{Z_{I1} = \frac{AZ_{I2} + B}{CZ_{I2} + D}} \text{ — ⑤}$$

Ily, if the network is terminated in Z_{i1} at port 1-1'



$V_1 = -I_1 Z_{i1}$!! $\boxed{Z_{i1} = -\frac{V_1}{I_1}}$ — ⑥

$\boxed{Z_{i2} = \frac{V_2}{I_2}}$ — ⑦

From ④ $\cancel{Z_{i1} = \frac{V_1}{I_1}} = \frac{AV_2 - BI_2}{CV_2 - DI_2}$

Divide by I_2 $\cancel{Z_{i1} = \frac{A\frac{V_2}{I_2} - B}{C\frac{V_2}{I_2} - D}} = \frac{AZ_{i2} + B}{CZ_{i2} + D} =$

~~From ⑤ $Z_{i1} = \frac{AZ_{i2}+B}{CZ_{i2}+D}$~~

~~$CZ_{i1}Z_{i2} + Z_{i1}D = AZ_{i2} + B$~~

~~$CZ_{i1}Z_{i2} - AZ_{i2} = -Z_{i1}D + B$~~

~~$Z_{i2}[CZ_{i1} - A] = -(Z_{i1}D + B)$~~

~~$Z_{i2} = -\frac{Z_{i1}D + B}{CZ_{i1} - A}$~~

$AZ_{i2} - CZ_{i1}Z_{i2} = Z_{i1}D - D$

$Z_{i2}[A - CZ_{i1}] = Z_{i1}D - D$

$Z_{i2} = \frac{Z_{i1}D - D}{A - CZ_{i1}}$

$\frac{+6-7}{5-4}$

$V_1 = AV_2 - BI_2$ —⑧

$I_1 = CV_2 - DI_2$ —⑨

$Z_{i2} = \frac{V_2}{I_2}$

From ⑧ $V_2 = \frac{V_1 + BI_2}{A}$ —⑩

From ⑨ $I_2 = \frac{CV_2 - I_1}{D}$ —⑪

put ⑪ in ~~⑨~~10

$V_2 = \frac{V_1 + B\left(\frac{CV_2 - I_1}{D}\right)}{A}$

$V_2 = \frac{V_1 + \frac{BCV_2 - BI_1}{D}}{A}$

$V_2 = \frac{DV_1 + BCV_2 - BI_1}{AD}$

$V_2AD - BCV_2 = DV_1 + BI_1$

$$\boxed{V_2 = \frac{DV_1 - BI_1}{AD - BC}}$$ —⑫

Ily From ~~⑧~~⑩

$V_2 = \frac{V_1 + BI_2}{A}$ —⑩

put ⑩ in ⑨

$I_1 = C\left(\frac{V_1 + BI_2}{A}\right) - DI_2$

$I_1 = \frac{CV_1 + BCI_2}{A} - DI_2$

$I_1 = \frac{CV_1 + BCI_2 - ADI_2}{A}$

$AI_1 = CV_1 + BCI_2 - ADI_2$

$I_2(AD - BC) = CV_1 - AI_1$

$$\boxed{I_2 = \frac{CV_1 - AI_1}{AD - BC}}$$ —⑬

From ⑦ $Z_{i2} = \frac{V_2}{I_2} = \frac{DV_1 - BI_1}{CV_1 - AI_1}$ —⑭

$Z_{i1} = -\frac{V_1}{I_1}$ ∴ $\boxed{V_1 = -Z_{i1}I_1}$ —⑮

put ⑮ in ⑭

$Z_{i2} = \frac{D(-Z_{i1}I_1) - BI_1}{C(-Z_{i1}I_1) - AI_1}$

$= \frac{-DZ_{i1}I_1 - BI_1}{-CZ_{i1}I_1 - AI_1} = \frac{\cancel{I_1}(-DZ_{i1} - B)}{\cancel{I_1}(-CZ_{i1} - A)}$

$$\boxed{Z_{i2} = \frac{DZ_{i1} + B}{CZ_{i1} + A}}$$ —⑯

We knew that

$$Z_{i1} = \frac{AZ_{i2}+B}{CZ_{i2}+D} \quad - ⑦$$

$$Z_{i2} = \frac{DZ_{i1}+B}{CZ_{i1}+A} \quad - ⑯$$

substitute ⑦ in ⑯

$$Z_{i2} = \frac{D\left[\frac{AZ_{i2}+B}{CZ_{i2}+D}\right]+B}{C\left[\frac{AZ_{i2}+B}{CZ_{i2}+D}\right]+A}$$

$$Z_{i2} = \frac{ADZ_{i2}+BD+BCZ_{i2}+BD}{ACZ_{i2}+BC+ACZ_{i2}+AD}$$

$$Z_{i2} = \frac{ADZ_{i2}+BCZ_{i2}+2BD}{Z_{i2}[2AC]+BC+AD}$$

$$Z_{i2}^2(2AC)+\cancel{Z_{i2}BC}+\cancel{Z_{i2}AD} = \cancel{ADZ_{i2}}+\cancel{BCZ_{i2}}+2BD$$

$$Z_{i2}^2 = \frac{\cancel{2}BD}{\cancel{2}AC}$$

$$\boxed{Z_{i2} = \sqrt{\frac{BD}{AC}}} \quad - ⑰$$

To solve Z_{i1} put ⑯ in ⑦

$$Z_{i1} = \frac{A\left[\frac{DZ_{i1}+B}{CZ_{i1}+A}\right]+B}{C\left[\frac{DZ_{i1}+B}{CZ_{i1}+A}\right]+D}$$

$$Z_{i1} = \frac{ADZ_{i1}+AB+BCZ_{i1}+AB}{CDZ_{i1}+BC+CDZ_{i1}+AD}$$

$$Z_{i1} = \frac{ADZ_{i1}+2AB+BCZ_{i1}}{Z_{i1}[2CD]+BC+AD}$$

$$Z_{i1}^2(2CD)+\cancel{BCZ_{i1}}+\cancel{ADZ_{i1}} = 2AB+\cancel{BCZ_{i1}}+\cancel{ADZ_{i1}}$$

$$Z_{i1}^2 = \frac{\cancel{2}AB}{\cancel{2}CD}$$

$$\boxed{Z_{i1} = \sqrt{\frac{AB}{CD}}} \quad - ⑱$$

III Locus diagrams, Resonance and Magnetic Circuits

Locus means path.

→ In AC electrical circuits the magnitude and phase of the current vector depends upon the values of R, L & C when the applied voltage and frequency are kept constant.

→ The path traced by the terminus (tip) of the current vector when the parameters R, L & C are varied is called the current "Locus diagram."

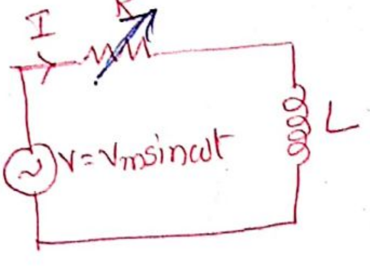
→ Locus diagrams are useful in studying and understanding the behavior of the RLC circuits when one of these parameters is varied, keeping voltage and frequency constant.

NOTE: The magnitude and phase of current phasor in circuit depends upon the values of R, L and C and fre. of supply.

→ Locus diagrams is used for design and analyzing of RLC circuits.

→ Locus diagram can be also drawn for reactance, impedance, susceptance and admittance when frequency is variable.

(i) Series RL circuit with varying R



$$I = \frac{V}{R+j\omega L}$$

Magnitude of I

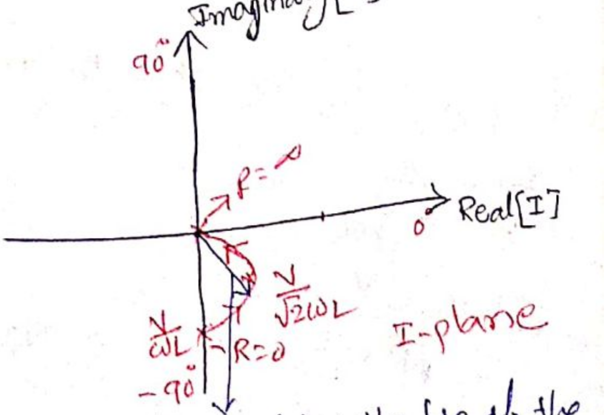
$$|I| = \frac{V}{\sqrt{R^2+\omega^2L^2}}$$

Phase of I

$$\angle I = -\tan^{-1}\left(\frac{\omega L}{R}\right)$$

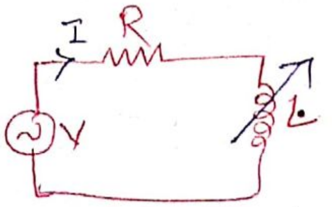
varying with R

R	$\lvert I\rvert$	$\angle I$
0	$\frac{V}{\omega L}$	$-90°$
ωL	$\frac{V}{\sqrt{2}\omega L}$	$-45°$
∞	0	$0°$



path traced by the tip of the current vector; like a semicircle

(ii) Series RL circuit with varying L



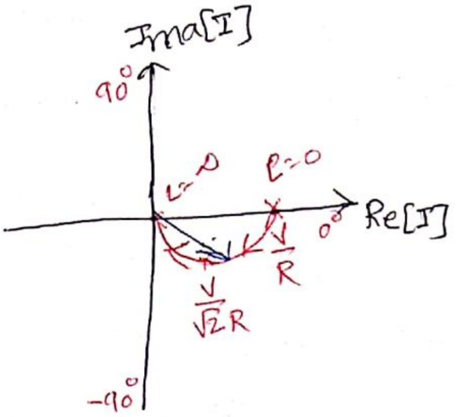
$$I = \frac{V}{R+j\omega L}$$

Magnitude $|I| = \dfrac{V}{\sqrt{R^2+\omega^2L^2}}$

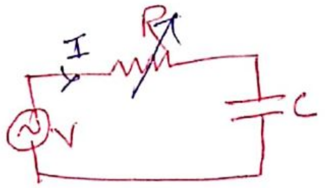
phase $\angle I = -\tan^{-1}\left(\dfrac{\omega L}{R}\right)$

varying L

L	$\lvert I\rvert$	$\angle I$
0	$\frac{V}{R}$	0°
$\frac{R}{\omega}$	$\frac{V}{\sqrt{2}R}$	-45°
∞	0	-90



(iii) Series RC circuit with varying R



$$I = \frac{V}{R+\frac{1}{j\omega C}} = \frac{V}{R-j\omega C}$$

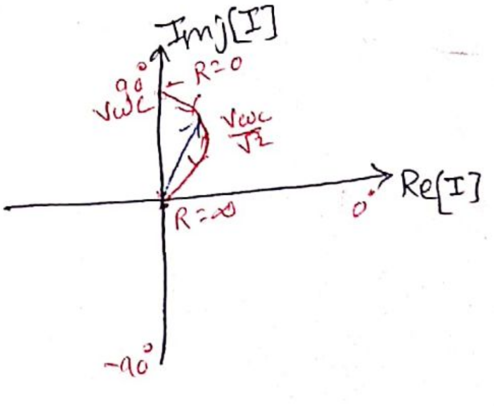
$$|I| = \frac{V}{\sqrt{R^2+\left(\frac{1}{\omega C}\right)^2}}$$

$$\angle I = 0^\circ-\left(-\tan^{-1}\left(\frac{1}{R\omega C}\right)\right)$$

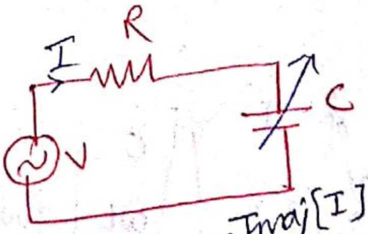
$$\angle I = \tan^{-1}\left(\frac{1}{R\omega C}\right)$$

Varying R

R	$\lvert I\rvert$	$\angle I$
0	$V\omega C$	90°
$\frac{1}{\omega C}$	$\frac{V\omega C}{\sqrt{2}}$	45°
∞	0	0°



(iv) Series RC Circuit with varying C



$$I = \frac{V}{R+\frac{1}{j\omega C}}$$

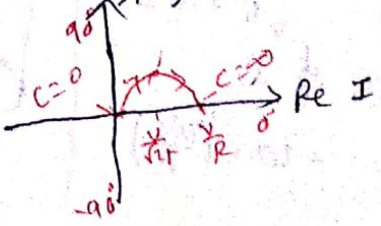
$$|I| = \frac{V}{\sqrt{R^2+\left(\frac{1}{\omega C}\right)^2}}$$

$$\angle I = 0-\left(-\tan^{-1}\left(\frac{1}{R\omega C}\right)\right)$$

$$\angle I = \tan^{-1}\left(\frac{1}{R\omega C}\right)$$

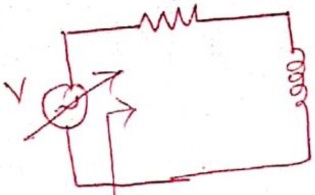
Varying C

C	$\lvert I\rvert$	$\angle I$
0	0	90°
$\frac{1}{R\omega}$	$\frac{V}{\sqrt{2}R}$	45°
∞	$\frac{V}{R}$	0°



Series RL varying ω

(v)



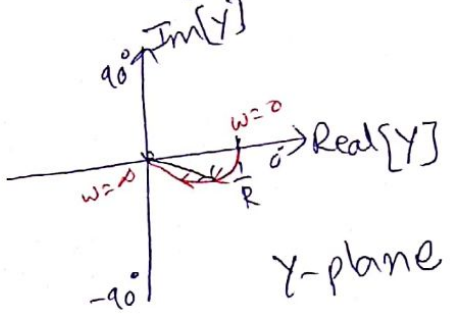
Y: admittance locus diagram

$$Y = \frac{1}{Z} = \frac{1}{R+j\omega L}$$

$$|Y| = \frac{1}{\sqrt{R^2+\omega^2 L^2}}$$

$$\angle Y = -\tan^{-1}\frac{\omega L}{R}$$

ω	|Y|	∠Y
0	$\frac{1}{R}$	0°
∞	0	-90°



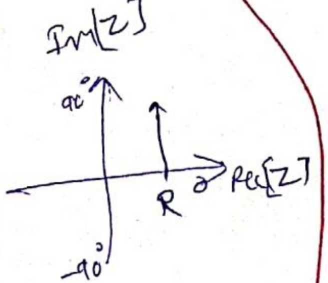
Z- Impedance locus diagram

$$Z = R+j\omega L$$

$$|Z| = \sqrt{R^2+\omega^2 L^2}$$

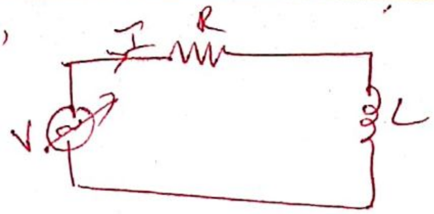
$$\angle Z = \tan^{-1}\left(\frac{\omega L}{R}\right)$$

ω	|Z|	∠Z
0	R	0°
∞	∞	90°



NOTE: Magnitude is ∞, hence networks avoids impedance locus diagram

(vi)

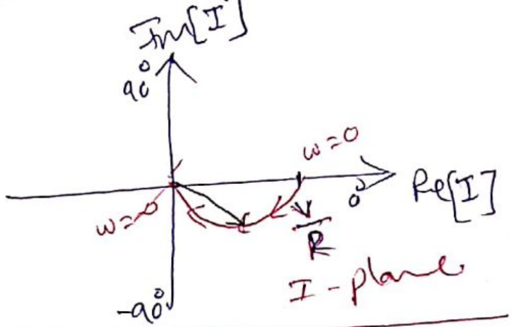


$$I = \frac{V}{R+j\omega L}$$

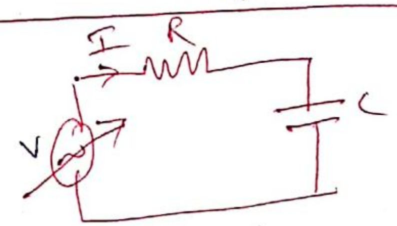
$$|I| = \frac{V}{\sqrt{R^2+\omega^2 L^2}}$$

$$\angle I = -\tan^{-1}\left(\frac{\omega L}{R}\right)$$

ω	|I|	∠I
0	$\frac{V}{R}$	0°
∞	0	-90°



(2)

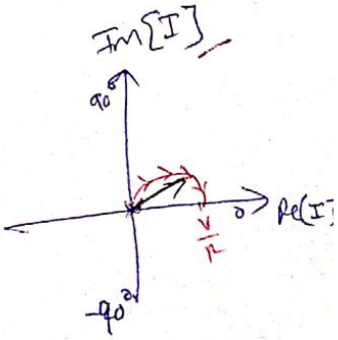


$$I = \frac{V}{R+\frac{1}{j\omega C}} = \frac{V}{R-j\left(\frac{1}{\omega C}\right)}$$

$$|I| = \frac{V}{\sqrt{R^2+\left(\frac{1}{\omega C}\right)^2}}$$

$$\angle I = 0-\left(-\tan^{-1}\left(\frac{1}{R\omega C}\right)\right) = \tan^{-1}\left(\frac{1}{R\omega C}\right)$$

ω	|I|	∠I
0	0	90°
∞	$\frac{V}{R}$	0°



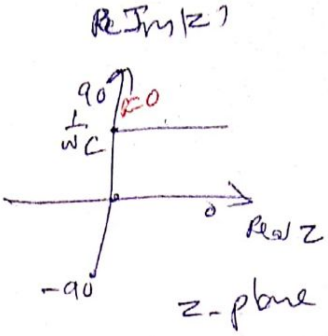
Z - Impedance locus diagram

$$Z = R + \frac{1}{j\omega C}$$

$$|Z| = \sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}$$

$$\angle Z = \tan^{-1}\left(\frac{1}{R\omega C}\right)$$

R	$|Z|$	$\angle Z$
0	$\frac{1}{\omega C}$	90°
∞	∞	0°



Resonance

→ An a.c. Circuit is said to be in "resonance" when the applied voltage and the circuit current are in phase.

→ Thus at resonance the equivalent complex impedance of the circuit consists of only the resistance. The power factor of the circuit is unity

→ Resonance circuit are formed by the combinations of inductance and capacitances which may be connected in series(or) in parallel as series Resonance and Parallel Resonance.

NOTE: Resonant circuits are also known as tuned circuits.

→ Parallel resonant circuit is also known as anti-resonance(or) tank ckt

Power Factor(p.f)

→ Power factor is defined as cosine of the angle of lead(or) lag between voltage and current. $\boxed{p.f = \cos\phi}$ ϕ - Phase angle between V & I

→ Power factor can also be defined as the ratio of true power (Active Power) to the apparent power.

$$\boxed{p.f = \frac{\text{true Power}}{\text{Apparent power}} = \frac{VI\cos\phi}{VI} = \cos\phi}$$

→ Power factor is also be defined as the ratio of the resistance and impedance of the current.

$$\boxed{p.f = \frac{\text{Resistance}}{\text{Impedance}} = \frac{R}{Z}}$$

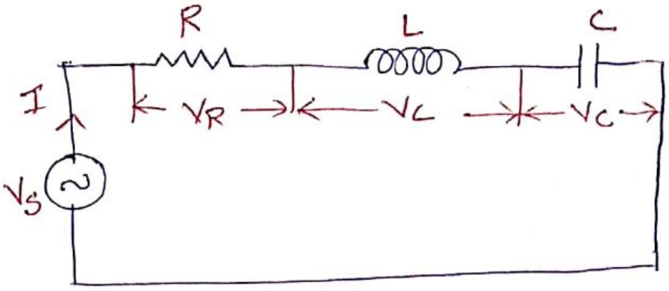
NOTE: To make the angle $\phi \to 0$ i.e. $p.f \to 1$, is termed as power factor correction.

→ For the usual case of inductive load, it is often possible to improve the power factor by placing capacitors in parallel with the load. Since the power factor is increased, the current and apparent power decrease. and a more efficient utilization of the power distribution system is obtained.

→ Quality factor 'Q' of an Inductor is $\boxed{Q_0 = \frac{\omega_0 L}{R}}$

→ Quality factor Q of a capacitor is $\boxed{Q_0 = \frac{1}{\omega_0 RC}}$

Series Resonance:



In a series RLC circuit, the current lags behind (or) leads the voltage applied depending upon the values of X_L and X_C.

→ X_L causes the total current to lag behind the applied voltage.

→ X_C causes the total current to lead the applied voltage.

→ when $X_L > X_C$, the circuit is predominantly inductive.

→ when $X_L < X_C$, the circuit is predominatly capacitive.

If one of the parameters of the series RLC circuit is varied in such a way that the current in the circuit is in phase with the applied voltage, then the circuit is said to be in resonance.

From series RLC circuit shown above

The total impedance 'z' is

$$Z = R + j\omega L + \frac{1}{j\omega C} = R + j\omega L - \frac{j}{\omega C}$$

$$\boxed{Z = R + j\left(\omega L - \frac{1}{\omega C}\right)} \qquad \boxed{Z = R + j(X_L - X_C)}$$

At Resonance $X_L = X_C$: $2\pi f_0 L = \frac{1}{2\pi f_0 C}$

where f_0/f_r = Resonance fre.

$$f_0^2 = \frac{1}{2\pi \times 2\pi \times LC} = \frac{1}{4\pi^2 LC} ; \quad \boxed{f_0 = \frac{1}{2\pi\sqrt{LC}}}$$

In a series RLC circuit, Resonance may be produced by varying the fre., keeping L and C constat otherwise resonance may be produced by varying either L (or) C for a fixed fre.

Scanned by CamScanner

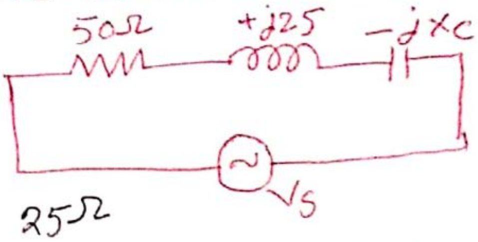
The current at any instant in a series resonance circuit is

$$I = \frac{V}{\sqrt{R^2+(\omega L-\frac{1}{\omega C})^2}}.$$

current at resonance i.e. when reactance is zero.will be

$$\boxed{I_0 = \frac{V}{R}}$$

Q1. Determine the value of capacitive reactance and impedance at resonance.

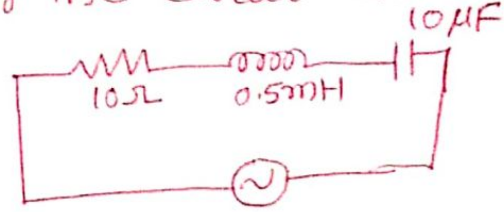


At resonance $X_L = X_C$

Since $X_L = 25\Omega$, then X_C is also 25Ω

The value of impedance at resonance is $Z = R = 50\Omega$.

Q2. Determine the resonant fre. for the circuit shown.



$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

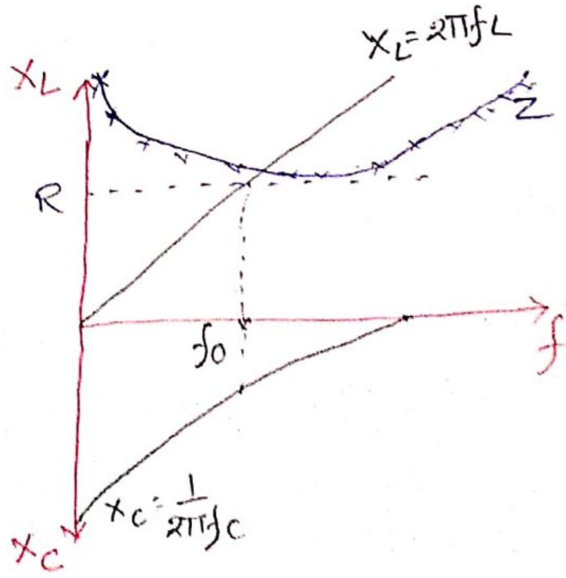
$$f_0 = \frac{1}{2\pi\sqrt{0.5\times10^{-3}\times10\times10^{-6}}}$$

$$\boxed{f_0 = 2.25\,KHz}$$

Impedance and Phase angle of a series Resonance circuit

The impedance of a series RLC circuit is

$$|Z| = \sqrt{R^2+(\omega L-\frac{1}{\omega C})^2}.$$



The variation of X_C and X_L with fre. $X_L = 2\pi f L$; $X_C = \frac{1}{2\pi f C}$

At $f = 0$, $X_L - 0$; $X_C = \infty$

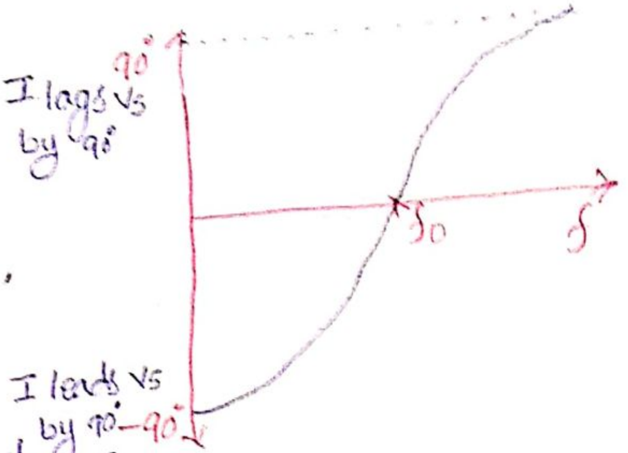
At $f = \infty$; $X_L = \infty$; $X_C = 0$

As f increases from 0 to ∞

X_L increases from 0 to ∞

X_C decreases from ∞ to 0

Phase Angle



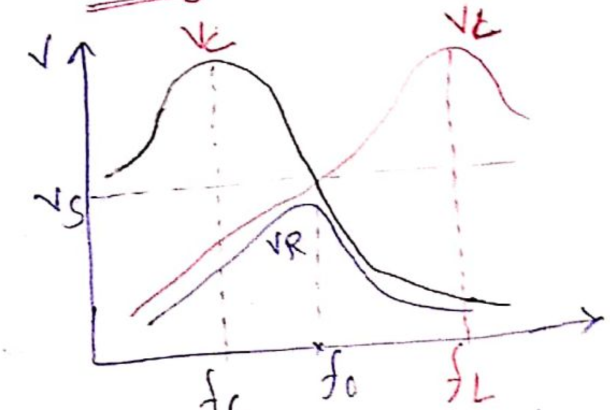
At Zero fre. both X_C and Z are infinitely large and X_L is '0'

At a fre. below the resonant fre. f_0, current leads the source voltage because the capacitive reactance is greater than the inductive reactance.

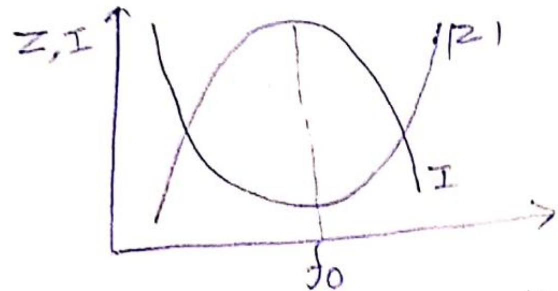
The phase phase angle decreases as the fre. approaches the resonant value, and is 0° at resonance.

At frequencies above resonance, the current lags behind the source voltage, because the inductive reactance is greater than capacitive reactance. As the fre. goes higher, the phase angle approaches 90°.

Voltages and currents in a series Resonant circuit



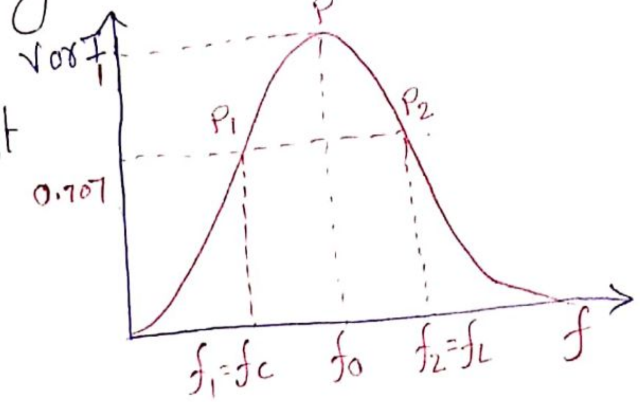
At $f = 0$, the capacitor acts as an open circuit and blocks current, complete source voltage appears across the capacitor.
As the fre. increases, X_C decreases and X_L increases, causing total reactance $X_C - X_L$ to decrease. As a result, the $|Z|$ decreases and current increases, V_R increases and both V_C and V_L increase.



At resonance fre. f_0, the capacitive reactance is equal to inductive reactance, and hence the impedance is minimum. Because of min. Z maximum current flows.

Bandwidth of an RLC circuit

The bandwidth of any system is the range of frequencies for which the current or output voltage is equal to 70.7% of its value at the resonant frequency.



The frequency f_1 is the fre. at which the current is 0.707 times the current (or) voltage at resonant value, and called the "lower cut-off frequency."

The frequency f_2 is the frequency at which the current (or) voltage is 0.707 times the current (or) voltage at resonant value (maximum value) and is the "upper cut-off fre."

The Bandwidth BW is defined as the frequency difference between f_2 and f_1 i.e. $\boxed{BW = f_2 - f_1}$ units are Hertz

If the current at P_1 is $0.707 I_{max}$, the impedance of the circuit at this point is $\sqrt{2}R$ and hence

$$\frac{1}{\omega_1 C} - \omega_1 L = R \quad - ①$$

lly $$\omega_2 L - \frac{1}{\omega_2 C} = R \quad - ②$$

① - ②

$$\frac{1}{\omega_1 C} - \omega_1 L = \omega_2 L - \frac{1}{\omega_2 C}$$

$$(\omega_1 + \omega_2) L = \frac{1}{\omega_1 C} + \frac{1}{\omega_2 C}$$

$$(\omega_1 + \omega_2) L = \frac{1}{C}\left(\frac{1}{\omega_1} + \frac{1}{\omega_2}\right)$$

$$\cancel{(\omega_1 + \omega_2)} L = \frac{1}{C}\left(\frac{\cancel{\omega_1 + \omega_2}}{\omega_1 \omega_2}\right)$$

$$\omega_1 \omega_2 = \frac{1}{LC} \quad - ③$$

we knew that $\omega_0^2 = \frac{1}{LC}$ - ④

④ in ③ $\boxed{\omega_0^2 = \omega_1 \omega_2}$ - ⑤

① + ②

$$\frac{1}{\omega_1 C} - \omega_1 L + \omega_2 L - \frac{1}{\omega_2 C} = 2R$$

$$(\omega_2 - \omega_1) L + \frac{1}{C}\left(\frac{1}{\omega_1} - \frac{1}{\omega_2}\right) = 2R$$

$$(\omega_2 - \omega_1) L + \frac{1}{C}\left(\frac{\omega_2 - \omega_1}{\omega_1 \omega_2}\right) = 2R$$

From ④ $\omega_0^2 = \frac{1}{LC}$; $\frac{1}{C} = \omega_0^2 L$

From ⑤ $\omega_0^2 = \omega_1 \omega_2$

$$(\omega_2 - \omega_1) L + \omega_0^2 L\left(\frac{\omega_2 - \omega_1}{\cancel{\omega_0^2}}\right) = 2R$$

$$\cancel{2}(\omega_2 - \omega_1) L = \cancel{2} R$$

$$\omega_2 - \omega_1 = \frac{R}{L}$$

$$2\pi f_2 - 2\pi f_1 = \frac{R}{L}$$

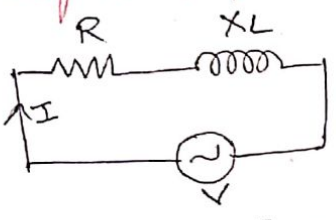
$$\boxed{f_2 - f_1 = \frac{R}{2\pi L}} \qquad \boxed{BW = \frac{R}{2\pi L}}$$

Quality factor(Q) and its effect on Bandwidth

Q factor of a resonant circuit is a measure of "goodness" (or) quality of a resonant circuit.

→ Quality factor is also called as Figure of Merit

Q-factor of Inductor



$$Q = 2\pi \frac{\text{Max. Energy stored}}{\text{Energy dissipated per cycle}}$$

→ In an Inductor max. energy stored is $\frac{1}{2} L I_{max}^2$

→ Energy dissipated per cycle is given by the product of average power in the resistor i.e.

$$I_{rms}^2 RT = \left(\frac{I_{max}}{\sqrt{2}}\right)^2 RT$$

$$T = \frac{1}{f_0}$$

So $$Q = 2\pi \frac{\frac{1}{2} L I_{max}^2}{\left(\frac{I_{max}}{\sqrt{2}}\right)^2 R \cdot \frac{1}{f_0}}$$

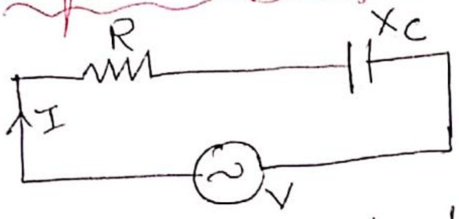
$$Q = 2\pi \times \frac{1}{\cancel{2}} L \cancel{I_{max}^2} \times \frac{2f_0}{\cancel{I_{max}^2} \cdot R}$$

$$Q = \frac{2\pi f_0 L}{R}$$

$$\boxed{Q_L = \frac{\omega_0 L}{R}}$$

where $\omega_0 = 2\pi f_0$

Q-factor of capacitor



$$Q = 2\pi \frac{\text{Max. Energy stored}}{\text{Energy dissipated per cycle}}$$

→ In capacitor max. energy stored is $\frac{1}{2} C V_{max}^2$

→ Energy dissipated per cycle is

$$I_{rms}^2 RT = \left(\frac{I_{max}}{\sqrt{2}}\right)^2 R \cdot \frac{1}{f_0}$$

$$Q = 2\pi \frac{\frac{1}{2} C V_{max}^2}{\left(\frac{I_{max}}{\sqrt{2}}\right)^2 R \cdot \frac{1}{f_0}}$$

$$Q = 2\pi \times \frac{1}{\cancel{2}} C V_{max}^2 \times \frac{2f_0}{I_{max}^2 R}$$

$$V_{max} = I_{max} X_C \; ; \; X_C = \frac{1}{\omega_0 C}$$

$$Q = \cancel{2}\pi \cancel{C}\, \cancel{I_{max}^2} \times \frac{1}{\cancel{2}\,\omega_0^2 C^{\cancel{2}}} \times \frac{2f_0}{\cancel{I_{max}^2} R}$$

$$Q = \frac{2\pi f_0}{\omega_0^2 RC} \qquad 2\pi f_0 = \omega_0$$

$$Q = \frac{\cancel{\omega_0}}{\omega_0^{\cancel{2}} RC}$$

$$\boxed{Q_C = \frac{1}{\omega_0 RC}}$$

(or)

$$Q = \frac{\text{Reactive Power in Inductor (or) capacitor at resonance}}{\text{Average power at Resonance.}}$$

Reactive Power in Inductor at resonance $= I^2 X_L$

Reactive Power in capacitor at resonance $= I^2 X_C$

Average Power at resonance $= I^2 R$

Q-factor for an Inductor is	Q-factor for ~~an~~ capacitor is
$Q = \frac{I^2 X_L}{I^2 R}$ $\quad \because X_L = \omega_0 L$	$Q = \frac{I^2 X_C}{I^2 R}$ $\quad X_C = \frac{1}{\omega_0 C}$
$\boxed{Q_L = \frac{\omega_0 L}{R}}$	$\boxed{Q_C = \frac{1}{\omega_0 RC}}$

Q interms of R,L,C is $\omega_0 = \frac{1}{\sqrt{LC}}$; $Q = \frac{1}{\sqrt{LC}} \times \frac{L}{R} = \frac{1}{\sqrt{L}\sqrt{C}} \times \frac{\sqrt{L}\sqrt{L}}{R} = \frac{1}{R}\sqrt{\frac{L}{C}}$; $\boxed{Q = \frac{1}{R}\sqrt{\frac{L}{C}}}$

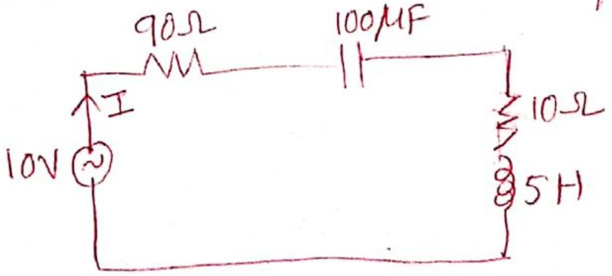
The relation between Q-factor and Bandwidth is

$$\boxed{Q = \frac{f_0}{BW}} \quad \therefore \quad Q \propto \frac{1}{BW}$$

A higher value of circuit Q results in a smaller Bandwidth.

A lower value of circuit Q results in a higher Bandwidth.

(Q1) Determine the value of Q at resonance and BW of the ckt



The resonant fre, f_0 is

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

$$= \frac{1}{2\pi\sqrt{5 \times 100 \times 10^{-6}}}$$

$$\boxed{f_0 = 7.12\,Hz}$$

Quality factor, $Q = \frac{X_L}{R} = \frac{2\pi f_0 L}{R} = \frac{2\pi \times 7.12 \times 5}{100} = 2.24$

$$\boxed{Q = 2.24}$$

Bandwidth $BW = \frac{f_0}{Q} = \frac{7.12}{2.24} = 3.178\,Hz$

$$\boxed{BW = 3.178\,Hz}$$

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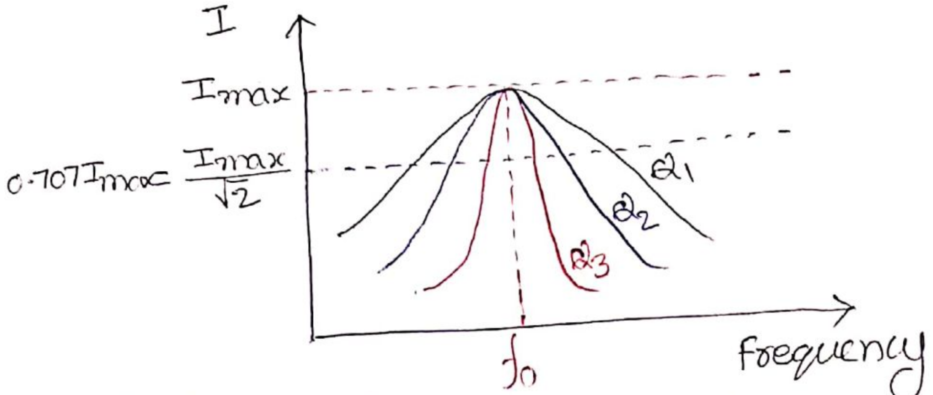
Selectivity

Selectivity of a series RLC circuit indicates how well the given circuit responds to a given resonant frequency and how well it rejects all other frequencies. selectivity is the ratio of Resonant fre. f_0, to Bandwidth BW

Selectivity ∝ Quality factor.

$$\text{Selectivity} = \frac{f_0}{BW}$$

A circuit with good selectivity (high Q-factor) will have max. gain at the resonant fre. and will have min. gain at other fre.



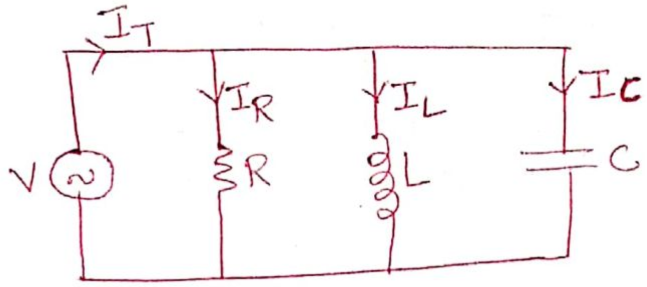
* For Good Selectivity Q-factor should be higher.

Properties of Series Resonance:

1. The supply voltage, V_s, & the resulting current I are in phase to eachother.
2. The net rectance is zero at resonance $X = X_L - X_C = 0$
3. The impedance have resistive part only, $Z = R$; Z is minimum.
4. The current in the circuit is maximum. $I = \frac{V}{Z} = \frac{V}{R} A$

   Since at resonance, the line current in the series RLC circuit is maximum hence it is called "Acceptor circuit."
5. Series resonance frequency $f_0 = \frac{1}{2\pi\sqrt{LC}}$ Hz
6. The Power factor at resonance is unity i.e. $\cos\phi = \frac{R}{Z} = \frac{R}{R} = 1$
7. At resonance, the circuit has got minimum Impedance & Max. admittance
8. The magnitudes of the capacitive Reactance & Inductive reactance becomes equal.
9. The voltage V_C becomes equal to V_L at resonance and is Q times higher than V_R
10. The Q-factor of Inductor $Q = \frac{\omega_0 L}{R}$, capacitor is $Q = \frac{1}{\omega_0 RC}$
11. The Q-factor of RLC is $Q = \frac{1}{R}\sqrt{\frac{L}{C}}$.

Parallel Resonance:



$$Y = \frac{1}{R} + \frac{1}{j\omega L} + \frac{1}{\frac{1}{j\omega C}}$$

$$Y = \frac{1}{R} + \frac{1}{j\omega L} + j\omega C$$

$$Y = \frac{1}{R} + j\omega C - \frac{j}{\omega L}$$

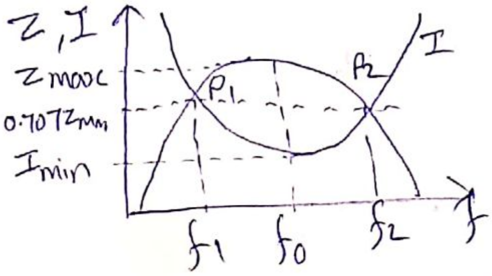
$$Y = \frac{1}{R} + j\left[\omega C - \frac{1}{\omega L}\right]$$

For Resonance imajinary part should be zero

$$\omega_0 C - \frac{1}{\omega_0 L} = 0$$

$$2\pi f_0 C = \frac{1}{2\pi f_0 L} ; \quad f_0^2 = \frac{1}{4\pi^2 LC} ; \quad \boxed{f_0 = \frac{1}{2\pi\sqrt{LC}}}$$

BW Bandwidth



Amplitude of admittance at point P_1 is $Y = \sqrt{\frac{1}{R^2} + \left(\frac{1}{\omega_1 L} - \omega_1 C\right)^2}$ and equating this to $\frac{\sqrt{2}}{R}$ we get

$$\frac{1}{\omega_1 L} - \omega_1 C = \frac{1}{R} \quad -①$$

||ly amplitude of admittance at point P_2 is $Y = \sqrt{\frac{1}{R^2} + \left(\omega_2 C - \frac{1}{\omega_2 L}\right)^2}$ and equating to $\sqrt{2}/R$ we get $\omega_2 C - \frac{1}{\omega_2 L} = \frac{1}{R}$ —②

① - ②

$$\frac{1}{\omega_1 L} - \omega_1 C - \omega_2 C + \frac{1}{\omega_2 L} = 0$$

$$(\omega_1 + \omega_2) C = \frac{1}{L}\left[\frac{1}{\omega_1} + \frac{1}{\omega_2}\right]$$

$$\cancel{(\omega_1 + \omega_2)} C = \frac{1}{L}\left[\frac{\cancel{\omega_1 + \omega_2}}{\omega_1 \omega_2}\right]$$

$$\boxed{\omega_1 \omega_2 = \frac{1}{LC}} \quad -③$$

① + ② $\frac{1}{\omega_1 L} - \omega_1 C + \omega_2 C - \frac{1}{\omega_2 L} = \frac{2}{R}$

$$(\omega_2 - \omega_1) C + \frac{1}{L}\left(\frac{1}{\omega_1} - \frac{1}{\omega_2}\right) = \frac{2}{R}$$

$$(\omega_2 - \omega_1) C + \frac{1}{L}\left(\frac{\omega_2 - \omega_1}{\omega_1 \omega_2}\right) = \frac{2}{R}$$

from ③ $\frac{1}{L} = \omega_1 \omega_2 C$

$$(\omega_2 - \omega_1) C + \cancel{\omega_1 \omega_2} C\left(\frac{\omega_2 - \omega_1}{\cancel{\omega_1 \omega_2}}\right) = \frac{2}{R}$$

$$\cancel{2}(\omega_2 - \omega_1) C = \frac{\cancel{2}}{R} ; \quad \omega_2 - \omega_1 = \frac{1}{RC}$$

$$f_2 - f_1 = \frac{1}{2\pi RC} ; \quad \boxed{BW = \frac{1}{2\pi RC}}$$

Quality factor of a Parallel resonance circuit

$$Q = 2\pi \times \frac{\text{maximum energy stored}}{\text{Energy dissipated per cycle.}}$$

In case of Inductor

max. energy stored: $\frac{1}{2} L I^2$

Energy dissipated per cycle is

$$\left(\frac{I}{\sqrt{2}}\right)^2 R \times T \qquad T = \frac{1}{f_0}$$

$$Q = 2\pi \times \frac{\frac{1}{2} L I^2}{\left(\frac{I}{\sqrt{2}}\right)^2 R \cdot \frac{1}{f_0}}$$

$$\boxed{\cancel{= 2\pi \times \frac{1}{2} L I^2 \times \frac{2f_0}{I^2 R}}}$$

$$Q = 2\pi \times \frac{\frac{1}{2} L \left(\frac{V}{\omega_0 L}\right)^2 R}{\frac{V^2}{2} \times \frac{1}{f}}$$

$$Q = \frac{2\pi f L R}{\omega^2 L^2} = \frac{R}{\omega_0 L}$$

$$\boxed{Q_L = \frac{R}{\omega_0 L}}$$

In case of capacitor

Maximum energy stored $= \frac{1}{2} C V^2$

Energy dissipated per cycle $= P \times T = \frac{V^2}{2R} \times \frac{1}{f}$

$$Q = 2\pi \times \frac{\frac{1}{2} C V^2}{\frac{V^2}{2R} \times \frac{1}{f_0}}$$

$$Q = 2\pi \times \frac{1}{\cancel{2}} C \cancel{V^2} \times \frac{2 R f_0}{\cancel{V^2}}$$

$$Q = 2\pi f_0 RC$$

$$\boxed{Q = \omega_0 RC}$$

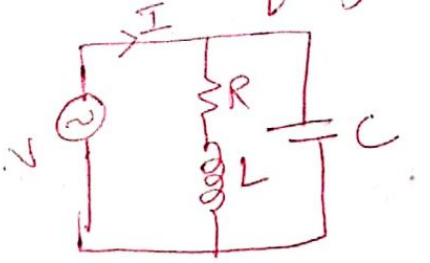
Q interms of RLC in parallel resonance is

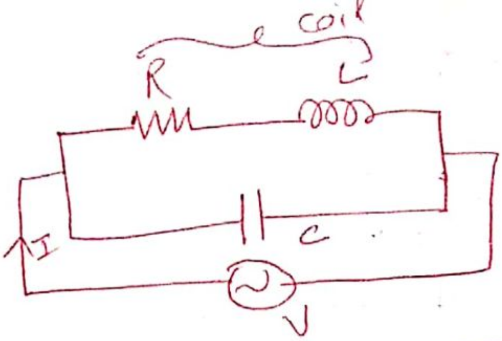
$$Q = \omega_0 RC \, ; \quad \omega_0 = \frac{1}{\sqrt{LC}}$$

$$Q = \frac{1}{\sqrt{LC}} RC = \frac{1}{\sqrt{L}\cancel{\sqrt{C}}} \times R\cancel{\sqrt{C}}\sqrt{C} = R\sqrt{\frac{C}{L}}$$

$$\boxed{Q_{\text{in terms of } RLC} = R\sqrt{\frac{C}{L}}}$$

Derivation of fo





$$f_0 = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \left(\frac{R}{L}\right)^2}$$

$Z_1 = R + j\omega L \;;\; Z_2 = -j\omega C$

$Y = \frac{1}{Z_1} + \frac{1}{Z_2}$

$= \frac{1}{R+j\omega L} + \frac{1}{-j\omega C}$

Rationalize with $R - j\omega L$

$Y = \frac{R-j\omega L}{(R+j\omega L)(R-j\omega L)} + \frac{1}{j\omega C}$

$= \frac{R-j\omega L}{R^2+\omega^2 L^2} + \frac{1}{j\omega C}$

$= \frac{R}{R^2+\omega^2L^2} - \frac{j\omega L}{R^2+\omega^2L^2} + \frac{j}{j\omega C}$

$= \frac{R}{R^2+\omega^2L^2} - j\left[\frac{1}{\omega C} - \frac{\omega L}{R^2+\omega^2L^2}\right]$

At resonance imaginary part is '0'

$\frac{1}{\omega C} = \frac{\omega L}{R^2+\omega^2L^2}$

$\omega_L \omega_C = R^2 + \omega^2 L^2$

$2\pi f_r L \times \frac{1}{2\pi f_r C} = R^2 + (2\pi f_r)^2 L^2$

$\frac{L}{C} = R^2 + 4\pi^2 f_0^2 L^2$

$4\pi^2 L^2 f_0^2 = \frac{L}{C} - R^2$

$f_0^2 = \frac{1}{4\pi^2 L^2}\left(\frac{L}{C} - R^2\right)$

$f_0 = \frac{1}{2\pi L}\sqrt{\frac{L}{C} - R^2}$

$= \frac{1}{2\pi}\sqrt{\frac{L}{L^2 C} - \frac{R^2}{L^2}}$

$$\boxed{f_0 = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \left(\frac{R}{L}\right)^2}}$$

~~Starts from here~~ ↓

$Z_1 = R + jX_L$

$Y_1 = \frac{1}{Z_1} = \frac{1}{R+jX_L} \times \frac{R-jX_L}{R-jX_L}$

$Y_1 = \frac{R-jX_L}{R^2-(jX_L)^2} = \frac{R-jX_L}{R^2+X_L^2}$

$Y_1 = \frac{R}{R^2+X_L^2} - j\frac{X_L}{R^2+X_L^2}$ — ①

$Z_2 = -jX_C$

$Y_2 = \frac{1}{Z_2} = \frac{1}{-jX_C} = \frac{j}{X_C}$ — ②

$Y_T = Y_1 + Y_2$

$Y_T = \frac{R}{R^2+X_L^2} - j\frac{X_L}{R^2+X_L^2} + \frac{j}{X_C}$

$\frac{R}{R^2+X_L^2} + j\left[\frac{1}{X_C} - \frac{X_L}{R^2+X_L^2}\right]$ At resonance j part is '0'

$\frac{1}{X_C} = \frac{X_L}{R^2+X_L^2}$

$R^2 + X_L^2 = X_L X_C$

$R^2 + (2\pi f_0 L)^2 = \cancel{2\pi f_0} L \times \frac{1}{\cancel{2\pi f_0} C}$

$(2\pi f_0 L)^2 = \frac{L}{C} - R^2$

$4\pi^2 f_0^2 L^2 = \frac{L}{C} - R^2$

Important points in parallel RLC circuit at Resonance.

1. The impedance of the circuit becomes resistive and maximum.
2. The current in the circuit becomes minimum.
3. The magnitudes of the capacitive Reactance and inductive Reactance become equal.
4. The current through the capacitor becomes equal and opposite to the current through the inductor at resonance and is Q times higher than the current through the resistor.

NOTE: Series resonance circuit draws maximum current & hence it is called Acceptor circuit, But parallel resonance circuit draws minimum current & is called rejector circuit (or) anti resonance.

Magnetic Circuits

Magnetic field: Magnetic fields are the fundamental medium through which energy is converted from one form to another in motors, generators and transformers.

Important Definations in Magnetic field circuits:

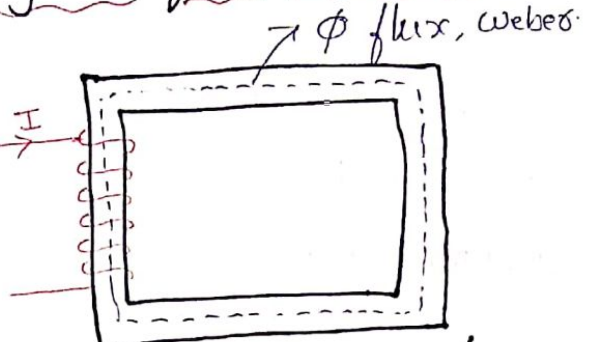
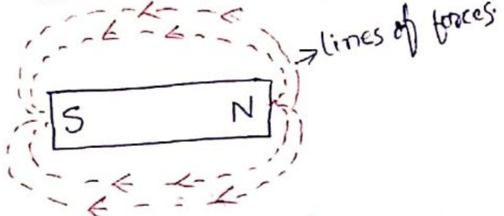


Fig: Magnetic Circuit

(1) **Magnetic flux: φ:**

Magnetic flux is produced due to the flow of a current in a wire.

Flux of 1 weber = 10^8 lines of forces.

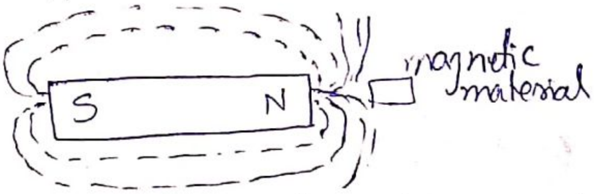


(2) **Magnetic flux density: B:** Magnetic flux per unit Area.

$$B = \frac{\phi\ (wb)}{A\ (m^2)}$$; i.e. Units of B is wb/m² (or) Tesla

NOTE: In Electric circuit current density $J = \frac{I}{A}$

(3) **Magnetic field (or) flux Intensity:**



A force on magnetic material excited by magnetic system from magnetic material North pole is called Magnetic field Intensity. (or) magnetic flux intensity. $H = \frac{NI}{l}$, N - no. of turns
I - current
l - length.

(or)

The magnetic flux intensity is the mmf per unit length along the path of the flux. $H = \frac{mmf}{l} = \frac{NI}{l}$

H unit is Ampertuns/meter.

*) Magneto Motive force mmf

It is the ability of a coil to produce magnetic flux.

$\boxed{mmf = NI}$

N - no. of turns of winding in a coil

I - current passing through the coil.

unit of mmf is Ampere turns

Driving force required for the magnetic flux to be transfered in any magnetic circuit is mmf

emf in electric field.

electro motive force.

Applied force (V) on electrons to be in motion

stable charge, q.

After applying force $\frac{dq}{dt} = i$

$emf = V$ } electric circuit

$V = IR$ }

$\boxed{mmf = \phi S}$ — magnetic circuit.

(or) $\boxed{mmf = Hl}$

$mmf = \phi S$

$S = \frac{l}{A \mu_0 \mu_r}$

$mmf = \frac{\phi}{A} \frac{l}{\mu_0 \mu_r}$

$mmf = B \frac{l}{\mu_0 \mu_r}$

$mmf = \frac{Bl}{\mu}$

$B \propto H$

$B = \mu H$

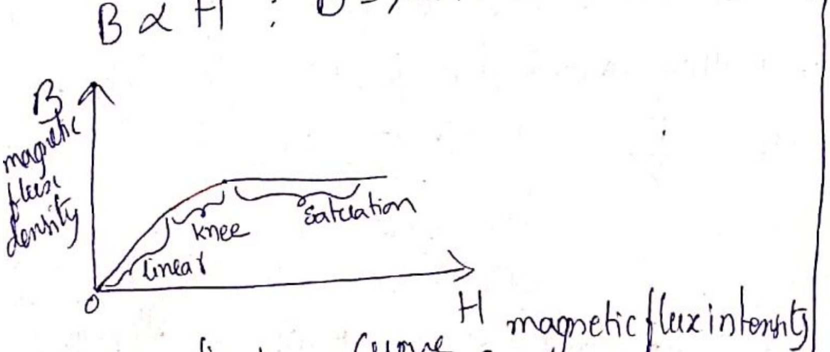
$mmf = \frac{\cancel{\mu} H l}{\cancel{\mu}}$

$\boxed{mmf = Hl}$

$mmf = \frac{NI}{l}$

BH curve

$B \propto H \; : \; B = \mu H$



Magnetization ~~curve~~ curve (BH curve)

B H curve is the property of magnetic material.

(5) **Reluctance** (R) Opposition offered by magnetic material to the flow of magnetic flux.

Reluctance is same as that of in electric circuit as Resistor but in magnetic circuit it is Reluctance (S)

$$S = \frac{l}{A\mu} = \frac{l}{A\mu_0\mu_r} \text{ (Ampere turns/wb)}$$

(6) **Permeability : μ :** How much flux can be taken in magnetic material that figure of merit is Permeability : μ

$$\boxed{\mu = \mu_0\mu_r}$$

μ_0 - Permeability ~~of air~~ / Absolute permeability

μ_r - Relative permeability of medium

μ_0 - is fixed for [illegible] $4\pi \times 10^{-7}$

Permitivity in electric ckt

$\varepsilon = \varepsilon_0\varepsilon_r$

μ_r - Relative permeabity of medium

having value = 1 { only for air (or) Vaccuum }

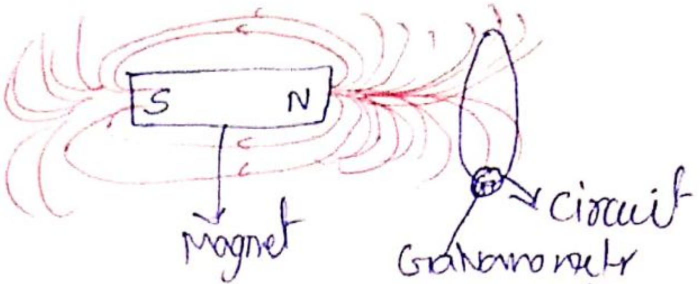
(7) **Permiance:** Reciprocal of Reluctance i.e. conductivity of flux

$$P = \frac{1}{S} = \frac{A\mu}{l} = \frac{A\mu_0\mu_r}{l} \text{ (wb/Amper turn)}$$

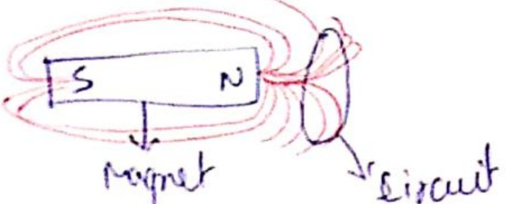
Faraday's law of electromagnetic ~~[illegible]~~ Induction.

First Law: States that whenever magnetic flux linked with a circuit changes, induced e.m.f. is produced.

No emf if the circuit is stable.



North to South lines are moving.
If a circuit crosses no. of field lines then e.m.f. is induced.



As circuit is moving towards magnet more no. of field lines crosses the circuit hence more e.m.f is induced.

As circuit is moving away from magnet less no. of field lines crosses the circuit. hence less e.m.f is induced

To identify the magnitude of induced emf ~~in~~ is determined by Faraday's second law. How much emf induced is by Faraday's second law

Second Law: The magnitude of induced e.m.f is directly proportional to the time rate of change in magnetic flux linked with the ckt.

(or)

The emf induced across the coil is equal to the rate of change of flux in the coil.

$\boxed{e = -N\frac{d\phi_B}{dt}}$ · negative sign shows that emf induced always oppose the change in flux.

$e = \frac{\phi_2 - \phi_1}{dt}$ If $\phi_1 = 20$ with in 5 sec. ϕ changes to 30 then induced emf in galvanometr is $e = \frac{30-20}{5} = \frac{10}{5} = 2V$

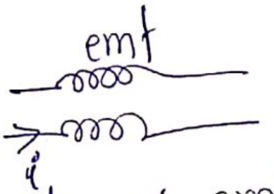
N is no. of turns should be equal to constant 1

$e = -\frac{d\phi_B}{dt}$. – sign specifies "Lenz's law"

Self Inductance:

Inductance:- It is the property of the electric conductor by which the change in current produces an emf.

The rate of change of current in this coil produces the emf. within same coil is "Self Inductance"



The rate of change of current in one coil produces emf in other coil is "Mutual Inductance"

Self Inductance

$v \propto \frac{di}{dt}$

$v = -L\frac{di}{dt}$ ⇒ Lenz's Law - ①

L - self Inductance.

$\frac{di}{dt}$ $v = -N\frac{d\phi}{dt}$ - ②

① = ②

$\cancel{-}L\frac{di}{\cancel{dt}} = \cancel{-}N\frac{d\phi}{\cancel{dt}}$ $\boxed{L = \frac{Nd\phi}{di}}$ magnitude of self Inductance.

$L\,i = N\phi$ (or)

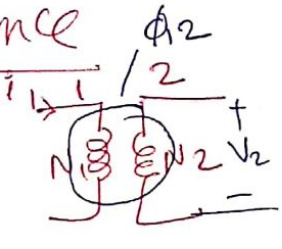
$N\phi$ - flux linkage ψ

$$\boxed{L = \frac{N\phi}{i} = \frac{\psi}{i}}$$

Inductance interms of magnetic flux.

Lenz's law:- The generated emf (or) voltage opposes the rate of change of current through which it been generated

Mutual Inductance



$v_2 = -N_2\frac{d\phi_{12}}{dt}$ - ①

$v_2 \propto \frac{di_1}{dt}$

$v_2 = -M\frac{di_1}{dt}$ - ②

① = ②

$\cancel{-}M\frac{di_1}{\cancel{dt}} = \cancel{-}N_2\frac{d\phi_{12}}{\cancel{dt}}$

$\boxed{M = \frac{N_2 d\phi_{12}}{di_1}}$

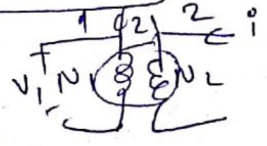
⇒ $M di_1 = N_2 d\phi_{12}$

when the coils are linked with air as medium, M is $M = N_2\frac{\phi_{12}}{i_1}$

$M i_1 = N_2\phi_{12}$ ✓

$\boxed{M = \frac{N_2\phi_{12}}{i_1}}$. Mutual inductance in coil 2 with i in coil 1

lly.



$\boxed{M = \frac{N_1\phi_{21}}{i_2}}$

$$\boxed{M = \frac{N_2 \phi_{12}}{i_1} = \frac{N_1 \phi_{21}}{i_2}}$$

So Mutual inductance is the bilateral property of the linked coils.

Mutual inductance is a property which is associated mutually with two (or) more coils that are physically close together.

Mutual inductance results from a slight extension of self inductance. i.e. a current flowing in ~~one~~ first coil establishes a magnetic flux about that coil and also about a second coil which is sufficiently close to first coil.

Coefficient of coupling (K):

It is defined as the fraction of total flux that links the coils.

i.e. $\boxed{K = \frac{\phi_{12}}{\phi_1} = \frac{\phi_{21}}{\phi_2}}$

since $\phi_{12} < \phi_1$ & $\phi_{21} < \phi_2$, the maximum value of "K" is unity

we know that

$$M = N_2 \frac{\phi_{12}}{i_1} \quad -① \quad ; \quad M = N_1 \frac{\phi_{21}}{i_2} \quad -②$$

$$① \times ② \quad M^2 = N_1 N_2 \frac{\phi_{12}\phi_{21}}{i_1 i_2} = N_1 N_2 \frac{\phi_{12}}{\phi_1} \frac{\phi_{21}}{\phi_2} \times \frac{\phi_1 \phi_2}{i_1 i_2} \qquad \Big| \; K = \frac{\phi_{12}}{\phi_1} = \frac{\phi_{21}}{\phi_2}$$

$$M^2 = \frac{N_1 N_2 \, K\phi_1 \, K\phi_2}{i_1 i_2} \; ; \quad M^2 = K^2 N_1 \frac{\phi_1}{i_1} N_2 \frac{\phi_2}{i_2} \qquad \Big| \; L_1 = \frac{N\phi}{i}$$

$$M^2 = K^2 L_1 L_2$$

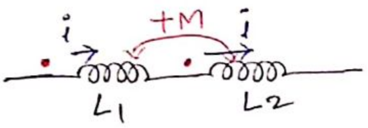
$$M = K\sqrt{L_1 L_2} \; ; \quad X_M = K\sqrt{X_1 X_2}$$

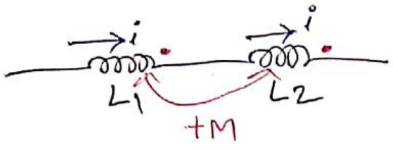
$$\boxed{K = \frac{M}{\sqrt{L_1 L_2}}}$$

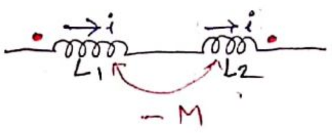
Dot convention

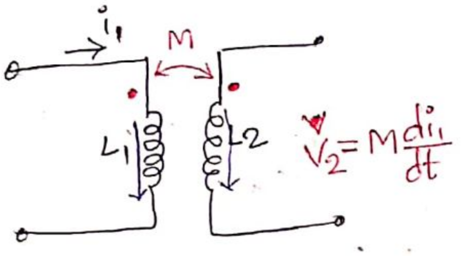
- → To determine the relative polarity of the induced voltage in the coupled ~~circuit~~ coil, the coils are marked with dots.
- → On each coil, a dot is placed at the terminals which are instantaneously of the same polarity on the basis of Mutual inductance.
- → When the currents through each of the mutually coupled coils are going away from the dot (or) towards the dot, the Mutual inductance is "positive"
- → When the current through the coil is leaving the dot for one coil ~~&~~ entering the other, the Mutual inductance is "Negative"

→

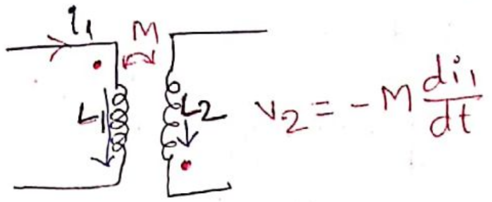




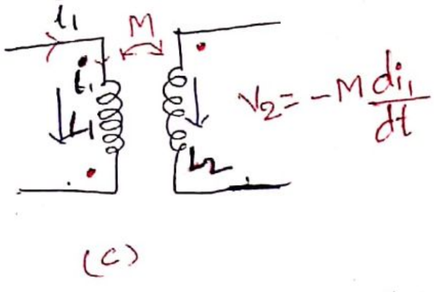




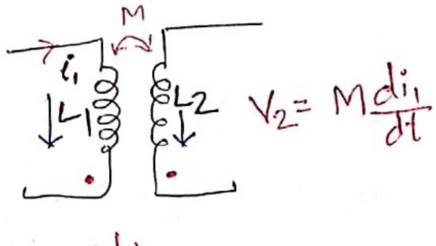
(a)



(b)



(c)



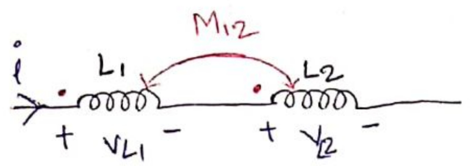
(d)

(a) Current entering the dotted terminal of one coil produces a voltage that is sensed positively at the dotted terminal of the second coil.

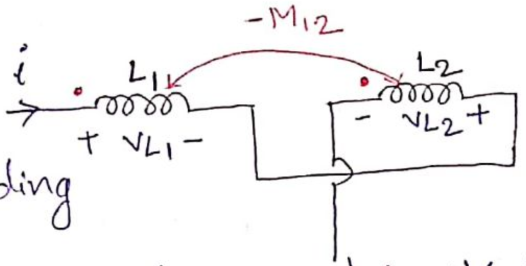
(d) Current entering the undotted terminal of one coil produces a voltage that is sensed positively at the undotted terminal of the second coil.

Analysis of Series and parallel magnetic circuits

Series magnetic circuits



(a) Mutually coupled coils in series adding
(Flux of both the coils mutually assist eachother)



(b) Mutually coupled coils in series opposition.
(Flux of both coils mutually oppose each other)

For fig.(a)

Let two coils of self-inductance L_1 & L_2 are connected in series, when a current i flows through them, the voltage induced in coil 1 is v_{L_1} & that in coil-2 is v_{L_2}.

Let M_{12} be the Mutual inductance.

$$v_{L_1} = L_1 \frac{di}{dt} + M_{12}\frac{di}{dt} = (L_1 + M_{12})\frac{di}{dt}$$

$$v_{L_2} = L_2 \frac{di}{dt} + M_{12}\frac{di}{dt} = (L_2 + M_{12})\frac{di}{dt}$$

$$v_L = v_{L_1} + v_{L_2} = (L_1 + M_{12})\frac{di}{dt} + (L_2 + M_{12})\frac{di}{dt} = \frac{di}{dt}(L_1 + L_2 + 2M_{12})$$

$\therefore M = M_{12}$

$$\boxed{v_L = (L_1 + L_2 + 2M)\frac{di}{dt}}$$ series adding.

total inductance $L = L_1 + L_2 + 2M$

For fig(b) $v_{L_1} = L_1 \frac{di}{dt} - M_{12}\frac{di}{dt} = (L_1 - M_{12})\frac{di}{dt}$

$$v_{L_2} = L_2 \frac{di}{dt} - M_{12}\frac{di}{dt} = (L_2 - M_{12})\frac{di}{dt}$$

$$v_L = v_{L_1} + v_{L_2} = (L_1 - M_{12})\frac{di}{dt} + (L_2 - M_{12})\frac{di}{dt}$$

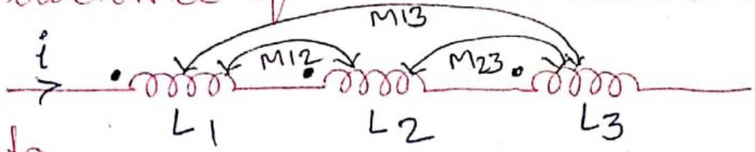
$$= \frac{di}{dt}(L_1 + L_2 - 2M_{12})$$

$M_{12} = M$

$$\boxed{v_L = (L_1 + L_2 - 2M)\frac{di}{dt}}$$ series opposition.

Total inductance $L = L_1 + L_2 - 2M$

(Q1) Find the total inductance of the three series connected coupled coils with the following given data.



$L_1 = 1H$; $L_2 = 2H$; $L_3 = 5H$; $M_{12} = 0.5H$; $M_{23} = 1H$; $M_{13} = 1H$.

For coil-1, $L_1 + M_{12} + M_{13} = 1 + 0.5 + 1 = 2.5H$

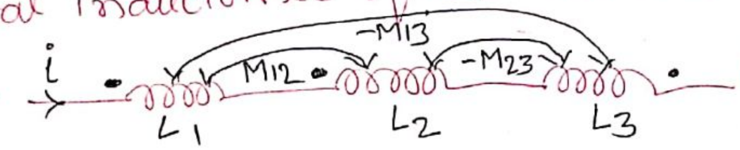
coil-2. $L_2 + M_{23} + M_{12} = 2 + 1 + 0.5 = 3.5H$

coil-3, $L_3 + M_{13} + M_{23} = 5 + 1 + 1 = 7H$

The net inductance, $L = (L_1 + M_{12} + M_{13}) + (L_2 + M_{23} + M_{12}) + (L_3 + M_{13} + M_{23})$

$$\boxed{L = 2.5 + 3.5 + 7 = 13H}$$

(Q2) Find the total inductance of the three series connected coupled coils for same data.



For coil-1 $L_1 + M_{12} - M_{13} = 1 + 0.5 - 1 = 0.5H$

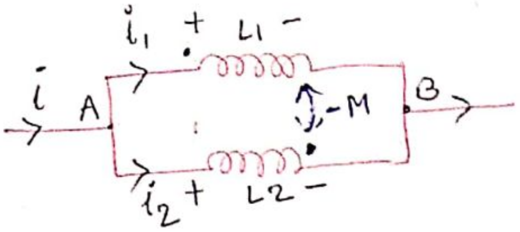
coil-2 $L_2 + M_{12} - M_{23} = 2 + 0.5 - 1 = 1.5H$

coil-3 $L_3 - M_{23} - M_{13} = 5 - 1 - 1 = 3H$

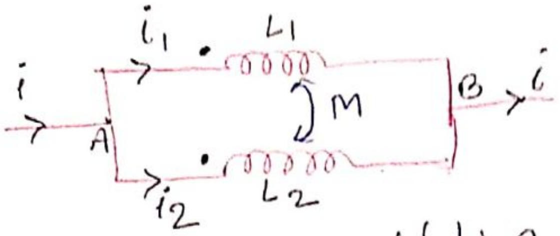
Net inductance $= L = (L_1 + M_{12} - M_{13}) + (L_2 + M_{12} - M_{23}) + (L_3 - M_{23} - M_{13})$

$$\boxed{L = 0.5 + 1.5 + 3 = 5H}$$

Parallel Magnetic Circuit



Parallel opposition.



parallel Addition

Let L_1 & L_2 = self inductance of two coils connected in parallel

M = Co-efficient of mutual inductance.

i = Main supply current

i_1 & i_2 = Branch currents

$$i = i_1 + i_2$$

$$\frac{di}{dt} = \frac{di_1}{dt} + \frac{di_2}{dt} \quad - ①$$

In each coil both self & mutually induced emfs are produced, since the coils are in parallel, these e.m.f s are equal.

For a case when self induced emf assist the mutually induced emf we get

$$e = L_1\frac{di_1}{dt} + M\frac{di_2}{dt} = L_2\frac{di_2}{dt} + M\frac{di_1}{dt}$$

$$L_1\frac{di_1}{dt} + M\frac{di_2}{dt} = L_2\frac{di_2}{dt} + M\frac{di_1}{dt}$$

$$L_1\frac{di_1}{dt} - M\frac{di_1}{dt} = L_2\frac{di_2}{dt} - M\frac{di_2}{dt}$$

$$(L_1 - M)\frac{di_1}{dt} = (L_2 - M)\frac{di_2}{dt}$$

$$\frac{di_1}{dt} = \left(\frac{L_2 - M}{L_1 - M}\right)\frac{di_2}{dt} \quad - ②$$

substitute ② in ①

$$\frac{di}{dt} = \left(\frac{L_2 - M}{L_1 - M}\right)\frac{di_2}{dt} + \frac{di_2}{dt}$$

$$\frac{di}{dt} = \left[\frac{L_2 - M}{L_1 - M} + 1\right]\frac{di_2}{dt} \quad - ③$$

If L is equivalent inductance, then

$$e = L\frac{di}{dt}$$

= induced emf in the Parallel combination

= induced e.m.f in any one coil

$$= L_1\frac{di_1}{dt} + M\frac{di_2}{dt}$$

$$\Rightarrow \frac{di}{dt} = \frac{1}{L}\left[L_1\frac{di_1}{dt} + M\frac{di_2}{dt}\right] - ④$$

Substitute ② in ④

$$\frac{di}{dt} = \frac{1}{L}\left[L_1\left(\frac{L_2 - M}{L_1 - M}\right)\frac{di_2}{dt} + M\frac{di_2}{dt}\right]$$

$$\frac{di}{dt} = \frac{1}{L}\left[L_1\left(\frac{L_2 - M}{L_1 - M}\right) + M\right]\frac{di_2}{dt} \quad - ⑤$$

③ = ⑤

$$\cancel{d}\left[\frac{L_2 - M}{L_1 - M} + 1\right]\cancel{\frac{di_2}{dt}} = \frac{1}{L}\left[L_1\left[\frac{L_2 - M}{L_1 - M}\right] + M\right]\cancel{\frac{di_2}{dt}}$$

$$\frac{L_2-M}{L_1-M}+1=\frac{1}{L}\left[L_1\left(\frac{L_2-M}{L_1-M}\right)+M\right]$$

$$\frac{L_2-M+L_1-M}{L_1-M}=\frac{1}{L}\left[\frac{L_1L_2-L_1M}{L_1-M}+M\right]$$

$$\frac{L_1+L_2-2M}{L_1-M}=\frac{1}{L}\left[\frac{L_1L_2-\cancel{L_1M}+\cancel{L_1M}-M^2}{L_1-M}\right]$$

$$\frac{L_1+L_2-2M}{\cancel{L_1-M}}=\frac{1}{L}\left[\frac{L_1L_2-M^2}{\cancel{L_1-M}}\right]$$

$$\boxed{L=\frac{L_1L_2-M^2}{L_1+L_2-2M}}$$

Parallel Adding, two fluxes assist eachother. M is +ve.

When two fluxes oppose each other M is −ve then

$$L=\frac{L_1L_2-(-M)^2}{L_1+L_2-2(-M)}$$

$$\boxed{L=\frac{L_1L_2-M^2}{L_1+L_2+2M}}$$

Parallel opposition.

IV Transmission Lines-1

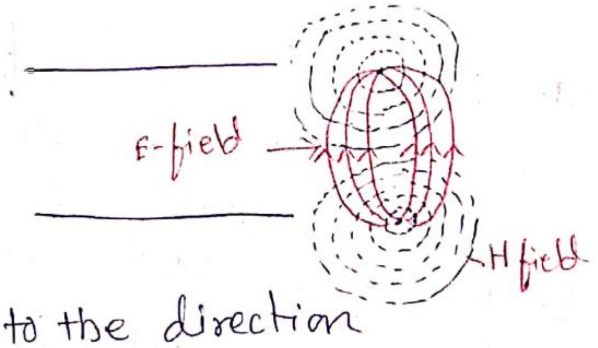
Energy can be transmitted either by the radiation of free electromagnetic waves as in the radio or it can be constrained to move or carried in various conductor arrangement known as "Transmission Line".

Thus a transmission line is a conductive method of guiding electrical energy from one place to another. Transmission lines are employed, not only to transmit energy, but also as circuit elements like inductors, capacitors resonant circuits, filters, transformers and even insulators at very high frequencies. They are also used as measuring devices and as an aid to obtain impedance matching etc.

Basically there are four types of transmission lines:
1. Parallel wire type 2. Coaxial 3. Wave guide 4. Optical fibers.

1. Parallel wire type: A common form of transmission line also known as open wire line because of its construction.

Electric energy propagating through these lines set up electric fields between line conductors. These fields are at right angles to each other and to the direction of propagation and are shown in figure.



This type of energy transmission is commonly known as "Transverse electromagnetic mode of propagation".

②

Advantages: 1. Easy to construct and are cheaper, capable of handling high power

2. Since insulation between line conductors is normally air, the dielectric loss is extremly small.

Disadvantages: 1. There is significant energy loss due to radiation.

2. Not suitable for frequencies above 100MHz, because it will generate skin effect

Applications: 1. They are commonly employed as Telephone lines, telegraphy line and power lines.

2. Short runs of these lines are also used as antenna feeders and impedance matching purpose.

Characteristic impedance

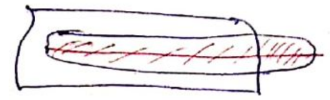
$Z_0 = 276 \log_{10} \frac{2D}{d}$

D - distance b/w two wires, d = diameter of wire

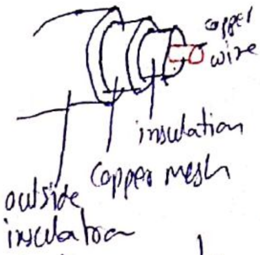
2. Coaxial type:

In order to avoid severe radiation losses taken place in open wire lines at frequencies beyond 100MHz, a closed field configuration is employed in coaxial cable by surrounding the inner conductor with an outer cylindrical hallow conductor.

The dielectric may be solid or gaseous.



Advantages:



1. Electric and magnetic fields remain confined within the outer conductor and cannot leak into free space.
2. Radiation is totally eliminated.
3. The outer conductor also provides a highly effective electromagnetic shielding against external electromagnetic signals usually have a continuous dielectric, protected from dust, rust etc.
4. Flexibility, less space occupied.

Disadvantags

1. costlier as compared to open wire lines, Difficult to designs
2. Losses in the dielectric increases as the signal frequency is increased. These losses becomes excessive at frequencies above 1GHz by and which, these cannot be used. 3. handles low power transmissions.

Applications: 1. Coaxial cables are extensively used in the frequency range, extending upto 1GHz.

2. Computer network (e.g. Ethernet) connections.
3. Digital audio.
4. Distribution of cable television signals.

NOTE: The most common impedances that are widely used are 50 (or) 52Ω and 75Ω.

Characteristic impedance $Z_0 = \overset{138}{\boxed{233}}\log_{10}\frac{D}{d}$

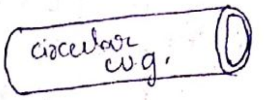
D - internal wire diameter of the braded wire.

d - diameter of the inner conductor.

3. waveguide:

A transmission line consisting of a suitable shaped hallow conductor, which may be filled with a dielectric material and is used to guide electromagnetic waves of UHF propagated along its length is called a "waveguide"

The transmitted wave is reflected back by the internal walls of waveguide and the resulting distribution associated with the wave causes the transmission mode.



a) TE wave (Transverse electric wave)

b) TM wave (Transverse magnetic wave)


Rectangular waveguide

c) TEM wave (Transverse electromagnetic wave)

A waveguide in which no reflected wave occurs at any of the transverse section is called a "matched waveguide"

Advantages: 1. waveguides are simpler to manufacture.

2. Higher power handling capacity than coaxial cable.
3. Power loss is low due to the fact that propagation of energy is by means of reflection from the walls.
4. lower attenuation for given cutoff wavelength.
5. Easy to install waveguides in a microwave transmission systems due to simpler structure on both the ends.

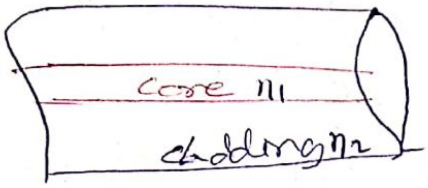
Disadvantages:

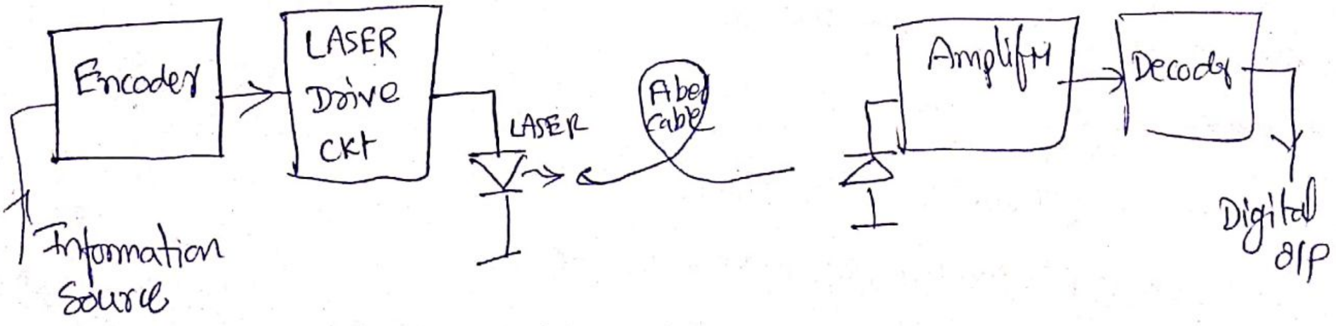
1. It is not suitable for operations at lower frequencies due to increased dimensions.
2. It is very bulky in size and weight
3. It is not economical
4. Narrow band of operation
5. TEM mode of propagation is not possible in the waveguide.

Applications:

1. waveguides are widely used in communication network for the transmission of EM waves.

4. Optical fibres:





optical fiber communication is interms of light. sothat information is converted into light using optical source (LED or LASER) and inserted to an optical fiber and at the end of optical fiber photo detector is used to convert light information into original ~~transmit~~ electrical data.

Advantages

(1) High Bandwidth
(2) Small size and weight
(3) Electrical isolation
(4) Immunity to interference and cross talk.
(5) Signal secuirity
(6) Low transmission loss
(7) Ruggedness and flexibility
(8) system reliability and ease of maintenance
(9) Potential low cost

Fundamental Quantities

If a long line consisting of two parallel uniform conductors is carrying current, there is a magnetic field around conductors and voltage drop along them.

The magnetic field, which is proportional to current, indicates that the line has series inductance 'L', the voltage drop indicates the presence of series resistance 'R'.

Voltage applied across the conductor produces an electric field between the conductor and charges on them. This indicates that the lines contains shunt capacitance 'C' and since capacitance is never lossless (or) perfect, it will have some shunt conductance 'G', as well.

When R, L, C, and G are uniformly distributed along the entire length of a transmission line. it is termed as "uniform Transmission line".

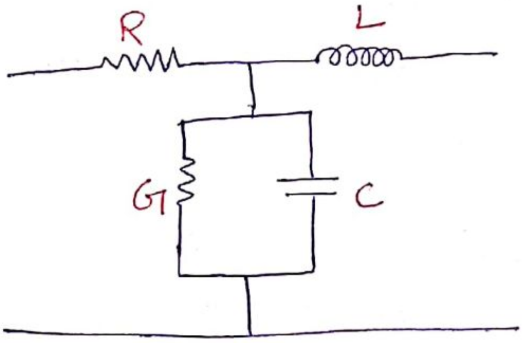


Fig: Equivalent circuit of a unit length of transmission line.

Primary constants of Transmission Line:

The four line parameters R, L, C, and G are termed as primary constants of the transmission line.

1. Resistance 'R' is defined as loop resistance per unit length of line. Thus, it is sum of resistance of both the wires for unit line length. Its unit is ohms per km.
2. Inductance 'L' is defined as loop inductance per unit length of line. Thus it is sum of inductance of both wires for unit line length. Its unit is Henrys per km.
3. Conductance 'G' shunt conductance between the two wires per unit line length. Its unit is mhos per km.

4. Capacitance 'C' Shunt capacitance between the two wires per unit line length. Its unit is Farad per km.

Since G and C are present between the two wires, the loop notation is not necessary.

NOTE: Although R, L, C & G are referred to as primary constants but in general all will vary with frequency. However, for the purpose of transmission line theory, they will be assumed to be independent of frequency.

Thus the series impedance 'Z' and shunt admittance 'Y' of the line per unit length are

$$\boxed{Z = R+j\omega L \; ; \; Y = G+j\omega C}$$

Transmission Line Equations

Let the line be for length 'l' and primary constants of the line be R, L, C and G per km. Assume, they do not vary with fre.

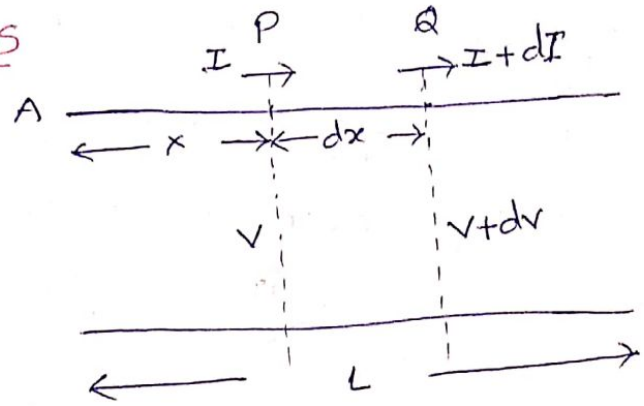


Fig: Short section PQ distance x from the sending end of transmission line.

Consider a short section of line PQ of length dx, at a distance x from the sending end A as shown in figure. By making dx very small, the current may be considered constant for voltage calculations and voltage constant for current calculations.

At P, Let the voltage be V and current I

At Q, the voltage will be V+dV and current I+dI

The series impedance of small section dx will be $(R+j\omega L)dx$

Ily the shunt admittance of small section dx will be $(G+j\omega C)dx$.

Since dx is very small, the voltage drop from P to Q may be considered to be due to the current I flowing through the series Impedance $(R+j\omega L)dx$. The decrease in current from P to Q may be considered to be due to the voltage v applied to the shunt admittance $(G+j\omega C)dx$.

Potential difference between P and Q is due to current flowing through series impedance $(R+j\omega L)dx$

Thus $$v-(v+dv) = I(R+j\omega L)dx$$

$$\cancel{v}-\cancel{v}-dv = (R+j\omega L)I dx$$

$$\boxed{-\frac{dv}{dx} = (R+j\omega L)I} \quad \text{—} ①$$

current difference between P and Q is due to voltage applied to shunt admittance $(G+j\omega C)dx$

Thus $$I-(I+dI) = v(G+j\omega C)dx$$

$$\cancel{I}-\cancel{I}-dI = (G+j\omega C)v dx$$

$$\boxed{-\frac{dI}{dx} = (G+j\omega C)v} \quad \text{—} ②$$

To make only one independant variable in ① & ② differential I - ① w.r.t to x and differential v - ② w.r.t. x

From ①
$$-\frac{d^2v}{dx^2} = (R+j\omega L)\frac{dI}{dx} \text{—} ③ ;$$
From ②
$$-\frac{d^2I}{dx^2} = (G+j\omega C)\frac{dv}{dx} \text{—} ④$$

Substitute ② in ③ and ① in ④

$$\cancel{-}\frac{d^2v}{dx^2} = (R+j\omega L)(\cancel{-}(G+j\omega C)v))$$

$$\frac{d^2v}{dx^2} = (R+j\omega L)(G+j\omega C)v$$

$$\cancel{-}\frac{d^2I}{dx^2} = (G+j\omega C)(\cancel{-}(R+j\omega L)I))$$

$$\frac{d^2I}{dx^2} = (R+j\omega L)(G+j\omega C)I$$

Assume $(R+j\omega L)(G+j\omega C) = P^2$, complex constant for a given fre.

$$\boxed{\frac{d^2v}{dx^2} = P^2 v} \text{—} ⑤ \quad \text{and} \quad \boxed{\frac{d^2I}{dx^2} = P^2 I} \text{—} ⑥$$

⑤ & ⑥ are referred to as differential equations of the transmission line, fundamental to circuit of distributed constants. These equations are standard linear differential equations with constant coefficients whose solutions are

$$V = ae^{Px} + be^{-Px} \quad — ⑦$$

$$I = ce^{Px} + de^{-Px} \quad — ⑧$$

a, b are constants with the dimensions of voltage.
c, d are constants with the dimensions of current.

Substituting the values of $e^{Px} = \cosh Px + \sinh Px$; $e^{-Px} = \cosh Px - \sinh Px$ in ⑦ & ⑧

$$V = a(\cosh Px + \sinh Px) + b(\cosh Px - \sinh Px)$$

$$V = a\cosh Px + a\sinh Px + b\cosh Px - b\sinh Px$$

$$= a\cosh Px + b\cosh Px + a\sinh Px - b\sinh Px$$

$$= (a+b)\cosh Px + (a-b)\sinh Px$$

$$V = (a+b)\cosh Px + (a-b)\sinh Px$$

$$\boxed{V = A\cosh Px + B\sinh Px} \quad — ⑨$$

$A = a+b$
$B = a-b$

$$I = c(\cosh Px + \sinh Px) + d(\cosh Px - \sinh Px)$$

$$= c\cosh Px + c\sinh Px + d\cosh Px - d\sinh Px$$

$$= c\cosh Px + d\cosh Px + c\sinh Px - d\sinh Px$$

$$= (c+d)\cosh Px + (c-d)\sinh Px$$

$$\boxed{I = C\cosh Px + D\sinh Px} \quad — ⑩$$

$C = c+d$
$D = c-d$

Instead of four constants A, B, C and D ⑨ & ⑩ can be simplified to only two unknown constants, by substituting the values of V from ⑨ in ①

$$-\frac{d}{dx}(A\cosh Px + B\sinh Px) = (R+j\omega L)I$$

$$-(A.P\sinh Px + B.P\cosh Px) = (R+j\omega L)I$$

$$-P(A\sinh Px + B\cosh Px) = (R+j\omega L)I \quad — ⑪$$

we knew that $p^2 = (R+j\omega L)(G+j\omega C)$

$$p = \sqrt{(R+j\omega L)(G+j\omega C)}$$

Substitute p in ⑪

$$-\frac{\sqrt{(R+j\omega L)(G+j\omega C)}}{(R+j\omega L)}(A\sinh Px + B\cosh Px) = I$$

$$-\frac{\sqrt{(\cancel{R+j\omega L})}\sqrt{(G+j\omega C)}}{\sqrt{(\cancel{R+j\omega L})}\sqrt{(R+j\omega L)}}(A\sinh Px + B\cosh Px) = I$$

$$-\sqrt{\frac{G+j\omega C}{R+j\omega L}}(A\sinh Px + B\cosh Px) = I$$

Therefore $\boxed{I = -\frac{1}{Z_0}(A\sinh Px + B\cosh Px)}$ - ⑫

where $Z_0 = \sqrt{\frac{R+j\omega L}{G+j\omega C}}$, which is also a complex constant for a given frequency.

⑨ & ⑫ may be written in the form

$$\boxed{V = A\cosh Px + B\sinh Px} \quad — ⑬$$

$$\boxed{I = -\frac{1}{Z_0}(A\sinh Px + B\cosh Px)} \quad — ⑭$$

Again these equations can also be expressed in exponential form. substituting the values of

$$\cosh Px = \frac{e^{Px} + e^{-Px}}{2} \; ; \; \sinh Px = \frac{e^{Px} - e^{-Px}}{2} \text{ in } ⑭$$

and writting equation ⑦ without any change, we get

$$\boxed{V = ae^{Px} + be^{-Px}} \quad — ⑮$$

$$\boxed{I = \frac{1}{Z_0}(be^{-Px} - ae^{-Px})} \quad - ⑯$$

where a and b are old constants described earlier.

The relations between the old and new constants are

$a+b=A$ and $a-b=B$ (or) $a = \frac{A+B}{2}$ and $b = \frac{A-B}{2}$

⑬, ⑭, ⑮ & ⑯ are general equations of a transmission line.

Secondary constants of Transmission Line

1. Propagation Constant, P. = $\sqrt{(R+j\omega L)(G+j\omega C)} = \sqrt{Z \times Y}$

2. Characteristic Impedance, $Z_0 = \sqrt{\frac{R+j\omega L}{G+j\omega C}} = \sqrt{\frac{Z}{Y}}$

1. Propagation constant:

The propagation constant per unit length of a uniform line may be defined as the natural logarithm of the steady state vector ratio of the current (or) voltage at any point, to that at a point unit distance further from the source, when the line is infinitely long.

$$P = \log_e \frac{I_1}{I_2} = \log_e \frac{V_1}{V_2}$$

(i) P interms of sending end current/voltage to the receiving end current/Voltage.

$$\boxed{P = 20\log_{10}\frac{I_S}{I_R} = 20\log_{10}\frac{V_S}{V_R}}$$

(ii) Propagation constant P interms of Primary constants R, L, C and G

$$\boxed{P = \sqrt{(R+j\omega L)(G+j\omega C)}}$$

(iii) P interms of Attenuation (α) and Phase constant (β).

$$\boxed{P = \alpha + j\beta}$$

Attenuation constant, α: determines the reduction (or) attenuation in voltage and current along the line and higher its value the quicker the reduction. Its unit is Neper/km.

Relation between neper and decibel is 1 neper = 8.686 db

Phase constant, β: determines the variation in phase position of voltage and current along the line. radians/km.

Relation between radian and degrees is 1 rad = 57.3°

NOTE: Propagation constant should have a positive angle when expressed in its polar form, hence α and β both should be +ve. However, in some open and short circuit measurements. Pl is found to have negative angle, which has to be converted to positive by adding the least multiple of 2π to β.

Phase constant when multiplied by the length of transmission line is termed as electrical length of the line. Ily attenuation constant when multiplied by the length of line is termed as total attenuation (or) simply "Line Attenuation."

$$P = \alpha + j\beta = \sqrt{(R+j\omega L)(G+j\omega C)}.$$

Squaring both sides and equating the real parts we get,

$$(\alpha+j\beta)^2 = (R+j\omega L)(G+j\omega C)$$

$$\alpha^2+(j\beta)^2+2j\alpha\beta = RG+\underline{j\omega RC}+\underline{j\omega LG}+j^2\omega^2LC$$ only real parts.

$$\boxed{\alpha^2-\beta^2 = RG-\omega^2LC} \quad — ①$$

Also $|P| = \sqrt{\alpha^2+\beta^2}$

$$\boxed{\alpha^2+\beta^2 = \sqrt{(R^2+\omega^2L^2)(G^2+\omega^2C^2)}} \quad — ②$$

① + ②

$$\alpha^2-\cancel{\beta^2}+\alpha^2+\cancel{\beta^2} = RG-\omega^2LC+\sqrt{(R^2+\omega^2L^2)(G^2+\omega^2C^2)}$$

$$2\alpha^2 = (RG-\omega^2LC)+\sqrt{(R^2+\omega^2L^2)(G^2+\omega^2C^2)}$$

$$\boxed{\alpha = \sqrt{\tfrac{1}{2}\left[(RG-\omega^2LC)+\sqrt{(R^2+\omega^2L^2)(G^2+\omega^2C^2)}\right]}} \quad — ③$$

① − ②

$$\cancel{\alpha^2}-\beta^2-\cancel{\alpha^2}-\beta^2 = (RG-\omega^2LC)-\sqrt{(R^2+\omega^2L^2)(G^2+\omega^2C^2)}$$

$$\boxed{\beta = \sqrt{\tfrac{1}{2}\left[(\omega^2LC-RC)-\sqrt{(R^2+\omega^2L^2)(G^2+\omega^2C^2)}\right]}} \quad — ④$$

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Characteristic Impedance, Z_0

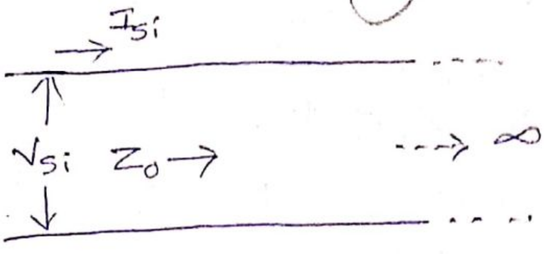


fig: Infinite line.

$$I = I_{si} e^{-Px}$$

$$V = V_{si} e^{-Px}$$

$$-\frac{dV}{dx} = (R+j\omega L) I.$$

$$-\frac{d}{dx} V_{si} e^{-Px} = (R+j\omega L) I_{si} e^{-Px}$$

$$P V_{si} e^{-Px} = (R+j\omega L) I_{si} e^{-Px}$$

$$\frac{V_{si}}{I_{si}} = \frac{R+j\omega L}{P}$$

we knew that $P = \sqrt{(R+j\omega L)(G+j\omega C)}$

$$\frac{V_{si}}{I_{si}} = \frac{R+j\omega L}{\sqrt{(R+j\omega L)(G+j\omega C}} = \frac{\sqrt{\cancel{R+j\omega L}}\sqrt{(R+j\omega L)}}{\sqrt{\cancel{R+j\omega L}}\sqrt{G+j\omega C}}$$

$$\frac{V_{si}}{I_{si}} = Z_0$$

$$\boxed{Z_0 = \sqrt{\frac{R+j\omega L}{G+j\omega C}}} \qquad Z_0 = \sqrt{\frac{Z}{Y}}$$

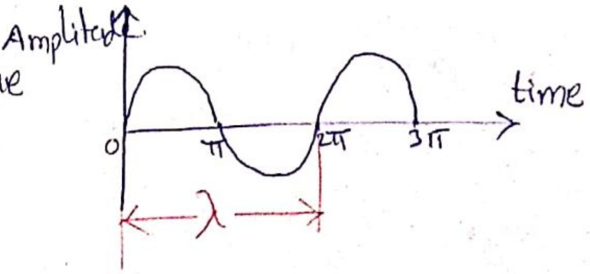
Characteristic Impedance of a uniform transmission line may be defined as the steady-state vector ratio of the voltage to the current at the input of an infinite line.

Alternately; it can simply be defined as the impedance looking into an infinite length of the line. Its unit is ohms. It is also known as "Surge Impedance".

wavelength, λ

Distance that a wave travels along the line inorder that the total shift is 2π radians Units are metre. $\beta\lambda = 2\pi$

where β is Phaseshift $\boxed{\lambda = \frac{2\pi}{\beta}}$



Phase Velocity v_p

Velocity of propagation is defined as the velocity with which a signal of single frequency propagates along the line at a particular frequency f. v_p, km/sec.

Since the change of 2π in phase angle represents one cycle in time 't' and occurs in a distance of one wave length λ, then

$\lambda = v_p \times t$, t is the time period that is the time taken by one complete cycle, and time period is the reciprocal of frequency.

$$\lambda = v_p \times \frac{1}{f} \quad ; \quad v_p = \lambda f$$

we knew that $\lambda = \frac{2\pi}{\beta}$.

$$v_p = \frac{2\pi f}{\beta} \quad ; \quad \boxed{v_p = \frac{\omega}{\beta}}$$

Group velocity: v_g

In case of a distortionless (or) lossless line β is not a constant multiple of 'ω'. As a result of this the components in a complex waveform normally shift in phase relation during propagation. This phenomenon is known as dispersion which results in distortion. When dispersion exists, the significant value of 'v_p' is often difficult to define in complex wave.

In small dispersion, a significant velocity of propagation is "group velocity." small dispersion take place when the maximum difference in the frequencies of the components in a given signal is small.

Thus group velocity is defined as the velocity of the envelope of a complex signal. 'v_g'

Let ω_1 and ω_2 be the two close angular frequencies being transmitted and β_1 and β_2 be the corresponding phase constants, then group velocity v_p will be given as

$$\boxed{v_g = \frac{\omega_2 - \omega_1}{\beta_2 - \beta_1}}$$

$$\boxed{v_g = \frac{d\omega}{d\beta}}$$

Relation between the group velocity and phase velocity.

$$v_p = \frac{\omega}{\beta}$$

Differentiating with respect to ω, we get

$$\frac{dv_p}{d\omega} = \frac{1\beta - \omega\frac{d\beta}{d\omega}}{\beta^2} = \frac{1 - \frac{\omega}{\beta}\left(\frac{d\beta}{d\omega}\right)}{\beta}$$

$$\beta . \frac{dv_p}{d\omega} = 1 - v_p\frac{1}{v_g}$$

$$\beta . \frac{dv_p}{d\omega} = 1 - \frac{v_p}{v_g}$$

$$\frac{v_p}{v_g} = 1 - \beta\frac{dv_p}{d\omega}$$

$$\boxed{v_g = \frac{v_p}{1 - \beta\frac{dv_p}{d\omega}}} \quad (\text{or}) \quad \boxed{v_g = \frac{v_p}{1 - \frac{\omega}{v_p}\left(\frac{dv_p}{d\omega}\right)}}$$

when $\frac{dv_p}{d\omega} = 0$, then $v_g = v_p$.

Infinite Line A signal fed into a line of infinite length could not reach the far end in a finite time. Consequently the condition of the far end (i.e. open and shorted termination) can have no effect at the input end.

For this reason transmission line analysis begins with an infinite line in order to separate input conditions from output conditions.

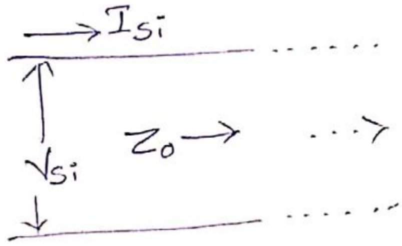


Fig. Infinite line.

When an A.C. voltage is applied to the sending end of an infinite line, a finite current will flow due to the capacitance 'c' and the leakage conductance 'G' between the two wires of the line.

The ratio of the voltage applied to the current flowing will give the input impedance of an infinite line. This input impedance is known as characteristic impedance of the line, Z_o. $Z_o = \frac{V_{si}}{I_{si}}$ — ①

V_{si} & I_{si} are respectively the sending end voltage and current of an infinite line, as shown in figure.

current at any point distance x from the sending end

$I = ce^{Px} + de^{-Px}$ — ②

At the sending end of the infinite line $x=0$ and $I = I_{si}$

$I_{si} = ce^{0} + de^{0} = I_{si} = c+d$

At the receiving end of the infinite line $x = \infty$ and $I = 0$

From ② $0 = c \times \infty + d \times 0$ $e^{\infty} = \infty$, $e^{-\infty} = 0$

$0 = c \times \infty$ Thus either $c = 0$ (or) $\infty = 0$.

But $\infty = 0$ cant possible, therefore $c = 0$.

when $c=0$, $I_{S_i} = d$

from ② $\boxed{I = I_{S_i} \bar{e}^{Px}}$ — ③

This equation gives current at any point of an infinite line.

lly the voltage at any point of an infinite line can be deduced to be $\boxed{V = V_{S_i} \bar{e}^{Px}}$ — ④

By definition of propagation constant for a unit length

$e^{P} = \frac{I_S}{I_1}$, I_1 is the current at a unit distance from sending end. Then a distance x from sending end

$\boxed{e^{Px} = \frac{I_S}{I_R}}$ — ⑤, I_R is the current at distance x.

But $P = \alpha + j\beta$

From ⑤ $\frac{I_S}{I_R} = e^{Px} = e^{\alpha x} \angle \beta x$

$$\boxed{I_R = I_S e^{-\alpha x} \angle -\beta x} \text{ — ⑥}$$

⑥ represents the equation of the current at a distance x down the line.

lly, the voltage V_R at any point x down the line is

$$\boxed{V_R = V_S e^{-\alpha x} \angle -\beta x} \text{ — ⑦}$$

Hence the graphical representation of a current and voltage at any point along the finite line is.

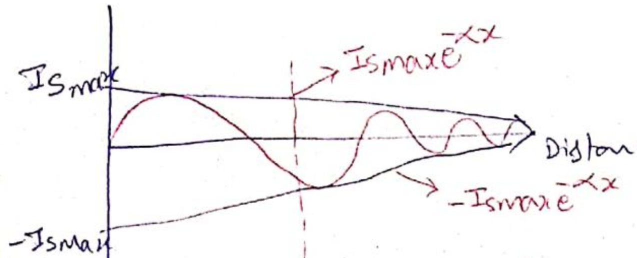


fig a. current at any point along the length of line

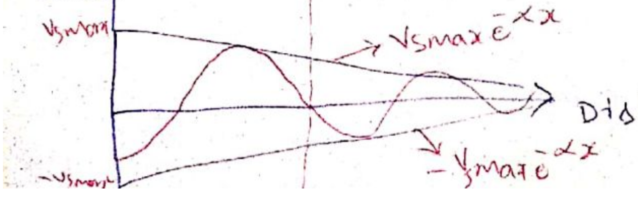
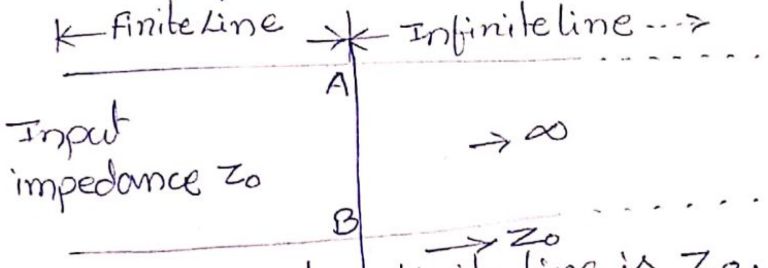


fig b. voltage at any point along the length of the line.

***Q. Prove that a finite line terminated in it's characteristic Impedance behaves as an Infinite line.**

When a finite length line and infinite lines are attached, it results in one infinite line and the input impedance of this total line is equal to the input impedance of infinite line itself.

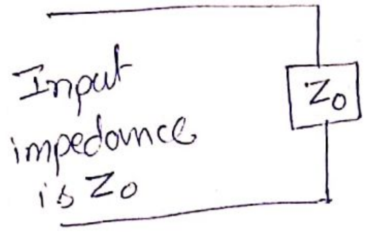


The input impedance of a finite line is Z_0.

A finite line has an input impedance Z_0, When it is terminated in Z_0 (or) a finite line terminated by it's Z_0 behaves as an infinite line.

Proof:

Consider a finite line of length 'l' is terminated in it's characteristic impedance.



Assume that V_R and I_R is the voltage and current at the termination. Impedance, $Z_0 = \frac{V_R}{I_R}$

The derived and standard equations of voltage and current is

$$V = V_s \cosh Px - I_s Z_0 \sinh Px \quad —①$$

$$I = I_s \cosh Px - \frac{V_s}{Z_0} \sin Px \quad —②$$

Substituting $x = l$, $V = V_R$ and $I = I_R$ we get

$$V_R = V_s \cosh Pl - I_s Z_0 \sinh Pl \quad —③$$

$$I_R = I_s \cosh Pl - \frac{V_s}{Z_0} \sinh Pl \quad —④$$

To obtain Z_0, dividing V_R by I_R then

$$\frac{V_R}{I_R} = Z_0 = \frac{V_s \cosh pl - I_s Z_0 \sinh pl}{I_s \cosh pl - \frac{V_s}{Z_0} \sinh pl}$$

Multiplying the numerator and denominator with Z_0 at the right hand side of the equation.

$$Z_0 = \frac{Z_0 (V_s \cosh pl - I_s Z_0 \sinh pl)}{Z_0 \left[I_s \cosh pl - \frac{V_s}{Z_0} \sinh pl \right]}$$

$$1 = \frac{V_s \cosh pl - I_s Z_0 \sinh pl}{Z_0 I_s \cosh pl - V_s \sinh pl}$$

$$Z_0 I_s \cosh pl - V_s \sinh pl = V_s \cosh pl - I_s Z_0 \sinh pl$$

$$Z_0 I_s \cosh pl + Z_0 I_s \sinh pl = V_s \cosh pl + V_s \sinh pl$$

$$Z_0 I_s \left[\cancel{\cosh pl + \sinh pl} \right] = V_s \left[\cancel{\cosh pl + \sinh pl} \right]$$

$$\boxed{Z_0 = \frac{V_s}{I_s}, \ \frac{V_s}{I_s} \text{ is input impedance } Z_{in}}$$

$$\boxed{\therefore \ Z_0 = Z_{in}}$$

The input impedance of finite line that is terminated by Z_0 is equal to the characteristic impedance of the line. The impedance of both finite and infinite line is same.

Hence, it is prooved that a finite line terminated in its characteristic impedance behaves as an infinite line.

condition for minimum attenuation

we knew that Propagation constant $P = \alpha + j\beta$

where α - Attenuation constant

$$\alpha = \sqrt{\frac{1}{2}\left[(RG-\omega^2LC)+\sqrt{(R^2+\omega^2L^2)(G^2+\omega^2C^2)}\right]}$$

β - Phase constant

$$\beta = \sqrt{\frac{1}{2}\left[(\omega^2LC-RC)-\sqrt{(R^2+\omega^2L^2)(G^2+\omega^2C^2)}\right]}$$

In addition to the frequency the value of 'α' will depend on the four primary constants (R, L, C and G).

(i) Value of L for minimum attenuation.

To determine the value of 'L' for minimum attenuation so that the other three line parameters, R, C, and G should Keep in constant including α that its L may be varied.

$$\alpha = \sqrt{\frac{1}{2}\left[(RG-\omega^2LC)+\sqrt{(R^2+\omega^2L^2)(G^2+\omega^2C^2)}\right]} \quad — ①$$

Differentiate ① w.r.t. 'L' and equating to zero, then

$$\frac{d\alpha}{dL} = \frac{1}{2}\,\frac{\frac{1}{2}\left\{\frac{2\omega^2L(G^2+\omega^2C^2)}{\sqrt{(R^2+\omega^2L^2)(G^2+\omega^2C^2)}} - \omega^2C\right\}}{\sqrt{\frac{1}{2}\left[(RG-\omega^2LC)+\sqrt{(R^2+\omega^2L^2)(G^2+\omega^2C^2)}\right]}}$$

$$\Rightarrow \frac{1}{2}\left[\frac{1}{2}\left\{\frac{2\omega^2L(G^2+\omega^2C^2)}{\sqrt{(R^2+\omega^2L^2)(G^2+\omega^2C^2)}} - \omega^2C\right\}\right] = 0$$

$$\Rightarrow \frac{\omega^2L(G^2+\omega^2C^2)}{\sqrt{(R^2+\omega^2L^2)(G^2+\omega^2C^2)}} - \omega^2C = 0$$

4.11

$$\frac{\omega^2 L\sqrt{G^2+\omega^2C^2}\times\sqrt{\cancel{G^2+\omega^2C^2}}}{\sqrt{R^2+\omega^2L^2}\sqrt{\cancel{G^2+\omega^2C^2}}} - \omega^2 C = 0$$

$$\cancel{\omega^2} L\sqrt{\frac{G^2+\omega^2C^2}{R^2+\omega^2L^2}} = \cancel{\omega^2} C$$

$$L\sqrt{G^2+\omega^2C^2} = C\sqrt{R^2+\omega^2L^2}$$

Squaring on both sides

$$L^2(G^2+\omega^2C^2) = C^2(R^2+\omega^2L^2)$$

$$L^2G^2+\cancel{L^2\omega^2C^2} = R^2C^2+\cancel{\omega^2L^2C^2}$$

$$\boxed{LG = RC} \quad (\text{or}) \quad \boxed{\frac{R}{G} = \frac{L}{C}}$$

NOTE: The above condition is also same for distortion less line, therefore the value of α, β, V_p and Z_0 the value obtain in distortion lessline is same for the line with minimum attenuation.

(i) From the condition the value of 'L' for minimum attenuation is $L = \frac{RC}{G}$ henries/km

In practice the value 'L' is lessthan the desired value. Hence the attenuation of a line can be reduced by increasing the value of 'L'

(ii) Value of 'C' for minimum attenuation, from the condition

$C = \frac{LG}{R}$ Farad/km

In practice the value of 'C' is greater than the desired value. Hence to reduce the attenuation the value of 'C' should be decrease.

iii) Value of R and G for minimum Attenuation

To determine the value of R and G there will be either R (or) G varies, then there is no minimum attenuation will occur while we are differentiating or equating to zero.

When R=0 and G=0, then the attenuation constant α is also zero. Hence the value of R and G should keep small.

Lossless Transmission Line:

A transmission line is known to be a lossless transmission line provided it satisfies the following two conditions.

(i) The conductors of the transmission line perfect ($\sigma_c = \infty$)

(ii) The dielectric medium between the transmission lines is lossless $\sigma_d = 0$.

A transmission line is also said to be lossless if R=G=0.

then propagation constant $P = \alpha + j\beta$ (or) $\gamma = \alpha + j\beta$

but $\alpha = 0$, P (or) $\gamma = j\beta = \sqrt{(R+j\omega L)(G+j\omega C)}$

$R=G=0$, $P = \sqrt{j\omega L j\omega C} = j\omega\sqrt{LC}$

$P = \alpha + j\beta = j\omega\sqrt{LC}$

$\alpha = 0$, then $\boxed{\beta = \omega\sqrt{LC}}$ Phase constant

The expression for characteristic impedance is given by

$Z_0 = \sqrt{\frac{R+j\omega L}{G+j\omega C}}$, but $R=G=0$ $\boxed{Z_0 = \sqrt{\frac{L}{C}}}$

The velocity of Propagation is given by the following relation,

$V_p = \frac{\omega}{\beta} = \frac{\omega}{\omega\sqrt{LC}}$; $\boxed{V_p = \frac{1}{\sqrt{LC}}}$

4.12

Distortionless Line

Distortionless Line

A transmission line is said to be distortionless, if it satisfies

(i) The attenuation constant(α) is independent of frequency.

(ii) The Phase constant(β) is linearly dependent on the fre.

A transmission line is also said to be distortionless if

$$\frac{R}{L}=\frac{G}{C} \quad (or) \quad LG=RC$$

Then, the expression for propagation constant P is given by

$$P=\sqrt{(R+j\omega L)(G+j\omega C)}$$

$$=\sqrt{R\left[1+\frac{j\omega L}{R}\right]G\left[1+\frac{j\omega C}{G}\right]}$$

$$=\sqrt{RG\left[1+\frac{j\omega L}{R}\right]\left[1+\frac{j\omega C}{G}\right]} \qquad \frac{R}{L}=\frac{G}{C} \quad (or) \quad \frac{L}{R}=\frac{C}{G}$$

$$=\sqrt{RG\left[1+\frac{j\omega C}{G}\right]\left[1+\frac{j\omega C}{G}\right]}$$

$$=\sqrt{RG\left[1+\frac{j\omega C}{G}\right]^2}$$

$$P=\sqrt{RG}\left[1+\frac{j\omega C}{G}\right]=\sqrt{RG}+\sqrt{RG}\,j\frac{\omega C}{G}$$

$$P=\sqrt{RG}+j\omega C\frac{\sqrt{R}\cancel{\sqrt{G}}}{\sqrt{G}\cancel{\sqrt{G}}}=\sqrt{RG}+j\omega C\frac{\sqrt{R}}{\sqrt{G}}$$

put $R=\frac{LG}{C}$ in imaginary part

$$P=\sqrt{RG}+j\omega C\sqrt{\frac{L\cancel{G}}{C\cancel{G}}}=\sqrt{RG}+j\omega\sqrt{C}\cancel{\sqrt{C}}\frac{\sqrt{L}}{\cancel{\sqrt{C}}}$$

$$\boxed{P=\sqrt{RG}+j\omega\sqrt{LC}}$$

$\alpha=\sqrt{RG}$, Attenuation constant

$\beta=\omega\sqrt{LC}$, Phase constant.

The expression for characteristic impedance is ㉔

$$Z_0 = \sqrt{\frac{R+j\omega L}{G+j\omega C}}$$

$$Z_0 = \sqrt{\frac{R(1+j\omega \frac{L}{R}}{G(1+j\omega \frac{C}{G})}} \qquad \frac{R}{L} = \frac{G}{C} \text{ (or) } \frac{L}{R} = \frac{C}{G}$$

$$Z_0 = \sqrt{\frac{R\,\cancel{(1+j\omega \frac{C}{G})}}{G\,\cancel{(1+j\omega \frac{C}{G})}}}$$

$$\boxed{Z_0 = \sqrt{\frac{R}{G}} = \sqrt{\frac{L}{C}}}$$

The velocity of propagation is given by

$$\boxed{v_p = \frac{\omega}{\beta} = \frac{\cancel{\omega}}{\cancel{\omega}\sqrt{LC}} = \frac{1}{\sqrt{LC}}}$$

Loading

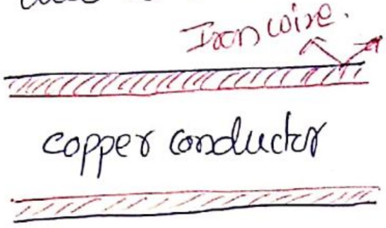
To minimise the attenuation and to reduce the distortion sothat we have to increase the value of inductance i.e. increasing the value of inductance by inserting the inductance in series with a line is called as loading and that lines are called as loaded lines.

There are three types of loading

(i) Continuous loading
(ii) Patch loading
(iii) Lumped loading.

(i) Continuous Loading

This method is used to increase the value of inductance upto 65mH per km but it is expensive due to laborious construction.



Here a type of iron (or) some other magnetic material is used and are wound around the conductor to increase the permeability of the surrounding medium and also to increase the value at inductance.

The eddy current and hysteresis losses in the magnetic material will increase the primary constant 'R' and there will be a small additional difference in mechanical components or pressure between tape and the conductor will cause a large variation in primary constant since the continuous loading is used only in ocean cable.

The continuously loaded cable has the advantage over a lumped loaded cable that is the value of 'α' will increase uniformly with the increase in frequency and there will be no cut-off fre.

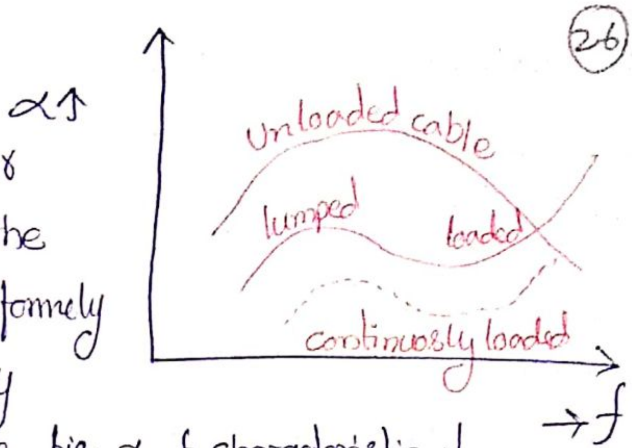


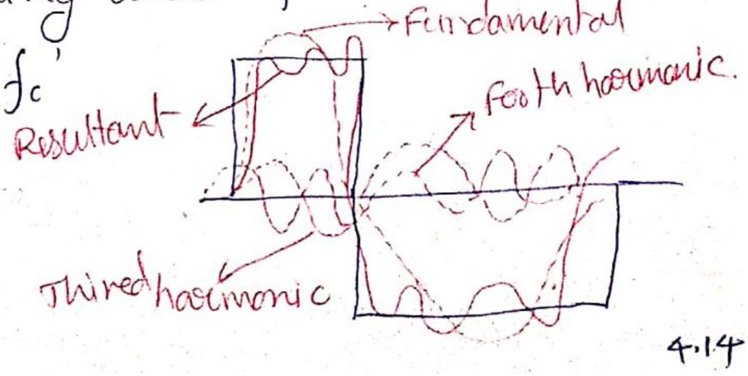
fig. α–f characteristic of continuously loaded cable.

(ii) Patch Loading:

This is normally known as continuously loaded cable which separates the section of unloaded cable. But the cost is reduced. In submarine cable there will be no use of continuous loading over the ~~whole length of the~~ entire cable, since to obtain the reduction in attenuation and a desired result without the continuous loading over the whole length of the cable. Therefore the typical length for the section is normally a quarter kilometer.

(iii) Lumped Loading:

In lumped loading the inductance of a line can also be increase by inserting a loading coil of uniform intervals. This Phenomenon is known as lumped loading and the lumped loaded lines will behave as low-pass filter and this method of loading is more convenient than the continuous loading which provides a limited frequency range upto 'f_c'



The loading coil have a certain resistance thus it will increase the total effecting inductance and there will be a practical limit of the amount by which the inductance of the line can be increased to reduce the attenuation since the eddy current losses and hysteresis will occur in the resulted loading coil.

Illustrative Problems

① A telephone line has $R = 30\Omega/km$, $L = 0.1H/km$ $C = 20\mu F/m$ and $G = 0$. At fre. $f = 10KHz$. find the secondary constants and phase velocity.

Given data: $R = 30\Omega/km$

$L = 0.1H/km$

$C = 20\mu F/m = 20mF/km$

$G = 0$

$f = 10KHz$

To find: $Z_0 = ?$, P (or) $\gamma = ?$ and $V_p = ?$

Formulae & Procedure: $Z_0 = \sqrt{\frac{Z}{Y}} = \sqrt{\frac{R+j\omega L}{G+j\omega C}}$;

$$Z = R+j\omega L, \quad Y = G+j\omega C$$

$$Z = R+j\omega L = 30+j2\pi\times10\times10^{3}\times0.1 = 30+j6.283\times10^{3}$$

$$Z = 6.283\times10^{3}\angle 89.72^\circ \; ; \; \boxed{Z = 6283.07\angle 89.72\,\Omega}$$

$$Y = G+j\omega C = 0+j2\pi\times10\times10^{3}\times20\times10^{-3} = j1.256\times10^{3} = 1.256\times10^{3}\angle 90^\circ$$

$$\boxed{Y = 1256.64\angle 90^\circ\ ℧}$$

$$Z_0 = \sqrt{\frac{Z}{Y}} = \sqrt{\frac{6283.07\angle 89.72}{1256.64\angle 90}} = 2.236\angle \frac{89.72-90^\circ}{2}$$

$$\boxed{Z_0 = 2.236\angle -0.14^\circ\,\Omega = 2.236+j0.0055\,\Omega}$$

Propagation constant P (or) $\gamma = \sqrt{ZY} = \sqrt{(R+j\omega L)(G+j\omega C)}$.

$Z = 6283.07\angle 89.72$; $Y = 1256.64\angle 90^\circ$

$$P = \sqrt{ZY} = \sqrt{(6283.07\angle 89.72)(1256.64\angle 90^\circ)}$$

$$= 2809.9 \angle \frac{89.72^\circ + 90^\circ}{2}$$

$$\boxed{P = 2809.9\angle 89.86^\circ = 6.744 + j\,2809.62 = \alpha + j\beta}$$

$$\boxed{\alpha = 6.794 \;,\; \beta = 2809.62}$$

$$\text{Phase velocity } (V_p) = \frac{\omega}{\beta} = \frac{2\pi f}{\beta} = \frac{2\pi \times 10 \times 10^3}{2809.62}$$

$$\boxed{V_p = 22.363\,\text{m/sec.}}$$

② A Distortion less transmission line operating at 500 Hz has $Z_0 = 80\Omega$, $\alpha = 0.04$ Np/m, $\beta = 1.5$ rad/m. Find the line Parameters R, L, C & G.

Given data: $f = 500$ Hz
$Z_0 = 80\Omega$
$\alpha = 0.04$ Np/m
$\beta = 1.5$ rad/m

To find: R, L, C and G = ?

Formulae & Procedure:

For distortionless line - $\frac{R}{G} = \frac{L}{C}$; $G = \frac{RC}{L}$ — ①

Characteristic impedance, $Z_0 = \sqrt{\frac{L}{C}}$ — ②

Attenuation constant, $\alpha = \sqrt{RG}$ — ③

Substituting G value ① in ③ $\alpha = \sqrt{R\frac{RC}{L}} = R\sqrt{\frac{C}{L}}$

From ② $\alpha = R\left(\frac{1}{Z_0}\right)$; $\boxed{R = \alpha Z_0}$ $R = 0.04 \times 80 = 3.2$.

$$\boxed{R = 3.2\,\Omega/m}$$

From ③ $\alpha^2 = RG$; $G = \frac{\alpha^2}{R} = \frac{(0.04)^2}{3.2} = 500 \times 10^{-6}$

$$\boxed{G = 500\,\mu s/m}$$

$Z_0 = \sqrt{\frac{L}{C}}$, from $v_p = \frac{\omega}{\beta} = \frac{\cancel{\omega}}{\cancel{\omega}\sqrt{LC}} = \frac{1}{\sqrt{LC}} = \cancel{\frac{2\pi f}{1.5}}$

$v_p^2 = \frac{1}{LC}$; $C = \frac{1}{L v_p^2}$

put c in Z_0, $Z_0 = \sqrt{\frac{L}{\frac{1}{L v_p^2}}} = \sqrt{L^2 v_p^2} = L v_p$

$v_p = \frac{2\pi f}{\beta} = \frac{2\pi \times 500}{1.5} = 2.09\,km/s$

$$\boxed{v_p = 2.09\,km/s}$$

$Z_0 = L v_p$; $L = \frac{Z_0}{v_p} = \frac{80}{2.09 \times 10^3} = 38.27\,mH$

$$\boxed{L = 38.27\,mH}$$

$Z_0 = \sqrt{\frac{L}{C}}$; $Z_0^2 = \frac{L}{C}$; $C = \frac{L}{Z_0^2} = \frac{38.27 \times 10^{-3}}{80 \times 80}$

$$\boxed{C = 5.96\,\mu F}$$

(Q3) An open wire telephone line has $R = 10\Omega/km$, $L = 0.0037 h/km$ $C = 0.0083\times10^{-6}$ F/km and $G = 0.4\mu℧/km$. Determine (10) Z_0, α and β at 1000 Hz.

Given data:

$R = 10\Omega/km$

$L = 0.0037 h/km$

$C = 0.0083\times10^{-6}$ F/km

$G = 0.4\mu℧/km$

$f = 1000$ Hz

To find: Z_0, α and β = ?

Formulae & Procedure:

$$Z_0 = \sqrt{\frac{Z}{Y}} = \sqrt{\frac{R+j\omega L}{G+j\omega C}}$$

$$Z = R+j\omega L = 10 + j2\pi\times1000\times0.0037 = 10+j23.2$$

$$\boxed{Z = 25.3\angle 66.8^\circ}$$

$$Y = G+j\omega C = 0.4\times10^{-6} + j2\pi\times1000\times0.0083\times10^{-6}$$

$$= 0.4\times10^{-6} + j52.1\times10^{-6} = 52.1\times10^{-6}\angle 89.6^\circ$$

$$\boxed{Y = 52.1\times10^{-6}\angle 89.6^\circ}$$

$$Z_0 = \sqrt{\frac{Z}{Y}} = \sqrt{\frac{25.3\angle 66.8^\circ}{52.1\times10^{-6}\angle 89.6^\circ}} = \sqrt{\frac{25.3}{52.1\times10^{-6}}}\angle \frac{66.8-89.6}{2}$$

$$\boxed{Z_0 = 697\angle -11.4^\circ = (683 - j138)\Omega}$$

$$P = \alpha + j\beta = \sqrt{ZY} = \sqrt{25.3\angle 66.8^\circ \times 52.1\times10^{-6}\angle 89.6^\circ} =$$

$$= \sqrt{25.3\times52.1\times10^{-6}}\angle \frac{66.8+89.6}{2} =$$

$$\boxed{P = 0.0363\angle 78.2^\circ = 0.0074 + j0.0356}$$

$$P = \alpha + j\beta = 0.0074 + j0.0356$$

$$\boxed{\alpha = 0.0074 \text{ neper/km}}$$

$$\boxed{\beta = 0.0356 \text{ radians/km}}$$

Q4) A telephone line has resistance of 20Ω, inductance of 10mH, capacitance of 0.1μF, and insulation resistance of 0.1 MΩ/km. Find the input impedance at angular fre. of 5000 radians/sec. if the line is very long.

(31)

Given data: $R = 20\Omega$

$L = 10\times10^{-3}\,H$

$G = \cancel{\frac{1}{R}}\,\frac{1}{0.1\times10^{6}} = 10^{-5}\,\mho$

$C = 0.1\times10^{-6}$

$\omega = 5000$ radians/see

To find: $Z_{in} = Z_0 = ?$

$Z = R + j\omega L = 20 + j\,5000\times10\times10^{-3} = 20 + j\,50$

$$\boxed{Z = 53.85\angle 68.2^\circ}$$

$Y = G + j\omega C = 10^{-5} + j\,5000\times0.1\times10^{-6}$

$$\boxed{Y = 10^{-6}(10 + j\,500) = 500\times10^{-6}\angle 88.9^\circ}$$

$$Z_0 = \sqrt{\frac{Z}{Y}} = \sqrt{\frac{53.85\angle 68.2^\circ}{500\times10^{-6}\angle 88.9^\circ}}$$

$$= \sqrt{10.77\times10^{4}}\;\angle\frac{68.2^\circ - 88.9^\circ}{2}$$

$$\boxed{Z_0 = 3283\angle -10.4^\circ\ \Omega}$$

(Q4) The primary constants of a line per loop km are (32) $R = 196\Omega$, $C = 0.09\mu F$, $L = 0.71 mH$ and leakage conductance is negligible. Calculate the characteristic impedance and the propagation constant at a fre. of $\frac{5000}{2\pi}$ Hz.

Given data:

$R = 196\Omega$

$L = 0.71 mH$

$C = 0.09 \mu F$

$G = 0$

$f = \frac{5000}{2\pi}$ Hz

To find: $Z_0, P = ?$

Formulae & Procedure:

$$\omega = 2\pi f = \cancel{2\pi} \times \frac{5000}{\cancel{2\pi}} = 5000$$

$$Z = R + j\omega L = 196 + j5000 \times 0.71 \times 10^{-3} = 196 + j35.5$$

$$\boxed{Z = 199.2 \angle 10.5^\circ}$$

$$Y = G + j\omega C = 0 + j5000 \times 0.09 \times 10^{-6} = 0.45 \times 10^{-3} \angle 90^\circ$$

$$\boxed{Y = 0.45 \times 10^{-3} \angle 90^\circ}$$

$$Z_0 = \sqrt{\frac{Z}{Y}} = \sqrt{\frac{199.2 \angle 10.5^\circ}{0.45 \times 10^{-3} \angle 90^\circ}} = 10.4 \angle -39.75^\circ$$

$$\boxed{Z_0 = 10.4 \angle -39.75^\circ}$$

$$P = \sqrt{ZY} = \sqrt{199.2 \angle 10.5^\circ \times 0.45 \times 10^{-3} \angle 90^\circ} = 2,999 \times 10^{-4} \angle 50.25^\circ$$

$$\boxed{P = 0.290 \angle 50.25^\circ}$$

4.17

V Transmission Lines - II

The voltage and current at any point in a transmission line are dependent on the load at the end of the line and on the distance of the point from the load.

Since the impedance at any point is the ratio of the voltage to the current at that point, the impedance then must also be dependent on the load and the distance from it.

Thus in any transmission line the load i.e., the termination establishes the current and voltage relations; while the relation at the generator terminals determines the input impedance.

Therefore various ways in which the voltage and current may be distributed along a transmission line can be understood by considering

(i) When the load end i.e. the terminating end is open

(ii) When the load end is shorted and

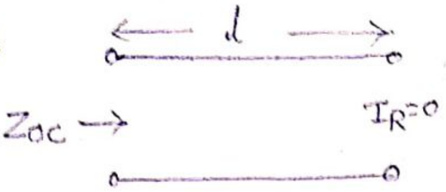
iii) When the load is equal to the characteristic impedance.

Open circuited line is defined as a transmission line whose far end i.e terminating end is open.

Short circuited line is defined as a transmission line whose far end is shorted.

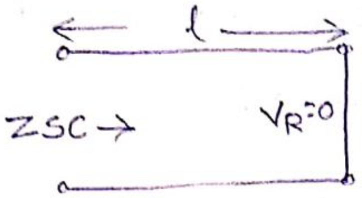
Input Impedance of open and short circuited lines.

Input Impedance of an open circuited line is the impedance measured at the input of a finite length of line when its far end is open. (Zoc)



(a) fig. open circuited lines

Input Impedance of a short-circuited line is the impedance measured at the input end of the finite length of line when its far end is shorted. (Zsc).



(b) fig. short circuited line

we knew that $V = V_s \cosh Px - I_s Z_0 \sinh Px$ —①

$I = -\frac{1}{Z_0}(-I_s Z_0 \cosh Px + V_s \sinh Px)$ -②

consider a length of line l, having far end voltage and current V_R and I_R respectively.

when $x = l$, $V = V_R$ and $I = I_R$, From ① or ②

$$V_R = V_s \cosh Pl - I_s \overset{Z_0}{} \sinh Pl \quad -③$$

$$I_R = I_s \cosh Pl - \frac{V_s}{Z_0} \sinh Pl \quad -④$$

In an open circuited line, $I_R = 0$ from fig (a) eqn ④ will become $0 = I_s \cosh Pl - \frac{V_s}{Z_0} \sinh Pl$

$$\frac{V_s}{Z_0} \sinh Pl = I_s \cosh Pl$$

$$\frac{V_s}{I_s} = Z_0 \frac{\cosh Pl}{\sinh Pl} = Z_0 \coth Pl$$

$$\boxed{\frac{V_s}{I_s} = Z_{oc} = Z_0 \coth Pl} -⑤$$

Ily in a short circuited line $V_R = 0$ from fig (b) eqn ③ ③ will become $0 = V_S \cosh Pl - I_S^{Z_0} \sinh Pl$

$$V_S \cosh Pl = I_S^{Z_0} \sinh Pl$$

$$\frac{V_S}{I_S} = Z_0 \frac{\sinh Pl}{\cosh Pl} = Z_0 \tanh Pl$$

$$\boxed{\frac{V_S}{I_S} = Z_{SC} = Z_0 \tanh Pl} \quad - ⑥$$

* For an infinite length of line $l = \infty$, from ⑤ & ⑥ both tanhpl and cothpl will become 1. Thus Zoc and Zsc will each become to Z_0. $Z_{oc} = Z_{sc} = Z_0$.
Therefore it is proved again that input impedance of an infinite line is its characteristic impedance.

Multiplying ⑤ & ⑥ $Z_{oc} \times Z_{sc} = Z_0 \coth Pl \times Z_0 \tanh Pl$

$$Z_{oc} \times Z_{sc} = Z_0^2$$

$$\boxed{Z_0 = \sqrt{Z_{oc} Z_{sc}}}$$

characteristic Impedance interms of ZocZsc.

Reflection co-efficient: 'K'

Reflection of energy occures when there is an impedance irregularity. Reflection is normally undesirable on transmission line.

The reflection will be maximum when the line is open or short circuited and will be zero when $Z_R = Z_0$.

Reflection co-efficient is defined as the ratio of the reflected voltage/current to the incident voltage/current.

Let V_i & V_r be the incident and reflected voltage respectively, then

$$\frac{V_r}{V_i} = k$$

Ily Let I_i and I_r be the incident and reflected current

$$\frac{I_r}{I_i} = k$$

Fundamental equations for voltage and current at any point of a transmission line are

$$V = be^{-Px} + ae^{Px}$$

$$I = \underbrace{\frac{b}{Z_0}e^{-Px}}_{\text{Incident voltage/current}} - \underbrace{\frac{a}{Z_0}e^{Px}}_{\text{Reflected voltage/current}}$$

If y is the distance measured from the termination Z_R, simply by putting $x = -y$

$$V = be^{Py} + ae^{-Py}$$

$$I = \frac{b}{Z_0}e^{Py} - \frac{a}{Z_0}e^{-Py}$$

At the termination Z_R, $y = 0$, $V = V_R$ and $I = I_R$

$$V_R = b + a \quad - ①$$

$$I_R = \frac{b}{Z_0} - \frac{a}{Z_0} = \frac{b-a}{Z_0}; \quad I_R Z_0 = b - a \quad - ②$$

① + ② : $b + \cancel{a} + b - \cancel{a} = V_R + I_R Z_0$

$$2b = V_R + I_R Z_0; \quad \boxed{b = \frac{V_R + I_R Z_0}{2}}$$

① − ② $\cancel{b} + a - \cancel{b} + a = V_R - I_R Z_0$

$$2a = V_R - I_R Z_0; \quad \boxed{a = \frac{V_R - I_R Z_0}{2}}$$

Voltage reflection coefficient, $K = \frac{V_r}{V_i} = \frac{a e^{-Py}}{b e^{Py}} = \frac{a}{b} e^{-2Py}$ ⑤

At the termination Z_R. $y=0$

Therefore $\boxed{K = \frac{a}{b}}$

substituting values of a, b

$$K = \frac{V_R - I_R Z_0}{V_R + I_R Z_0}$$

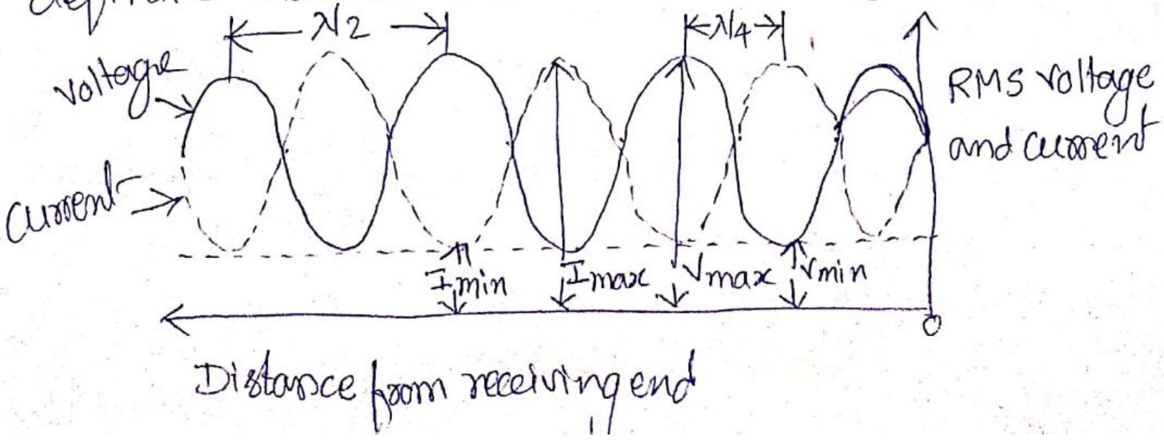
Dividing numerator and denominator by I_R and substituting $\frac{V_R}{I_R}$ as Z_R

$$K = \frac{\frac{V_R}{I_R} - Z_0}{\frac{V_R}{I_R} + Z_0} = \frac{Z_R - Z_0}{Z_R + Z_0}$$

$$\boxed{K = \frac{Z_R - Z_0}{Z_R + Z_0}}$$

Standing Wave Ratio

Reflection takes place when the line is not terminated in its characteristic impedance. This cause reflection of waves. The combination of incident and reflected waves of current and voltage with give rise to "standing waves" of current and voltage with definite maxima and minima along the line.



When even reflection takes place in line transmission at same points, the incident and reflected signals are in phase and both the components add together.

On the other hand, at some other points, the two components may oppose each other. The net or resultant graphical profile of both these incident and reflected wave is called standing waves as shown in figure.

$$|V_{max}| = |V_i| + |V_r|$$

$$|V_{min}| = |V_i| - |V_r|$$

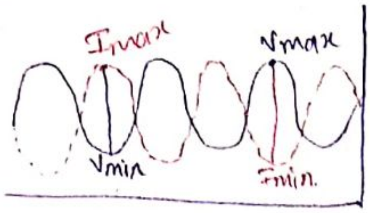
$$|I_{max}| = |I_i| + |I_r|$$

$$|I_{min}| = |I_i| - |I_r|$$

The magnitude of standing waves provides an idea of the amount of reflection.

The ratio of the maximum and minimum magnitude of current or voltage on a line having standing waves is called the standing wave ratio 'S'.

$$VSWR = \frac{|V_{max}|}{|V_{min}|} = \frac{|V_i| + |V_r|}{|V_i| - |V_r|}$$



$$\frac{\cancel{|V_i|}\left[1 + \frac{|V_r|}{|V_i|}\right]}{\cancel{|V_i|}\left[1 - \left|\frac{V_r}{V_i}\right|\right]} = \frac{1+|K|}{1-|K|}$$

$V_{max} = I_{min}$

$V_{min} = I_{max}$

$$\boxed{VSWR = \frac{1+|K|}{1-|K|}}$$ Always >1

$$CSWR = \frac{|I_{min}|}{|I_{max}|} = \frac{|I_i| - |I_r|}{|I_i| + |I_r|} = \frac{\cancel{|I_i|}\left[1 - \left|\frac{I_r}{I_i}\right|\right]}{\cancel{|I_i|}\left[1 + \left|\frac{I_r}{I_i}\right|\right]} = \frac{1-|K|}{1+|K|}$$

$$\boxed{CSWR = \frac{1-|K|}{1+|K|}}$$ Always <1

$$\boxed{|K| = \frac{|S|-1}{|S|+1}}$$

When VSWR is equal to 1, the line is correctly terminated and there is no reflection.

VSWR is more popular since it is easy to measure VSWR at different points of the line.

UHF Lines:

Ultra High Frequency lines normally abbreviated as UHF lines, covers fre. range from 300 to 3000MHz, whose wavelengths are from 100cm to 10cm

Characteristic Impedance Z_0

At UHF range $\omega L >> R$ as ω will be very large. In addition UHF lines are physically short and hence the resistance will be very small as compared with the reactance. llly $\omega C >> G$, G can be assumed nearly zero.

$$Z = R + j\omega L = j\omega L$$

$$Y = G + j\omega C = j\omega C$$

$$\boxed{Z_0 = \sqrt{\frac{Z}{Y}} = \sqrt{\frac{L}{C}}}$$

Input Impedance (Z_{in}) interms of secondary constants

Input impedance of transmission line is defined as the impedance measured across the input terminals of the transmission line. $Z_{in} = \frac{V_S}{I_S}$

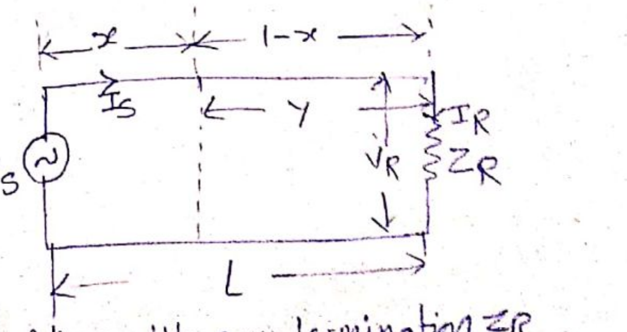


Fig. A line with any termination ZR

5.4

$$V_S = V_R \cosh pl + I_R Z_0 \sinh pl$$

$$I_S = \frac{V_R}{Z_0} \sinh pl + I_R \cosh pl$$

$$Z_{in} = \frac{V_S}{I_S} = \frac{V_R \cosh pl + I_R Z_0 \sinh pl}{\frac{V_R}{Z_0} \sinh pl + I_R \cosh pl}$$

Multiplying numerator & denominator by $\frac{Z_0}{I_R}$,

$$Z_{in} = \frac{\frac{Z_0}{I_R} V_R \cosh pl + \frac{Z_0}{\cancel{I_R}} \cancel{I_R} Z_0 \sinh pl}{\frac{\cancel{Z_0}}{I_R} \cdot \frac{V_R}{\cancel{Z_0}} \sinh pl + \frac{Z_0}{\cancel{I_R}} \cancel{I_R} \cosh pl}$$

$$Z_{in} = \frac{Z_0 \left(\frac{V_R}{I_R} \cosh pl + Z_0 \sinh pl\right)}{\frac{V_R}{I_R} \sinh pl + Z_0 \cosh pl}$$

$$Z_R = \frac{V_R}{I_R},$$

$$Z_{in} = Z_0 \times \frac{Z_R \cosh pl + Z_0 \sinh pl}{Z_R \sinh pl + Z_0 \cosh pl}$$

$$\boxed{Z_{in} = Z_0 \frac{Z_R \cosh pl + Z_0 \sinh pl}{Z_0 \cosh pl + Z_R \sinh pl}}$$

Since high frequency lines can be considered loss free,

$\alpha = 0$ resulting $P = j\beta$

$$Z_{in} = Z_0 \frac{Z_R \cosh j\beta l + Z_0 \sinh j\beta l}{Z_0 \cosh j\beta l + Z_R \sinh j\beta l} = Z_0 \frac{Z_R \cos\beta l + jZ_0 \sin\beta l}{Z_0 \cos\beta l + jZ_R \sin\beta l}$$

$\cosh j\beta l = \cos\beta l$

$\sinh j\beta l = j \sin\beta l$

Divinding $\cos\beta l$

$$Z_{in} = Z_0 \frac{Z_R \frac{\cancel{\cos\beta l}}{\cancel{\cos\beta l}} + jZ_0 \frac{\sin\beta l}{\cos\beta l}}{Z_0 \frac{\cancel{\cos\beta l}}{\cancel{\cos\beta l}} + jZ_R \frac{\sin\beta l}{\cos\beta l}}$$

$$\boxed{Z_{in} = Z_0 \left[\frac{Z_R + jZ_0 \tan\beta l}{Z_0 + jZ_R \tan\beta l}\right] = Z_0 \left[\frac{Z_R + jZ_0 \tan\frac{2\pi}{\lambda} l}{Z_0 + jZ_R \tan\frac{2\pi}{\lambda} l}\right]}$$

When length $l = \lambda/8$.

If a transmission line of length $\frac{\lambda}{8}$ and characteristic impedance 'Z_0' is terminated by a load impedance 'Z_R' then the impedance at the sending end is

$$Z_{in} = Z_0\left[\frac{Z_R + jZ_0 \tan\left(\frac{\cancel{2}\pi}{\cancel{\lambda}} \times \frac{\cancel{\lambda}}{\cancel{8}4}\right)}{Z_0 + jZ_R \tan\left(\frac{\cancel{2}\pi}{\cancel{\lambda}} \times \frac{\cancel{\lambda}}{\cancel{8}4}\right)}\right] = Z_0\left[\frac{Z_R + jZ_0 \tan\frac{\pi}{4}}{Z_0 + jZ_R \tan\frac{\pi}{4}}\right]$$

$$\boxed{Z_{in} = Z_0\left[\frac{Z_R + jZ_0}{Z_0 + jZ_R}\right]} \quad -①$$

$$Z_{in} = Z_0\left[\frac{\frac{Z_R}{Z_R} + \frac{jZ_0}{Z_R}}{\frac{Z_0}{Z_R} + j\frac{Z_R}{Z_R}}\right] = Z_0\left[\frac{1 + \frac{jZ_0}{Z_R}}{\frac{Z_0}{Z_R} + j}\right] -(ii)$$

(i) when $Z_R = 0$, then $Z_{in} = jZ_0$

(ii) when $Z_R = \infty$, then $Z_{in} = -jZ_0$

when length $l = \lambda/4$

For a quarter wavelength line

$$Z_{in} = Z_0\left[\frac{Z_R + jZ_0 \tan\left(\frac{\cancel{2}\pi}{\lambda} \times \frac{\lambda}{\cancel{4}2}\right)}{Z_0 + jZ_R \tan\left(\frac{\cancel{2}\pi}{\lambda} \times \frac{\lambda}{\cancel{4}2}\right)}\right] = Z_0\left[\frac{Z_R + jZ_0 \tan\frac{\pi}{2}}{Z_0 + jZ_R \tan\frac{\pi}{2}}\right]$$

$$Z_{in} = Z_0\left[\frac{\frac{Z_R}{\tan\frac{\pi}{2}} + j\frac{Z_0 \cancel{\tan\frac{\pi}{2}}}{\cancel{\tan\frac{\pi}{2}}}}{\frac{Z_0}{\tan\frac{\pi}{2}} + jZ_R\frac{\cancel{\tan\frac{\pi}{2}}}{\cancel{\tan\frac{\pi}{2}}}}\right]$$

$$\tan\frac{\pi}{2} = \infty$$

$$Z_{in} = Z_0\left[\frac{jZ_0}{jZ_R}\right] = \frac{Z_0^2}{Z_R}$$

$$\boxed{Z_{in} = \frac{Z_0^2}{Z_R}}$$

(i) The quarter-wavelength can be used for impedance inversion i.e. the normalized impedance of a quarter wavelength line is equal to the normalized admittance at the receiving end.

ii) The quarter wavelength line can be used for impedance matching $Z_{in} = \frac{Z_0^2}{Z_R}$

when length $l = \lambda/2$

The input impedance of a half wavelength line($\lambda/2$) with characteristic impedance 'Z_0' terminated with impedance 'Z_R' is given by

$$Z_{in} = Z_0\left[\frac{Z_R + jZ_0 \tan\left(\frac{2\pi}{\lambda} \times \frac{\lambda}{2}\right)}{Z_0 + jZ_R \tan\left(\frac{2\pi}{\lambda} \times \frac{\lambda}{2}\right)}\right] = Z_0\left[\frac{Z_R + jZ_0 \tan\pi}{Z_0 + jZ_R \tan\pi}\right]$$

$$Z_{in} = Z_0 \frac{Z_R}{Z_0}$$

$$\boxed{Z_{in} = Z_R}$$

* Thus the input impedance of a $\lambda/2$ line is equal to the load impedance independent of 'Z_0'

Stab Matching or (tuning stub)

Sections of open or short circcuited line called "stub."

.To connect sections of open or short circcuited line in shunt with the main line at some point or points to effect impedance matching. This is called stub matching.

A Section of Transmission line use as a matched section insesting b/w the load and the source.

Advantages:

(1) The length and characteristic impedance of the line remain unaltered.

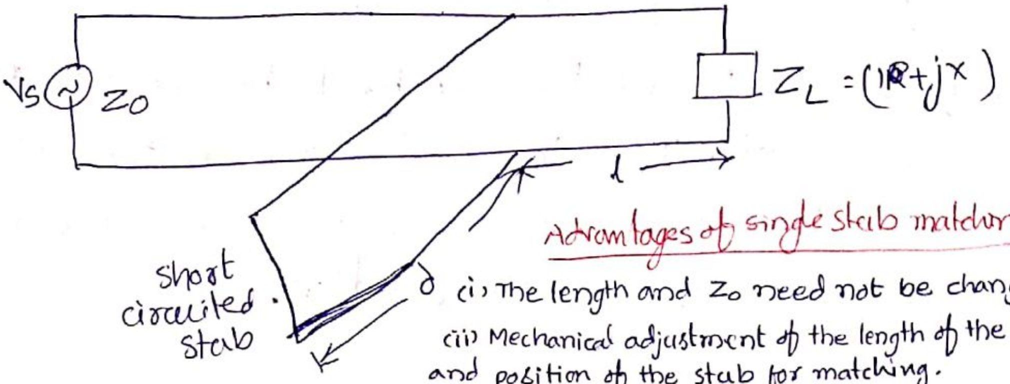
(2) From mechanical stand point, adjustable susceptance are added in shunt with the line.

Basically stub matching are of two types.

a) Single stub matching.
b) Double stub matching

Single stub Matching

Stub matching is nothing but Impedance matching. Impedance matching can be done if load impedance only real part. If load impedance is complex $(R \pm jx)$ then stub matching is used to get max. power transfered.



Advantages of single stub matching

(i) The length and Z_0 need not be changed.

(ii) Mechanical adjustment of the length of the stub and position of the stub for matching.

Let the normalized load of the transmission line is $Z_L = 1+jx$ i.e. the source and load is having different impedance (Mismatched load) so the total power will not be absorbed by the load and a part of the signal travel back towards source termed as reflected signal.

The reflection is occuring due to $+jx$ component to avoid it add $-jx$ component to the main transmission line in the form of stub (secondary transmission line).

$$Z_L = 1+jx-jx$$

$\boxed{Z_L = 1}$ If anything multiplied to 1, the value will be same as it multiplied. i.e. if Z_0 is multiplied to 1, the value will be Z_0 only. Hence $Z_L = 1 \times Z_0$

$\boxed{Z_L = Z_0}$ Impedance matching with stub is done and max. power can be transferred.

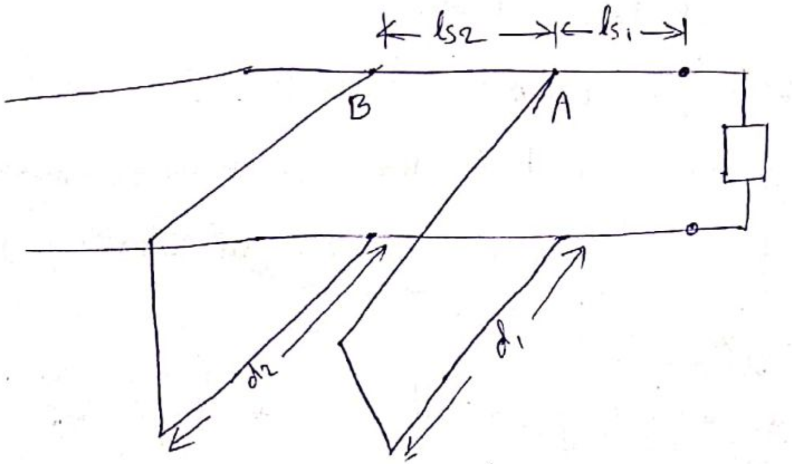
The short-circuit stub is invariably used because
(i) it radiates less power and
(ii) It's effective length may be varied by means of a shorting bar which normally takes the shapes of shorting plugs.

Disadvantages of Single Stub Matching:

(i) Single stub matching is a narrow band system.
(ii) As the fre. changes, the location of the stub will have to be changed, therefore single stub matching system is useful for a fixed frequency only.
(iii) For final adjustment the stub has to be moved along the line slightly. This is possible only in open wire lines and therefore, on coaxial lines single stub matching may become inaccurate in practice.

Double Stub matching:

To overcome the ~~two~~ disadvantages of single stub matching two short-circuited stubs whose lengths are adjustable independently but whose positions are fixed.



usually these stubs are separated by a length $\frac{\lambda}{4}$ or 0.375λ

Smith Chart (circular chart)

The smith chart was developed by Philip Smitch at Bell Telephone's Radio Research Lab during the 1939s.

Smith chart is a plot of complex reflection overlaid with an impedance and/or admittance grid referenced to a 1-ohm characteristic impedance.

The smith chart is plotted on the complex reflection coefficient plane in two dimensions and is scaled in normalised admittance (or) normalised impedance (most common) (or) both. using different colours to distinguish between them. These are often known as Y, Z, and YZ smitch charts.

→ Smith chart is the representation of reflection coefficient interms of normalized impedance.

→ It is used to determine reflection coefficient, VSWR, input impedance, location of maxima & minima.

→ It is a polar plot of real part of reflection coefficient verses imaginary part of reflection ~~re~~ coefficient.

→ There are two families of circle in smitch chart.

(i) Constant resistance circle

(ii) constant ~~resis~~ reactance circle.

Properties (1) Normalising Impedance

(2) Plotting of an impedance

(3) Determination of SWR

(4) Determination of K (reflection coefficient) and direction & Magnitude.

(5) Location of voltage maximum and minimum.

(6) open and short circulated line

(7) Movement along the periphery of the chart

(8) Matched load.

Applications of Smith's charts

(i) Smith Chart is used as a admittance diagram
(ii) It is used for converting a impedance into admittance.
(iii) It is used to determine the input impedance.
(iv) used to determine the load impedance.
v) To determine the input impedance and the admittance of a short-circuited lines.
vi) Smith chart is used to determine the input impedance and the admittance of an open circuited lines.

① The terminating load of UHF transmission line ($Z_0 = 50\angle 0^\circ$) ohms working at 300 MHz is $(50 + j50)\Omega$ calculate VSWR?

Given data: $Z_0 = 50\Omega$

$f = 300 MHz$

$Z_R = 50 + j50$

To find: VSWR = ?

Formulae & Procedure: $K = \frac{Z_R - Z_0}{Z_R + Z_0} = \frac{50 + j50 - 50}{50 + j50 + 50}$

$K = \frac{j50}{100 + j50} = \frac{\cancel{50}(j)}{\cancel{50}(2 + j)}$ rationalize

$K = \frac{j}{2+j} \times \frac{2-j}{2-j} = \frac{j2 + 1}{4 + 1} = \frac{1 + j2}{5}$

$|K| = \frac{1}{5} + j\frac{2}{5} = 0.2 + j0.4 = 0.4472\angle 63.5^\circ$

$\boxed{K = 0.4472\angle 63.5^\circ}$

Now $VSWR = \frac{1 + |K|}{1 - |K|} = \frac{1 + 0.4472}{1 - 0.4472} = 2.62$

$\boxed{VSWR = 2.62}$

② A certain low loss line has a characteristic impedance of 400 ohms. Determine VSWR with the following receiving end impedance.

(a) $Z_R = 70 + j0.0$ (b) $Z_R = 800 + j0.0$ (c) $Z_R = 650 - j475$