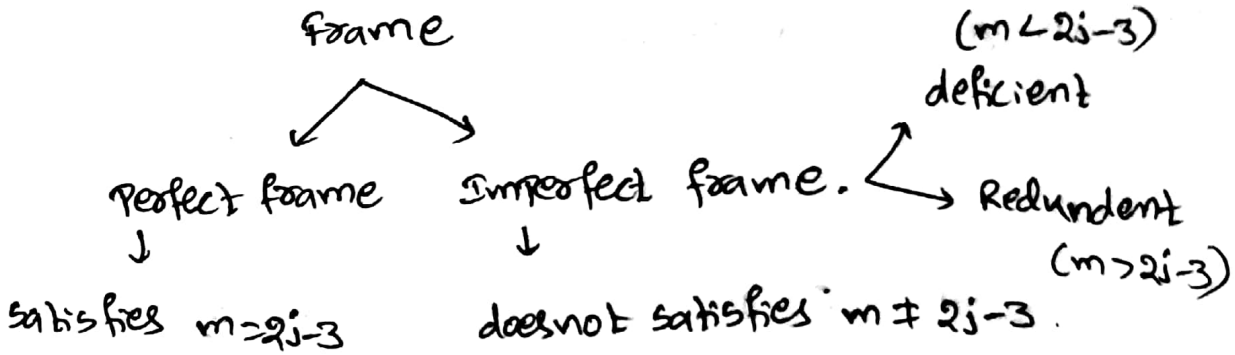
07/12/19

Module-I.

Analysis of Frames and Energy Theorem

Types of Frames:-

Frame



- Perfect frame → satisfies $m = 2j-3$
- Imperfect frame → does not satisfies $m \neq 2j-3$.
  - deficient ($m < 2j-3$)
  - Redundent ($m > 2j-3$)

Analysis of Frames:-

By 3 methods. Frames can be analysed.

1) method of joints
2) method of section.
3) Graphical method.

Assumptions of method of joints:-

1) self weight of member Ignored.
2) All are Pinpointed.
3) loads are acting at joints.
4) All members are lying in sample.

* Frames → Pinjointed, Regid jointed.

$D_{sc} = m - (2j-3)$

m → no. of members

j → no. of joints.

3 → no. of equilibrium

$D_{sc} = 3c$ or $6c$.

c → no. of closed Paths

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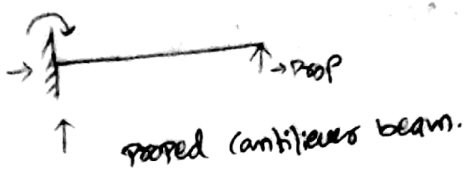
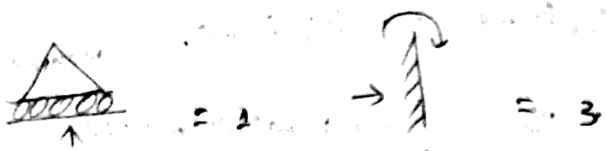
Static Indeterminacy (D_s):-



propped cantilever beam.

Calculating no. of unknowns:-



roller support — fixed



Hinged — plane

Total static Indeterminacy (D_s)

$$D_s = D_{se} - D_{si} - \text{Releases}.$$

$D_{se} = r - R$

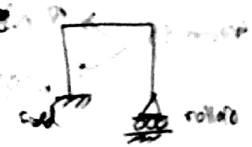
$r \rightarrow$ no. of unknowns.

$R \rightarrow$ no. of equilibrium

plane — 3 equilibrium

space — 6 equilibrium.

eg: Calculate D_s for following figure.



$r = 3$(fixed) $+ 1$(roller)

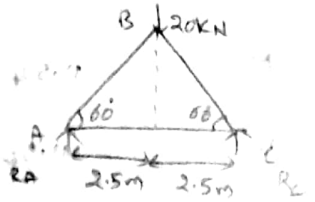
$\therefore r = 4$. , $R = 3$

$D_{se} = r - R = 4 - 3$. $\boxed{D_{se} = 1}$

D_{si} = no. of closed paths $= 0 = 3(0) = 0$ (rigid frame)

$D_s = D_{se} - D_{si} = 1 - 0 = 1$.

1) Calculate forces in member for following cases.



$R_A + R_C = 20\,kN$. $\rightarrow$ (1)

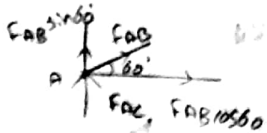
$\Sigma M_A = 0$.

$R_C \times 5 - 20 \times 2.5 = 0$

$\boxed{R_C = 10\,kN}$, $\boxed{R_A = 10\,kN}$.

choose a joint where the unknowns are not more than 2.

Consider joint "A".



$\Sigma H = 0$.

$F_{AC} + F_{AB}\cos 60° = 0$ $\rightarrow$ (2)

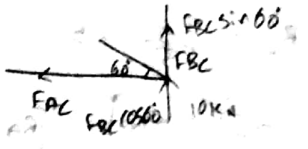
$\Sigma V = 0$, $10 + F_{AB}\sin 60° = 0$

$10 + F_{AB}\frac{\sqrt{3}}{2} = 0$, $\boxed{\therefore F_{AB} = -11.54\,kN}$ (compressive -ve)

sub F_{AB} in (2) eqn.

$\boxed{\therefore F_{AC} = +5.77\,kN}$ (tensile)

Consider joint "C"



$\Sigma H = 0$,

$-F_{AC} - F_{BC}\cos 60° = 0$

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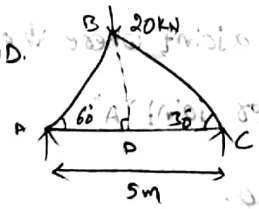
$-5.77 - F_{BC}\cos 60° = 0$

$\therefore F_{BC} = -11.54\,kN$ (compressive).

member	force	Nature.
AB	11.54	compressive
BC	11.54	compressive
AC	5.44	Tensile

2) Calculate forces in member.



calculate distance AD.

$R_A + R_C = 20\,kN$

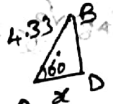
from ΔABC.



$\cos 60° = \frac{BA}{5}$

$BA = 5\cos 60° \Rightarrow 4.33\,m.$

from ΔABD:



$\cos 60° = \frac{AD}{AB} \Rightarrow AD = AB\cos 60°$

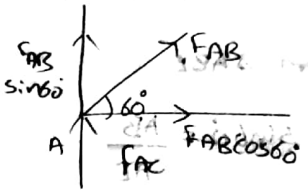
$AD = 4.33\cos 60° = 1.25\,m.$

$R_C \times 5 = 20 \times 1.25$

$R_C = \frac{25}{5} = 5$

$R_C = 5\,KN.$ $R_A = 15\,KN.$

Consider joint A:-



$\Sigma H = 0$

$F_{AC} + F_{AB}\cos 60° = 0$

$\Sigma V = 0.$

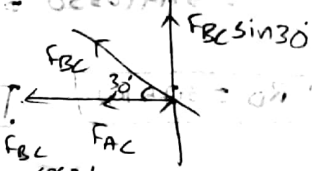
$F_{AB}\sin 60° + 15 = 0.$

$F_{AB} = \frac{-15}{\sin 60°} = -17.32\,KN$ (compression)

$F_{AC} = 17.32\cos 60° = 8.66\,KN.$

Consider joint C:-

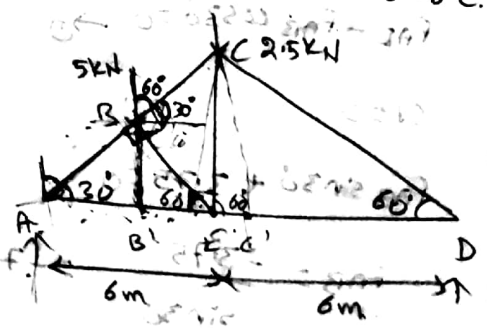
$\Sigma H = 0$



$-F_{AC} - F_{BC}\cos 30° = 0$

$F_{BC} = \frac{-F_{AC}}{\cos 30°} = \frac{-8.66}{\cos 30°} = -9.99\,KN.$

3) Calculate the forces in the member for following figure.



$R_A + R_D = 5 + 2.5.$

$R_A + R_D = 7.5\,KN.$

$\Sigma M_A = 0.$

$R_D \times 12 - 2.5 \times 9 - 5 \times 4.5 = 0.$

$R_D \times 12 = 22.5 + 22.5$

$R_D = \frac{45}{12} = 3.7\,KN.$

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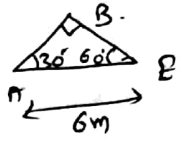
from ΔABE.

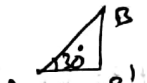


$\sin 60^\circ = \frac{AB}{AE}$

$AB = AE \sin 60^\circ$

$AB = 6 \sin 60^\circ = 5.19 m.$

from $\Delta le\ ABB'$



$\cos 30^\circ = \frac{AB'}{AB}$

$AB' = AB \cos 30^\circ$

$= 5.19 \cos 30^\circ = 4.5 m.$

$\therefore R_D = 3.75 KN$ $\quad R_A = 3.75 KN$

Consider joint A.



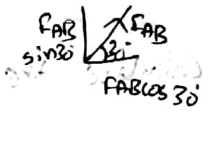
$\Sigma H = 0.$

$f_{AE} + f_{AB} \cos 30^\circ = 0 \rightarrow (1)$

$\Sigma V = 0$

$f_{AB} \sin 30^\circ + 3.75 = 0$

$f_{AB} = \frac{-3.75}{\sin 30^\circ} = -7.5 KN.$

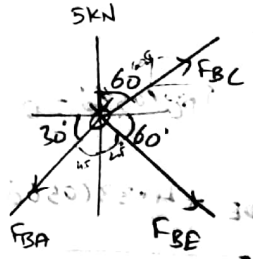


sub f_{AB} in (1) eqn.

$f_{AE} - 7.5 \cos 30^\circ = 0$

$\therefore f_{AE} = 6.45 KN$

Consider joint B.



$\Sigma V = 0.$

$-5 + F_{BC} \cos 60^\circ - f_{BE} \sin 60^\circ - f_{AB} \sin 30^\circ = 0$

$-5 + F_{BC} \times 0.5 - f_{BE} (0.866) + 7.5 (0.5) = 0$

$0.5 f_{BC} - 0.866 f_{BE} = 1.25 \rightarrow (3)$

$\Sigma H = 0.$

$f_{BC} \sin 60^\circ + f_{BE} \cos 60^\circ - f_{AB} \cos 30^\circ = 0.$

$0.866 f_{BC} + 0.5 f_{BE} + 7.5 (0.866) = 0$

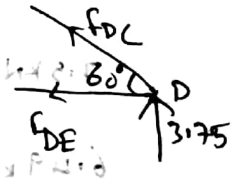
$0.866 f_{BC} + 0.5 f_{BE} = -6.49 = 0 \rightarrow (4)$

By solving (3) & (4)

$f_{BC} = -5 KN.$

$f_{BE} = -4.32 KN.$

Consider joint D.



$\Sigma V = 0.$

$3.75 + f_{DC} \sin 60^\circ = 0$

$f_{DC} = \frac{-3.75}{\sin 60^\circ}.$

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$\therefore f_{DC} = -4.33\,KN$.

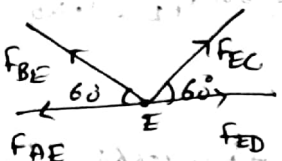
$\Sigma H = 0$.

$-f_{DE} - f_{DC}\cos 60^\circ = 0$.

$-f_{DE} + 4.33\cos 60^\circ = 0$

$\therefore f_{DE} = 2.165\,KN$

Consider joint E.



$\Sigma H = 0$.

$f_{ED} + f_{EC}\cos 60^\circ - f_{AE} - f_{BE}\cos 60^\circ = 0$

$2.165 + f_{EC}\cos 60^\circ - 6.45 + 4.33\cos 60^\circ = 0$

$f_{EC}\cos 60^\circ - 2.12 = 0$.

$\therefore f_{EC} = \frac{2.12}{\cos 60^\circ} = 4.24\,KN$.

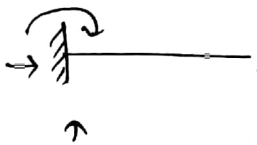
$\therefore f_{EC} = 4.24\,KN$.

Member	force	Nature.
AB	7.5 KN	Compressive
AE	6.49 KN	Tensile
BC	5 KN	Compressive
BE	4.33 KN	Compressive
EC	4.33 KN	Tensile
ED	2.16 KN	Tensile
CD	4.33 KN	Compressive.

Static Determinant structure:-

The structure which is analyzed by the equilibrium equation is known as static determinant structure.

eg:



(i) No. of unknowns = 3.

(ii) No. of equilibrium eqn = 3.

$= r - R$

$= 3 - 3 = 0$

$\therefore$ Cantilever beam is static determinant

Static Indeterminant structure:-

The structure which can't be analyzed by

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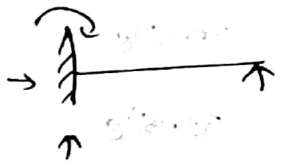
equilibrium equation alone.

eg:



No. of unknown = 4

No. of equilibrium = 3

$= r - R$

$= 4 - 3 = 1.$

* Compatability equation are required for analyzing the indeterminant structures.

Static Indeterminacy :-

The additional no. of equation required for analyzing the indeterminant members or structure is known as static Indeterminacy.

$D_s = D_{se} + D_{si} - \text{Releases}$

D_{se} = Degree of external Indeterminacy.

D_{si} = Degree of Internal static indeterminacy.

* Propped cantilever beam is 1° of determinacy.

* fixed beam is 3° of indeterminacy.

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Frame

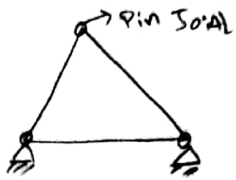
A frame or a truss is a structure which is form by connected different members or bars.

Classification of Truss based on connections:-

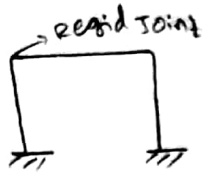
There are 2 types of truss/frames.

1) Pin-Jointed Truss/Frame.

2) Regid Jointed Truss/Frame



Pin Jointed Truss



Regid Jointed Truss

* In truss the loads are acting at the joints. whereas in frames loads are acting on the members.

* In trusses the members are subjected axial forces (Tensile or Compressive).

* In frames the members are subjected to axial force, shear force and Bending moment.

Classification based on stability:-

1. Perfect frame.
2. Imperfect frame.

1. Perfect frame:- If a frame or truss satisfies the eqn $m = 2j-3$. is known as perfect frame.

$m \rightarrow$ no. of members. $j \rightarrow$ no. of Joints.

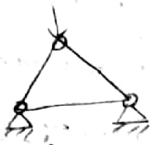
eg:

$m = 3$, $J = 3$, equilibrium $= 3$

$m = 2j-3$

$3 = 2(3) - 3$

$3 = 3$.



Pin Joint Truss.

2. Imperfect frame:- The frame or truss which doesn't satisfy the eqn. $m = 2j-3$. is known as Imperfect frame.

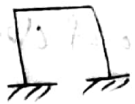
$m = 3$

$J = 4$, equi $= 3$.

$m = 2j-3$

$3 = 8-3$

$3 \neq 5$



Regid Jointed Truss

* Imperfect frame may be classified into 2 types.

1. Deficient frame 2) Redundent frame.

1. Deficient frame:- When the no. of members less than (m) $2j-3$. i.e., $m < 2j-3$ is known as deficient frame

2. Redundent frame:- When the no. of members(m) greater than $2j-3$. ie, $m > 2j-3$ is known as Redunded frame

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$D_s = D_{se} + D_{si}$ – Releases.

$D_{se} = r - R$.

$D_{si} = 0$ for beams.

$D_{si} = m - (2j - 3)$ for Pin Jointed frames.

$D_{si} = 3c$ for Regid Jointed frame

r = no. of unknowns.

R = no. of equilibrium equation.

m = no. of members.

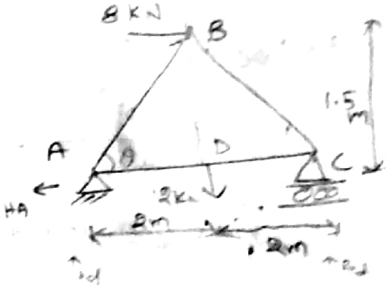
J = no. of Joints.

c = no. of closed Path.

Assumptions:

1) All the members are Pin jointed.
2) Frame is loaded at joints only.
3) Assume the frame is perfect frame.
4) All the members are lying in a same Plane
5) self weight of members Ignored.

1) A frame of 4m Pan and 1.5m height subjected to 2 Point loads at B & D. Find the forces in members. Calculate force in AB, BC & CA.



$R_A + R_C = 12$ → (1)

$\Sigma V = 0$.

$\Sigma H = 0$

$H_A + 8 = 0$

$\boxed{H_A = -8KN}$

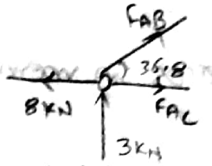
$\Sigma M_A = 0$

$R_C \times 4 - 12 \times 2 - 8 \times 1.5 = 0$

$4R_C - 24 - 12 = 0$

$\boxed{R_C = 9}$ $\boxed{\therefore R_A = 3}$

Consider a joint "A":



$\tan\theta = \frac{1.5}{2} = 36.8$

$\Sigma V = 0$

$3 + F_{AB} \sin 36.8 = 0$

$F_{AB} = \frac{-3}{\sin 36.8}$ $\boxed{\therefore F_{AB} = -5KN}$

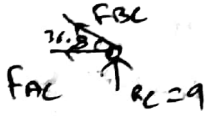
$\Sigma H = 0$

$-8 + F_{AB} \cos 36.8 + F_{AC} = 0$

$-8 - 5 \cos 36.8 + F_{AC} = 0$

$\boxed{\therefore F_{AC} = +12\,KN}$

Consider Joint "C"



$\Sigma V = 0$.

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$9 + f_{BC} \sin 36.8 = 0.$

$$f_{BC} = \frac{-9}{36.8} = -15\,KN.$$

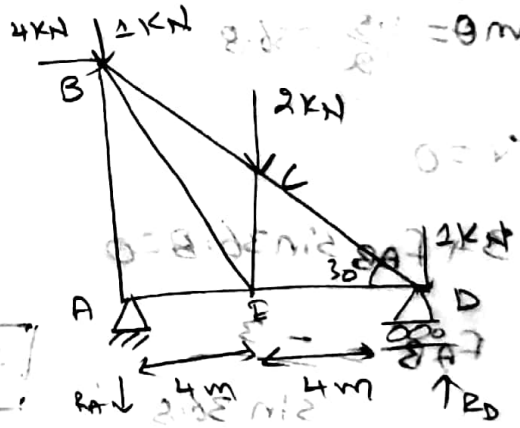
$\therefore f_{BC} = -15\,KN.$

member	force	Nature.
AB	5 KN	Compressive
AC	12 KN	Tensile.
BC	15 KN	Compressive.

2. Calculate forces in members for following figure.

$\Sigma V = 0$

$R_A + R_D = 1 + 2 + 1 = 4\,KN.$



$\Sigma H = 0$

$4 + H_A = 0$

$H_A = -4\,KN.$

$\Sigma M_A = 0.$

$R_D \times 8 - 1 \times 8 - 2 \times 4 - 4 \times AB = 0$

$R_D \times 8 - 8 - 8 - 4 \times 4.61 = 0$

$\therefore R_D = 4.305\,KN$

$R_A + R_B = 4$

$R_A = 4 - 4.305$

$\therefore R_A = -0.305\,KN$

In $\triangle le\ ABD$, $\tan 30 = \frac{AB}{8}$

$AB = 4.61m$

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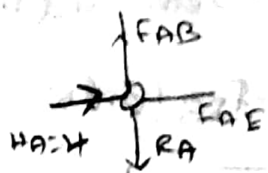
Consider ajoint "A"

$\Sigma V = 0.$

$-R_A + f_{AB} = 0$

$f_{AB} = 0.305\ kN.$



$\Sigma H = 0.$

$-H_A + f_{AE} = 0$

$\boxed{f_{AE} = 4\ kN}$

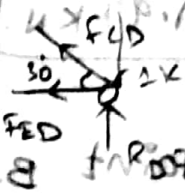
Consider ajoint "D"

$\Sigma V = 0.$

$4.305 - 1 + f_{CD} \sin 30^\circ = 0$

$f_{CD} = \frac{-4.305 + 1}{\sin 30^\circ}$

$\boxed{\therefore f_{CD} = -6.61\ kN.}$

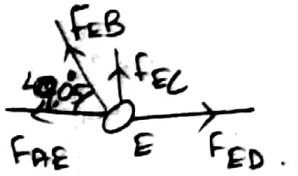


$\Sigma H = 0.$

$f_{ED} + f_{CD} \cos 30^\circ = 0.$

$f_{ED} = +6.61 \times \cos 30^\circ$, $\boxed{\therefore f_{ED} = 5.72\ kN.}$

Consider joint "E".



$\Sigma V = 0$ $\tan\theta =$

$f_{EC} + f_{EB} \sin 49.05^\circ = 0 \rightarrow (1)$

$\Sigma H = 0.$

$f_{ED} - f_{AE} - f_{EB} \cos 49.05 = 0$

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A ~~frame~~ 4m span and 1.3m depth

$5.7 - 4 - f_{EB}(0.65) = 0$

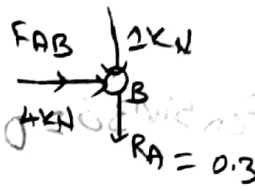
$\boxed{F_{EB} = 2.82\ KN}$

sub F_{EB} in ① eqn.

$F_{EC} = -F_{EB} \sin 49.05$

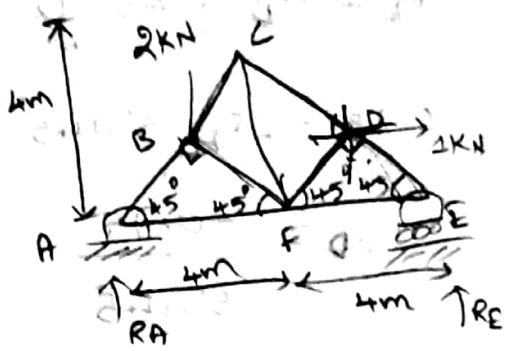
$\boxed{\therefore F_{EC} = -1.97\ KN.}$

Consider a joint B.



13/12/19

3) Calculate the forces in member for following figure.



$\Sigma V = 0$.

$R_A + R_E = 2 + 0 = 2KN$.

$\Sigma H = 0$.

$1 + H_A = 0$

$\boxed{H_A = -1\ KN}$

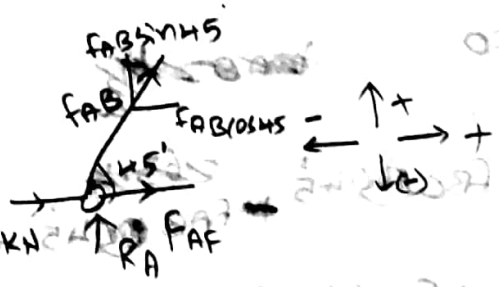
$\Sigma M_A = 0$,

$R_E \times 8 - 1 \times 2 - 2 \times 2 = 0$

$8R_E = 6$

$\boxed{R_E = 0.75\ KN.}$, $\boxed{R_A = 1.25\ KN}$

Consider a joint "A".



$\Sigma V = 0$.

$R_A + F_{AB} \sin 45 = 0$

$1.25 + F_{AB} \times 1/\sqrt{2} = 0$, $\boxed{F_{AB} = -1.76KN.}$

$\Sigma H = 0$.

$-1 + F_{AF} + F_{AB} \cos 45 = 0$.

$-1 + F_{AF} - 1.76 \cos 45 = 0$

$-1 + F_{AF} - 1.24 = 0$

$\boxed{\therefore F_{AF} = +2.24KN.}$

Consider a joint E.

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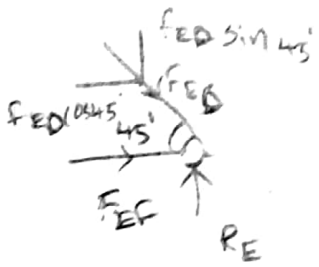
$\Sigma V = 0$.



$R_E + f_{ED}\sin 45^\circ = 0$

$0.75 = -f_{ED}\sin 45^\circ$

$f_{ED} = \dfrac{-0.75}{\sin 45^\circ}$ $\therefore f_{ED} = -1.06$ kN.

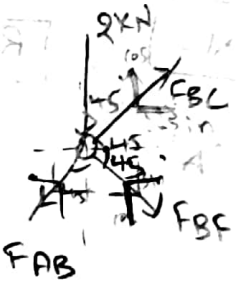
$\Sigma H = 0$

$\rightarrow + f_{ED}\cos 45^\circ + f_{EF} = 0$

$-1.06\cos 45^\circ + f_{EF} = 0$

$f_{EF} = 0.743$ kN.

Consider ajoint B.



$\Sigma V = 0$.

$-2 + f_{BC}\sin 45^\circ + f_{AB}\sin 45^\circ - f_{BF}\sin 45^\circ = 0$

$-2 + f_{BC}\sin 45^\circ - (1.76)\sin 45^\circ - f_{BF}\sin 45^\circ = 0$

$-2 + 1.244 + f_{BC}\sin 45^\circ - f_{BF}\sin 45^\circ = 0$

$-0.756 + \sin 45^\circ (f_{BC} - f_{BF}) = 0$.

$f_{BC} - f_{BF} = \dfrac{0.756}{\sin 45^\circ} = 1.069$ kN. $\rightarrow$ (1)

$\Sigma H = 0$.

$f_{BC}\cos 45^\circ + f_{BF}\cos 45^\circ - f_{AB}\cos 45^\circ = 0$

$f_{BC} \times 1/\sqrt{2} + f_{BF} \times 1/\sqrt{2} - f_{AB} \times 1/\sqrt{2} = 0$

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$$F_{BC} + F_{BF} - F_{AB} = 0$$

$$1.069 + F_{BC} + F_{BF} + 1.76 = 0$$

$$-2.829 + 2F_{BF} = 0$$

$$\boxed{F_{BF} = 1.4\ KN.}$$

$$F_{BC} = +(-1.76) - F_{BF}.$$

$$\boxed{\therefore F_{BC} = -0.35\ KN.}$$

Consider joint D.

$\Sigma V = 0.$



$$+F_{DC}\sin 45 - F_{DF}\cos 45 - F_{DE}\sin 45 = 0$$

$$+F_{DC}\sin 45 - F_{DF} \times \frac{1}{\sqrt{2}} - (-1.06) \times \frac{1}{\sqrt{2}} = 0$$

$$+(F_{DC} + F_{DF}) = +1.06\ KN. \quad \rightarrow (1)$$

$\Sigma H = 0$

$$1 + F_{DC}\cos 45 + F_{DF}\sin 45 + F_{DE}\cos 45 = 0$$

$$1 + (-1.06) - F_{DC} - F_{DF} + \sqrt{2} = 0$$

$$F_{DC} + F_{DF} - 0.354 = 0. \quad \rightarrow (2)$$

$$\boxed{\therefore F_{DC} = 0.353\ KN} \quad , \quad \boxed{F_{DF} = 0.707\ KN.}$$

Consider joint B:-

$\Sigma V = 0.$



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$-2 - F_{BF}\sin 45^\circ + F_{BC}\cos 45^\circ - F_{AB}\cos 45^\circ = 0$

$F_{BC} - F_{BF} = 1.07 \rightarrow ①$

$\Sigma H = 0.$

$F_{BC}\sin 45^\circ + F_{BF}\cos 45^\circ - F_{AB}\sin 45^\circ = 0$

$F_{BC} + F_{BF} = -1.76 \rightarrow ②.$

$F_{BC} = -0.35\,KN \;,\; F_{BF} = -1.4\,kN.$

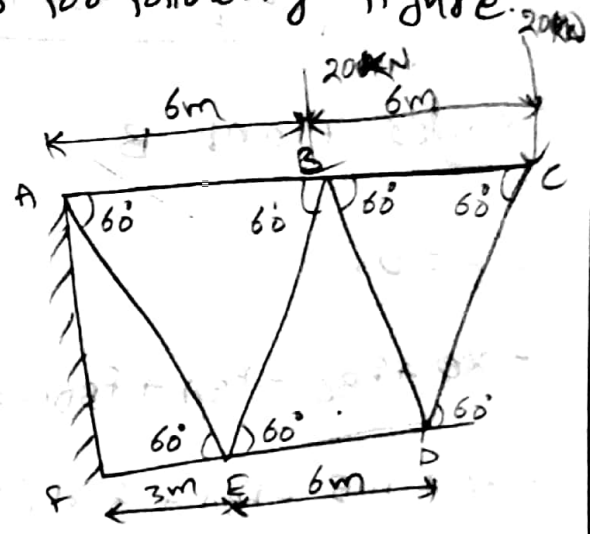
Consider joint F:-



$\Sigma V = 0.$

$F_{FB}\sin 45^\circ + F_{FC} + F_{FD}\sin 45^\circ = 0$

4) Calculate forces in all members for following figure.



consider joint C.

$\Sigma V = 0$.



$-20KN - F_{CD}\sin 60° = 0$

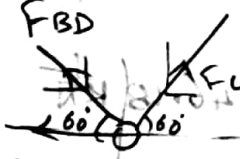
$F_{CD} = \frac{-20}{\sin 60°} = -23.1\,KN.$

$\Sigma H = 0$.

$-F_{BC} - F_{CD}\cos 60° = 0$

$-F_{BC} = -23.1 \times \cos 60° = 11.5\,KN.$

consider joint D.



$\Sigma V = 0$.

$+F_{CD}\sin 60° + F_{BD}\sin 60° = 0$

$+(-23.1)\sin 60°) + F_{BD}\sin 60° = 0$

$-20 + F_{BD}\sin 60° = 0$

$F_{BD} = \frac{20}{\sin 60°}$

$F_{BD} = 23.1\,KN.$

$\Sigma H = 0$.

$-F_{DE} - F_{BD}\cos 60° + F_{CD}\sin 60° = 0$

$-F_{DE} - 11.55 + (-23.1\cos 60°) = 0$

$\therefore F_{DE} = -23.1$.

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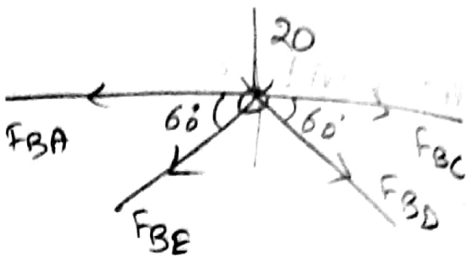
$\therefore F_{DE} = -23.1\ KN$.

Consider a joint B.



$\Sigma H = 0$,

$-20 + F_{BC} - F_{BA} + F_{BD}\cos 60° - F_{BE}\cos 60° = 0$.

$-20 + 11.5 - F_{BA} + 23.1(0.5) - F_{BE}\cos 60 = 0$

$-F_{BA} - F_{BE}\cos 60° = 20 - 11.5 - 11.5$

$F_{BA} + F_{BE}\cos 60° = 23.09\ kN$.

$\Sigma V = 0$.

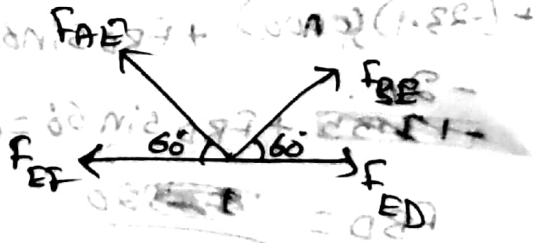
$-20 - F_{BE}\sin 60° - F_{BD}\sin 60° = 0$

$-20 - F_{BE}\sin 60 - 23.09\sin 60° = 0$

$\therefore F_{BE} = -46.18\ KN$.

$\therefore F_{AB} = 46.175\ KN$.

Consider joint 'E'



$\Sigma V = 0$

$F_{AE}\sin 60° + F_{BE}\sin 60° = 0$

$\therefore F_{AE} = 46.18\ KN$.

$\Sigma H = 0$.

$-F_{EF} + F_{ED} - F_{AE}\cos 60° + F_{BE}\cos 60° = 0$

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$$-f_{EF} + (-23.09) - 46.18\cos 60^\circ - 46.18\cos 60^\circ = 0$$

$$\therefore f_{EF} = -69.27 \text{ kN}$$

member	force	nature

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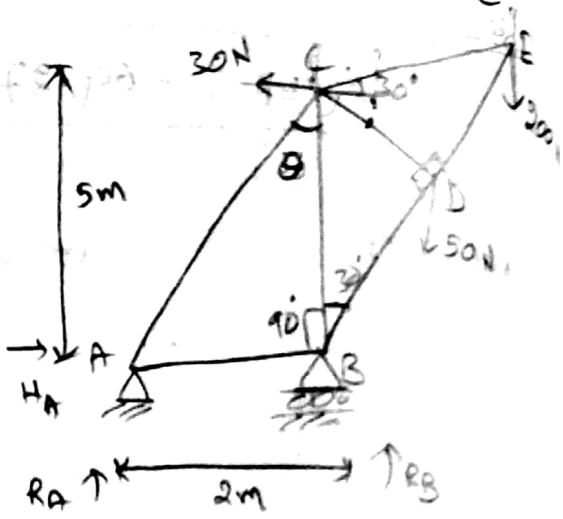
5) calculate the forces in member for following figure



$\Sigma H = 0$.

$H_A - 30 = 0$

$H_A = 30$ KN. (

Consider joint E.

$\Sigma V = 0$.

$-200 - F_{EC}\sin\theta - F_{ED}\sin 30 = 0$

$F_{EC}\sin\theta + F_{ED}\sin 30 = 200$ → ①

$\tan\theta = \frac{2}{5}$, $\theta = 21.8$



$\Sigma H = 0$.

$-F_{ED}\sin 30 - F_{EC}\cos 30 = 0$

$F_{ED}\sin 30 + F_{EC}\cos 30 = 0$ → ②

solving ① & ②.

① ⇒ $F_{EC}\sin 30 + F_{ED}\cos 30 = -200$

② ⇒ $F_{EC}\cos 30 + F_{ED}\sin 30 = 0$.

multiply ① × 0.5.

multiply ② × 0.866

$F_{ED} = -346.43$ N.

$F_{EC} = +200$ N.

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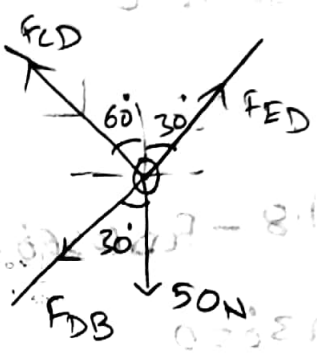
consider joint D.



$\Sigma V = 0$.

$-50 - F_{DB}\cos 30° + F_{ED}\cos 30° + F_{CD}\sin 60° = 0$

$-50 - 0.866F_{DB} + 346.43(0.866) + 0.5F_{CD} = 0.$

$0.5F_{CD} - 0.866F_{BD} = 350.48 \longrightarrow ③.$

$\Sigma H = 0.$

$F_{ED}\sin 30° - F_{CD}\sin 60° - F_{BD}\sin 30° = 0.$

$0.866F_{ED} - F_{BD}\,0.5 = -173.21 \longrightarrow ④.$

③ × 0.866 ⇒ ④ × 0.5.

$(0.25F_{CD} = 0.433F_{BD} - 175.24).$

⑤ ⇒ $0.433F_{CD} - 0.749F_{BD} = 303.51.$

⑥ ⇒ $0.433F_{CD} - 0.25F_{BD} = -86.60$

$-0.499F_{BD} = 390.1$

$\therefore F_{BD} = -\frac{390.1}{0.499}$

$F_{CD} = +25.24$

$\therefore F_{BD} = -390.14$ kN.

$F_{BD} = -390.14$

$F_{CD} = 25.24$ kN.

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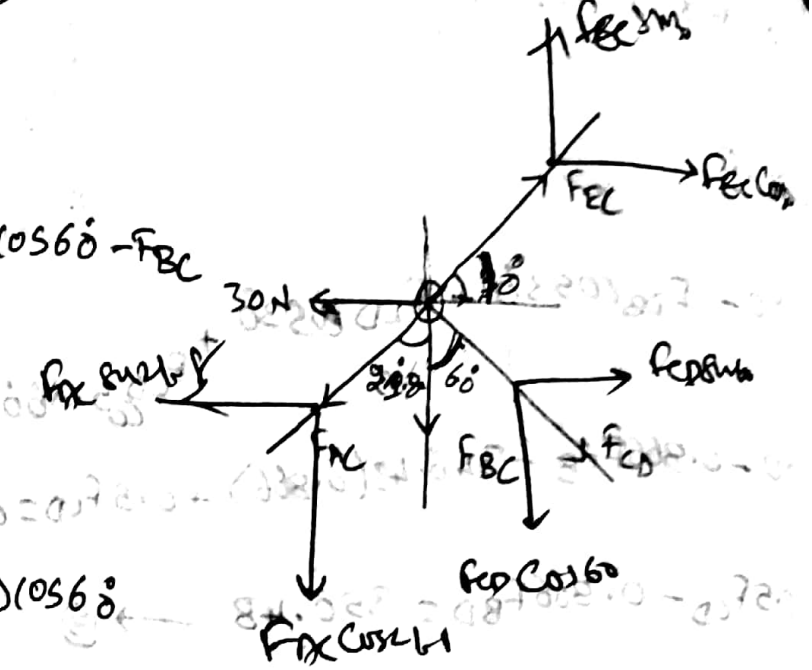
Consider joint C.



$\Sigma V = 0$.

$-F_{AC}\cos 21.8 - F_{CD}\cos 60^\circ - F_{BC} + F_{EC}\sin 30^\circ = 0$

$-F_{AC}\cos 21.8 - (425)\cos 60^\circ$

$-F_{BC} + 200\sin 30^\circ = 0$

$200 \times \frac{1}{2} + 25 \times 0.5 = F_{BC} + F_{AC} \times 0.92$

$100 - 112.5 = F_{BC} + F_{AC}\,0.92 \longrightarrow (5)$

$F_{BC} + F_{AC}\,0.92 = 87.5 \longrightarrow (5)$

$\Sigma H = 0$.

$-30 - F_{AC}\sin 21.8 + F_{CD}\sin 60^\circ + F_{EC}\cos 30^\circ = 0$

$-30 - F_{AC}\,0.37 + 25 \times 0.86 + 173.2 = 0$

$164.7 = F_{AC} \times 0.37$

$\therefore F_{AC} = 445.1$ N.

(5) $\Rightarrow$ $F_{BC} = 87.5 - 445.1 \times 0.92$

$\therefore F_{BC} = -321.9$ N.

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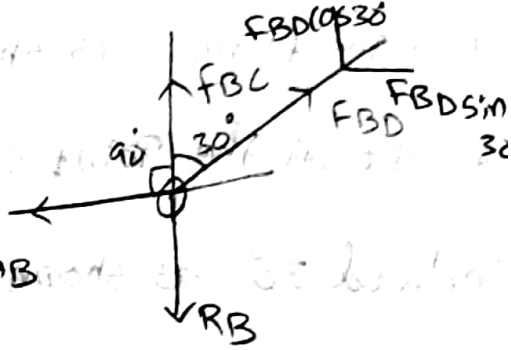
Consider joint B.



$\Sigma V = 0$.

$-R_B + F_{BD}\cos 30^\circ + F_{BC} = 0$

$-R_B + (-390)\cos 30^\circ + (-321.9) = 0$

$-R_B = 659.6$, $\therefore R_B = -660\ N$.

$\Sigma H = 0$.

$-F_{AB} + F_{BD}\sin 30^\circ = 0$

$-F_{AB} - 390 \times \sin 30^\circ = 0$

$-F_{AB} = 195\ N$.

$R_A + R_B = 250$.

$\therefore R_A = -412\ N$.

Member	Force	Nature
AB	195 N	Compressive
BC	321.9 N	Compressive
AC	445.1 N	Tensile
BD	390.44 N	Compressive
CD	25.24	Tensile.
EC	200	Tensile.
ED	346.43	Compressive

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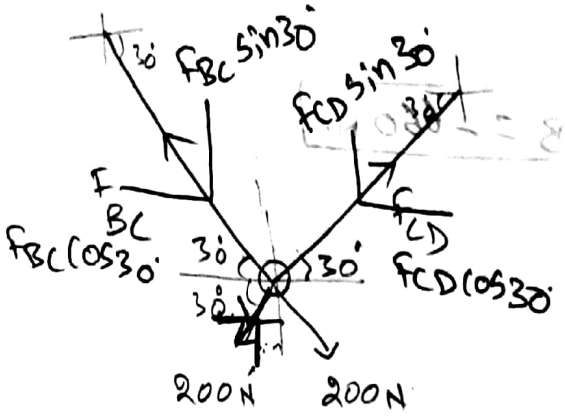
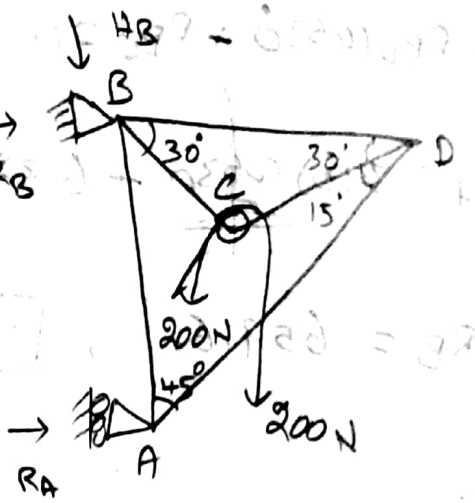
6) In the frame as shown in figure, there is a small weight at the point C. A weight 200 N is held by a force inclined 30°, as shown in figure. Find the forces in all members.

Consider joint C.





$\Sigma V = 0$

$-200 - 200\cos 30° + f_{CD}\sin 30° + f_{BC}\sin 30° = 0$

$-400 + f_{CD} \times \frac{1}{2} + f_{BC} \times \frac{1}{2} = 0$

$\frac{1}{2}(f_{CD} + f_{BC}) = 400 \rightarrow (1)$

$\Sigma H = 0.$

$f_{CD}\cos 30° - f_{BC}\cos 30° = 0$

$f_{CD} - f_{BC} = 0 \rightarrow (2)$

From (1) & (2)

$f_{CD} + f_{BC} = 400$ sub $f_{BC} = f_{CD}$ in eq.

$f_{CD} - f_{BC} = 0$

$\frac{1}{2}(2f_{CD}) = 400$

$\therefore f_{CD} = 400$

$\therefore f_{CD} = 400$

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$\frac{1}{2}(F_{CD} + F_{BC}) = 400$, $400 + F_{BC} = 8$

$\therefore F_{BC} = 200$ T

Consider joint C.

$\Sigma V = 0$

$-200 + F_{CD}\sin 30 + F_{BC}\sin 30 - 200\cos 30 = 0$

$-200 + \frac{1}{2}F_{CD} + \frac{1}{2}F_{BC} - 100 = 0$

$0.5F_{CD} + 0.5F_{BC} = 300$ ... ①

$\Sigma H = 0$

$F_{CD}\cos 30 - F_{BC}\cos 30 + 200\sin 30 = 0$

$0.86F_{CD} - 0.86F_{BC} + 173.2 = 0$ ... ②

multiply 1.732 to eqn ①

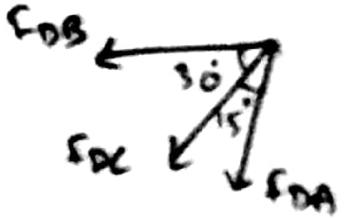
$0.86F_{CD} + 0.86F_{BC} = 519.6$

$0.86F_{CD} - 0.86F_{BC} = +173.2$

$1.72F_{CD} = 692.8$

$\therefore F_{CD} = 400$

Consider joint D:



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$\Sigma V = 0$

$-F_{DC}\sin 30 - F_{DA}\sin 45 = 0$

$0.5(400) + F_{DA}(1/\sqrt{2}) = 0$

$\therefore F_{DA} = -282.84\ N.$

$\Sigma H = 0.$

$-F_{DB} - F_{DC}\cos 30 - F_{DA}\cos 45 = 0$

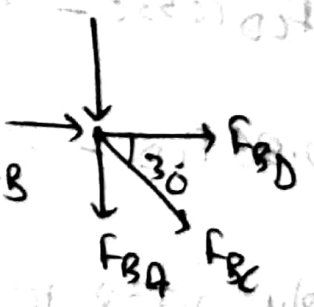
$-F_{DB} - 400\cos 30 + 282.84\cos 45 = 0$

$\therefore F_{DB} = 146.41\ N.$

Consider joint B:-

$\Sigma V = 0.$

$-V_B - F_{BA} - F_{BC}\sin 30 = 0$



16/9/19

7) Calculate the forces in the member for following figure



From $\Delta ACC'$.

$\cos 30° = \frac{AC'}{AC}$

$\cos 30° = \frac{7.5}{AC}$, $\therefore AC = 8.66m$

$AC = CE = 8.66m$.

$\Delta ACC'$.

$AC' = 7.5m$, $AC = 8.66m$, $CC' = \sqrt{8.66^2 - 7.5^2}$

$CC' = 4.32m$.

$R_A + R_E = 1 \times \sin 60° + 2 \times \sin 60° + 1 \times \sin 60°$.

$R_A + R_E = 3.46\,kN$.

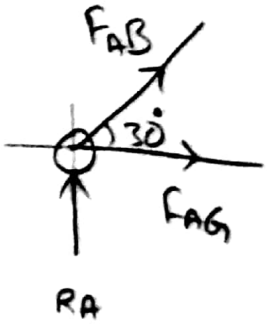
$\Sigma M_E = 0$.

$R_A \times 15 - 1 \times 8.66 - 2 \times 4.33 = 0$

$\therefore R_A = 1.15\,kN$.

$R_E = 3.46 - 1.15 = 2.31\,kN$.

Consider Joint "A"



$\Sigma V = 0$.

$1.15 + F_{AB} \sin 30° = 0$

$\therefore F_{AB} = -2.30\,kN$.

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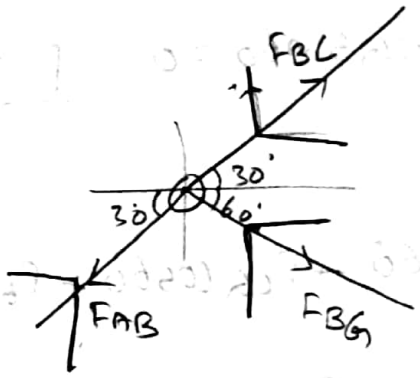
$\Sigma H = 0$

$F_{AG} + F_{AB}\cos 30^\circ = 0$

$\therefore F_{AG} = 2\,KN.$

Consider joint B.



$\Sigma V = 0$

$+ F_{BC}\sin 30^\circ - F_{BG}\sin 60^\circ$

$- F_{AB}\sin 30^\circ = 0$

$-F_{BC}\,0.5 + F_{BG} \times 0.866 + (-2.30 \times 0.5)$

$-F_{BC}\,0.5 + F_{BG}\,0.866 = -1.15 \rightarrow (1)$

$\Sigma H = 0.$

$F_{BC}\cos 30^\circ - F_{AB}\cos 30^\circ + F_{BG}\cos 60^\circ = 0$

$F_{BC} \times 0.866 + 2.30 \times 0.866 + F_{BG}\,0.866 = 0$

$F_{BC} \times 0.866 + F_{BG} \times \frac{0.866}{5} = -1.99 \rightarrow (2).$

$0.866 \times (1) \Rightarrow -0.433\,F_{BC} + 0.749\,F_{BG} = -0.99$

$0.5 \times (2) \Rightarrow 0.433\,F_{BC} + 0.25\,F_{BG} = -0.99$

$0.99\,F_{BG} = 1.98$

$\therefore F_{BG} = -0.004\,KN \simeq 0\,KN.$

$F_{BC} = -2.3\,KN.$

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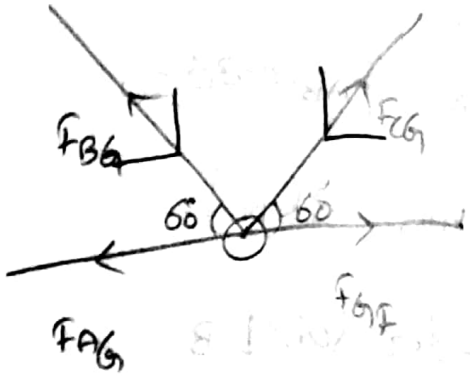
Consider joint "G".



$\Sigma V = 0$.

$F_{BG}\sin 60° + F_{CG}\sin 60° = 0$

$0 + F_{CG}\sin 60° = 0$ $\quad \therefore F_{CG} = 0$

$\Sigma H = 0$.

$-F_{BG}\cos 60° + F_{CG}\cos 60° + F_{GF} - F_{AG} = 0$.

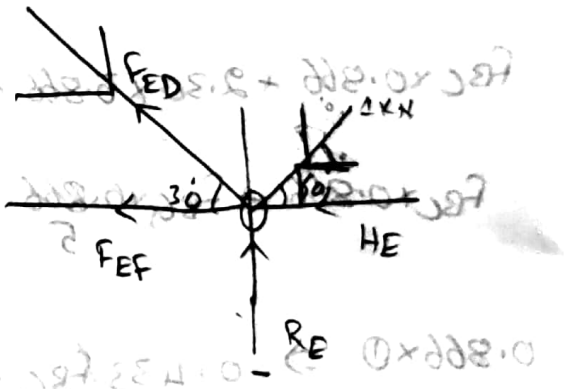
$0 + 0 + F_{GF} - 2 = 0$ $\quad \therefore F_{GF} = 2\,KN$

Consider joint E.

$\Sigma H = 0$.

$1\cos 60° + 2\cos 60° + 1\cos 60° + H_E = 0$

$H_E = -2\,KN$



$\Sigma V = 0$.

$F_{ED}\sin 30° + 1\times\sin 60 + R_E = 0$

$F_{ED}\,0.5 - 0.866 + 2.31 = 0$

$F_{ED} = -2.88\,KN$

$\Sigma H = 0$.

1KN is inclined

$-F_{ED}\cos 30° + 1\cos 60° - F_{EF} + H_E = 0$

$F_{EF} = 4\,KN$.

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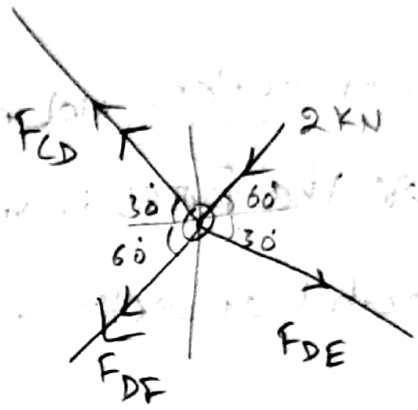
Consider joint D:



$\Sigma V = 0.$

$-2\sin 60 + F_{CD}\sin 30 - F_{DF}\sin 60$

$-F_{ED}\sin 30 = 0.$

$F_{CD}(0.5) - F_{DF}(0.866) = 0.298.$

$\Sigma H = 0.$

$-2\cos 60 - F_{CD}\cos 30 - F_{DF}\cos 60 + F_{DE}\cos 30 = 0$

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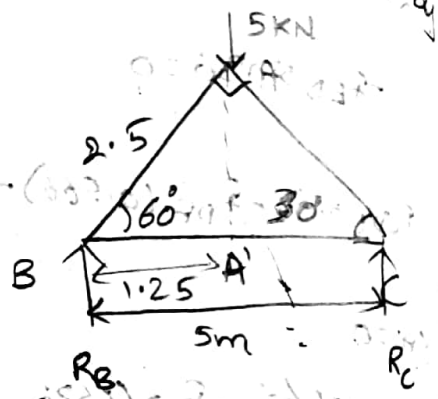
18/12/19

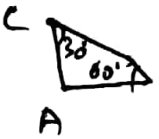
Method of section:- not more than 3 members.

1) calculate the forces in member for following figure by using method of section.



$R_B + R_C = 5\,KN$.

from ΔABC,



$\cos 60° = \frac{AB}{BC}$

$AB = 5 \cos 60° = 2.5\,m$

from $\Delta ABA'$



$\cos 60° = \frac{BA'}{AB}$

$BA' = AB \cos 60°$

$BA' = 1.25\,m$.

$AA' = \sqrt{2.5^2 - 1.25^2}$

$AA' = 2.16\,m$.

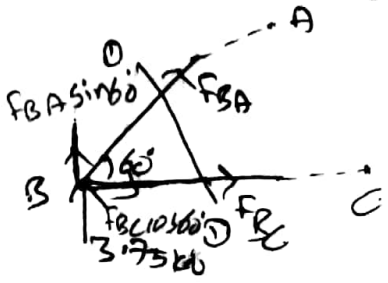
consider any fixed point from the member within direction is zero and resolve forces at fixed point and taking moments of at a point (Force x ⊥ dis)

$\Sigma M_A = 0$. $\quad R_C \times 5 - 5 \times 1.25 = 0$

$R_C = \frac{5 \times 1.25}{5} = 1.25$.

$R_B = 5 - 1.25 = 3.75\,KN$.

Consider a section 1-1 which is passing across members AB, BC



Taking moment about Point C.

$3.75 \times 5 + f_{BA} \sin 60° \times 5 = 0$

$f_{BA} = \frac{-3.75 \times 5}{5 \times \sin 60°} = -4.34\,KN$.

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taking moment about Point 'A'

$\Sigma M_A = 0, \quad 3.75 \times 1.25 - F_{BC} \times 2.16 = 0$

$F_{BC} = \frac{3.75 \times 1.25}{2.16} = 2.17 \text{ KN}.$

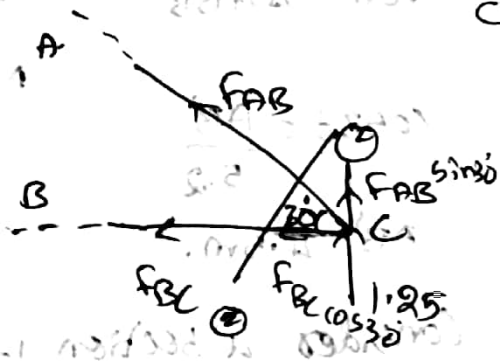
consider a section ②-② which is passing through AC & BC

taking moment about Point B:

$1.25 \times 5 + F_{AB} \sin 30 \times 5 = 0$

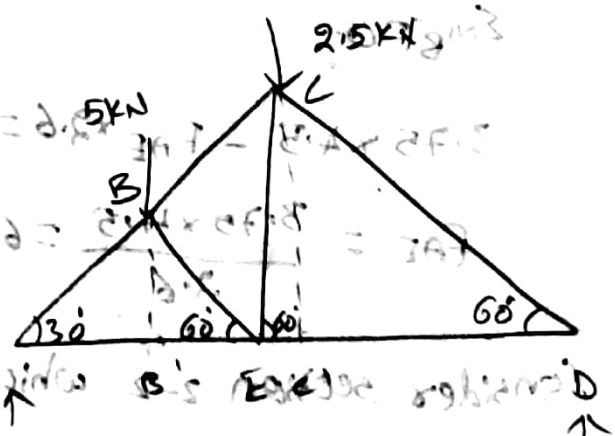
$6.25 + F_{AB} 2.5 = 0$

$F_{AB} = \frac{-6.25}{2.5} = -2.5 \text{ KN}.$



member	force	Nature.
AB	2.34 KN	compressive.
BA	4.34 KN	compressive
BC	2.17 KN	Tensile

2) Calculate the force in member for following truss by method of sections.



$R_A + R_D = 2.5 + 5 = 7.5 \text{ KN}.$

$\Sigma M_A = 0.$

$R_D \times 12 - 2.5 \times 9 - 5 \times 4.5 = 0$

$12 R_D = 45$

$\therefore R_D = 3.7 \text{ KN}.$

$R_A = 3.7 \text{ KN}$

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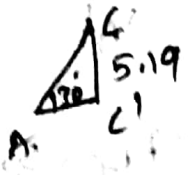
from $\Delta ACC'$



$AC = \sqrt{5.19^2 + 9^2}$

$AC = 10.4m.$

$\tan 30° = \frac{CC'}{9}$

$CC' = 5.19m$,

from $\Delta ABB'$



$\cos 30° = \frac{AB'}{5.2}$

$AB' = 4.5m.$

from $\Delta BB'E$



$\cos 30° = \frac{B'E}{BE}$

$BE = \frac{1.5}{\cos 30°}$

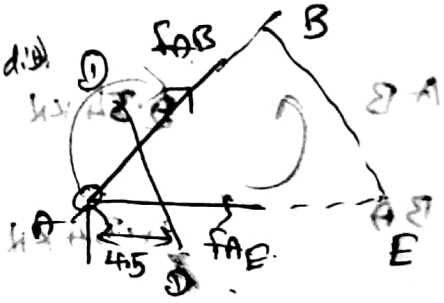
$B'E = 1.5$

Consider a section 1-1 which is passing across AB & AE.

taking moment about point E.

$\Sigma M_E = 0.$

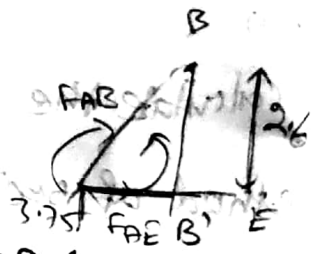
$3.75 \times 6 + F_{AB} \times BE$ → (F × ⊥r dist)



$F_{AB} = -\frac{3.75 \times 6}{3}$

$\boxed{\therefore F_{AB} = -7.5 KN}$

taking moment about point B.



$\Sigma M_B = 0.$

both acting upward clockwise (-ve).

$3.75 \times 4.5 - F_{AE} \times 2.6 = 0.$

$BB' = 2.6$

$F_{AE} = \frac{3.75 \times 4.5}{2.6} = 6.4 KN.$

$3.75 \times \perp$ rds

$- F_{AE} \times \perp$ rds

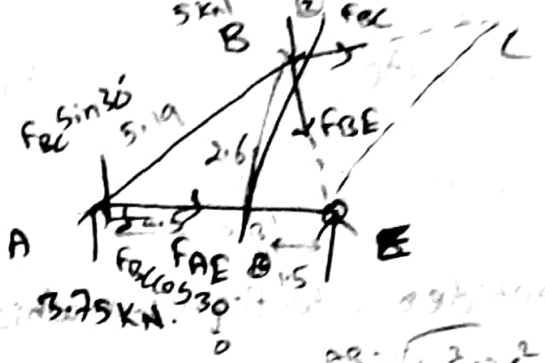
Consider section 2-2 which is passing across BC, BE & AE

take a moment about point A.

$\Sigma M_A = 0$

$5 \times 4.5 + F_{BE} \times 5.2 = 0.$

$\therefore F_{BE} = -4.33\,KN.$



taking moment about E.

$\Sigma M_E = 0$.

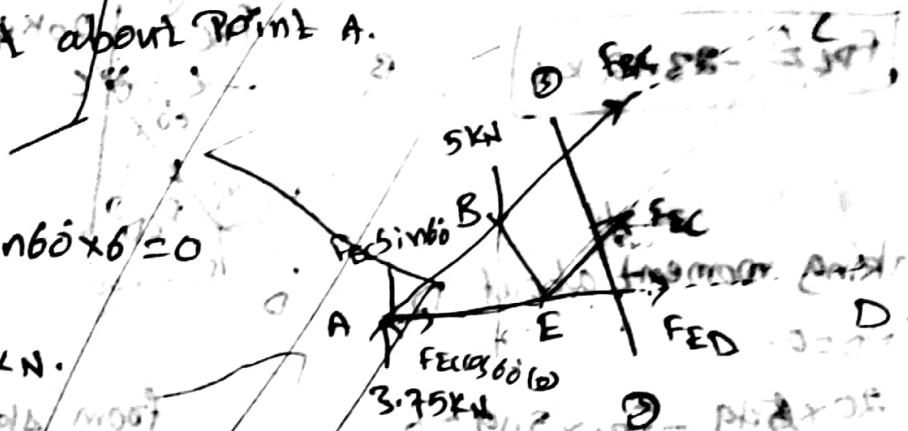
F_{BE} & F_{AE} are zeroes.

$3.75_{\times 6} - 5 \times 1.5 + F_{BC}\sin30^\circ = 0$ (×6)

$6F_{BC} = \frac{-15}{\sin30^\circ} = \frac{-30}{6} = -5\,KN.$

Consider a section 3-3 which is passing across BC, CE & ED.

taking moment about Point A.



$\Sigma M_A = 0$.

$5 \times 4.5 - F_{EC}\sin60 \times 6 = 0$

$F_{EC} = +4.33\,KN.$

taking moment about Point C.

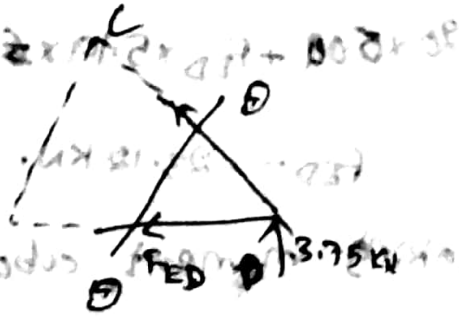
$\Sigma M_C = 0$.

$3.75 \times 9 - 5 \times 4.5 - F_{ED} \times 5.19 = 0$

$F_{ED} = 2.165\,KN.$

Consider a section 4-4 passing across. DC & DE

taking moment about Point E.



$\Sigma M_E = 0$, $3.75 \times 6 + F_{DC}\sin60^\circ \times 6 = 0$.

$F_{DC} = -4.33\,KN.$

member	force	nature.

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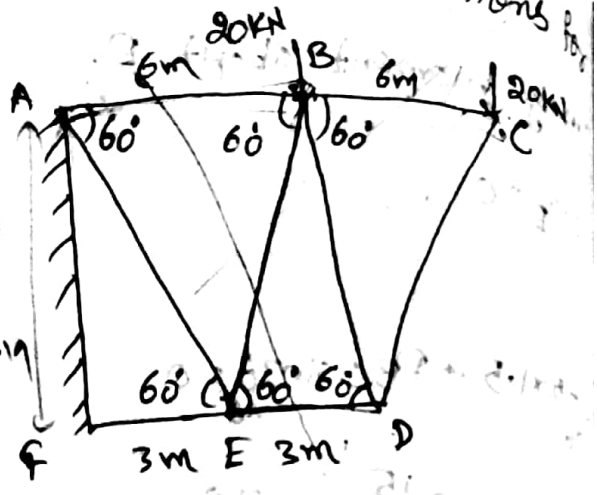
20/12/19

3) Calculate forces in members by method of sections for following figure.



Consider a section 1-1 which is passing across CB & CD.

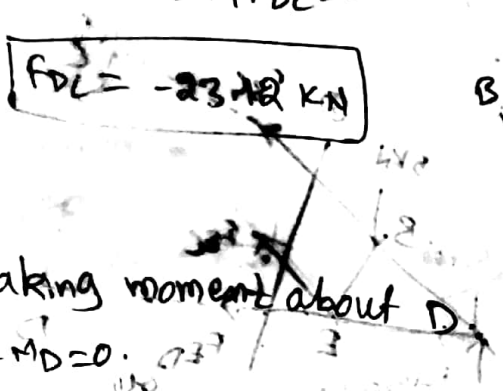
taking moment about B.

$\Sigma M_B = 0$

$20 \times 6 + F_{DC} \sin 60 \times 6 = 0$

$120 + 5.19 F_{DC} = 0$

$F_{DC} = -23.12$ KN

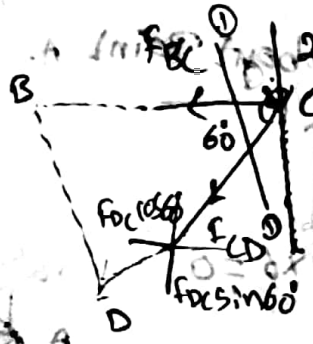


taking moment about D

$\Sigma M_D = 0$.

$20 \times \frac{5.19}{3} - F_{BC} \times 5.19 = 0$.

$\therefore F_{BC} = 11.5$ KN.



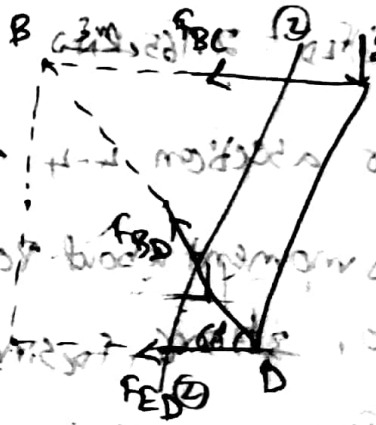
from $\triangle$le AFE

$\tan 60 = \frac{AF}{FE}$

$AF = 5.19$m

Consider a section 2-2 which is passing across BC, BD & ED.

taking moment about B.



$\Sigma M_B = 0$

$20 \times 5.19 + F_{ED} \times 5.19 \times \sin 60 = 0$

$F_{ED} = -23.12$ KN.

taking moment about E.

$\Sigma M_E = 0$

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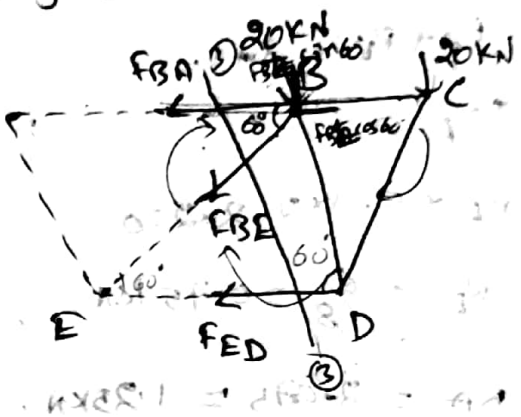
$20\times(6+3) - F_{BC} \times 5.19 - F_{BD} \times \sin 60^\circ \times 6 = 0$

$F_{BD} = 23.09\ KN.$

consider section 3-3 which is passing across AB, BE, ED



taking moment about E.

$\Sigma M_E = 0$

$20\times 9 + 20\times 3 + F_{BA}\times 5.19 = 0$

$\therefore F_{BA} = +46.24\ KN.$

taking moment about A.

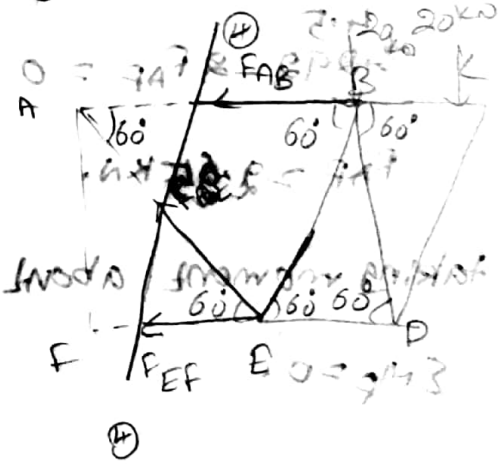
$\Sigma M_A = 0$

$20\times 6 + 20\times 12 + F_{BE}\sin 60^\circ \times 6 + F_{ED}\times 5.196 = 0$

$120 + 240 + F_{BE}\times 6\times \sin 60^\circ + 5.196\times F_{ED} = 0$

$\therefore F_{ED} = +46.16\ KN.$

consider section 4-4 which is passing across AE, AB, EF.



taking moment about A.

$\Sigma M_A = 0.$

$20\times 6 + 20\times 6 + F_{EF}\times 5.196 = 0$

$\therefore F_{EF} = 69.28\ KN.$

taking moment about E.

$\Sigma M_E = 0$

$20\times 12 + 20\times 6 - F_{AB}\times 5.196 - F_{AE}\times \sin 60^\circ \times 3 = 0$

$F_{AE} = 46.12\ KN.$

member	force	Nature

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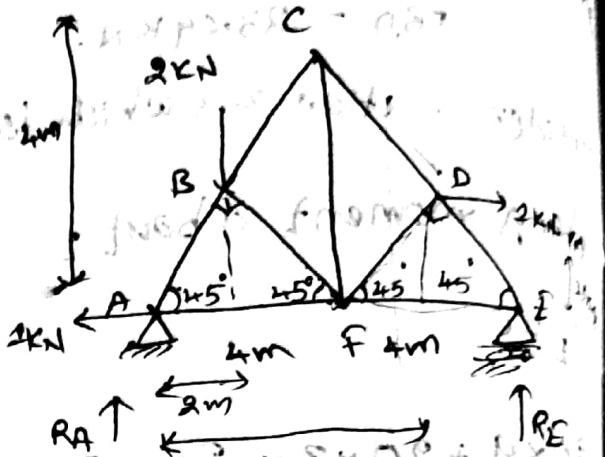
21/12/19

4) calculate the forces in members by method of sections for following figure.



$R_A + R_E = 3$

$\Sigma M_E = 0$

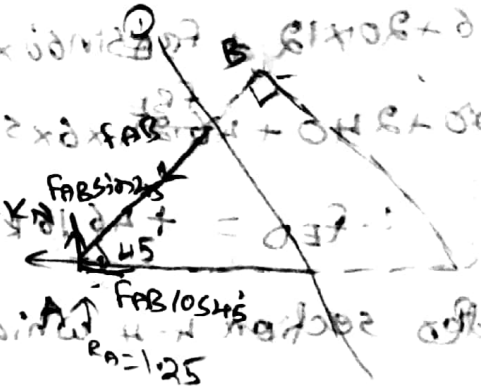
$R_E \times 8 - 1 \times 2 - 2 \times 2 = 0$

$R_E = 6/8 = 0.75$ KN

$R_A = 2 - 0.75 = 1.25$ KN.

Consider a section 1-1 which is passing across AB & AF

taking a moment about B.



$\Sigma M_B = 0$

$1.25 \times 2 - F_{AF} \times 2 + 1 \times \sin 45 \times 2 = 0$

$2.5 - 2F_{AF} + 1.414 = 0$

$3.914 - 2F_{AF} = 0$

$F_{AF} = 1.957$ KN.

taking moment about F.

$\Sigma M_F = 0$.

$R_A \times 4 + F_{AB} \sin 45 \times 4 = 0$

$4 \times 1.25 + F_{AB} \times 2.828 = 0$

$F_{AB} = -1.76$ KN.

Consider a section 2-2 which is passing across BC and BF and AF.

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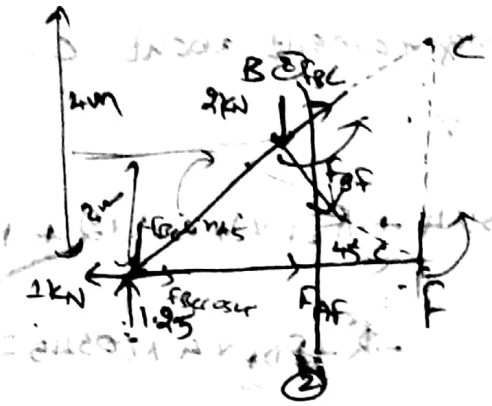
taking moment about C.

$\Sigma M_C = 0$.

$1.25 \times 4 + 1 \times 4 - 2 \times 2 - F_{BF} \times 4$

$- F_{BF} \times \cos 45 \times 4 = 0$

$F_{BF} = -1.414\,KN$.



taking moment about f.

$\Sigma M_F = 0$

$1.25 \times 4 - 2 \times 2 + F_{BC} \sin 45 \times 4 = 0$

$\therefore F_{BC} = -0.35\,KN$.

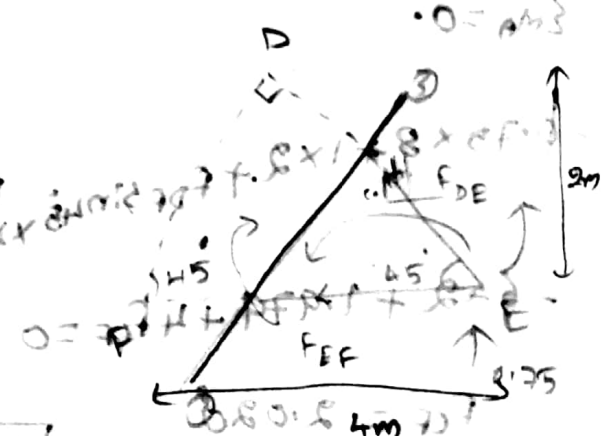
cosider a section 3-3 which is passing accoss through DE and FE.

taking moment about D.

$\Sigma M_D = 0$

$-0.75 \times 2 + F_{EF} \times 2 = 0$

$F_{EF} = 0.75\,KN$.



taking moment about F.

$\Sigma M_F = 0$

$-0.75 \times 4 - F_{DE} \sin 45 \times 4 = 0$

$F_{DE} = -1.06\,KN$.

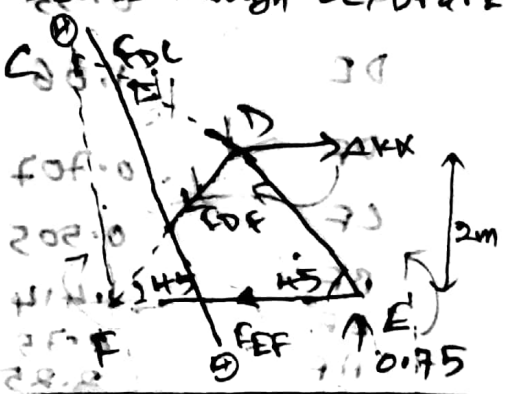
Consider a section 4-4 which is passing across through DC, DF & FE.

taking moment about F.

$\Sigma M_F = 0$

$-0.75 \times 4 - F_{DC} \sin 45 \times 4 - 1 \times 2 = 0$

$F_{DC} = -0.35\,KN$.



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taking moment about C.

$\Sigma M_C = 0$

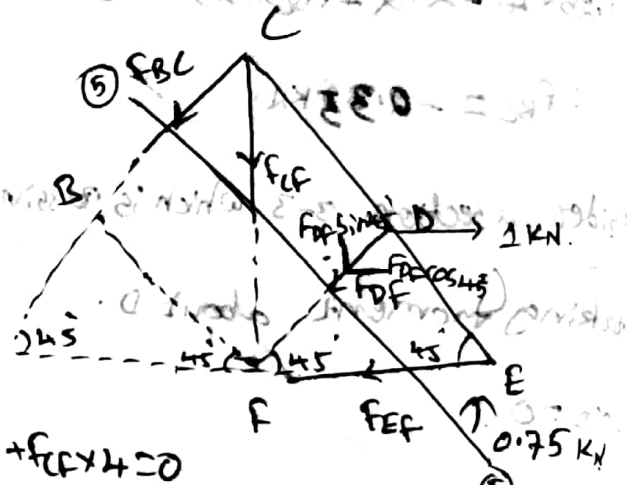
$-0.75 \times 4 + F_{EF} \times 4 - 1 \times 2 + F_{DF} \times 1.0545 \times 4 = 0$

$-2 + F_{DF} \times 4 \times 1.0545 = 0$

$F_{DF} = \frac{2}{4 \times 1.0545}$

$\therefore F_{DF} = 0.707\ KN.$

consider a section 5-5 which is passing across through BC, CF, DF and EF.



taking moment about A.

$\Sigma M_A = 0.$

$-0.75 \times 8 + 1 \times 2 + F_{DF} \sin 45 \times 4 + F_{CF} \times 4 = 0$

$-6 + 2 + 1.979 + 4F_{CF} = 0$

$F_{CF} = \frac{2.020}{4}$

$\therefore F_{CF} = 0.505\ KN$

Member	Force	Nature.
AB	1.76	Compressive
BC	0.35	Compressive
CD	0.35	Compressive
DE	1.06	Compressive
DF	0.707	Tensile
CF	0.505	Tensile
BF	1.414	Compressive
EF	0.79	Tensile
AF	2.25	Tensile.

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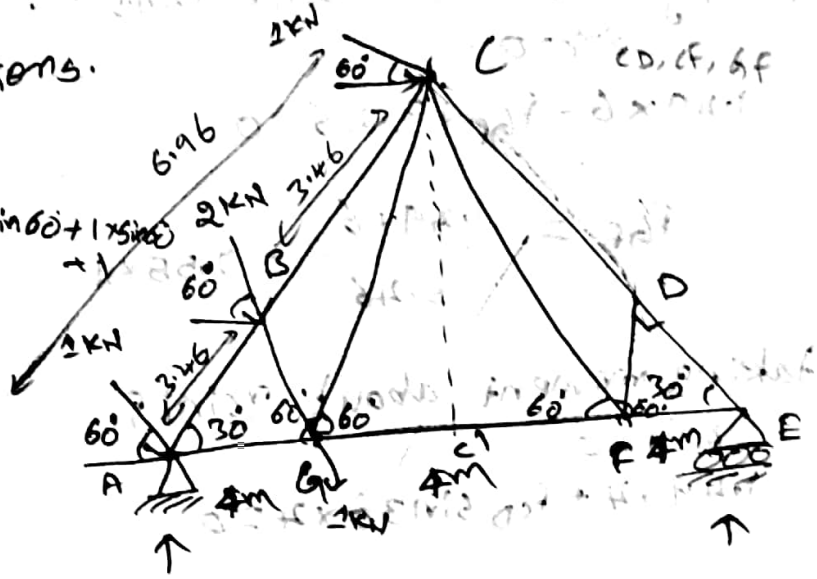
5) Calculate the forces in member for following figure by using method of sections.



$R_A + R_E = 2\times\sin60 + 1\times\sin60 + 1\times\sin60 + 1$

$R_A + R_E = 3.46 + 1$

$R_A + R_E = 4.46\,KN \rightarrow (1)$

$\Sigma M_A = 0$

$R_E \times 12 - 1\times\cos60\times AC - 2\times AB - 1\times 4$

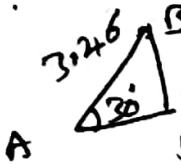
$\Delta ACC'$.



$\cos30 = \frac{AC'}{AC}$

$AC = \frac{6}{\cos30} = 6.92$.

From $\Delta ABB'$.



$\cos30 = \frac{AB'}{3.46}$

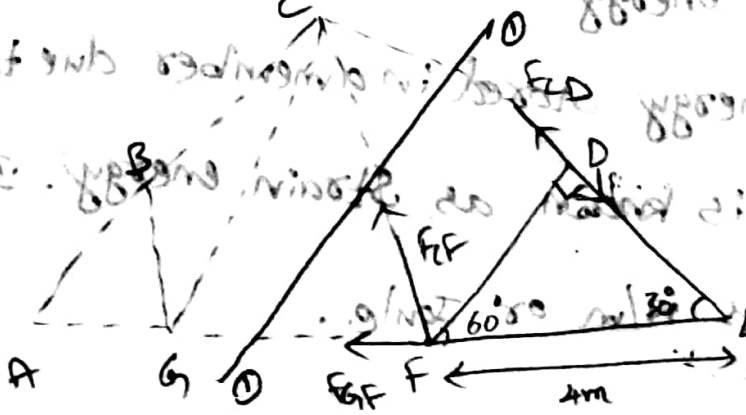
$AB' = 3m$.

$R_E \times 12 - 1\times 6.92 - 2\times 3.46 - 1\times 4 = 0$

$\therefore R_E = 1.49\,KN$.

$R_A = 2.97\,KN$.

Consider a section which is passing across through CD, CF & GF.



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taking moment about Point C.

$\Sigma M_C = 0$

$1.49 \times 6 - F_{GF} \times 3.46 = 0$

$F_{GF} = \frac{1.49 \times 6}{3.46} = 2.58\,KN.$

from $\triangle ACC'$

$\tan 30^\circ = \frac{CC'}{AC'}$

$CC' = 6 \tan 30^\circ$

$CC' = 3.46.$

taking moment about Point F.

$\Sigma M_F = 0$

$1.49 \times 4 + F_{CD} \sin 30^\circ \times 4 = 0$

$F_{CD} = -\frac{1.49 \times 4}{4 \times \sin 30^\circ} = -2.98\,KN.$

taking moment about E.

$\Sigma M_E = 0$

$F_{CF} \sin 60^\circ \times 4 = 0$

$\therefore F_{CF} = 0.$

Member	force	Nature

Strain Energy:-

~~The energy stor~~

When an elastic body subjected to external loads, due to external loads. the body will deforms and some internal energy will be stored in the body. Therefore, the energy stored in a member due to external applied loads is known as strain energy. It is denoted by 'U'

* units N/m or Joule.

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* for most of the elastic materials the elongation increases linearly with increasing in load. and therefore the load vs elongation diagram is a straight line with its elastic limit.

* Strain energy also defined as the area under load deformation curve.



strain energy (U) = $\frac{1}{2} P \cdot \Delta$.

Strain energy due to axial loading :-

Consider a member which is subjected to load P. and it deforms an amount of 'δ'. Hence then strain energy (U) is

$$U = \frac{1}{2} \cdot P \cdot \delta \longrightarrow (1)$$

$$= \frac{1}{2} P \left(\frac{PL}{AE}\right) \quad \left(\because \delta = \frac{PL}{AE}\right)$$

$$U = \frac{P^2 L}{2AE} \longrightarrow (2)$$

$$U = \frac{1}{2} P \delta l \quad \Rightarrow \left(\sigma = \frac{P}{A} \Rightarrow P = \sigma \cdot A\right)$$

$$U = \frac{1}{2} (\sigma \cdot A)(e \cdot l) \qquad e = \frac{\delta l}{l} \Rightarrow \delta l = e \times l.$$

$$U = \frac{1}{2} \times \sigma \times e \times \text{volume}.$$

$$U = \frac{1}{2} \times \text{stress} \times \text{strain} \times \text{volume} \longrightarrow (3)$$

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$$U = \frac{1}{2} \sigma \cdot e \cdot V$$

$$U = \frac{1}{2} \sigma \left(\frac{\sigma}{E}\right) V$$

$$U = \frac{\sigma^2}{2E} \times \text{volume} \rightarrow (4)$$

1) Steel bar of 4cm x 4cm in cross section and 3m long subjected to an axial pull of 128 kN. Calculate alteration in length and strain energy. take E = 200 GN/m²

G.T.

$$E = 200 \text{ GN/m}^2$$

$$E = 200 \times 10^9 \text{ N/m}^2$$

$$A = 0.04 \times 0.04 = 1.6 \times 10^{-3} \text{ m}^2$$

$$P = 128 \text{ kN} = 128 \times 10^3 \text{ N}$$

$$l = 3\text{m}, \quad \delta l = ?, \quad U = ?$$

$$U = \frac{P^2 l}{2AE}$$

$$U = \frac{(128 \times 10^3)^2 \times 3}{2 \times 1.6 \times 10^{-3} \times 200 \times 10^9}$$

$$\therefore U = 76.8 \text{ N/m}$$

W.K.T

$$U = \frac{1}{2} P \times \delta l$$

$$76.8 = \frac{1}{2} \times 128 \times 10^3 \times \delta l$$

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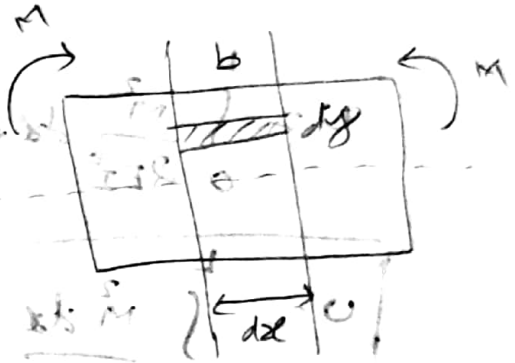
$$\delta l = \frac{76.8 \times 2}{128 \times 10^3}$$

$$\therefore \delta l = 1.2 \times 10^{-3}\ m.$$

27/12/10

Strain Energy due to Bending :-

Consider a small length of dx. of a beam where the B.M is "M"



Consider a small strip of thickness dy and width of strip "b".

The volume of the strip = dx.dy.b.

$$dU = \frac{\sigma^2}{2E} \times \text{volume}$$

$$dU = \frac{\sigma^2}{2E} \times dx.dy.b$$

$$\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}$$

$$\sigma = \frac{M.y}{I}$$

$$dU = \frac{(My/I)^2}{2E} \times dx \times dy \times b.$$

$$dU = \frac{M^2y^2}{2EI^2} \cdot dx.dy.b$$

Integrate above eqn to get total U.

$$U = \int_0^A \int_0^L \frac{M^2y^2}{2EI^2} dx.dy.b$$

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$$U = \int_0^L \frac{M^2}{2EI^2} dx \int_0^A dy\, by^2$$

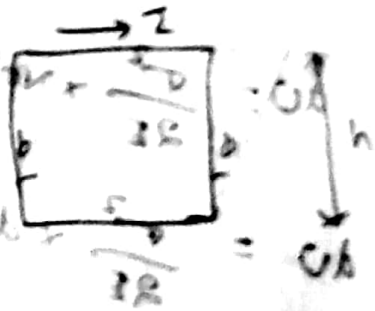
$$U = \int_0^L \frac{M^2}{2EI^2} dx \int_0^A y^2 \cdot dA \qquad \left[\because \int_0^A y^2 dA = I\right].$$

$$U = \int_0^L \frac{M^2}{2EI^2} dx \left(\frac{I}{I}\right)$$

$$\boxed{U = \int_0^L \frac{M^2 dx}{2EI}}$$

Strain Energy due to shear:-

Consider a block with dimensions l, b, h and which is subjected to shear stress "I" and angle of twist "Φ".



$U = \frac{1}{2} \times P \times \delta$

$U = \frac{1}{2} \times$ couple $\times$ angle of twist

$= \frac{1}{2} \times$ force $\times$ perpendicular distance (height) $\times \Phi$

$= \frac{1}{2} \times I \times$ Area $\times$ height $\times \Phi$ $\qquad \left[\because \Phi = \frac{I}{G}\right]$

$= \frac{1}{2} \times I \times$ Area $\times$ height $\times \frac{I}{G}$

$$\boxed{\therefore U = \frac{I^2}{2G} \times \text{volume.}}$$

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i) In steel specimen 1.5 cm² Area stretches 0.05 mm over 5 cm Gauge length ($l = 5.65\sqrt{A}$) under axial load of 30 kN. Calculate strain energy at this point.

ii) If the load at elastic limit is 50 kN. Calculate elongation at elastic limit.

area $= 1.5\,cm^2 = 1.5 \times 10^{-4}\,m^2$.

$\delta l = 0.05\,mm = 0.05 \times 10^{-3}\,m$.

$l = 5\,cm = 5 \times 10^{-2}\,m$.

$P = 30\,kN = 30 \times 10^{3}\,N$

$U = \frac{1}{2} \times P \times \delta = \frac{1}{2} \times 30 \times 10^{3} \times 0.05 \times 10^{-3}$

$\therefore U = 0.79\ N\text{-}m.$

$\delta l = \frac{PL}{AE} \Rightarrow E = \frac{30 \times 10^{3} \times 5 \times 10^{-2}}{1.5 \times 10^{-4} \times 0.05 \times 10^{-3}}$

$E = 2 \times 10^{11}\,N/m^2$

If load: $= 50\,kN = 50 \times 10^{3}\,N$.

$(\delta l)_{Elastic} = \frac{PL}{AE}$

$\delta l = \frac{50 \times 10^{3} \times 5 \times 10^{-2}}{1.5 \times 10^{-4} \times 2 \times 10^{11}}$

$(\delta l)_{elastic} = 8.33 \times 10^{-4}\,m = 8.33 \times 10^{-2}\,mm$

$\therefore \delta l = 0.0833\,mm$

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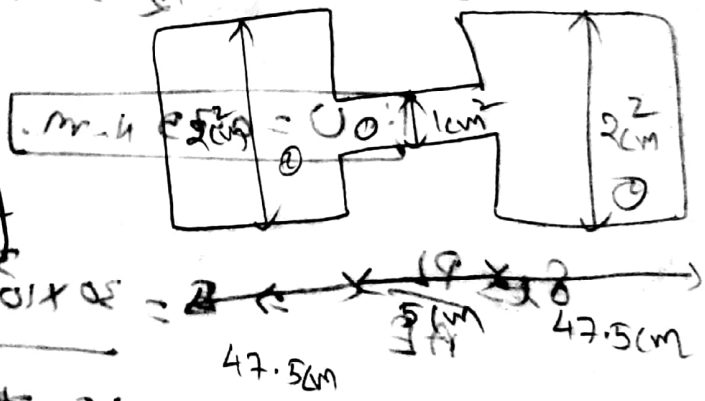
2) A bar of 100cm length is subjected to an axial pull such that the maximum stress is 150 MN/m². It's area of cross section is 2cm² over a length of 95cm. and 1cm² cross-section area over a length of middle 5cm length. Calculate strain energy. take E = 200 GN/m²

Sol $a_1 = 1cm^2 = 1\times10^{-4}m$, $l_1 = 5cm = 5\times10^{-2}m$

$a_2 = 2cm^2 = 2\times10^{-4}m$, $l_2 = 95cm = 95\times10^{-2}m$

Area × shear = σ_1 = 150 MN/m²



$\sigma_1 A_1 = \sigma_2 A_2$

$150\times1 = \sigma_2\times2$

$\therefore \sigma_2 = 75$ MN/m²

$U = U_1 + U_2$

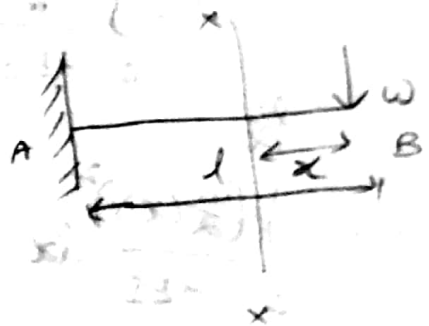
$U = \frac{\sigma_1^2}{2E}\times$ volume $+ \frac{\sigma_2^2}{2E}\times$ volume

$= \frac{(150\times10^6)^2}{2\times200\times10^9}\times1\times10^{-4}\times95\times10^{-2} + \frac{(75\times10^6)^2}{2\times200\times10^9}\times2\times10^{-4}\times95\times10^{-2}$

$\therefore U = 2.95$ kNm

28/12/19

3) Calculate the strainenergy for following figure.



$M_x = -wx$

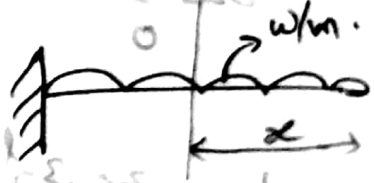
$$U = \int_0^L \frac{M^2}{2EI}\,dx$$

$$U = \int_0^l \frac{(-wx)^2}{2EI}\,dx.$$

$$U = \frac{w^2}{2EI}\int_0^l x^2\,dx, \quad \frac{w^2}{2EI}\left[\frac{x^3}{3}\right]_0^L$$

$$\boxed{U = \frac{w^2 L^3}{6EI}}$$

4) Calculate S.E for following figure.



$M_x = -w \times x \times \frac{x}{2}$

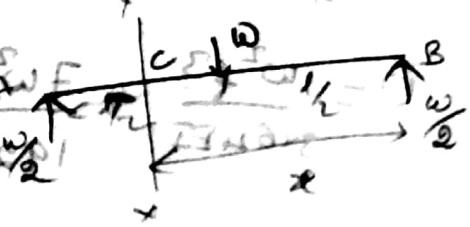
$M_x = -\frac{wx^2}{2}.$

$$U = \int_0^l \frac{M^2}{2EI}\,dx, \quad U = \int_0^l \frac{\left(-\frac{wx^2}{2}\right)^2}{2EI}\,dx.$$

$$U = \frac{w^2}{2EI}\int_0^l \frac{x^4}{4}\,dx, \quad U = \frac{w^2}{8EI}\left[\frac{x^3}{3}\right]_0^l$$

$$\boxed{\therefore U = \frac{w^2 l^5}{40EI}}$$

5) Calculate SE for following figure.



$M_x = +\frac{wx}{2} - w\left(x - \frac{l}{2}\right)$

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$$U = \int_0^{l} \frac{M_x^2}{2EI}\, dx.$$

$$U = \int_0^{l/2} \frac{\left(\frac{w}{2}(x)\right)^2}{2EI} dx + \int_{l/2}^{l} \frac{\left\{\frac{w}{2}x - w\left(x - \frac{l}{2}\right)\right\}^2}{2EI} dx.$$

$$U = \int_0^{l/2} \frac{\frac{w^2}{4}\cdot x^2}{2EI} dx + \int_{l/2}^{l} \frac{\left(\frac{wx}{2} - wx + \frac{wl}{2}\right)^2}{2EI} dx$$

$$U = \frac{1}{8EI}\int_0^{l/2} (w^2x^2)\, dx + \int_{l/2}^{l} \frac{\left(\frac{-wx}{2} + \frac{wl}{2}\right)^2}{2EI} dx$$

$$U = \frac{1}{8EI}\, w^2\left[\frac{x^3}{3}\right]_0^{l/2} + \int_{l/2}^{l} \frac{\frac{w^2x^2}{4} + \frac{w^2l^2}{4}x - 2\cdot\frac{w^2xl}{4}}{2EI}\, dx.$$

$$U = \frac{w^2}{3\times 64EI}\times l^3 + \frac{1}{2EI}\left[\frac{w^2x^3}{12} + \frac{w^2l^2}{4}x - \frac{2w^2x^2l}{8}\right]_{l/2}^{l}$$

$$U = \frac{w^2l^3}{3\times 64EI} + \frac{1}{2EI}\left[\frac{w^2l^3}{96} + \frac{w^2l^3}{8} - \frac{2w^2l^3}{16} - \frac{w^2l^3}{12} - \frac{w^2l^3}{4} + \frac{2w^2l^3}{8}\right]$$

$$U = \frac{w^2l^3}{3\times 64EI} + \frac{1}{2EI}\left[-\frac{7w^2l^3}{96} - \frac{w^2l^3}{8} + \frac{w^2l^3}{8}\right]$$

$$U = \frac{w^2l^3}{3\times 64EI} + \frac{7w^2l^3}{192EI}$$

$$\therefore U = \frac{w^2l^3}{64EI}$$

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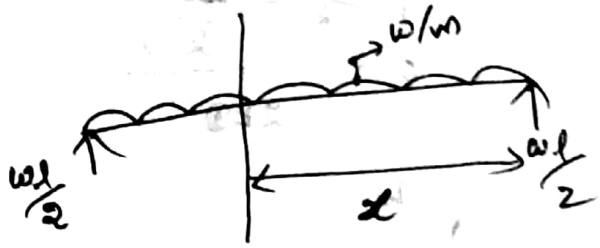
◆ Calculate S.E for following figure.



$$M_x = +\frac{wl}{2}(x) - \frac{wx^2}{2}.$$

$$U = \int_0^L \frac{M^2}{2EI}\,dx.$$

$$= \int_0^l \frac{\left(\frac{wlx}{2} - \frac{wx^2}{2}\right)^2}{2EI}\,dx.$$

$$= \frac{1}{2EI}\int_0^L \left[\frac{w^2l^2x^2}{4} + \frac{w^2x^4}{4} - \frac{2w^2lx^3}{4}\right]dx.$$

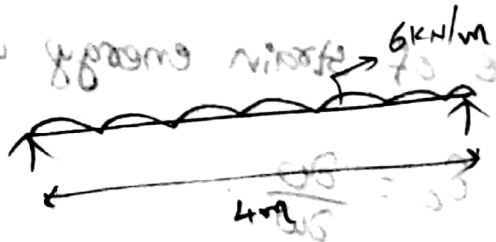
$$= \frac{1}{2EI}\left[\frac{w^2l^2x^3}{12} + \frac{w^2x^5}{20} - \frac{2w^2lx^4}{16}\right]_0^L$$

$$= \frac{1}{2EI}\left[\frac{w^2l^2l^3}{12} + \frac{w^2l^5}{20} - \frac{2w^2l^5}{16}\right]$$

$$\therefore U = \frac{w^2l^5}{240EI}.$$

1). A beam of 4m length simply supported at its ends and carries a UDL of 6KN/m over entire length. Find the length. Calculate Strain energy.

$$M_x = \frac{6\times4}{2}(x) - \frac{6}{2}x^2.$$



$$U = \int_0^4 \frac{M^2}{2EI}\,dx.$$

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$$= \int_0^4 \frac{\left(\frac{24x}{2} - \frac{36}{2}\right)^2}{2EI}$$

$$= \frac{1}{2EI} \int_0^4 \frac{(24)^2 x^2}{4} + \frac{36x^4}{4} - 12 \times 24 \frac{x^3}{4} \, dx$$

$$= \frac{1}{2EI} \left[\frac{(24)^2 x^3}{12} + \frac{(36)^2 x^5}{20} - \frac{6^2 \times 4 \times x^4}{8} \right]_0^4$$

$$U = \frac{1}{2EI} \left[\frac{6^2 \times 4^5}{12} + \frac{36 \times 4^5}{20} - \frac{6^2 \times 4^5}{8} \right]$$

$$U = \frac{(6 \times 10^3)^2 \times 4^5}{240EI}$$

$$\therefore U = \frac{153.6 \times 10^6}{EI} \text{ N-m}$$

02/03/2020

Castigliano's Theorem :-

The deflection or rotation at any point caused by an external force or moment is equals to partial derivative of strain energy wrt force or moment.

$$\delta_c = \frac{\partial U}{\partial W}$$

$$\theta_c = \frac{\partial U}{\partial M}$$



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It is applicable for statical determinant structures.

If no load or no moment is acting at a point where deflection or rotation is required, apply a imaginary load or fictious load, Imaginary moment or fictious moment. Calculate strain energy due to applied loads including fictious loads.

* Calculate the partial derivative of strain energy wrt Imaginary load and Imaginary moment and make Imaginary load into zero.

Calculate

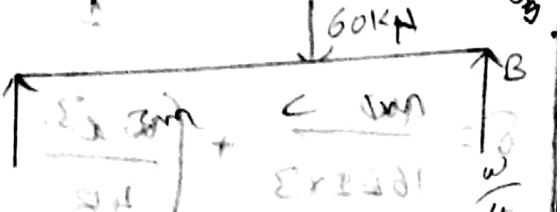
$$U = \int_0^L \frac{M^2}{2EI}\,dx \qquad \frac{\partial U}{\partial M} = \int_0^L \frac{2M}{2EI}\,dx$$

$$\frac{\partial U}{\partial M} = \int_0^L \frac{M}{EI}\,dx \qquad \delta = \frac{\partial U}{\partial \omega}$$

$$\delta = \frac{\partial U}{\partial M} \times \frac{\partial M}{\partial \omega}$$

$$\boxed{\delta = \int_0^L \frac{M}{EI}\,dx \times \frac{\partial M}{\partial \omega}}$$

Calculate the deflection under the load for following



$W = 60\,KN$

$$R_A = \frac{W \times 1}{4} = \frac{W}{4}$$

$$R_B = \frac{W \times 3}{4} = \frac{3W}{4}$$

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origin	span	limit	EI		$\frac{\partial M}{\partial \omega}$
B	BC	0-1	EI	$\frac{3}{4}\omega(x)$	$\frac{3}{4}x$
B	AB	1-4	EI	$\frac{3}{4}\omega(x) - \omega(x-1)$	$\frac{3}{4}x - (x-1)$

$$\delta = \int_0^1 \frac{M_1}{EI} dx \times \frac{\partial M_1}{\partial \omega} + \int_1^4 \frac{M_2}{EI} dx \times \frac{\partial M_2}{\partial \omega}$$

$$\delta = \int_0^1 \frac{\frac{3}{4}\omega(x) \cdot \left(\frac{3}{4}\right)x}{EI} dx + \int_1^4 \frac{\frac{3}{4}\omega(x) - \omega(x-1) \times \frac{3}{4}x - (x-1)}{EI} dx$$

$$\delta = \frac{9\omega}{16EI} \int_0^1 \frac{x^2}{1} dx + \int_1^4 \frac{\left(\frac{3}{4}\omega x - \omega x + \omega\right)\left(\frac{3}{4}x - x + 1\right)}{EI} \cdot dx$$

$$\delta = \frac{9\omega}{16EI}\left[\frac{x^3}{3}\right]_0^1 + \int_1^4 \left(\frac{-\omega x}{4} + \omega\right)\left(\frac{-x}{4} + 1\right) dx$$

$$\delta = \frac{9\omega}{16EI}\left(\frac{1}{3}\right) + \int_1^4 \left(\frac{+\omega x^2}{16} - \frac{\omega x}{4} - \frac{\omega x}{4} + \omega\right) dx$$

$$\delta = \frac{9\omega}{16EI \times 3} + \left(\frac{\omega x^3}{48} - \frac{\omega x^2}{4} + \omega x\right)_1^4$$

$$\delta = \frac{3\omega}{16EI} + \left(\frac{\omega \times 64}{48} - \frac{16\omega}{4} + 4\omega - \frac{\omega}{48} + \frac{\omega}{4} - \omega\right)$$

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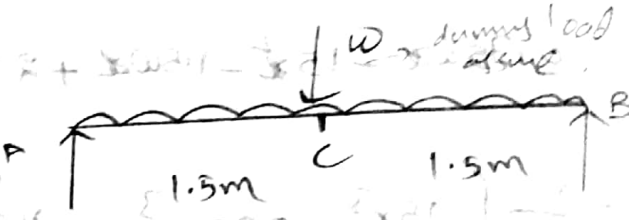
$$\delta = \frac{3\times60}{16EI} + \frac{60\times64}{48} - \frac{16\times60}{4} + 4\times60 - \frac{60}{48} + \frac{60}{4} - 60$$

$$\therefore \delta = \frac{45}{EI}, \quad EI = 2.2\times10^6 \text{ N-m}.$$

$$\delta = \frac{45\times10^3}{2.2\times10^6} = 0.020\text{m}.$$

$$\boxed{\therefore \delta = 20\text{mm}}$$

2) Calculate the deflection at point C. for following fig by using Castigliano's theorem.



$$R_A + R_B = 20\times3 + W$$

$$\Sigma m_A = 0.$$

$$R_B\times3 - 20\times3\times1.5 - \omega(1.5) = 0.$$

$$R_B = \frac{90 + 1.5\omega}{3}, \quad \boxed{R_B = 30 + 0.5\omega.}$$

origin	span	limit	EI	M	$\partial M/\partial\omega$
B	BC	0 to 1.5	EI	$(30+0.5\omega)x - \frac{20(x^2)}{2}$	$0.5x$
B	AC	1.5 to 3	EI	$(30+0.5\omega)x - \frac{20x^2}{2} - \omega(x-1.5)$	$0.5x-(x-1.5)$

$$\delta = \int_0^{1.5} \frac{m_1}{EI}\frac{\partial m_1}{\partial\omega}dx + \int_{1.5}^{3} \frac{m_2}{EI}\frac{\partial m_2}{\partial\omega}dx$$

$$\delta = \int_0^{1.5} \frac{\left((30+0.5\omega)x - 10x^2\right)}{EI} \times 0.5x\, dx + \int_{1.5}^{3} \frac{\left((30+0.5\omega)x - 10x^2 - \omega x + 1.5\omega\right)}{EI} \times (0.5x - x + 1.5)\, dx.$$

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$$\delta = \frac{1}{EI}\int_0^{1.5}(30x + 0.5\omega x - 10x^2)\,0.5x\,dx + \frac{1}{EI}\int_{1.5}^{3}\left[30x + 0.5\omega x - 10x^2 - \omega x + 1.5\omega\right]\left(0.5x - x + 1.5\right)dx$$

$$= \frac{1}{EI}\int_0^{1.5}(15x^2 + 0.25\omega x^2 - 5x^3)\,dx + \frac{1}{EI}\int_{1.5}^{3}\{(15x^2 + 0.25x^2 - 5x^3 - 0.5x^2\omega + 0.75\omega x) - (30x^2 - 0.5\omega x^2 + 10x^3 + \omega x^2 + 1.5\omega x) + 45x + 0.75\omega x - 15x^2 - 1.5\omega x + 2.25\omega\}\,dx.$$

$$= \frac{1}{EI}\left[\frac{15x^3}{3} + \frac{0.25\omega x^3}{3} - \frac{5x^4}{4}\right]_0^{1.5} + \frac{1}{EI}\int_{1.5}^{3}\{15x^2 + 0.25x^2 - 5x^3 - 0.5x^2\omega + 0.75\omega x - 30x^2 - 0.5\omega x^2 + 10x^3 + \omega x^2 + 1.5\omega x + 45x + 0.75\omega - 15x^2 - 1.5\omega x + 2.25\omega\}\,dx.$$

$$= \frac{1}{EI}\left[16.875 + 0.281\omega - 6.328\right] + \frac{1}{EI}\int_{1.5}^{3}\left(\frac{-119}{4}x^2 + 5x^3 + 45x + \frac{3}{2}\omega x + 2.25\omega\right)dx$$

$$= \frac{1}{EI}\left[10.547 + 0.281\omega\right) + \frac{1}{EI}\left[\frac{-119}{4}\cdot\frac{x^3}{3} + \frac{5\cdot x^4}{21} + \frac{45x^2}{2} + \frac{3}{2}\omega\frac{x^2}{2} + 2.25\omega\cdot x\right]_{1.5}^{3}.$$

$$= \frac{1}{EI}\left[10.547 + 0.281\omega\right) + \frac{1}{EI}\left\{\frac{-1071}{4} + \frac{405}{4} + \frac{405}{2} + \frac{27}{4}\omega + \frac{27}{4}\omega + \frac{1071}{32} - \frac{405}{64} - \frac{405}{8} - \frac{27\omega}{16} - \frac{27}{8}\omega\right\}.$$

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$= \frac{1}{EI}(10.547 + 0.281\omega) + \frac{1}{EI}\left(\frac{135}{64} + \frac{135}{16}\omega\right)$.

As we assume that ω is an fictious load.

i.e., $\omega = 0$

$$\delta = \frac{1}{EI}(10.547 + 0) + \frac{1}{EI}\left(\frac{135}{64} + 0\right)$$

$$\delta = \frac{1}{EI}\left(10.547 + \frac{135}{64}\right) \qquad \delta = \frac{21.09 \times 10^3}{EI}$$

$$\therefore \delta = \frac{23.06}{EI}$$

$EI = 2.2 \times 10^6$ N-m

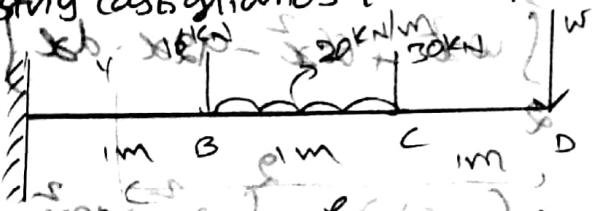
$= 9.37 \times 10^{-6}$ KN-m

$$\delta = \frac{23.06}{2.2 \times 18} = 0.010 \times 9.37 \times 10^{-3}$$ N-m

$\therefore \delta = 10$ mm

13/01/2020

3) Calculate deflection at point D using castigliano's theorem



Region	Span	limit	EI	M	$\frac{\partial M}{\partial W}$
D	DC	0 to 1	EI	$-Wx$	$-x$
C	BC	1 to 2	EI	$-Wx - 30(x-1) - 20\frac{(x-1)^2}{2}$	$-x$
B	AB	2 to 3	EI	$-Wx - 30(x-1) - 20\times 1(x-\frac{1}{2})$	$-x$

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$$\delta = \int_0^1 \frac{M_1}{EI}\frac{\partial M_1}{\partial \omega}dx + \int_1^2 \frac{M_2}{EI}\frac{\partial M_2}{\partial \omega}dx + \int_2^{} \frac{M_3}{EI}\frac{\partial M_3}{\partial \omega}dx$$

$$= \left\{\int_0^1 (-\omega x)(-x)\,dx + \int_1^2 \left(-\omega x - 30x + 30 - 10(x-1)^2\right)(-x)\cdot dx + \int_2^3 \left(-\omega x - 30x + 30 - 20x + 30 - 16x + 32\right)(-x)\cdot dx\right\}\cdot \frac{1}{EI}$$

$$= \frac{1}{EI}\left\{\int_0^1 \omega x^2\,dx + \int_1^2 \omega x^2 + 30x^2 + 30x + 10x(x-1)^2\cdot dx + \int_2^3 \omega x^2 + 66x^2 - 30x - 30x - 32x\cdot dx\right\}.$$

$$= \left\{\int_0^1 \omega x^2 dx + \int_1^2 \omega x^2 + 30x^2 + 30x + 10x(x^2+1-2x)\cdot dx + \int_2^3 \omega x^2 + 66x^2 - 92x\cdot dx\right\}\cdot \frac{1}{EI}$$

$$= \left(\omega\left[\frac{x^3}{3}\right]_0^1 + \int_1^2 \omega x^2 + 30x^2 + 30x + 10x^2 + 10x - 20x^2\cdot dx + \int_2^3 \omega x^2 + 66x^2 - 92x\cdot dx\right)\cdot \frac{1}{EI}$$

$$= \frac{1}{EI}\left(\omega\left[\frac{1}{3}\right] + \int_1^2 \omega x^2 + 10x^2 + 40x + 10x^3\cdot dx + \left[\omega\frac{x^3}{3} + 66\frac{x^3}{3} - \frac{92x^2}{2}\right]_2^3\right.$$

$$= \left[\frac{\omega}{3} + \left[\frac{\omega x^3}{3} + \frac{10x^3}{3} + \frac{40x^2}{2} + \frac{10x^4}{4}\right]_1^2 + \left\{\left(\omega\left[\frac{27}{3}\right] + \frac{66\times 27}{2} - \frac{92\times 9}{2}\right) - \left(\omega\left(\frac{8}{3}\right) + \frac{66\times 8}{3} - \frac{92\times 4}{2}\right)\right\}\right]\cdot \frac{1}{EI}$$

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$$= \left\{ \frac{w}{3} + \left[\left(w\left(\frac{8}{3}\right) + \frac{10\times 8}{3} + \frac{40\times 4}{2} + \frac{10\times 16}{4} \right) - \left(\frac{w}{3} + \frac{10}{3} + \frac{40}{2} + \frac{10}{4} \right) \right] + 90w + 891 - 444 - \frac{80w}{3} - 176 + 184 \right\} \frac{1}{EI}$$

$$= \left\{ \frac{w}{3} + \frac{8w}{3} + \frac{80}{3} + 80 + 40 - \frac{w}{3} - \frac{10}{3} - 20 - \frac{5}{2} + 90 - \frac{80w}{3} + 485 \right\} \frac{1}{EI}$$

As the load is an fictious load. take $w = 0$.

$$\delta = \left[\frac{80}{3} + 120 - \frac{10}{3} - 20 - \frac{5}{2} + 485 \right] \cdot \frac{1}{EI}$$

$$\delta = \frac{605.833}{EI}$$

$$EI = 2.2 \times 10^6 \text{ N-m}.$$

$$\delta = \frac{605.83 \times 10^3}{2.2 \times 10^6}$$

$$\therefore \delta = 0.275 \text{ m}$$

$$\boxed{\therefore \delta = 275 \text{ mm}}$$

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Unit load method:- It is used to findout the deflection at given/required point by applying unit load (1 kN).

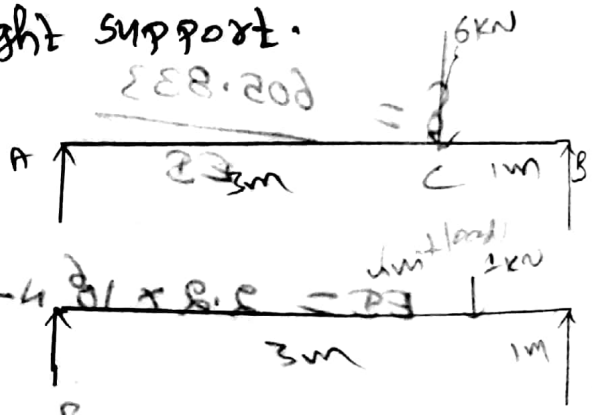
The deflection due to unit load,

$$\delta = \int_0^L \frac{M_x \cdot m_x}{EI} dx.$$

where. M_x = Bending moment due to external applied load

m_x = B.M due to unit load.

1) Calculate deflection under point load of a simply supported beam of span 4m subjected to a point load of 6kN at a distance of 1m from right support.



R_A & R_B for simply supported beam with external loads.

$$R_A = \frac{6 \times 1}{4} = 1.5 \text{ kN}.$$

$$R_B = \frac{6 \times 3}{4} = 4.5 \text{ kN}.$$

R_A and R_B with unit load.

$$R_A = \frac{1 \times 1}{4} = \frac{1}{4} \text{ kN}, \quad R_B = \frac{1 \times 3}{4} = \frac{3}{4} \text{ kN}.$$

origin	span	limit	EI	M_x	m_x
B	BC	0 to 1	EI	4.5x	$\frac{3}{4}x$
A	AC	0 to 3	EI	1.5x	$\frac{1}{4}x$.

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$$\delta = \int_0^L \frac{M_x m_x}{EI}\, dx$$

$$= \int_0^1 \frac{(4.5x)\cdot\left(\frac{3}{4}x\right)}{EI}\, dx + \int_0^3 \frac{(1.5x)\left(\frac{1}{4}x\right)}{EI}\, dx$$

$$= \int_0^1 \frac{3.37x^2}{EI}\, dx + \int_0^3 \frac{0.375x^2}{EI}\, dx$$

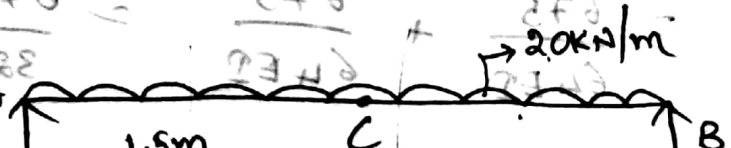
$$= \frac{3.37}{EI}\left(\frac{x^3}{3}\right)_0^1 + \frac{0.375}{EI}\left(\frac{x^3}{3}\right)_0^3 = \frac{4.5}{EI}$$

$$\delta = \frac{4.5\times10^3}{2.5\times10^6} = 1.8\times10^{-3}\ m$$

$$\therefore \delta = 1.8mm$$

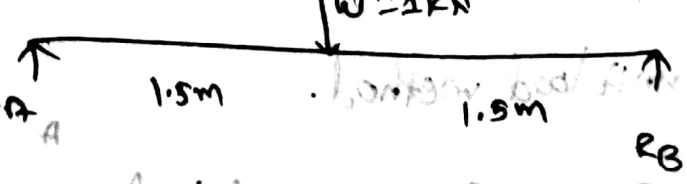
06/02/2020

2) Calculate deflection at point C for following figure by using unitload method.



R_A & R_B for external loads

$= R_A = R_B = 30KN$



R_A & R_B for unit load

$= R_A = R_B = 0.5KN$

origin	span	limit	EI	M_x	m_x
B	BC	0 to 1.5	EI	$30(x) - \frac{20x^2}{2}$	$0.5x$
C	AC	1.5 to 3	EI	$30(x) - 20x^2/2$	$0.5x - 1(x-1.5)$

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$$\delta = \int_0^L \frac{M_x m_x}{EI}\,dx$$

$$= \int_0^{1.5} \frac{(30x - 10x^2)(0.5x)}{EI}\,dx + \int_{1.5}^{3} \frac{(30x - 10x^2)(0.5x - x + 1.5)}{EI}$$

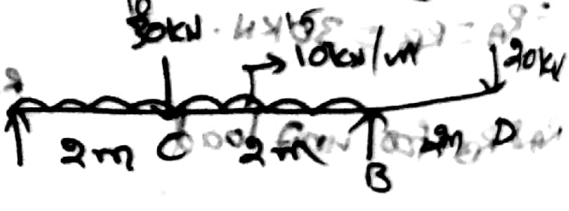
$$= \int_0^{1.5} \frac{15x^2 - 5x^3}{EI}\,dx + \int_{1.5}^{3} \frac{(30x - 10x^2)(-0.5x + 1.5)}{EI}\,dx$$

$$= \frac{1}{EI}\left[\frac{15x^3}{3} - \frac{5x^4}{4}\right]_0^{1.5} + \int_{1.5}^{3} \frac{-15x^2 + 5x^3 + 45x - 15x^2}{EI}\,dx$$

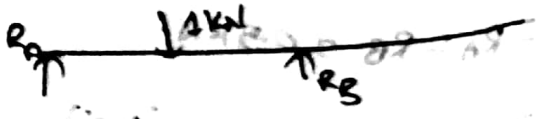
$$= \frac{1}{EI}\left[\frac{675}{64}\right] + \frac{1}{EI}\left[\frac{-15x^3}{3} + \frac{5x^4}{4} + \frac{45x^2}{2} - \frac{15x^3}{3}\right]_{1.5}^{3}$$

$$= \frac{675}{64EI} + \frac{675}{64EI} = \frac{675}{32EI}$$

3) Calculate deflection at point C for following fig. by unit load method.



R_A & R_B for external loading



$R_B \times 4 - 20 \times 6 - 10 \times 4 \times 2 - 30 \times 2 = 0$

$R_B = 65\,kN$.

$R_A + R_B = 30 + 40 + 20 = 90$.

$\therefore R_A = 25\,kN$

Calc R_B for unit load. $R_A = R_B = 0.5$ KN (due to symmetry)

origin	span	limit	EI	M_x	m_x
D	BD	0 to 2	EI	$-20x$	0
B	BC	2 to 4	EI	$-20x+65(x-2)-\frac{10}{2}(x-2)^2$	$0.5x$
A	AC	0 to 2	EI	$25x-\frac{10x^2}{2}$	$0.5x$

$$\delta = \int_0^L \frac{M_x m_x}{EI}\,dx$$

$$\delta = \int_0^2 \frac{(-20x)\cdot 0}{EI}\,dx + \int_2^4 \frac{(-20x+65x-130-5(x^2+4-4x))(0.5x-1)}{EI}\,dx$$

$$+ \int_0^2 \frac{(0.5x)(25x-5x^2)}{EI}\,dx$$

$$\delta = 0 + \frac{1}{EI}\int_2^4 10x^2+32.5x^2-65x-2.5(x^2+4-4x)+20x-65x+130$$
$$+5(x^2+4-4x)$$
$$+ \int_0^2 \frac{0.5x\cdot 25x+0.5\cdot 5x^2}{EI}\,dx$$

$$\delta = \frac{100}{3EI}$$

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R_A & R_B for unit load. $R_A = R_B = 0.5\ kN$ (due to symmetry)

origin	span	limit	EI	M_x	m_x
D	BD	0 to 2	EI	$-20x$	0
B	BC	2 to 4	EI	$-20x+65(x-2)-\frac{10}{2}(x-2)^2$	$0.5x$
A	AC	0 to 2	EI	$25x-\frac{10x^2}{2}$	$0.5x$

$$\delta = \int_0^L \frac{M_x m_x}{EI}\,dx$$

$$\delta = \int_0^2 \frac{(-20x)\cdot 0}{EI}\,dx + \int_2^4 \frac{(-20x+65x-130-5(x^2+4-4x))(0.5x-1)}{EI}\,dx$$

$$+ \int_0^2 \frac{(0.5x)(25x-5x^2)}{EI}\,dx$$

$$\delta = 0 + \frac{1}{EI}\int_2^4 10x^2+32.5x^2-65x-2.5(x^2+4-4x)+20x-65x+130$$
$$+5(x^2+4-4x)$$
$$+\int_0^2 \frac{0.5x+23x+0.5\times 5x^2}{EI}\,dx$$

$$\delta = \frac{100}{3EI}$$

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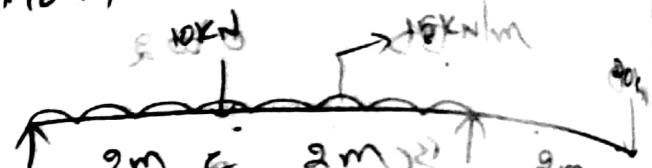
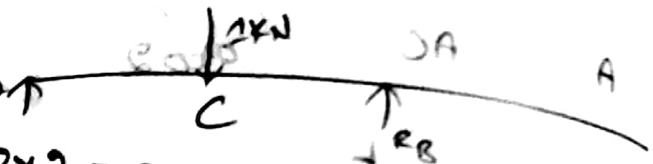
07/01/2020 ACT

1) calculate the deflection at Point C. for following fig by unit load method. $EI = 2.2 \times 10^6\ N/m^2$.



R_A & R_B for external loads



$R_B \times 4 - 20 \times 6 - (15 \times 4) \times 2 - 10 \times 2 = 0$

$4R_B - 120 - 120 - 20 = 0$

$\boxed{R_B = 65\ KN.}$

$R_A + R_B = 20 + 15 \times 4 + 10 = 90$

$\therefore \boxed{R_A = 25\ KN}$

R_A & R_B for unit loads. $R_A = R_B = 0.5\ KN$.

origin	span	limit	EI	M_x	m_x
D	BD	0 to 2	EI	$-20x$	0
B	BC	2 to 4	EI	$-20x + R_B(x-2)$	$0.5(x-2)$
				$+\frac{15}{2}(x-2)^2$	
A	AC	0 to 2	EI	$R_A x + \frac{15}{2}(x$	$0.5x$

$$\delta = \int_0 \frac{M_x \cdot m_x}{EI}\, dx.$$

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$$\delta = \int_0^2 \frac{(-20x)\cdot 0}{EI} + \int_2^4 \frac{0.5x(-20x+65x-130+7.5(x-2)^2)\cdot dx}{EI} + \int_0^2 \frac{(25x-7.5x^2)\,0.5x\cdot dx}{EI}$$

$$\delta = \int_2^4 \frac{-10x^2+32.5x^2-65x+3.75(x-2)^2}{EI}dx + \int_0^2 \frac{12.5x^2+3.75x^3}{EI}dx$$

$$\delta = \int_2^4 \frac{22.5x^2-65x+3.75(x-2)^2}{EI}dx + \left[\frac{12.5x^3}{3}+\frac{3.75x^4}{4}\right]_0^2 \frac{1}{EI}$$

$$\delta = \frac{1}{EI}\left[\frac{22.5x^3}{3}-\frac{65x^2}{2}+\frac{3.75(x-2)^3}{3}\right]_2^4 + \frac{1}{EI}\left[\frac{155}{3}\right]$$

$$\delta = \frac{1}{EI}\left[\frac{22.5(4)^3}{3}-\frac{65\times 16}{2}+\frac{3.75(2)^3}{3}-\left(\frac{22.5(2)^3}{3}\right)+\frac{65\times 4}{2}+\frac{3.75\times 8}{3}\right] + \frac{1}{EI}\left[\frac{155}{3}\right]$$

$$\delta = \frac{1}{EI}\left[-30+70\right] + \frac{1}{EI}\left(\frac{155}{3}\right), \quad \frac{40}{EI}+\frac{155}{3EI}$$

$$\delta = \frac{265}{3EI} = \frac{88.3333\times 10^3}{2.2\times 10^6}$$

$$\delta = 40.15\times 10^{-3}\ m = 40.15\ mm.$$

$$9.33\times 10^{-3} \qquad 9.33\ mm.$$

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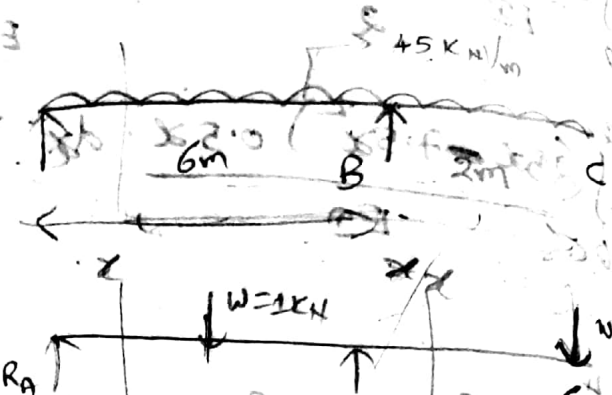
2) Calculate deflection at point for following fig by using unit load method.



$R_A + R_B = 45 \times 8 = 360$

$\Sigma M_A = 0$.

$R_B \times 6 = 45 \times 8 \times \frac{8}{2} = 0$

$6R_B = 1440$. $\therefore R_B = 240$ KN

$R_A = 120$ KN.

$R_A + R_B = 1$

R_A & R_B for unit loads $R_A = R_B = 0.5$ KN. $1 \times 8 = R_B \times 6$

$R_B = 1.33$

$R_A = 0.33$

origin	span	limit	ΣEI	M_x	m_x
C				$-\frac{45x^2}{2}$	0
B					$+1x$
	AB	$0 \to 6$	EI	$R_A \times x - 10 \times x \times \frac{x}{2}$	$-R_A \times x$
				$120x - \frac{45x^2}{2}$	$-0.33x$

$$\delta = \int_0^L \frac{m_x \cdot m_x}{EI}\, dx$$

$$= \int_0^2 \frac{\left(-\frac{45x^2}{2}\right)(-x)}{EI}\, dx + \int_0^6 \frac{\left(120x - \frac{45x^2}{2}\right)(-0.33x)}{EI}\, dx.$$

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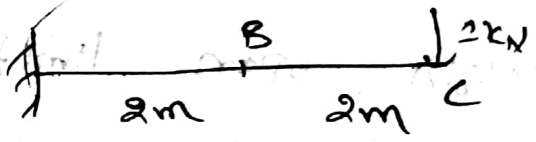
$$= \frac{1}{EI}\left[\frac{45\times 2^4}{8} - \frac{396\times 6^3}{3} + \frac{7.425\times 6^4}{4}\right]$$

$$= -\frac{359}{EI}, \quad \frac{355\times 10^3}{2.5\times 10^8} = \text{Nm} \rightarrow \text{mm}.$$

08/01/2020

1) Calculate deflection and rotation at free end of the cantilever beam of length 4m. carrying two Point loads of 20KN. each at free end. and 2m from free end. take $E = 2\times 10^5 \text{ N/mm}^2$. $I = 12\times 10^6 \text{ mm}^4$.





origin	span	limit	EI	M_x
C	BC	0 to 2	EI	$-20x$
C	AB	2 to 4	EI	$-20x - 20(x-2)$

$$\delta = \int_0^2 \frac{(-20x)(-x)}{EI}dx + \int_2^4 \frac{(-20x-20x+40)(-x)}{2EI}dx$$

$$\delta = \frac{1}{EI}\left[\frac{20x^3}{3}\right]_0^2 + \int_2^4 \frac{40x^2-40x}{2EI}dx$$

$$\delta = \frac{1}{EI}\left[\frac{20\times 8}{3}\right] + \left[\frac{20x^3}{3EI} - \frac{20x^2}{2EI}\right]_2^4$$

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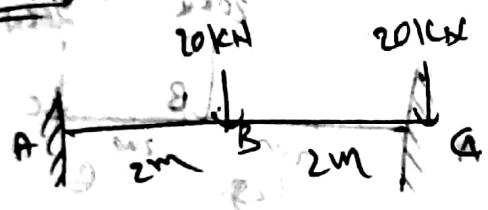
$$\delta = \frac{160}{3EI} + \left[\frac{20\times4^3}{3EI} - \frac{20\times4^2}{2EI} - \frac{20\times2^3}{3EI} + \frac{20\times2^3}{2EI}\right]$$

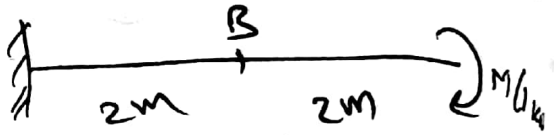
$$\delta = \frac{306.67}{EI}, \quad \delta = \frac{306.67\times10^3}{2\times10^5 \times 12\times10^6 \times10^{-12}}$$

$$\delta = 0.1277\ m, \quad \therefore \delta = 127\ mm$$

Rotation:



Apply unit moment at C.



origin	span	limit	EI	M_x	m_x
C	BC	0 to 2	EI	$-20x$	$-m$
C	AB	2 to 4	2EI	$-20x-20(x-2)$	$-m$

$$\delta = \int_0^2 \frac{(-20x)(-m)}{EI}dx + \int_2^4 \frac{(-20x-20(x-2))(-m)}{2EI}dx.$$

$$\delta = \int_0^2 \frac{20x(1)}{EI}dx + \int_2^4 \frac{(-40x+40)(-1)}{2EI}dx.$$

$$\delta = \left[\frac{20x^2}{2EI}\right]_0^2 + \int_2^4 \left[\frac{40x-40}{2EI}\right]dx$$

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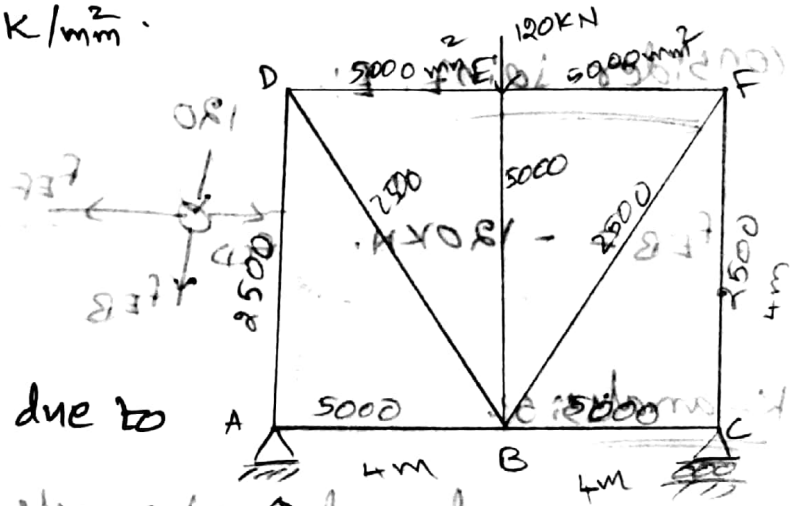
$$\delta = \frac{40EI}{} + \left[\frac{40x^2}{4EI} - \frac{40x}{2EI}\right]_2^4$$

$$\delta = 40EI + \frac{40\times16}{4EI} - \frac{40\times4}{2EI} - \frac{40\times4}{4EI} + \frac{40\times2}{2EI}$$

$$\therefore \delta = \frac{120}{EI} = \frac{120\times10^3}{\frac{2\times10^9}{10^{-6}}\times12\times10^6\times10^{-18}}$$

$$\boxed{\delta = 0.05m}$$

2) Calculate vertical deflection at joint B. for following truss take $E = 200\ KN/mm^2$.



$$\delta = \frac{\Sigma PKL}{AE}$$

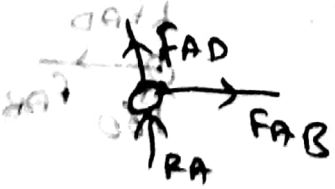
P = forces in the member due to applied loads

K = forces in the member due to unit load.

P-analysis:

$$R_A = R_C = \frac{120}{2} = 60\ KN.$$

Consider joint 'A'



$F_{AD} = -60\ KN.$

$F_{AB} = 0.$

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consider joint 'D'



$\Sigma V = 0$.

$-f_{AD} - f_{BD}\cos 45 = 0$

$60 = f_{BD}\cos 45$

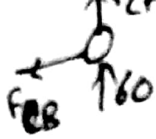
$f_{BD} = 60\sqrt{2}$.

$\Sigma H = 0$.

$f_{DE} + f_{DB}\sin 45 = 0$

$f_{DE} = -60$ kN.

consider joint 'C'



$f_{CF} = -60$

$f_{CB} = 0$

consider joint (F).

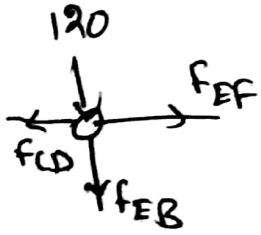


due to symmetry.

$f_{BF} = 60\sqrt{2}$

$f_{EF} = -60$.

consider joint. E:

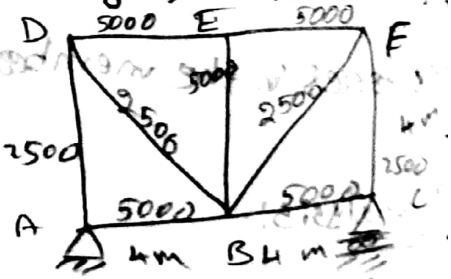


$f_{EB} = -120$ KN.

K-analysis:

no external load can be acts. neglected the load.

Apply unit load at B.



$R_A = R_C = 1/2 = 0.5$.

Consider joint 'A'.



$f_{AD} = -0.5$ KN

$f_{AB} = 0$.

Consider joint 'C'.

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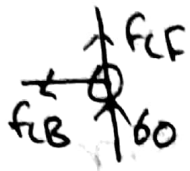


$f_{CF} = -0.5\,kN$.

$f_{CB} = 0$.

Consider joint "D"



$\Sigma V = 0; \quad -f_{AD} - f_{BD}\cos 45 = 0$

$0.5 = f_{BD}\cos 45$

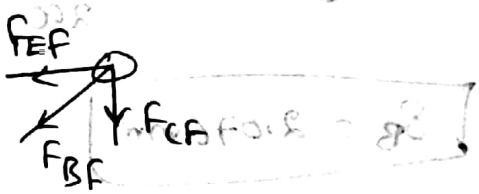
$f_{BD} = 0.5\sqrt{2}$.

$\Sigma H = 0, \quad f_{DE} + f_{DB}\sin 45 = 0$

$f_{DE} = -0.5$.

Consider joint F.

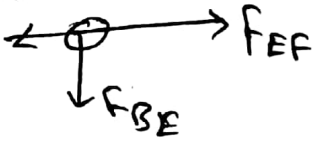
$f_{BF} = 0.5\sqrt{2}$.



$f_{EF} = -0.5$.

Consider joint "E".

f_{ED}.



$\Sigma V = 0$.

$f_{BE} = 0$.

member	length (mm)	area mm^2	P-analysis (KN)	K-analysis (KN)	$\frac{PKL}{A}$
AB	4000	5000	0	0	0
BC	4000	5000	0	0	0
DE	4000	5000	-60	-0.5	24
EF	4000	5000	-60	-0.5	24
AD	4000	2500	-60	-0.5	48
CF	4000	2500	-60	-0.5	48

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BE	4000	5000	-120	0°	0
BD	$4000\sqrt{2}$	2500	$60\sqrt{2}$	$0.5\sqrt{2}$	135.76
BF	$4000\sqrt{2}$	2500	$60\sqrt{2}$	$0.5\sqrt{2}$	135.76

$$\delta = \sum \frac{PKL}{AE} \rightarrow (1)$$

$$\frac{\sum PKL}{A} = 415.5$$

$$\delta = \frac{415.5}{E} = \frac{415.5}{200}$$

$$\boxed{\delta_B = 2.076 \text{ mm}}$$

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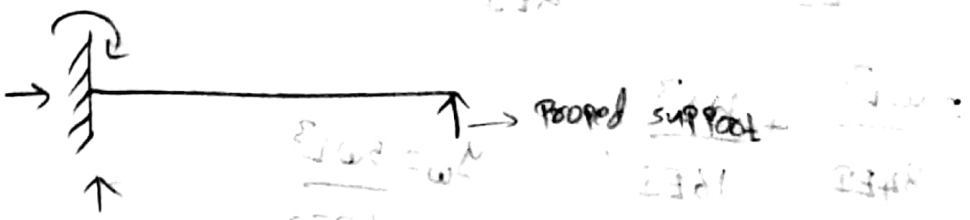
10/1/2020 Module-II Fixed and Continuous Beams.

Propped Cantilever beam:- The beam which is having fixed support at one end and prop at other end.

Propped Cantilever beam is a static indeterminate beam with 1 degree of static Indeterminacy.



$\gamma = 4$, R (no. of equilibrium) $= 3$.

$D_S = \gamma - R = 1$.

Standard case(i):-

Propped Cantilever beam subjected to point load at centre

Slope and deflections of various beams:-



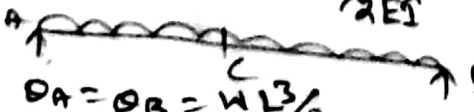
$\theta_A = 0$ $\quad \theta_B = WL^2/2EI$

$y_A = 0$ $\quad y_B = WL^3/3EI$



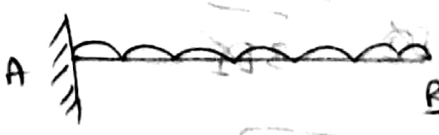
$\theta_A = 0$ $\quad \theta_B = ML/EI$

$y_A = 0$ $\quad y_B = ML^2/2EI$



$\theta_A = \theta_B = WL^3/24EI$, $\theta_C = 0$.

$y_A = y_B = 0$.



$\theta_A = 0$ $\quad \theta_B = wL^3/6EI$

$y_A = 0$ $\quad y_B = wL^4/8EI$



$\theta_A = \theta_B = WL^2/16EI$, $\theta_C = 0$.

$y_A = y_B = 0$. $y_C = wL^3/48EI$

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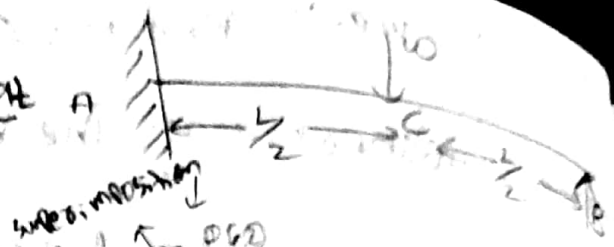
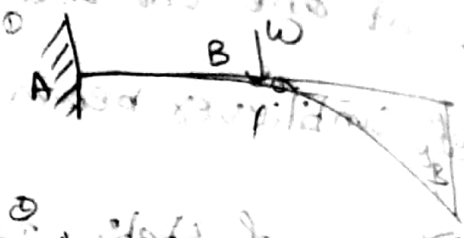
Case (i):-

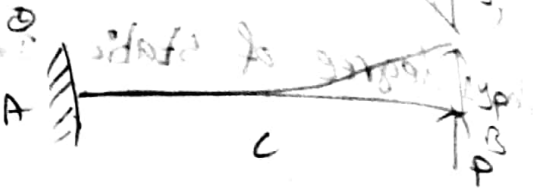
downward deflection at 'c' due to W:



$y_w = y_B + y_1$



$= \frac{W(L/2)^3}{3EI} + \frac{L}{2}\left(\frac{W(L/2)^3}{2EI}\right)$



$= \frac{WL^3}{24EI} + \frac{WL^3}{16EI}$ $\quad \therefore y_w = \frac{5WL^3}{48EI}$

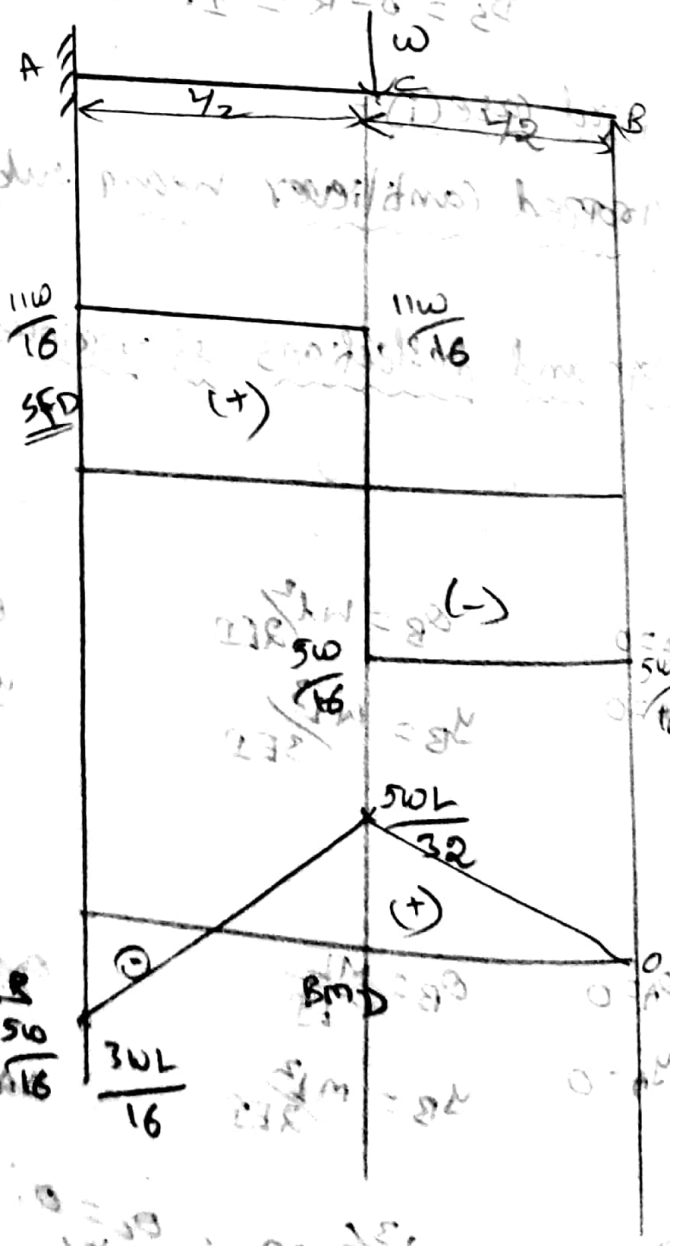
downward deflection at 'c' due to 'P'

$y_P = \frac{PL^3}{3EI}$

$\therefore y_w = y_P$

$\frac{5WL^3}{48EI} = \frac{PL^3}{3EI}$

$\therefore P = \frac{5W}{16}$



SFD:

Zone BC, ($x = 0$ to $L/2$).

$F_x = -\frac{5W}{16}$

$F_B = -\frac{5W}{16}$

$F_C = -\frac{5W}{16}$.

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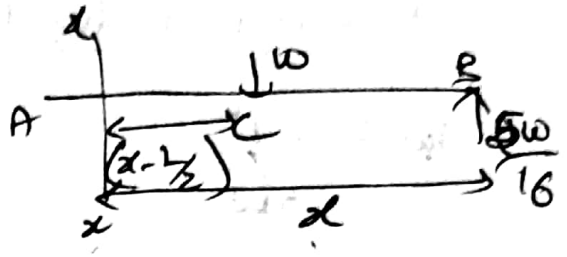
zone CA ($x = L/2$ to L).



$$F_x = -\frac{5w}{16} + w = \frac{11w}{16}$$

$$F_C = \frac{11w}{16}.$$

$$F_A = \frac{11w}{16}.$$

BMD:

zone BC ($x = 0$ to $L/2$).

$$M_x = \frac{5w}{16}(x), \quad M_B = \frac{5w}{16}(0) = 0, \quad M_C = \frac{5w}{16}(L/2) = \frac{5wL}{32}.$$

zone AC ($x = L/2$ to L)

$$M_x = \frac{5w}{16}(x) - w(x - L/2)$$

$$M_C = \frac{5w}{16}(L/2) - w(L/2 - L/2) = \frac{5wL}{32}.$$

$$M_A = \frac{5w}{16}(L) - w(L - L/2) = \frac{5wL}{16} - \frac{wL}{2} \Rightarrow \frac{-3wL}{16}.$$

Point of contraflexure.

Point of contraflexure is occured in zone AC at Pt. C,

the BM = 0. $\frac{5w}{16}(x) - w(x - L/2) = 0.$

$$\frac{5w}{16}(x) = w(x - L/2). \quad \therefore \frac{5x}{16} = x - L/2.$$

$$x = \frac{8L}{11} \text{ from B.}$$

10/02/2020 (class)

Propped cantilever beam subjected to UDL:

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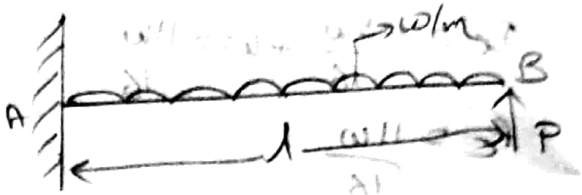
Downward deflection due to w/m:-

$$y_w = \frac{wL^4}{8EI}$$



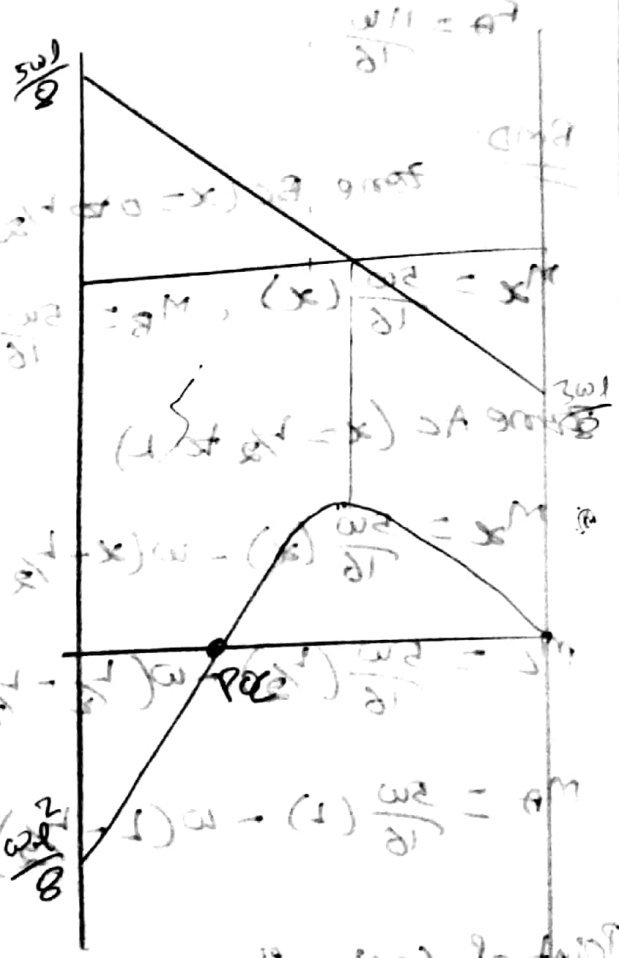
Upward deflection due to P:-

$$y_p = \frac{PL^3}{3EI}$$

$$\therefore y_w = y_p$$

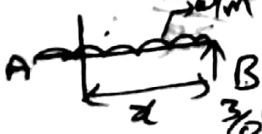
$$\frac{wL^4}{8EI} = \frac{PL^3}{3EI}$$

$$\therefore P = \frac{3}{8}wL$$



SFD:-

Zone AB (x=0 to L)



$$F_x = -\frac{3wl}{8} + w(x)$$

$$F_B = -\frac{3wl}{8} + w(0) = -\frac{3wl}{8}$$

$$F_A = -\frac{3wl}{8} + w(L) = \frac{3wl}{8}$$

BMD:-

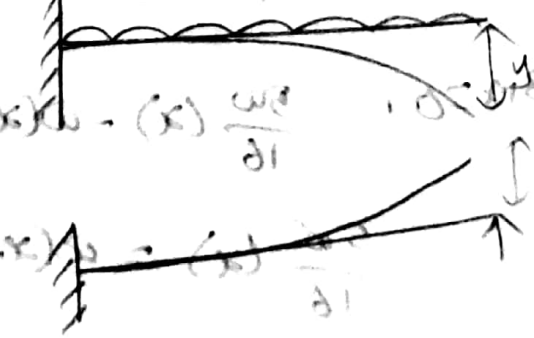
Zone AB (x= 0 to L)

$$M_x = \frac{3}{8}wL(x) - \frac{wx^2}{2}$$

$$M_B = \frac{3}{8}wl(0) - \frac{w(0)^2}{2} = 0$$

$$M_A = \frac{3}{8}wl(L) - \frac{w(L^2)}{2}$$

$$M_A = -wl^2/8.$$



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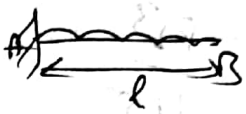
Point of contraflexure.

POC is occured in zone AB.

at POC, BM = 0.

$$\frac{3}{8}\omega l(x) - \frac{\omega x^2}{2} = 0, \quad \frac{3\omega l x}{8} = \frac{\omega x^2}{2}$$

$$\boxed{x = \frac{3}{4}l} \text{ from B.}$$



from A. $\rightarrow x = \frac{l}{4}$.

Max BM.

→ for cantilever & simply supported → no POCs

→ single overhang & propped beams → 1 POC

→ double overhang & fixed beam → 2 POCs

at point of maximum BM, SF = 0.

SF = 0, in zone AB.

$$-\frac{3\omega l}{8} + \omega(x) = 0, \quad -\frac{3\omega l}{8} = -\omega x$$

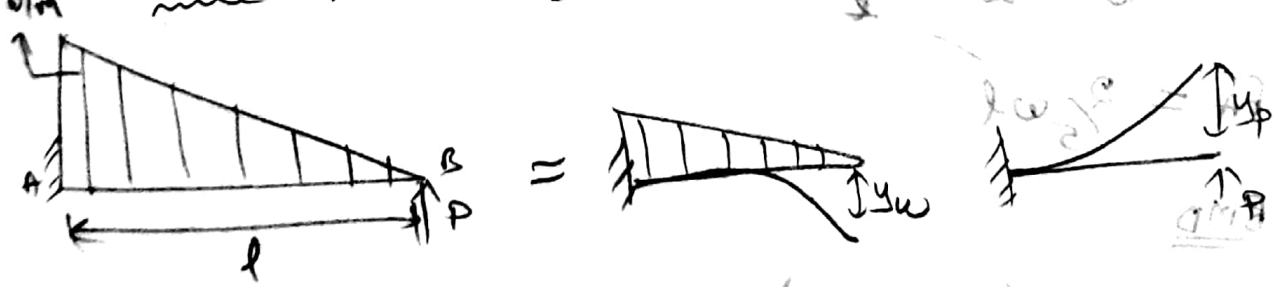
$$\boxed{\therefore x = \frac{3l}{8}}$$ subtitute $x = \frac{3l}{8}$ in $M_x = \frac{3}{8}\omega L(x) - \frac{\omega x^2}{2}$

* For continuous beam POC's are based on supports.

$$M_x = \frac{3}{8}\omega L\left(\frac{3l}{8}\right) - \frac{\omega}{2}\left(\frac{3l}{8}\right)^2$$

$$\boxed{\therefore M_x = \frac{9}{128}\omega l^2}$$

Case(ii) Propped cantilever beam subjected to UVL :-



downward deflection due to w :-

$$y_w = \frac{\omega L^4}{30EI}$$

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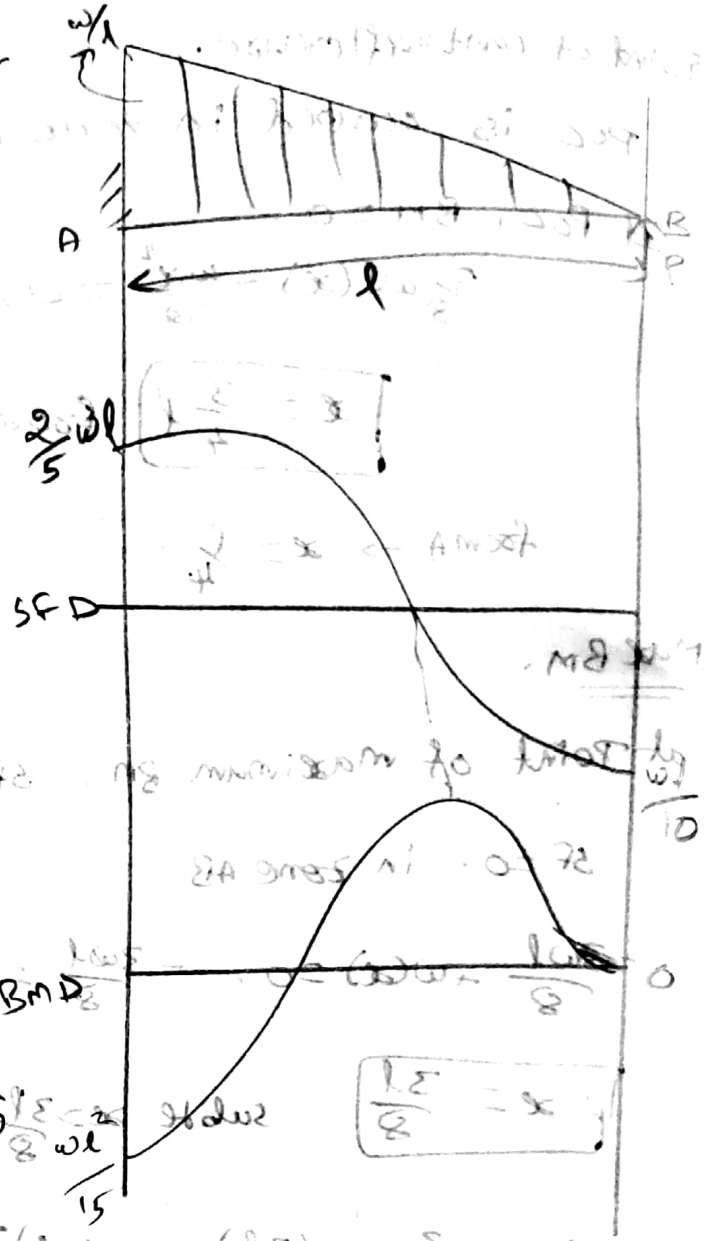
Upward deflection due to P:-

$$y_P = \frac{PL^3}{3EI}.$$

$$\therefore y_w = y_P$$

$$\frac{wL^4}{30EI} = \frac{PL^3}{3EI}$$

$$\boxed{\therefore P = \frac{wL}{10}}$$



SFD:-

Zone AB ($x = 0$ to L)



$$F_x = -\frac{wl}{10} + \frac{1}{2} \times x \times \frac{wx}{l}$$

$$F_B = -\frac{wl}{10} + 0 = -\frac{wl}{10}.$$

$$F_A = -\frac{wl}{10} + \frac{1}{2} \times L \times \frac{wl}{L}$$

$$F_A = \frac{2}{5} wl.$$

BMD:-

Zone AB ($x = 0$ to L)

$$M_x = \frac{wl}{10}(x) - \frac{1}{2} \times x \times \frac{wx}{L} \times \left(\frac{x}{3}\right)$$

$$M_B = 0, \quad M_A = \frac{wl}{10}(l) - \frac{1}{2} \times \frac{wl^3}{3l} = -\frac{wl^2}{15}.$$

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Point of contraflexure:-

POC is occured in zone AB.

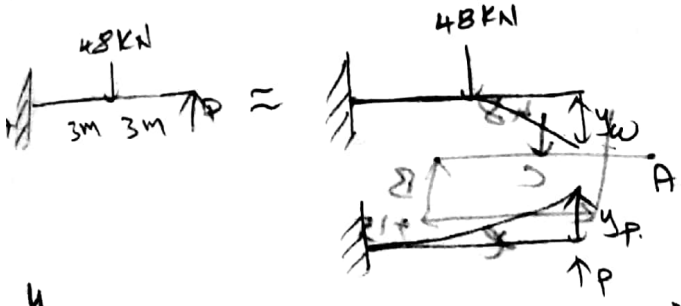
at POC, BM = 0.

$$M_x = \frac{wl}{10}(x) - \frac{1}{2} \times x \times \frac{wx}{l} \times \left(\frac{x}{3}\right) = 0$$

$$\frac{wlx}{10} = \frac{wx^3}{6l}$$

$$\therefore x = \sqrt{\frac{3}{5}}\, l$$

1) A cantilever beam of length 6m subjected to a point load of 48KN. at centre. The beam is propped at free end calculate propped reaction. ii) Draw SFD, BMD, iii) Locate POC.

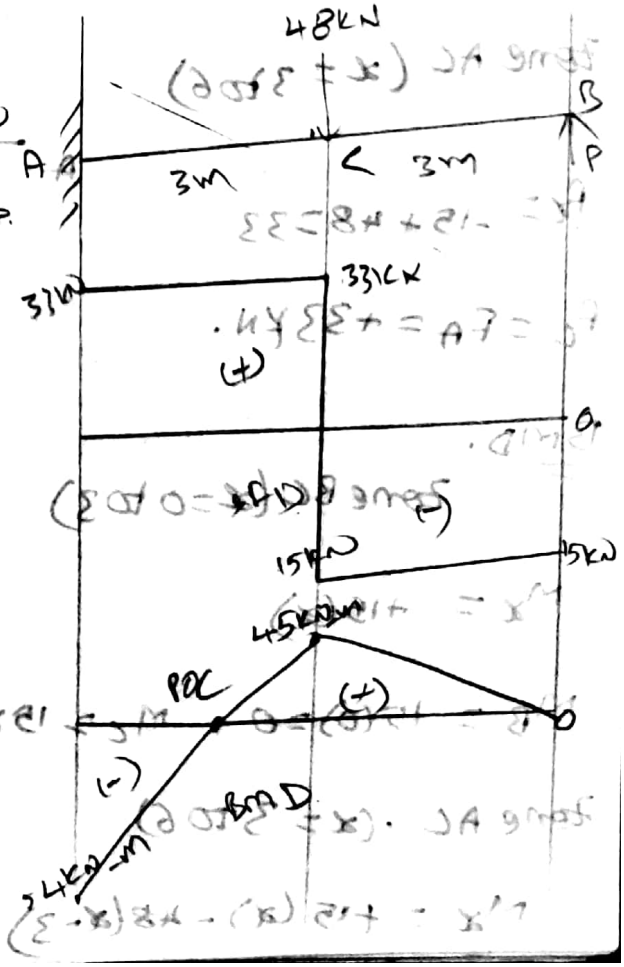


$$y_{BO} = y_B + y_P$$

$$= \frac{w(L/2)^3}{3EI} + \frac{1}{2}\left(\frac{w(L/2)^2}{2EI}\right)$$

$$= \frac{w(3)^3}{3EI} + \frac{6}{2}\left(\frac{w(3)^2}{2EI}\right)$$

$$= \frac{48 \times 3^3}{3EI} + 3\left(\frac{48 \times 3^2}{2EI}\right).$$



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$$\therefore y_w = \frac{1080}{EI}.$$

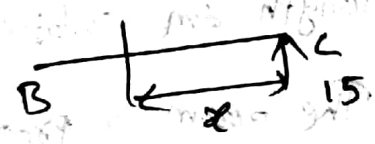
$$y_p = \frac{PL^3}{3EI} = \frac{P \times 6^3}{3EI} = \frac{72P}{EI}$$

$$y_w = y_p$$

$$\frac{1080}{EI} = \frac{72P}{EI} \quad \therefore \boxed{P = 15 \text{ KN}}.$$

SFD:

Zone BC ($x = 0$ to 3)

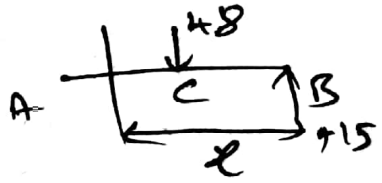


$F_x = -15$ kN

$F_C = -15$

$F_B = -15$.

Zone AC ($x = 3$ to 6)



$F_x = -15 + 48 = 33$

$F_C = F_A = +33$ KN.

BMD.

Zone BC ($x = 0$ to 3)

$M_x = +15(x)$

$M_B = 15(0) = 0$, $M_C = 15 \times 3 = 45$ KN-m.

Zone AC. ($x = 3$ to 6)

$M_x = +15(x) - 48(x-3)$

$M_C = 15(3) - 48(3-3) = 45$.

$M_A = 15(6) - 48(6-3) = -54$ kN-m.

POC: POC is occured in zone AC.

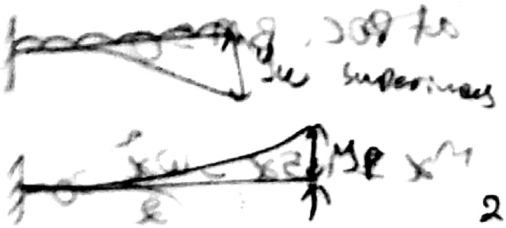
at POC, BM = 0.

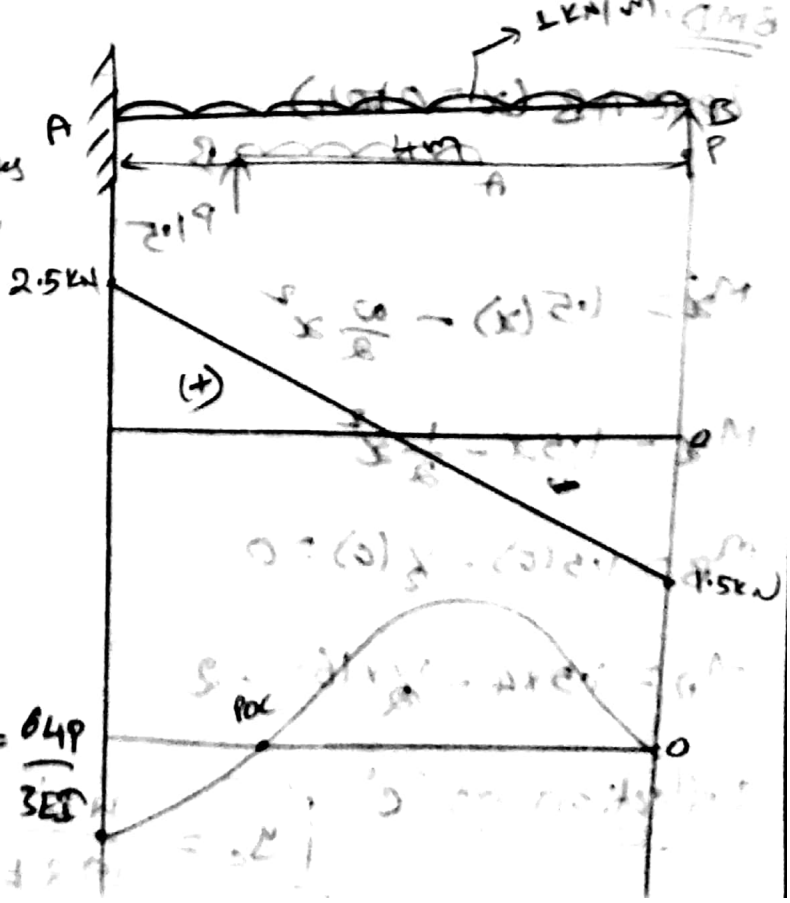
$M_x = 15(x) - 48(x-3) = 0$

$15x - 48x + 144 = 0$

$144 = 33x$. $\therefore x = 4.36$

2) A cantilever of length 4m, subjected to a UDL of 1kN/m throughout span. the cantilever is propped at free end. calculate propped reaction, Draw SFD, BMD. locate POC, calculate deflection at centre.





$y_{(w)} = \frac{wL^4}{8EI}$

$y_w = \frac{1 \times 4^4}{8EI} = \frac{256}{8EI}$

$y_p = \frac{PL^3}{3EI} = \frac{P \times 4^3}{3EI} = \frac{64P}{3EI}$

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$\therefore y_w = y_p$

$$\frac{256}{8EI} = \frac{64P}{3EI}, \quad P = \frac{256 \times 3}{8 \times 64}$$

$32 = 21.33P$ $\quad \therefore P = 1.5\ KN$

SFD:-

Zone AB ($x = 0$ to 4)



$f_x = -1.5 + 1(x)$

$f_B = -1.5 + 0 = -1.5$

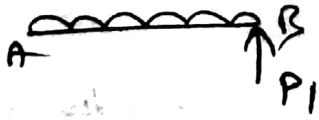
$f_A = -1.5 + 4 = 2.5$.

Point of contraflexure

POC occured in zone AB

BMD:

Zone AB ($x = 0$ to 4)



$M_x = 1.5(x) - \frac{w}{2}x^2$

$M_x = 1.5x - \frac{1}{2}x^2$

$M_B = 1.5(0) - \frac{1}{2}(0) = 0$

$M_A = 1.5 \times 4 - \frac{1}{2} \times 16 = -2$

at POC, $BM = 0$.

$M_x = 1.5x - \frac{wx^2}{2} = 0$

$1.5x - \frac{x^2}{2} = 0$

$3x = x^2$

$\therefore x = 3$

Deflection at "C"

$$y_c = \frac{WL^3}{192EI}$$

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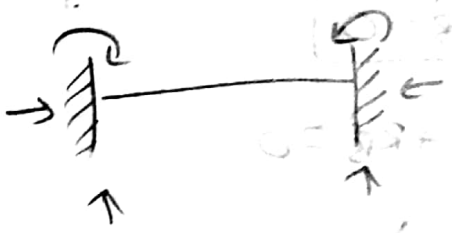
$$y_c = \frac{1 \times 4^4}{192 EI} = \frac{1.33}{EI}$$

fixed beam:- The beam which is having both ends are fixed support. is known as fixed beam.

* fixed beam is a 3rd degree of static indeterminacy

$r = 6, \quad R = 3$

$D_S = r - R = 3.$



(case i) fixed beam subjected to point load :-

R_A & R_B for simply supported beam with loading.

$$R_A = R_B = \frac{W}{2}.$$

NOTE

area of free BMD = area of fixed BMD

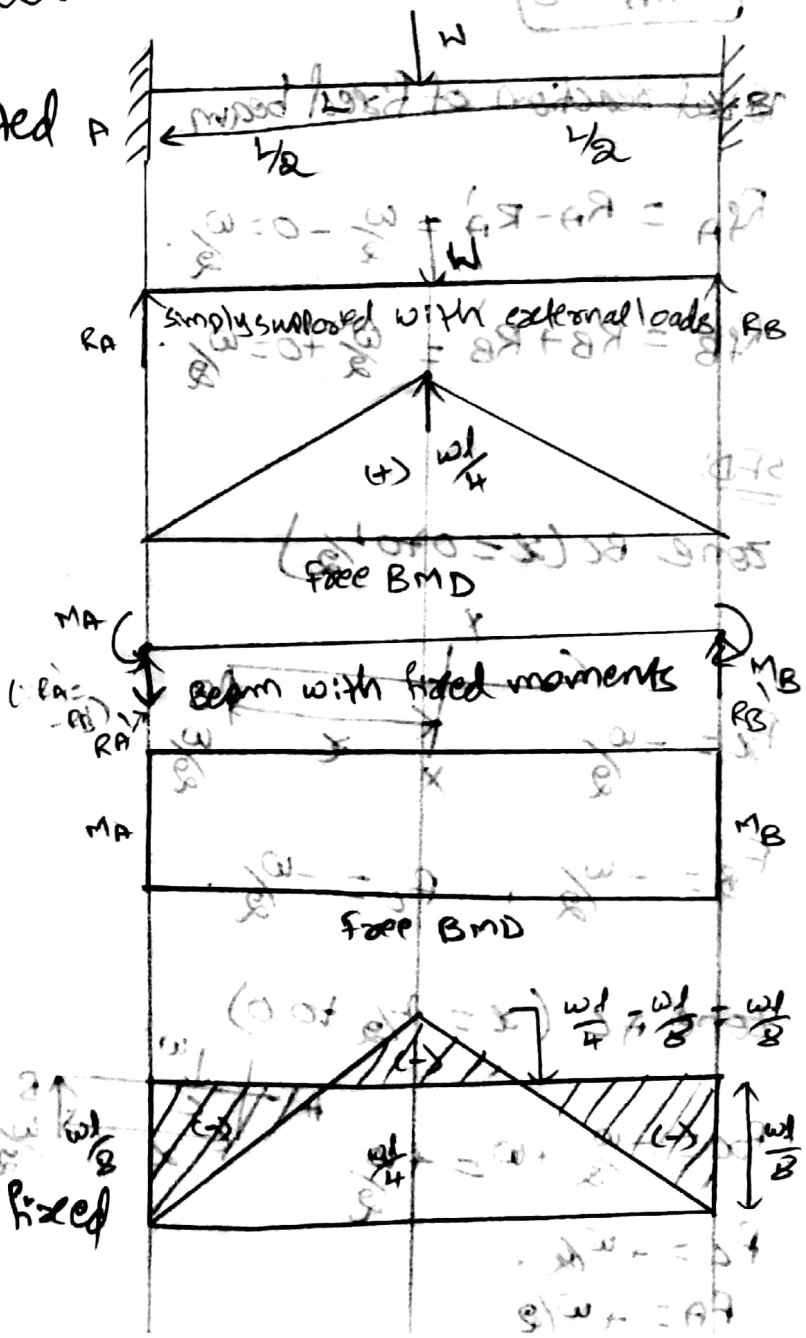
$$\frac{1}{2} \times l \times \frac{Wl}{4} = M_A \times l$$

$$\boxed{M_A = \frac{Wl}{8}} \quad \text{(Hogging)}$$

due to symmetry

$$\boxed{M_B = \frac{Wl}{8}}$$

R_A' & R_B' for beam with fixed moments



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$\Sigma M_A = 0$.

$R_B' \times l - M_B + M_A = 0$.

$R_B' = \frac{M_B - M_A}{l}$

$[\because M_B = M_A]$

$\therefore R_B' = 0$

$R_A' + R_B' = 0$

$\therefore R_A' = -R_B'$

$\therefore R_A' = 0$

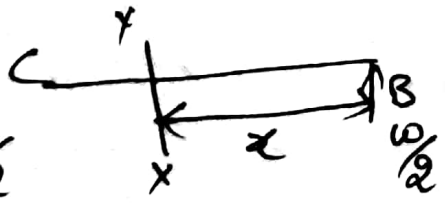
Total reaction of fixed beam.

$R_{fA} = R_A - R_A' = \frac{\omega}{2} - 0 = \frac{\omega}{2}$.

$R_{fB} = R_B + R_B' = \frac{\omega}{2} + 0 = \frac{\omega}{2}$.

SFD:

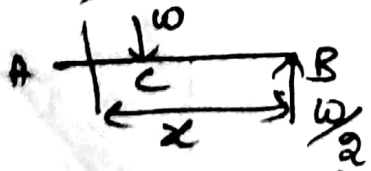
zone BC ($x = 0$ to $l/2$)



$F_x = -\omega/2$

$F_B = -\omega/2$, $F_C = -\omega/2$.

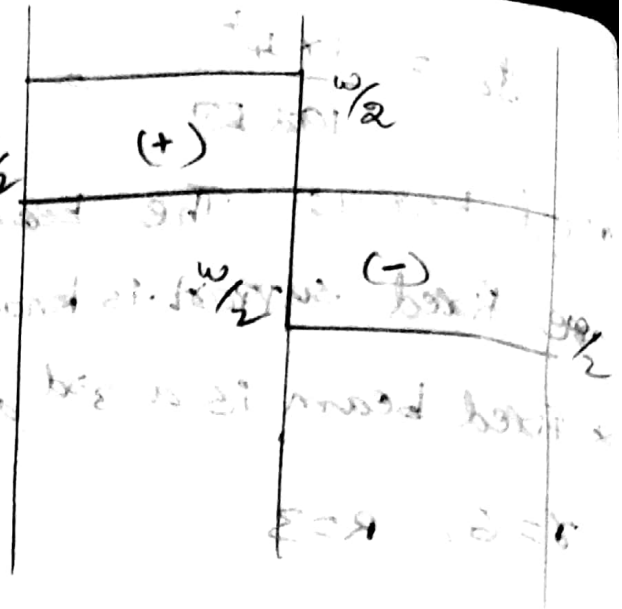
zone AC ($x = l/2$ to 0)



$F_x = -\frac{\omega}{2} + \omega = +\frac{\omega}{2}$

$F_C = +\omega/2$.

$F_A = +\omega/2$



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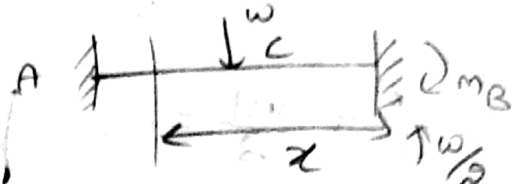
Max. deflection occurs under load at point C.

Macaulay's method:



$$EI\frac{d^2y}{dx^2} = M_x$$

$$EI\frac{d^2y}{dx^2} = \frac{w}{2}(x) - M_B - w(x-L/2) \rightarrow (1)$$

Integrate

$$EI\frac{dy}{dx} = \frac{wx^2}{4} - M_Bx + C_1 - \frac{w(x-L/2)^2}{2} \rightarrow (2)$$

Integrate.

$$EI.y = \frac{wx^3}{12} - M_B\frac{x^2}{2} + C_1x + C_2 - \frac{w(x-L/2)^3}{6} \rightarrow (3)$$

apply boundary conditions.

i) at $x=0$, $\frac{dy}{dx}=0$. ii) at $x=0$, $y=0$.

sub' in (2) & (3)

we get, $C_1=0$, $C_2=0$.

Max deflection occured at $x=L/2$.

sub $x=L/2$, in eq (3)

$$EI.y = \frac{w(L/2)^3}{12} - \frac{WL}{8}\left(\frac{(L/2)^2}{2}\right)$$

$$\therefore y_{max} = \frac{-wL^3}{192EI}$$

Point of contraflexure

$$\frac{w}{2}(x) - \frac{WL}{8} - w(x-L/2) = 0$$

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$$\frac{wx}{2} - \frac{wl}{8} - wx + \frac{wl}{2} = 0$$

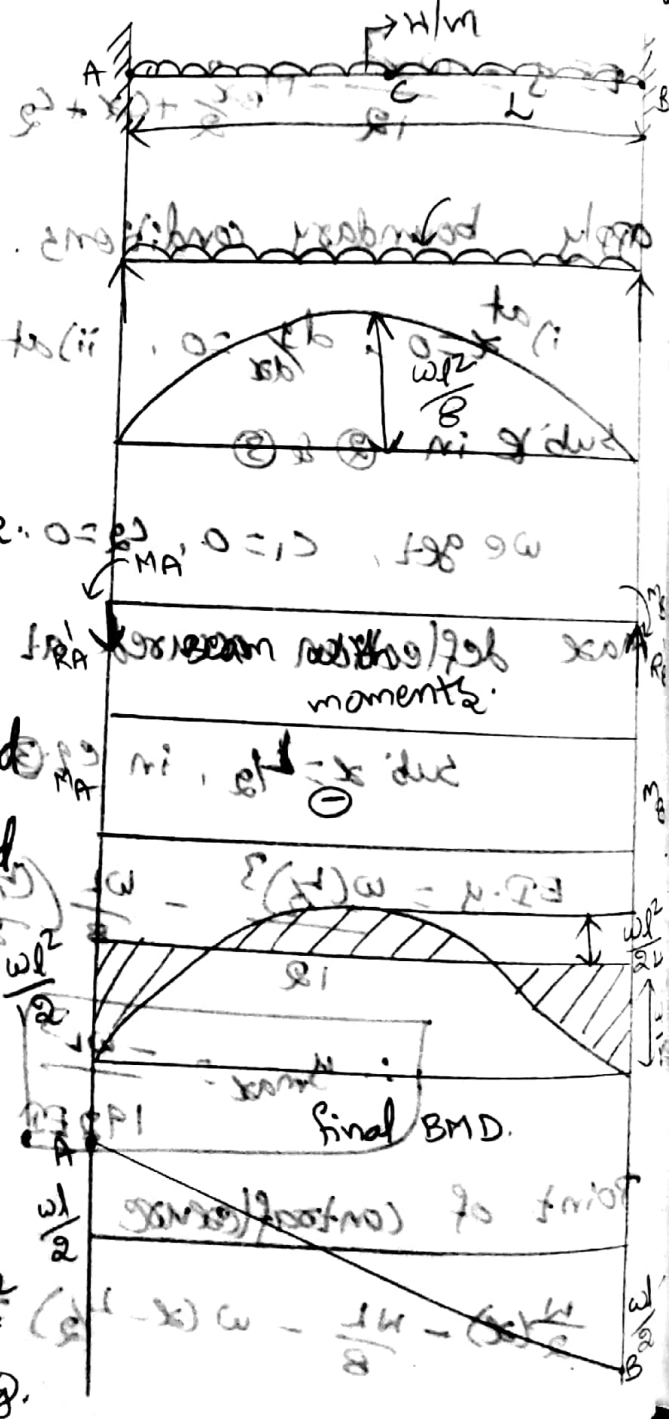
$$-\frac{wx}{2} + \frac{3wl}{8} = 0, \qquad \therefore x = \frac{3}{4}l.$$

17/01/2020

Fixed beam subjected to UDL loading

Simply supported beam with loading:-

R_A & R_B are the support reaction of simply supported beam with loading.



$R_A + R_B = wl.$

$R_A = R_B = wl/2.$

max B.M at 'C' $= \frac{wl^2}{8}$.

Beam with fixed moments:

R_A' & R_B' are the support reaction of beam with fixed moment M_A & M_B are fixed moments.

free BMD = fixed B.M.D

$$\frac{2}{3} \times l \times \frac{wl^2}{8} = M_A \times l$$

$$\frac{wl^2}{12} = M_A, \; M_B = \frac{wl^2}{12}$$

Hogging.

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Take a moment about point "A" of beam with fixed

moment $R_B' \times l - M_B + M_A = 0$.

$$R_B' = \frac{M_B - M_A}{l}, \quad R_B' = 0.$$

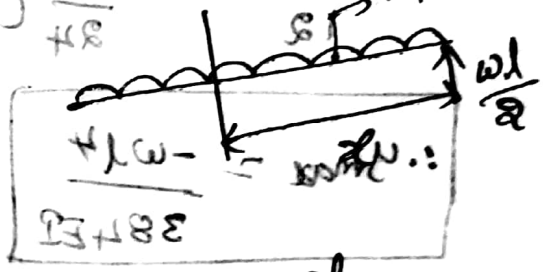
$$R_A' + R_B' = 0 \Rightarrow \boxed{R_A' = -R_B'}$$

Total reactions.

$$R_{fA} = R_A - R_A' = \frac{\omega l}{2} - 0 = \frac{\omega l}{2}.$$

$$R_{fB} = R_B + R_B' = \frac{\omega l}{2} + 0 = \frac{\omega l}{2}.$$

S.f.D



Zone AB ($x = 0$ to L)

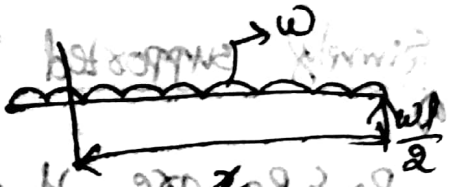
$$f_x = -\frac{\omega l}{2} - \omega x.$$

$$f_C = -\frac{\omega l}{2} - 0 \qquad f_A = -\frac{\omega l}{2} - \omega l = \frac{\omega l}{2}.$$

Max deflection:

$$EI\frac{d^2y}{dx^2} = M_x.$$

$$EI\frac{d^2y}{dx^2} = R_{fB}(x) - M_B - \frac{\omega(x^2)}{2}$$



$$EI\frac{d^2y}{dx^2} = \frac{\omega l}{2}(x) - \frac{\omega l^2}{12} - \frac{\omega x^2}{2}$$

Integrating on both sides.

$$EI\frac{dy}{dx} = \frac{wlx^2}{4} - \frac{wl^2x}{12} + C_1 - \frac{wx^3}{6} \rightarrow (2)$$

$$EIy = \frac{wlx^3}{12} - \frac{wl^2x^2}{24} + C_1x + C_2 - \frac{wx^4}{24} \rightarrow (3)$$

Applying boundary conditions:

at $x=0$, $\frac{dy}{dx}=0$, subt in (2) then $C_1=0$.

at $x=0$, $dy=0$. subt in (3) then $C_2=0$.

max. deflection occured at "C" i.e, $x=l/2$.

$$EI(y_{max}) = \frac{wl(l/2)^2}{12} - \frac{wl^2}{24}(l/2)^2 - \frac{w(l/2)^4}{24}$$

$$\therefore y_{max} = \frac{-wl^4}{384EI}$$

1) A fixed beam of length 6m subjected at a point load of 50kN at mid point. Calculate:
i) support reactions, ii) fixed end moments,
iii) Draw SF.D & BMD, iv) Calculate max deflection.

2) Simply supported beam with loading:-

R_A & R_B are the support reaction of SSB with loading.

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$R_A + R_B = 50KN$

$R_A = R_B = \frac{50}{2} = 25KN.$

Max BM at 'C' $= \frac{50 \times 6}{4} = 75KN\text{-}m.$

b) Beam with fixed moments.

R_A' & R_B' are support reaction of beam with fixed moment

M_A & M_B are fixed end moment.

Free BMD = fixed BMD

$\frac{1}{2} \times 6 \times 75 = M_A \times 6$

$M_A = 37.5\ KN\text{-}m$

$M_B = 37.5\ KN\text{-}m$

$R_A' + R_B' = 0.$

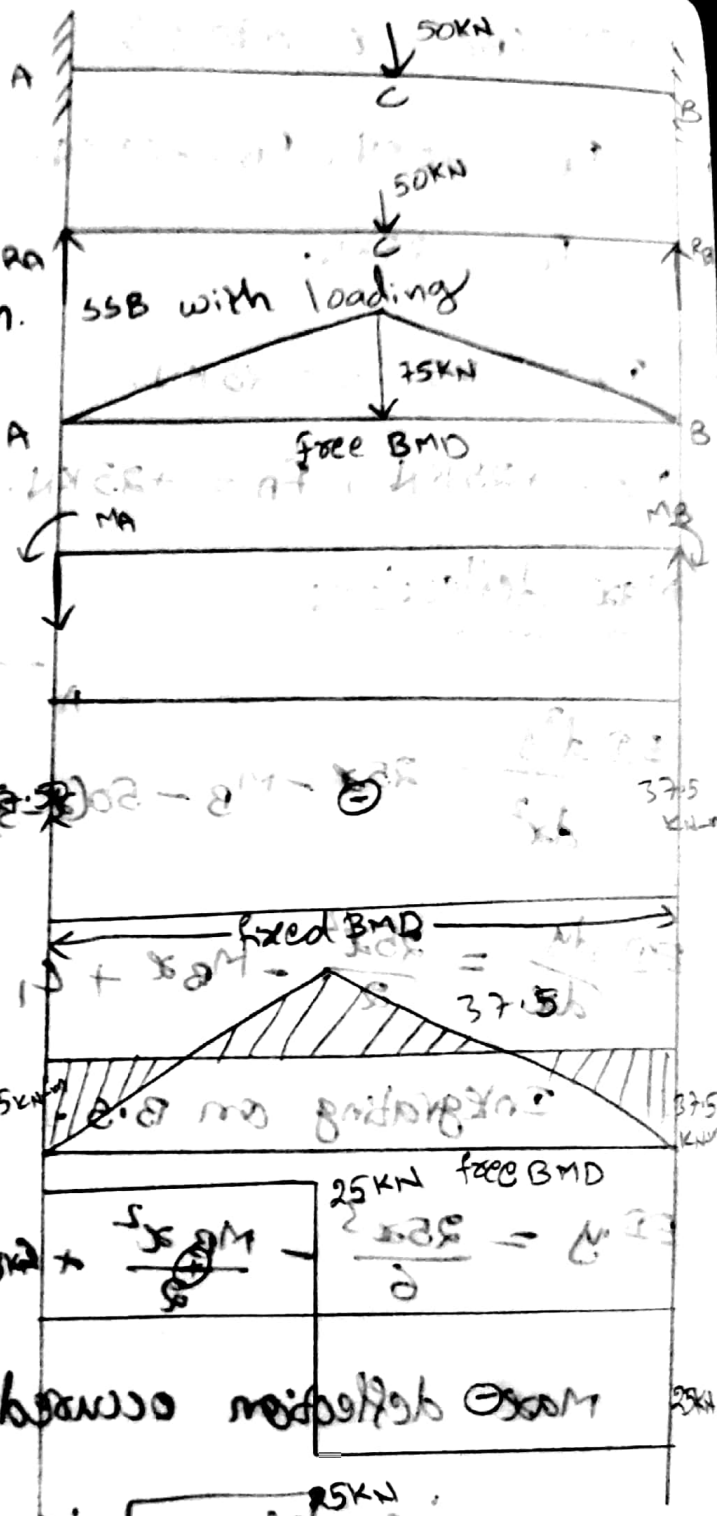
$M_A = 0, \quad R_B' \times 6 = 37.5 + 37.5 = 0$

$R_B' = 0 \Rightarrow \boxed{R_A' = -R_B'}$

Final reactions.

$R_{fA} = R_A + R_A' = 25 - 0 = 25KN.$

$R_{fB} = R_B + R_B' = 25 + 0 = 25KN.$



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$$\frac{1}{2} \times 3 \times 30 = \frac{M_A + M_B}{2} \times 3$$

$$M_A + M_B = 30 \longrightarrow (1)$$

Centroid of force BMD = Centroid of fixed BMD.

$$\frac{l+a}{3} = \left(\frac{M_A + 2M_B}{M_A + M_B}\right)\frac{l}{3}$$

$$\frac{3+2}{3} = \left(\frac{M_A + 2M_B}{M_A + M_B}\right)\frac{3}{3}$$

$$\frac{5}{3} = \frac{M_A + 2M_B}{M_A + M_B}, \quad 5M_A + 5M_B = 3M_A + 6M_B.$$

$$2M_A - M_B = 0 \longrightarrow (2).$$

Solving (1) & (2).

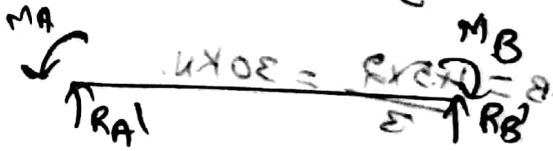
$$\boxed{M_A = 10\,KN\text{-}m}, \quad \boxed{M_B = 20\,KN\text{-}m}.$$

Taking a moment about Point of beam fixed moments.

$$R_B' \times 3 - M_B + M_A = 0$$

$$R_B' \times 3 - 20 + 10 = 0.$$

$$R_B' = 10/3 = 3.33\,KN, \quad \boxed{R_A' = -R_B' = -3.33\,KN.}$$



Final reaction:-

$$R_{fA} = R_A - R_A' = 15 - 3.33 = 11.67\,KN.$$

$$R_{fB} = R_B + R_B' = 30 + 3.33 = 33.33\,KN.$$

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SFD:- zone BC ($x=0$ to 1)

$F_x = -33.33$, $F_B = -33.33\,KN$, $F_C = -33.33\,KN$.

zone AC ($x=1$ to 3).

$F_x = -33.33 + 45$.

$F_C = 11.67\,KN$, $F_A = 11.67\,KN$.

Deflection:-

Take section in zone AC at a distance from x to B.

$$EI\frac{d^2y}{dx^2} = 33.33(x) - M_B - 45(x-1) \longrightarrow (1)$$

Integrate on B.S.

$$EI\frac{dy}{dx} = 33.33\frac{x^2}{2} - M_B x + C_1 - 45\frac{(x-1)^2}{2} \longrightarrow (2)$$

Integrate on B.S.

$$EI(y) = 33.33\frac{x^3}{6} - M_B\frac{x^2}{2} + C_1 x + C_2 - \frac{45(x-1)^3}{6} \longrightarrow (3)$$

at $x=0$, $\frac{dy}{dx}=0 \Rightarrow C_1 = 0$.

at $x=0$, $y=0$, $C_2 = 0$.

deflection under the load.

sub $x=1$, in eqn (3).

$$EI(y \text{ under load}) = \frac{33.33\times 1^3}{6} - \frac{20\times 1^3}{2}.$$

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$= \frac{-4.45}{EI}$, $EI = 1 \times 10^4$.

$= \frac{-4.45}{1 \times 10^4} = -4.45 \times 10^{-4}$ m.

$y_{under\ load} = -0.445$ mm.

Max deflection:-

at the point of max deflection, slope = 0.

$$\frac{33.33x^2}{2} - 20(x) - \frac{45(x-1)^2}{2} = 0$$

$$16.67x^2 - 20x - 22.5(x^2 - 2x + 1) = 0$$

$$16.67x^2 - 20x - 22.5x^2 + 45x - 22.5 = 0$$

$$-5.83x^2 + 25x - 22.5 = 0$$

$$5.83x^2 - 25x + 22.5 = 0$$

$$\boxed{x = 1.285 \text{ m}}$$

Subst $x = 1.285$ in eqn (3).

$$EI(y_{max}) = \frac{33.33(1.285)^3}{6} - \frac{20 \times (1.285)^2}{2} - \frac{45(1.285-1)^3}{6}$$

$$EI(y_{max}) = -4.89.$$

$$y_{max} = \frac{-4.89}{EI} = \frac{-4.89}{1 \times 10^4}$$

$$\boxed{y_{max} = -0.489 \text{ mm}}$$

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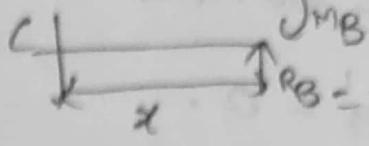
POC:

(POC)$_1$ occured in zone BC.



The BM in BC = 0.

$$33.33x - M_B = 0.$$

$$x = \frac{20}{33.33} = 0.6\text{m from B.}$$

(POC)$_2$ in from zone AC.

$$33.33x - M_B - 45(x-1) = 0.$$

$$-11.67x + 25 = 0 \quad , \quad x = \frac{25}{11.67} = 2.14\text{m from B.}$$

3) A fixed beam of span 6m carries two point loads. 150KN. each at a distance of 2m from the supports. Take $E = 2\times10^8$ KN/m^2. $I = 8\times10^8$ mm^4, Calculate support reaction. Find end moment, SFD & BMD and max deflection.

1) SSB with loading:-

$R_A = R_B = 150$KN.

moment at 'C' = 150×2 = 300 KN-m

moment at 'D' = 150×2 = 300 KN-m.

2) Beam with moment.

area of free BMD = area of fixed BMD.

$$\frac{2+6}{2} \times 300 = M_A \times 6.$$

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$M_A = 200\,KN\text{-}m$.

$M_B = 200\,KN\text{-}m$.

$R_A' + R_B' = 0$.

$\Sigma M_A = 0$.

$R_B' \times 6 - M_B + M_A = 0$

$R_B' = 0$, $R_A' = -R_B' = 0$.

Final reaction:-

$R_{fA} = 150 - 0 = 150KN$.

$R_{fB} = 150 + 0 = -150\,KN$.

SFD:

$R_B = -150KN$.

$f_D = 0$, $f_C = 0$.

$f_A = 150\,KN$.

Max deflection:-

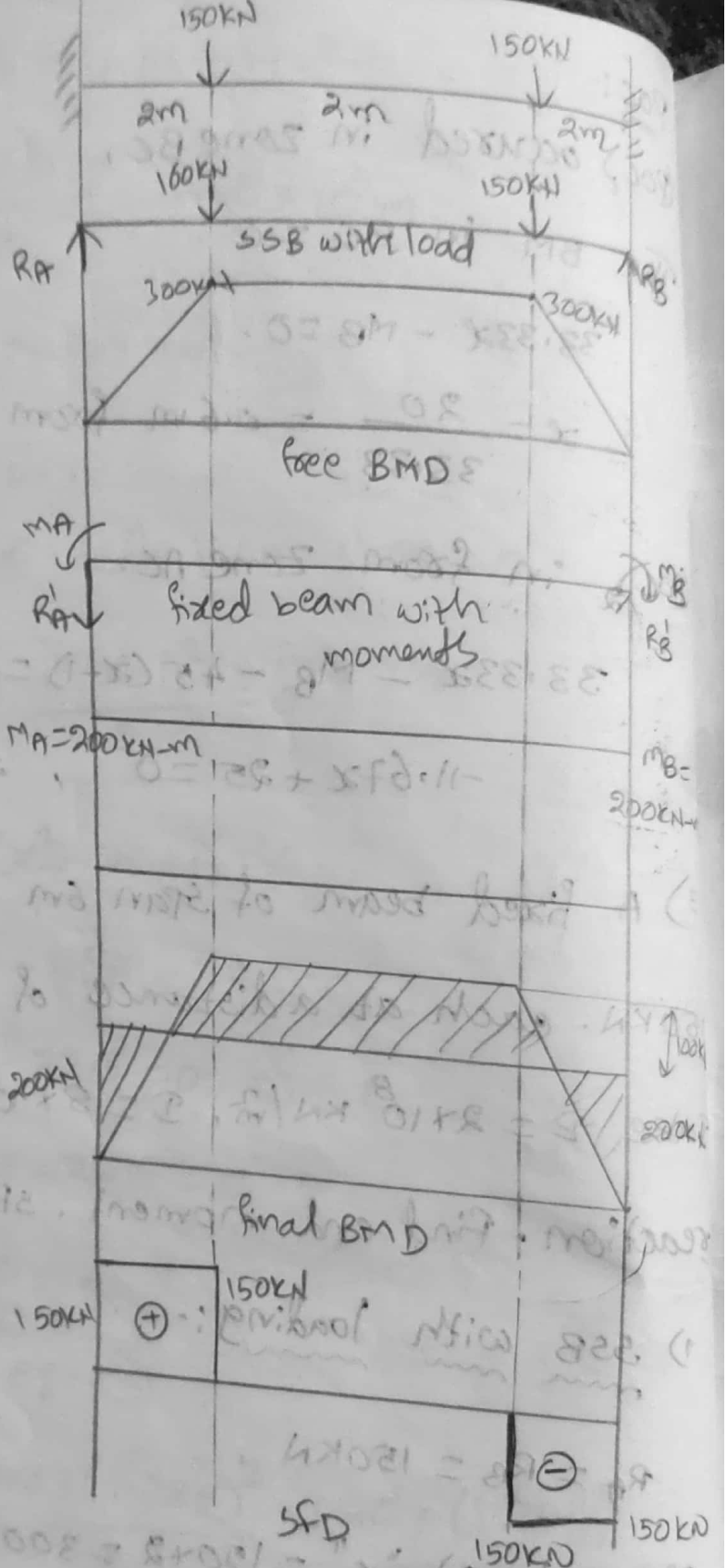
$$EI\frac{d^2y}{dx^2} = M.$$

$$EI\frac{d^2y}{dx^2} = 150x - M_B - 50(x-2) - 150(x-4) \rightarrow (1)$$

Integrating on B.S.

$$EI\cdot\frac{dy}{dx} = 150\frac{x^2}{2} - M_B x + C_1 - \frac{150(x-2)^2}{2} - \frac{150(x-4)^2}{2}$$

Integrating on B.S



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$$EI(y) = \frac{150x^3}{6} - \frac{M_B x^2}{2} + C_1 x + C_2 - \frac{150(x-2)^3}{6} - \frac{150(x-4)^3}{6} \rightarrow (3)$$

at $x=0$, $\frac{dy}{dx} = 0$, $\Rightarrow C_1 = 0$.

at $x=0$, $y=0$, $\Rightarrow C_2 = 0$.

and max deflection occured at $x=3$, sub $x=3$ in (3).

$$EI(y_{max}) = \frac{150 \times 3^3}{6} - \frac{200 \times 3^2}{2} + \frac{150(1)^3}{6}$$

$$EI \cdot y_{max} = -250.$$

$$y_{max} = \frac{-250}{EI} = \frac{-250}{2 \times 10^8 \times 8 \times 10^{-8} \times 10^{-12}}$$

$$\therefore y_{max} = -1.562 \times 10^{-3} \text{ m}.$$

$$\boxed{y_{max} = -1.562 \text{ mm}.}$$

4) A fixed beam of span 6m carrying point load of 75kN and 100 kN each at a distance of 2m from supports take $E = 2 \times 10^8$ KN/m^2, $I = 8 \times 10^{-8}$ m^4, find support reaction SFD & BMD, fixed end momentum max deflection..?

SSB with loading.

$R_A + R_B = 175$ KN,

$\Sigma M_A = 0$.

$R_B \times 6 - 75 \times 4 - 100 \times 2 = 0$, $R_B = \frac{300 + 200}{6} = 83.33$ KN.

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$R_A = 175 - 83.33 = 91.67\ KN$

$M_C = R_A \times 2 = 91.67 \times 2$

$M_C = 183.33\ KN-m$

$M_D = R_B \times 2 = 83.33 \times 2$

$\therefore M_D = 166.66\ KN-m$

Fixed end moment (M_A & M_B)

$$M_A = \frac{\Sigma W a b^2}{l^2} = \frac{100 \times 2 \times 4^2}{6^2}$$

$$+ \frac{75 \times 4 \times 2^2}{6^2} = 122.22\ KN-m.$$

$$M_B = \frac{\Sigma W a^2 b}{l^2}$$

$$= \frac{100 \times 2^2 \times 4}{6^2} + \frac{75 \times 4^2 \times 2}{6^2}$$

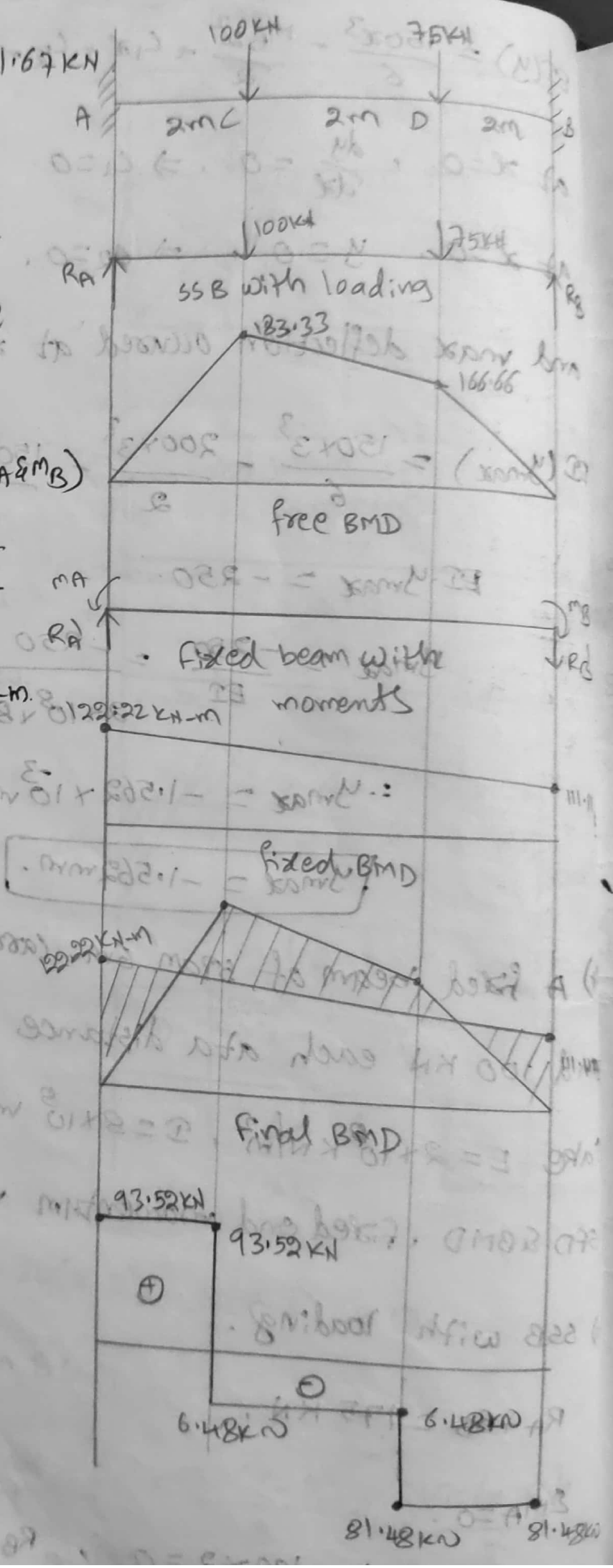
$\therefore M_B = 111.11\ KN$

$\Sigma M_A = 0$. for beam with fixed moments.

$R_B' \times 6 - M_B + M_A = 0$

$$R_B' = \frac{M_B - M_A}{6}$$

$$R_B' = \frac{111.11 - 122.22}{6}$$



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$R_B' = -1.85$ KN.

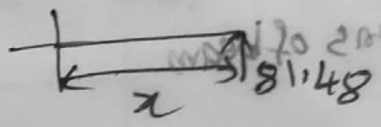
$R_A' + R_B' = 0$, $\therefore R_A' = -R_B' = 1.85$ KN.

final reaction.

$R_{fA} = R_A + R_A' = 91.67 + 1.85 = 93.52$ KN.

$R_{fB} = R_B - R_B' = 83.33 - 1.85 = 81.48$ KN.

SFD: (x 0 to 2)



$f_x = -81.48$ KN, $f_B = -81.48$ KN.

$f_J = -81.48$ KN., $f_D = -6.48$ KN, $f_C = -6.48$ KN.

$f_C = 93.52$ KN, $f_A = 93.52$ KN.

8/01/2020

5) A fixed beam of length 5m carries a UDL of 9 KN/m. over entire length take $I = 4.5 \times 10^{-4}$ m^4, $E = 1 \times 10^3$ KN/m^2. calculate support reactions. fixed end moments. Draw SFD and BMD and maximum deflection.

a) SSB with loading.

R_A & R_B are the reaction of SSB with loading.

$R_A = R_B = \frac{9 \times 5}{2} = 22.5$ KN.

Max B.M at C $= \frac{9 \times 5^2}{8} = 28.12$ KN-m.

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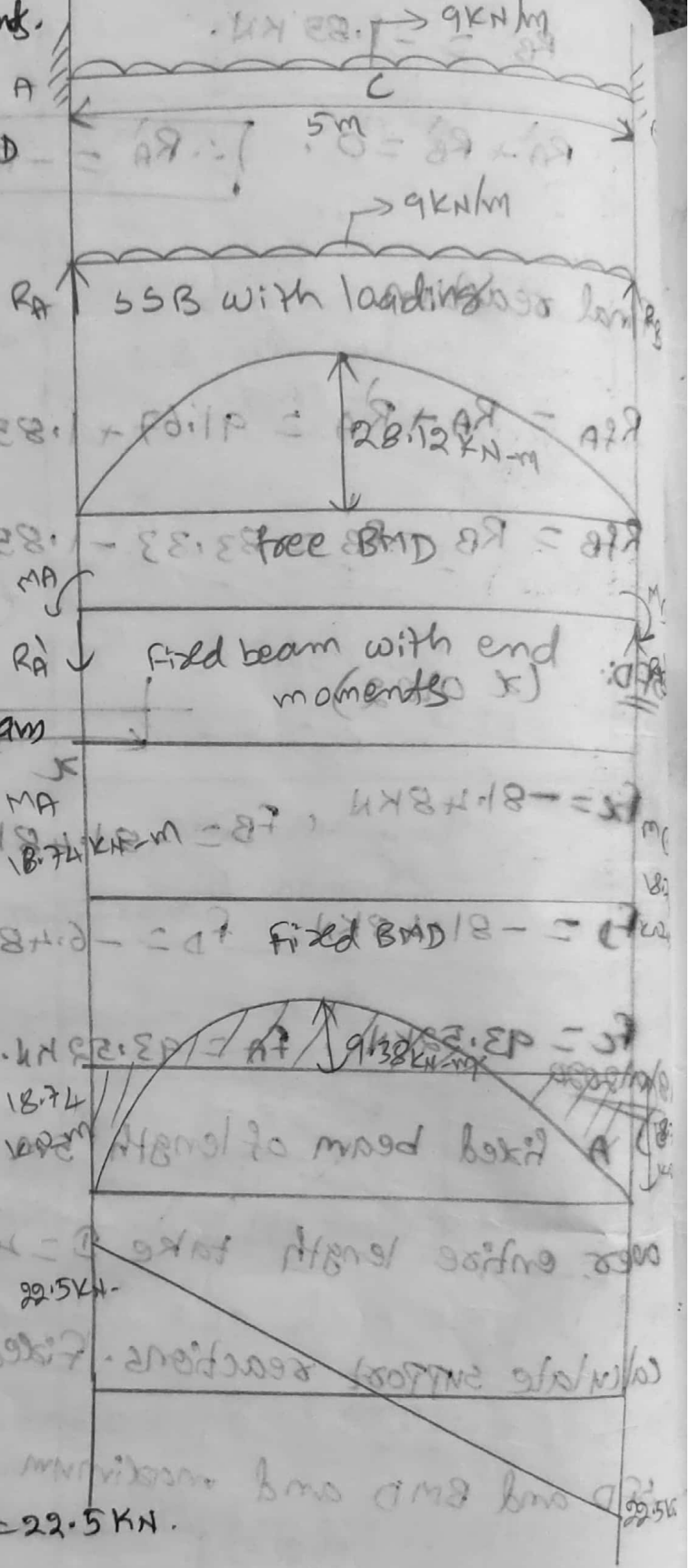
b) Beam with fixed end moments.



area of free BMD = area of fixed BMD

$$\frac{2}{3} \times l \times \frac{wl^2}{8} = M_A \times l.$$

$$\frac{2}{3} \times 28.12 = M_A$$

$$M_A = 18.74 \text{ KN-m}$$

$$M_B = 18.74 \text{ KN-m}$$

R_A' & R_B' are reactions of beam with moments.

$$R_A' + R_B' = 0$$

$$R_B' \times 5 - M_B + M_A = 0$$

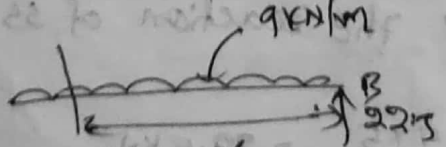
$$R_B' = \frac{M_B - M_A}{5} = 0$$

$$\therefore R_A' = -R_B' = 0$$

Final reactions:-

$$R_{fA} = R_A - R_A' = 22.5 - 0 = 22.5 \text{ KN}.$$

$$R_{fB} = R_B + R_B' = 22.5 + 0 = 22.5 \text{ KN}.$$



SFD: (x, 0 to 5)

$$F_x = -22.5 + 9x$$

$$F_B = -22.5 + 9(0) = -22.5 \text{ KN}.$$

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$F_A = -22.5 + 9(5) = 22.5\ KN$

max deflection:

max deflection occured at center i.e, $x = 2.5m$

$$EI\frac{d^2y}{dx^2} = RF_A(x) - M_B - 9\frac{x^2}{2} \rightarrow (1)$$

Integrating on B.S

$$EI\frac{dy}{dx} = 22.5\frac{x^2}{2} - 18.74x - \frac{9x^3}{6} + C_1 \rightarrow (2)$$ Integrate

$$EI.y = 22.5\frac{x^3}{6} - 18.74\frac{x^2}{2} - \frac{9x^4}{24} + C_1x + C_2 \rightarrow (3)$$

applying boundary conditions.

at $x=0$, $\frac{dy}{dx} = 0 \Rightarrow C_1 = 0$

$x=0$, $y=0$, $\Rightarrow C_2 = 0$

subst $x = 2.5$ in eqn (3)

$$EI(y_{max}) = \frac{22.5 \times (2.5)^3}{6} - \frac{18.74\,(2.5)^2}{2} - \frac{9\,(2.5)^4}{24}$$

$$EI.y_{max} = -14.64$$

$$y_{max} = \frac{-14.64}{EI} = \frac{-14.64}{14 \times 10^9 \times 45 \times 10^{-3}}$$

$$\therefore y_{max} = -3.25 \times 10^{-3}\ m$$

$$\boxed{\therefore y_{max} = -3.25\ mm}$$

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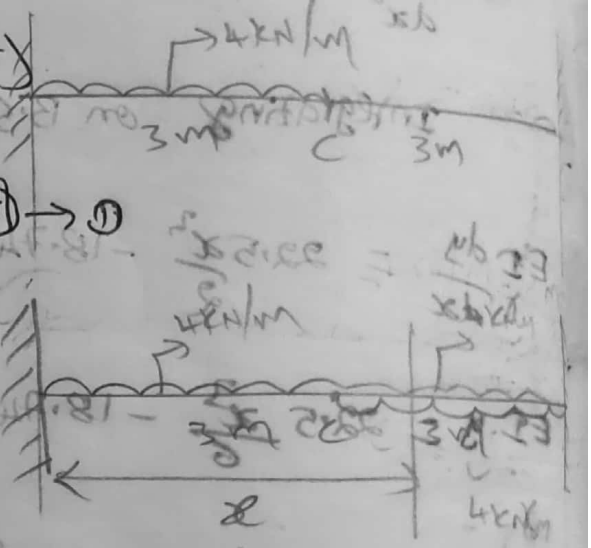
6) A fixed beam of length 6m is subjected to a UDL of 4 KN/m on left half of length. Calculate i) support reactions ii) fixed moments and draw SFD & BMD and max deflection

Take a section x-x at a distance x from B.



$$M_x = R_B(x) - M_B - 4\left(\frac{x^2}{2}\right) + 4(3)(x-1.5)$$

$$EI\frac{d^2y}{dx^2} = R_B(x) - M_B - \frac{4x^2}{2} + 12(x-1.5) \rightarrow (1)$$

add & delect the load of 4 KN/m over CB.

Take a section in zone BC at a distance of x from 'A'.

$$M_x = R_A(x) - M_A - \frac{4x^2}{2} + \frac{4(x-3)^2}{2}$$

Macaulay's method

$$EI\frac{d^2y}{dx^2} = R_A(x) - M_A - \frac{4x^2}{2} + \frac{4(x-3)^2}{2} \rightarrow (1)$$

$$EI\frac{dy}{dx} = R_A\frac{x^2}{2} - M_A x - \frac{4x^3}{6} + \frac{4(x-3)^3}{6} + C_1 \rightarrow (2)$$

$$EI\,y = R_A\frac{x^3}{6} - M_A\frac{x^2}{2} - \frac{4x^4}{24} + \frac{4(x-3)^4}{24} + C_1x + C_2 \rightarrow (3)$$

apply boundary conditions.

When $x=0$, $\frac{dy}{dx}=0 \Rightarrow C_1=0$

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at $x=0$, $y=0 \Rightarrow C_2=0$

at $x=6$, $\frac{dy}{dx}=0$. Sub in eq ②

$$EI(0) = \frac{R_A(6)^2}{2} - M_A \times 6 - \frac{4(6)^3}{6} + \frac{4(6-3)^3}{6}$$

$$18R_A - 6M_A - 126 = 0 \rightarrow ④$$

at $x=6$, $y=0$. sub in eq ③

$$EI(0) = R_A\frac{6^3}{6} - M_A\frac{6^2}{2} - \frac{4(6)^4}{24} + \frac{4(6-3)^4}{24}$$

$$36R_A - 18M_A - 202.5 = 0 \rightarrow ⑤$$

Solving ④ & ⑤:

$R_A = 9.75$ KN

$M_A = 8.25$ KNm

$R_A + R_B = 12$

$R_B = 12 - 9.75$

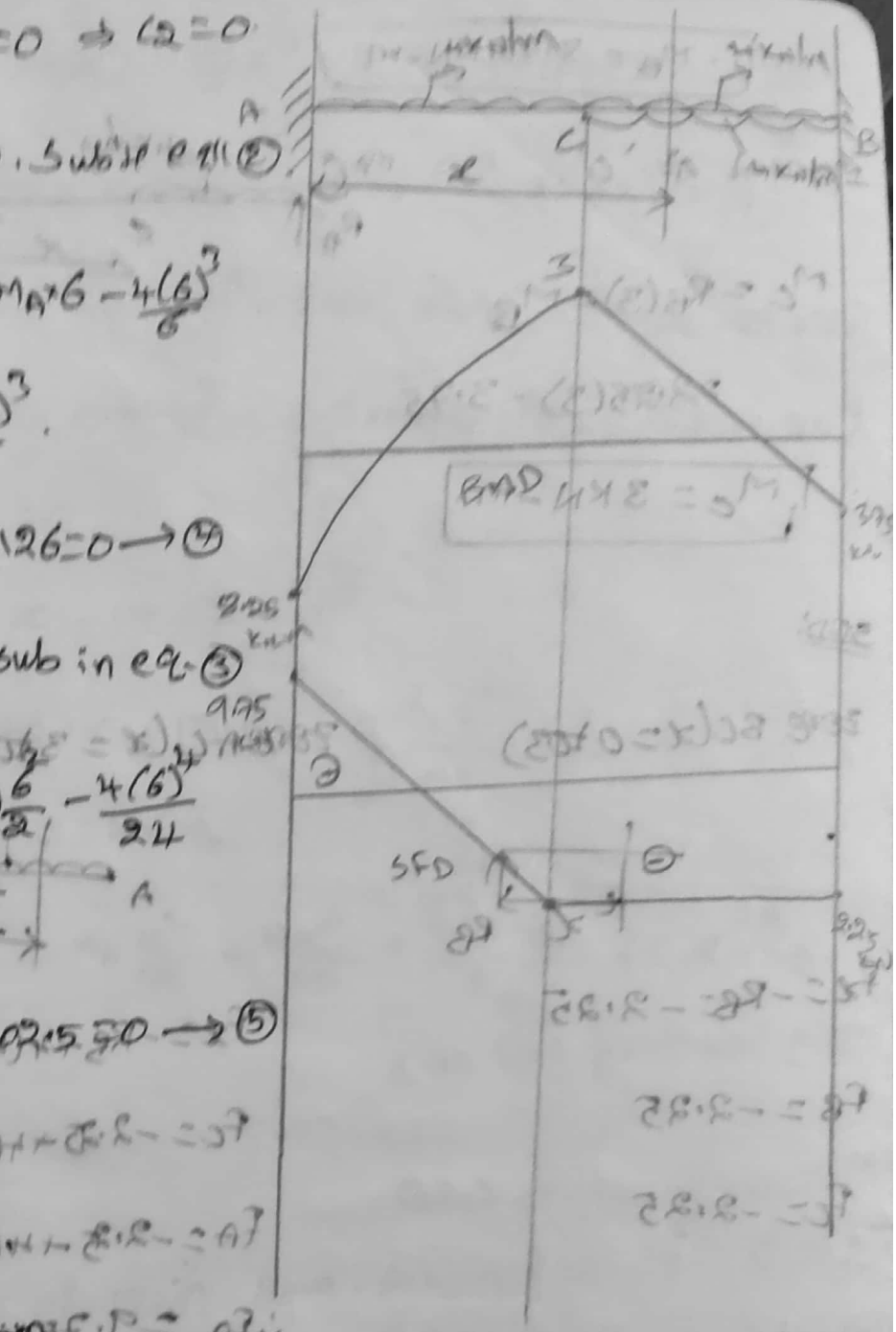
$R_B = 2.25$ KN.

$\Sigma M_B = 0$,

$R_A(6) - M_A + M_B - 4(3)(4.5) = 0$

$9.75 \times 6 - 8.25 - 54 = -M_B$

$-3.75 = -M_B$



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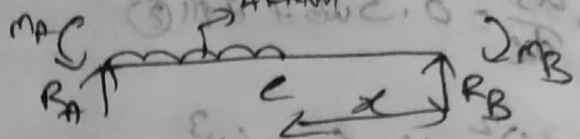
$\therefore M_B = 3.75\ KN-m$

Moment at 'C'.



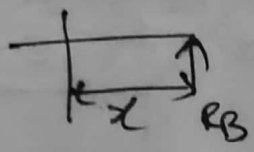
$M_C = R_B(3) - M_B$

$= 2.25(3) - 3.75$

$M_C = 3\ KN-m.$

SFD:

zone BC ($x = 0$ to 3)

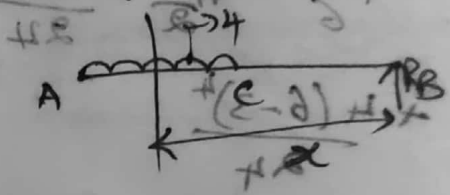


$f_x = -R_B = -2.25$

$f_B = -2.25$

$f_C = -2.25$

zone AC ($x = 3$ to 6)



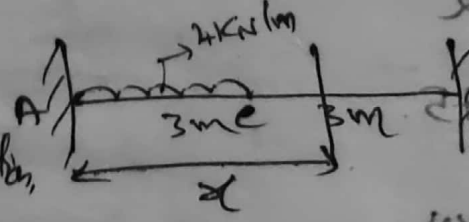
$f_x = -R_B + 4(x-3)$

$f_C = -2.25 + 4(3-3) = -2.25$

$f_A = -2.25 + 4(6-3) = 9.75\ KN$

$\therefore f_A = 9.75\ KN$

iv) Max deflection:



at point of max deflection,

slope = 0.

max deflection occured in zone BC.

The slope and in zone BC:

$$0 = \frac{R_A x^2}{2} - M_A x - \frac{4x^3}{6} + \frac{4(x-3)^3}{6}$$

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$$9.75\frac{x^2}{2} - 8.25(x) - \frac{2}{3}x^3 + \frac{2}{3}(x-3)^2(x-3) = 0.$$

$$4.87x^2 - 8.25x - 0.66x^3 + 0.66(x^2 + 9 - 6x)(x-3) = 0$$

$$4.87x^2 - 8.25x - 0.66x^3 + 0.66(x^3 + 9x - 6x^2 - 3x^2 - 27 + 18x) = 0$$

$$4.87x^2 - 8.25x - 0.66x^3 + 0.66x^3 + 3.96x^2 + 5.94x - 1.98x^2$$

$$-17.82 + 11.88x = 0$$

$$-1.07x^2 + 9.57x - 17.82 = 0$$

$$1.07x^2 - 9.57x + 17.82 = 0$$

$$x = 6.3\text{m}, \quad x = 2.6\text{m}$$

$$EI\,y_{max} = R_A\frac{x^3}{6} - M_A\frac{x^2}{2} - \frac{4x^4}{24} + \frac{4(x-3)^4}{24} + C_1x + C_2.$$

7) A fixed beam of span 20m. subjected to UDL of 8kN/m over left half of the portion and 120 kN. at a distance of 5m from right support. Calculate :i) support reactions ii) fixed end moments. iii) Draw SFD & BMD. iv) max. deflection. take $E = 2\times10^8$ kN/m^2, $I = 4\times10^{-4}$ m^4.

add and delect the load of 8 kN/m over CB.

Take a section in DB at a distance of x from A.

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$$EI\frac{d^2y}{dx^2} = R_A(x) - M_A - \frac{8x^2}{2}$$

$$-\frac{8(x-10)^2}{2} - 120(x-15) \rightarrow (1)$$

$$EI\frac{dy}{dx} = \frac{R_A x^2}{2} - M_A x - \frac{8x^3}{6} + \frac{8(x-10)^3}{6} - \frac{120(x-15)^2}{2} + C_1 \rightarrow (2)$$

$$EI\,y = \frac{R_A x^3}{6} - \frac{M_A x^2}{2} - \frac{8x^4}{24} + \frac{8(x-10)^4}{24} - \frac{120(x-15)^3}{6} + C_1 x + C_2 \rightarrow (3)$$

apply boundary conditions.

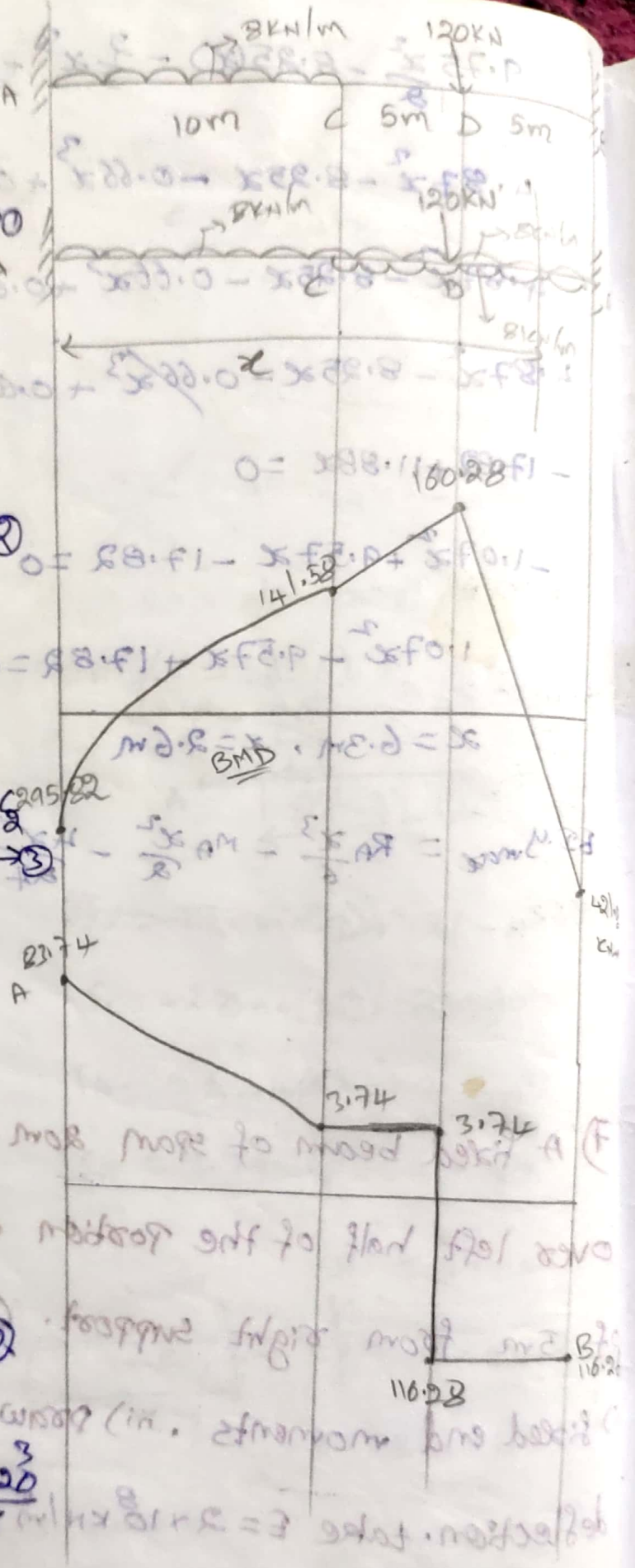
at $x=0, \frac{dy}{dx}=0 \Rightarrow C_1=0.$

at $x=0, y=0 \Rightarrow C_2=0.$

$x=20, \frac{dy}{dx}=0$. sub in (2)

$$EI(0) = \frac{R_A(20)^2}{2} - M_A \times 20 - \frac{8\times 20^3}{6} + \frac{8(10)^3}{6} - \frac{120(5)^2}{2}$$

$$200R_A - 20M_A - 10833.33 = 0 \rightarrow (4)$$



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at $x=20$, $y=0$. subst in eq. ③

$$EI(0) = \frac{R_A(20)^3}{6} + \frac{M_A \times 20^2}{2} - \frac{8(20)^4}{24} + \frac{8(10)^4}{24} - \frac{120(5)^3}{6}$$

$$1333.33 R_A - 200 M_A - 52500 = 0 \rightarrow ⑤$$

solving ④ & ⑤.

$$R_A = 83.74 \text{ KN}, \quad M_A = 295.82 \text{ KN-m}.$$

$$R_A + R_B = 200.$$

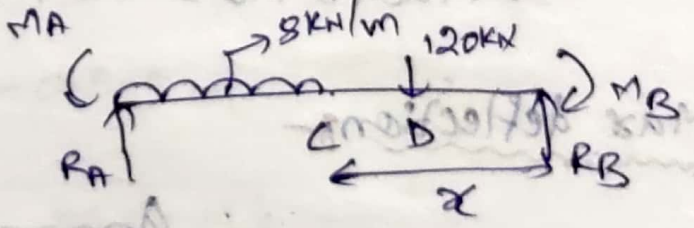
$$R_B = 200 - 83.74 = 116.26 \text{ KN}.$$

$\Sigma M_A = 0$.

$$R_B \times 20 - M_B + M_A - 120(15) - 8 \times 10 \times 5 = 0.$$

$$M_B = 421.02 \text{ KN-m}.$$

Moment at "D"

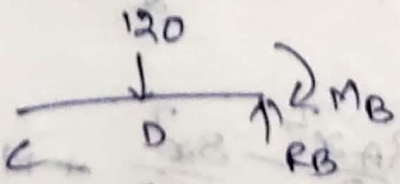


$$M_D = R_B \times 5 - M_B$$

$$= 116.26 \times 5 - 421.02$$

$$M_D = 160.28 \text{ KN-m}$$

Moment at "C"



$$M_C = R_B \times 10 - M_B - 120(5)$$

$$= 116.26 \times 10 - 421.02 - 120 \times 5 \quad \therefore M_C = 141.58 \text{ KN-m}$$

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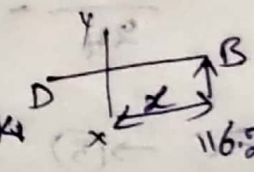
SFD:

Zone BD (x=0 to 5)

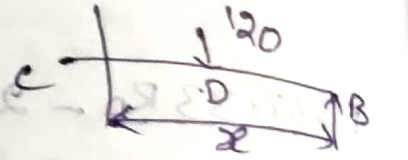


$f_x = -116.26$ KN

$f_B = -116.26$ KN.

$F_D = -116.26$ KN.

Zone CD (x = 5 to 10)

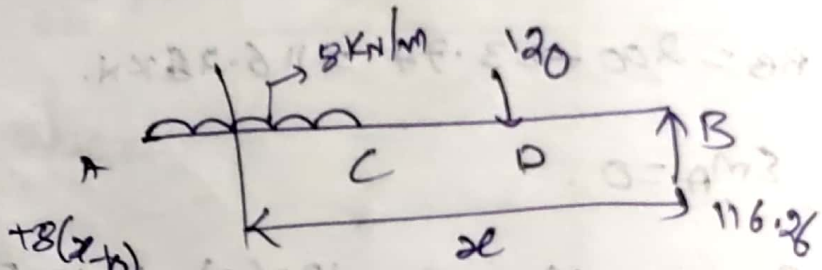


$f_x = -116.26 + 120 = 3.74$ KN.

$f_D = 3.74$ KN.

$f_C = 3.74$ KN.

Zone AC (x = 10 to 20)

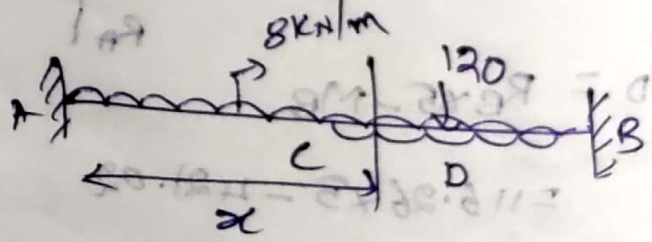


$f_x = -116.26 + 120 + 8(x-10)$

$f_C = -116.26 + 120 + 8(10-10) = 3.74$ KN.

$F_A = -116.26 + 120 + 8 \times 10 = 83.74$ KN.

iv) Max deflections



$$EI\frac{d^2y}{dx^2} = M_x = R_A(x) - \frac{8x^2}{2} - M_A + \frac{8(x-10)^2}{2}$$

$$EI\frac{dy}{dx} = \frac{R_A x^2}{2} - \frac{8x^3}{3\times 2} + M_A x + \frac{8(x-10)^3}{6} \rightarrow (2)$$

$$EI\,y = \frac{R_A x^3}{6} - \frac{8x^4}{24} - \frac{M_A x^2}{2} + 8(x-10)^4 \rightarrow (3)$$

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at the max deflection, slope = 0.

$$\frac{83.74(x^2)}{2} - \frac{Bx^3}{6} - 295.82(x) + \frac{B(x-10)^3}{6} = 0$$

$\boxed{x = 10.727\ m}$ from A.

for max deflection, subtitute $x = 10.727$ in eq (3)

$$EI(y_{max}) = \frac{R_A \times (10.727)^3}{6} - \frac{8(10.727)^4}{24} - \frac{295.82 \times (10.727)^2}{2} + \frac{8(10.727 - 10)^4}{24}$$

$$EI \cdot y_{max} = -4204.5$$

$$y_{max} = \frac{-4204.5}{2 \times 10^8 \times 4 \times 10^{-4}}$$

$$\therefore y_{max} = -0.5925\ m$$

$\boxed{y_{max} = -52.5\ mm}$

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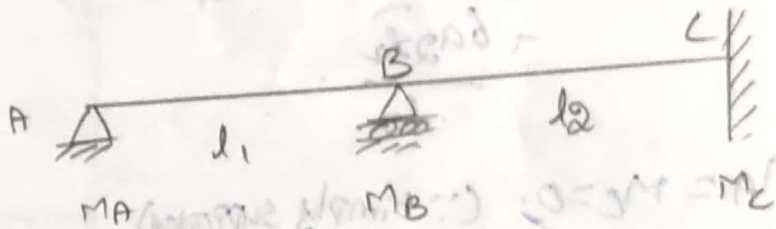
Continuous Beams:-

Clapeyron's theorem:- If a beam member having more than 2 supports is known as continuous beam.

Continuous beams are analyzed by clapeyron's free moment theorem.

Let ABC is a continuous beam subjected to some external loads. then the clapeyron's theorem is as follows.



$$M_A l_1 + 2M_B(l_1 + l_2) + M_C l_2 = \frac{6a_1\bar{x}_1}{l_1} + \frac{6a_2\bar{x}_2}{l_2}.$$

M_A, M_B and M_C are the moments at supports.

l_1, l_2 are the lengths of the member AB & BC.

a_1 = area of the BMD due to external loads on AB.

$\bar{x}_1$ = centroid of BMD on AB from A.

a_2 = area of the BMD due to external loads on BC.

$\bar{x}_2$ = Centroid of BMD on BC from C.

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1) A continuous beam of A,B,C carries a UDL of 10KN/m throughout the span. The length of AB 4m and BC 6m. Calculate i) support reactions, ii) moment at A, B & C, iii) Draw SFD & BMD. Assume A & C are simply supported.

$l_1 = 4m$, $l_2 = 6m$,

$w_1 = 10KN/m$, $w_2 = 10KN/m$

apply clapeyron's theorem on ABC

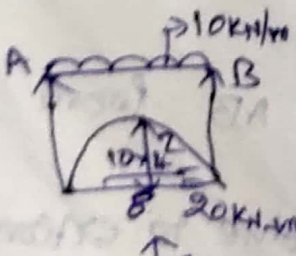
$$M_A l_1 + 2M_B(l_1 + l_2) + M_C l_2 = \frac{6a_1\bar{x}_1}{l_1} + \frac{6a_2\bar{x}_2}{l_2}$$

$M_A = M_C = 0$. (∵ simply supported)

$$2M_B(4+6) = \frac{6a_1\bar{x}_1}{4} + \frac{6a_2\bar{x}_2}{6}$$

$$20M_B = \frac{6a_1\bar{x}_1}{4} + \frac{6a_2\bar{x}_2}{6} \rightarrow (1)$$

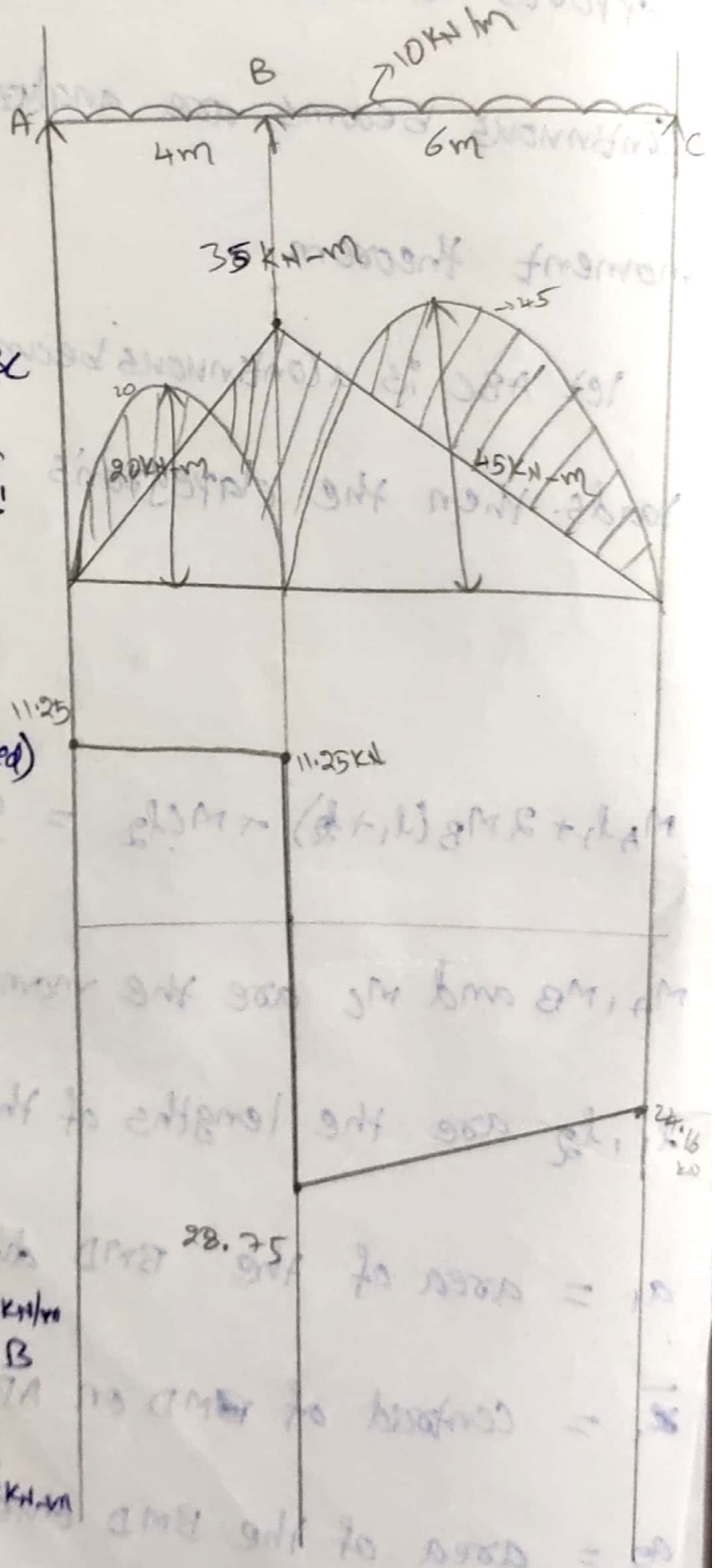
a_1 = area of BMD on AB.



$$a_1 = \frac{2}{3} \times 4 \times 20$$

$a_1 = 53.33$

$\bar{x}_1 = 4/2 = 2m$.



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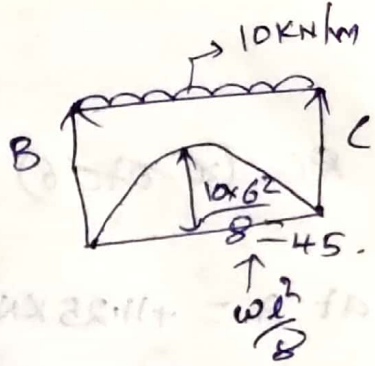
a_2 = area of BMD on BC.



$a_2 = \frac{2}{3} \times 6 \times 45$

$\therefore a_2 = 180$, $x_2 = 6/2 = 3m$.

Subst a_1, x_1 & a_2, x_2 in ① eqn.

$$20M_B = \frac{6 \times 53.33 \times 2}{4} + \frac{6 \times 180 \times 3}{6}$$

$$20M_B = 700. \quad \boxed{\therefore M_B = 35\ KN\text{-}m}$$

$R_A + R_B + R_C = 100.$

Take a member AB.

$\Sigma M_B = 0.$

$R_A \times 4 - 10 \times 4 \times 2 = M_B.$

$R_A \times 4 - 80 = -35$

$$R_A = \frac{-35 + 80}{4} = 11.25\ KN.$$

Take a member BC.

$\Sigma M_B = 0$, $R_C \times 6 - 10 \times 6 \times 3 = -35.$

$$R_C = \frac{-35 + 180}{6} = 24.16\ KN.$$

$R_B = 100 - 11.25 - 24.16$

$\boxed{\therefore R_B = 64.59\ KN.}$

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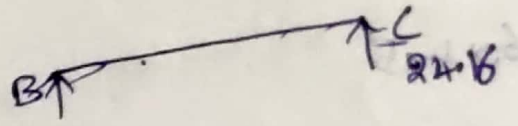
SFD:

Zone BC. (x: 0 to 6)



Sf at A = +11.25 KN.

Sf at C = −24.16 KN.

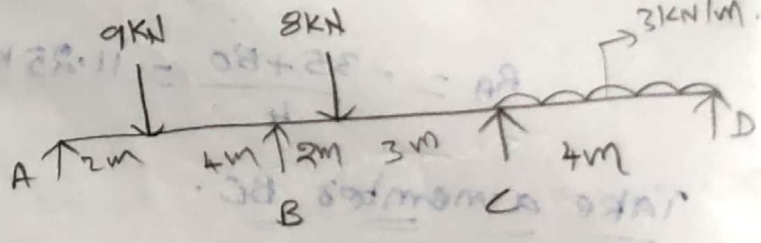
Sf @ left of B = +11.25 KN.

S.f @ right of B = +11.25 − 10×4

= −28.75 KN

23/01/2020.

2) A continuous beam ABCD simply supported at A, B, C, D. Calculate B.M, Reactions and Draco SFD, BMD. AB length 6m, Carries a Point load of 9KN. at a distance of 2m from A. The length of BC 5m subjected to a Point load of 8KN at a distance of 2m from B. The length of CD 4m subjected to a UDL of 3KN/m over the span of 4m.



Sol:

$l_1 = 6m$, $\omega_1 = 9KN$

$l_2 = 5m$, $\omega_2 = 8KN$

$l_3 = 4m$, $\omega_3 = 3KN/m$.

$M_A = M_D = 0$.

a) Apply Clapeyron's theorem for span ABC.

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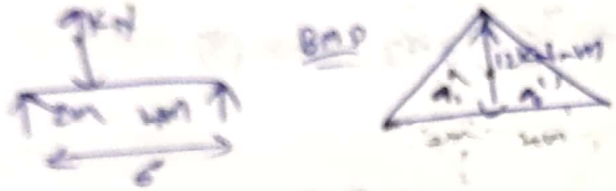
$$M_A l_1 + 2M_B(l_1+l_2) + M_C l_2 = \frac{6a_1\bar{x}_1}{l_1} + \frac{6a_2\bar{x}_2}{l_2}$$

$$0(6) + 2M_B(5+6) + M_C(5) = \frac{6a_1\bar{x}_1}{6} + \frac{6a_2\bar{x}_2}{5}$$

$$22M_B + 5M_C = a_1\bar{x}_1 + \frac{6}{5}a_2\bar{x}_2 \rightarrow (1)$$

a_1 = area of BMD due to 9KN on AB.



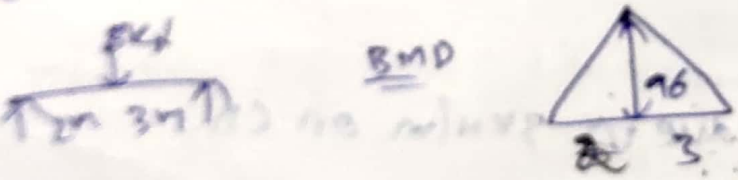
$$\text{max BM} = \frac{Wab}{l} = \frac{9\times2\times4}{6} = 12\text{KN-m}$$

$$a_1\bar{x}_1 = a_1'\bar{x}_1' + a_2''\bar{x}_1''$$

$$= \left(\tfrac{1}{2}\times2\times12\right)\left(\tfrac{2}{3}\times2\right) + \left(\tfrac{1}{2}\times4\times12\right)\left(2+\tfrac{4}{3}\right)$$

$$a_1\bar{x}_1 = 96.$$

a_2 = area of BMD due to 8KN on BC.



$$\text{max BM} = \frac{Wab}{l} = \frac{8\times2\times3}{5} = 9.6\text{ KN-m}$$

$$a_2\bar{x}_2 = \left(\tfrac{1}{2}\times3\times9.6\right)\left(\tfrac{2}{3}\times3\right) + \left(\tfrac{1}{2}\times2\times9.6\right)\left(3+\tfrac{2}{3}\right)$$

$$a_2\bar{x}_2 = 64.$$

$$\therefore 22M_B + 5M_C = 96 + \frac{6}{5}(64)$$

$$22M_B + 5M_C = 172.8 \rightarrow (2)$$

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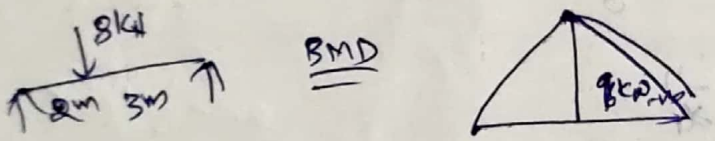
b) Apply clapeyron's theorem for BCD.

$$M_B l_2 + 2M_C(l_2+l_3) + M_D l_3 = \frac{6a_2\bar{x}_2}{l_2} + \frac{6a_3\bar{x}_3}{l_3}$$

$$M_B(5) + 2M_C(5+4) + 0 = \frac{6a_2\bar{x}_2}{5} + \frac{6a_3\bar{x}_3}{4}$$

$$5M_B + 18M_C = \frac{6}{5}a_2\bar{x}_2 + \frac{3}{2}a_3\bar{x}_3 \longrightarrow (3)$$

a_2 = area of BMD due to 8kN on BC.

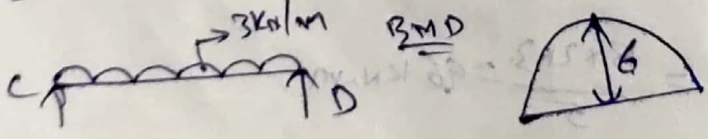


$$\text{Max B.M} = \frac{wab}{l} = \frac{8\times2\times3}{5} = 9.6 \text{ kN-m}$$

$$a_2\bar{x}_2 = \left(\tfrac{1}{2}\times2\times9.6\times\tfrac{2}{3}\times2\right) + \left(\tfrac{1}{2}\times3\times9.6\right)\left(2+\tfrac{3}{3}\right)$$

$$a_2\bar{x}_2 = 56.$$

a_3 = area of BMD due to 3kN/m on CD.



$$\text{Max BM} = \frac{wl^2}{8} = \frac{3\times4^3}{8} = 6 \text{ kN-m}$$

$$a_3\bar{x}_3 = \left(\tfrac{2}{3}\times4\times6\right)\left(\tfrac{4}{2}\right) = 32.$$

$$\therefore 5M_B + 18M_C = \tfrac{6}{5}(56) + \tfrac{3}{2}(32)$$

$$\therefore 5M_B + 18M_C = 115.2 \longrightarrow (4)$$

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solve ② & ④.

$\therefore M_B = 6.83\ KN\text{-}m$, $M_C = 4.50\ KN\text{-}m$.

$M_A = 0$, $M_D = 0$.

$R_A + R_B + R_C + R_D = 29\ KN$.

for span AB → ΣM_B.

$R_A \times 6 - 9 \times 4 = -M_B$.

$R_A \times 6 - 36 = -6.83$.

$R_A = 4.8\ KN$.

for span CD, ΣM_C.

$R_D \times 4 - 3 \times 4 \times 2 = -4.50$

$R_D = \frac{-4.50 + 24}{4} = 4.88\ KN$

for span ABC, ΣM_C.

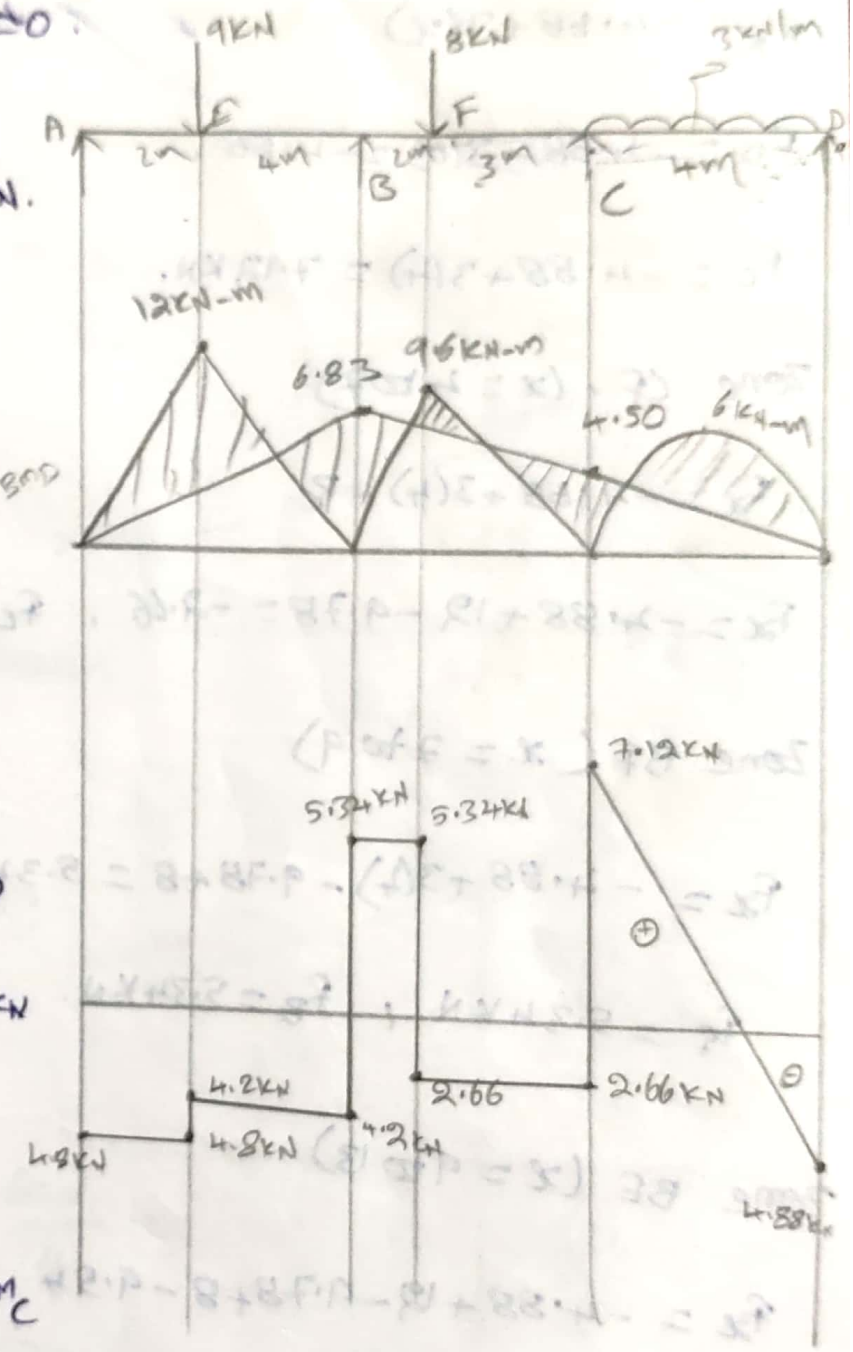
$R_A \times 11 - 9 \times 9 - 8 \times 3 + R_B \times 5 = -M_C$

$R_B = 9.54\ KN$.

$R_A + R_B + R_C + R_D = 29$

$R_C = 29 - 4.8 - 9.54 - 4.88$

$\therefore R_C = 9.78\ KN$.



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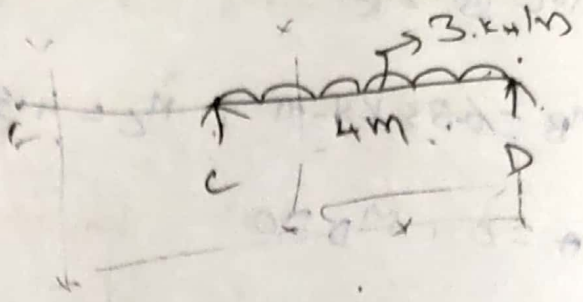
SFD:



Zone CD (x = 0 to 4)

$f_x = -4.88 + 3(x)$

$f_D = -4.88 + 3(0) = -4.88$.

$f_C = -4.88 + 3(4) = 7.12\ KN$.

Zone CF, (x = 4 to 7)

$f_x = -4.88 + 3(4) - R_C$.

$f_x = -4.88 + 12 - 9.78 = -2.66$, $f_C = -2.66$, $f_F = -2.66$.

Zone BF (x = 7 to 9)

$f_x = -4.88 + 3(4) - 9.78 + 8 = 5.34\ KN$.

$f_F = 5.34\ KN$, $f_B = 5.34\ KN$.

Zone BE (x = 9 to 13).

$f_x = -4.88 + 12 - 9.78 + 8 - 9.54$, $f_x = -4.2$.

$f_B = -4.2$ KN, $f_E = -4.2\ KN$

Zone AE (x = 13 to 18).

$f_x = -4.88 + 12 - 9.78 + 8 - 9.54 + 9$

$f_x = -4.8$.

$\therefore f_A = -4.8\ KN$, $\therefore f_E = +4.8\ KN$

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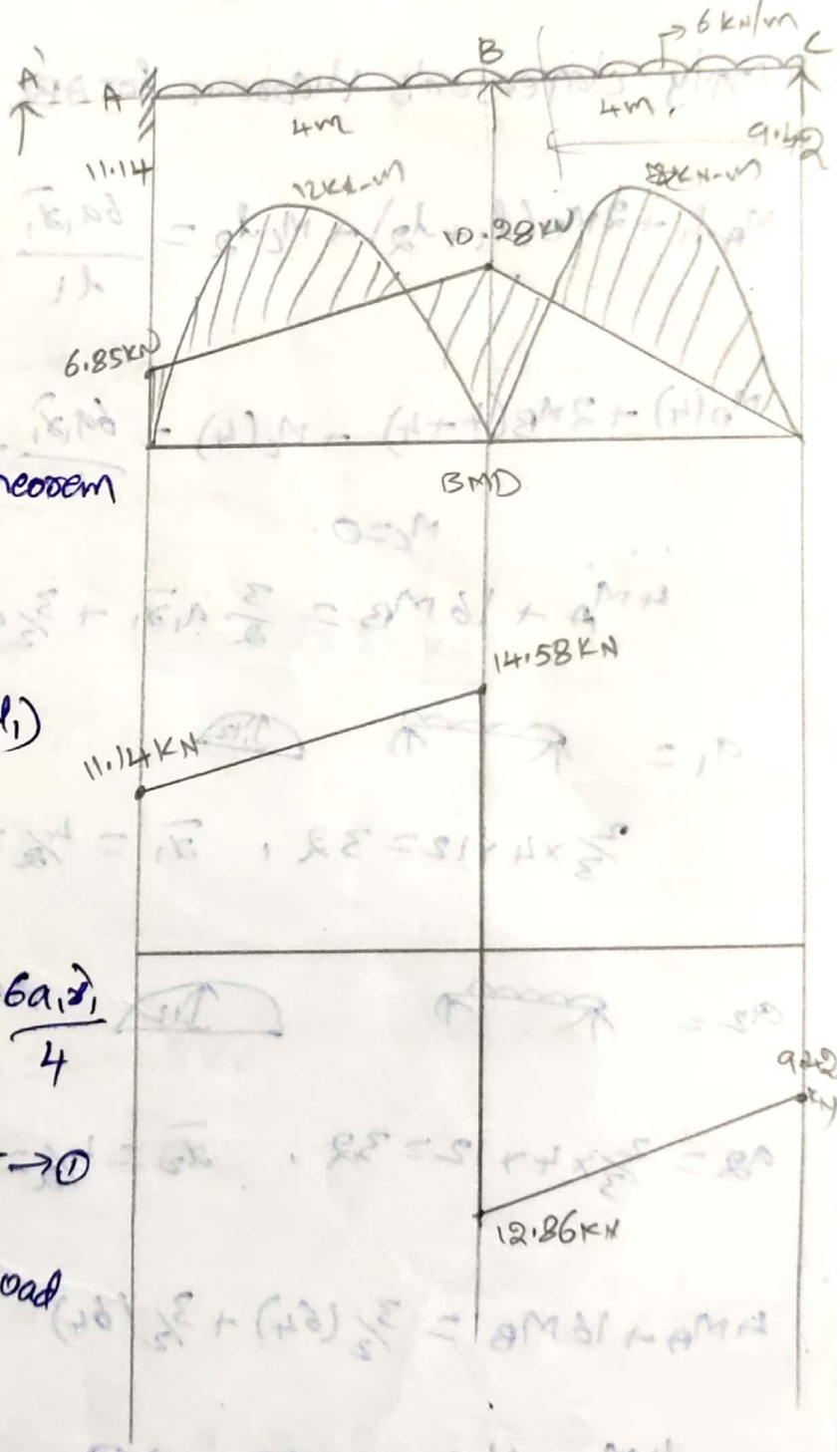
3) A continuous beam ABC of uniform section with AB 4m, BC = 4m. is fixed at A and simply support at C. The beam is carrying UDL of 6 KN/m throughout the length. Calculate support reactions and moments and Draw SFD, BMD.



let us assume a fixed beam Aa' with length l_0, and moment at $a' = M_0$.

a) Apply clapeyron's theorem for A'AB.

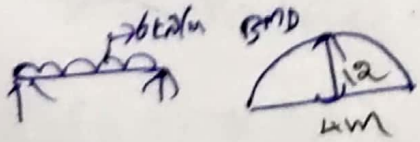
$$M_0 l_0 + 2M_A(l_0 + l_1) + M_B(l_1) = \frac{6a_0\bar{x}_0}{l_0} + \frac{6a_1\bar{x}_1}{l_1}$$

($M_0 l_0 = 0$, $\frac{6a_0\bar{x}_0}{l_0} = 0$)

$$2M_A(0+4) + M_B(4) = \frac{6a_1\bar{x}_1}{4}$$

$$8M_A + 4M_B = \frac{3a_1\bar{x}_1}{2} \rightarrow (1)$$

a_1 = area of BMD due to load 6 KN/m on AB.



$$\text{Max B.M} = \frac{6 \times 4^2}{8} = 12 \text{ KN-m}$$

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$a_1 = \frac{2}{3} \times 4 \times 12 = 32$, $\bar{x}_1 = \frac{4}{2} = 2m$.

$8M_A + 4M_B = \frac{3 \times 32 \times 2}{2} = 96 \rightarrow (2)$.

$\therefore 8M_A + 4M_B = 96 \rightarrow (2)$.

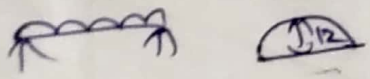
b) Apply clapeyon's theorem for ABC.

$$M_A l_1 + 2M_B(l_1 + l_2) + M_C l_2 = \frac{6a_1\bar{x}_1}{l_1} + \frac{6a_2\bar{x}_2}{l_2}$$

$$M_A(4) + 2M_B(4+4) + M_C(4) = \frac{6a_1\bar{x}_1}{4} + \frac{6a_2\bar{x}_2}{4}$$

$M_C = 0$.

$4M_A + 16M_B = \frac{3}{2} a_1\bar{x}_1 + \frac{3}{2} a_2\bar{x}_2 \rightarrow (3)$

$a_1 =$ 

$\frac{2}{3} \times 4 \times 12 = 32$, $\bar{x}_1 = \frac{4}{2} = 2m$.

$a_2 =$ 

Mod B.M $= \frac{6 \times 4^2}{8} = 12 KN\text{-}m$

$a_2 = \frac{2}{3} \times 4 \times 12 = 32$, $\bar{x}_2 = \frac{4}{2} = 2m$.

$4M_A + 16M_B = \frac{3}{2}(64) + \frac{3}{2}(64)$

$4M_A + 16M_B = 192 \rightarrow (4)$

Solve (2) & (4).

$M_A = 6.85 KN\text{-}m$, $M_B = 10.28 KN\text{-}m$.

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SFD: $R_A + R_B + R_C = 8 \times 6 = 48$.

for span BC. ΣM_B.

$$R_C \times 4 - 6 \times 4 \times \frac{4}{2} + M_B = 0$$

$$R_C = \frac{48 - 10.28}{4}, \quad \therefore R_C = 9.42\,KN$$

for span AB : ΣM_B.

$$R_A \times 4 - M_A - 6 \times 4 \times 2 + M_B = 0$$

$$R_A = \frac{6.85 + 48 - 10.28}{4} = 11.14.$$

$$R_B = 48 - 9.42 - 11.14, \quad \therefore R_B = 27.44\,KN$$

SFD:

Zone BC $(x = 0 \text{ to } 4)$.

$$f_x = -R_C + 6(x)$$

$$f_C = -9.42 + 6(0) = -9.42\,KN$$

$$f_B = -9.42 + 6(4) = 14.58\,KN$$

Zone AB. $(x = 4 \text{ to } 8)$

$$f_x = -R_C + 6(x) - R_B. = -9.42 + 24 - 27.44$$

$$f_B = -12.86\,KN$$

$$f_A = -R_C + 6 \times 8 - 27.44 = 11.14\,KN$$

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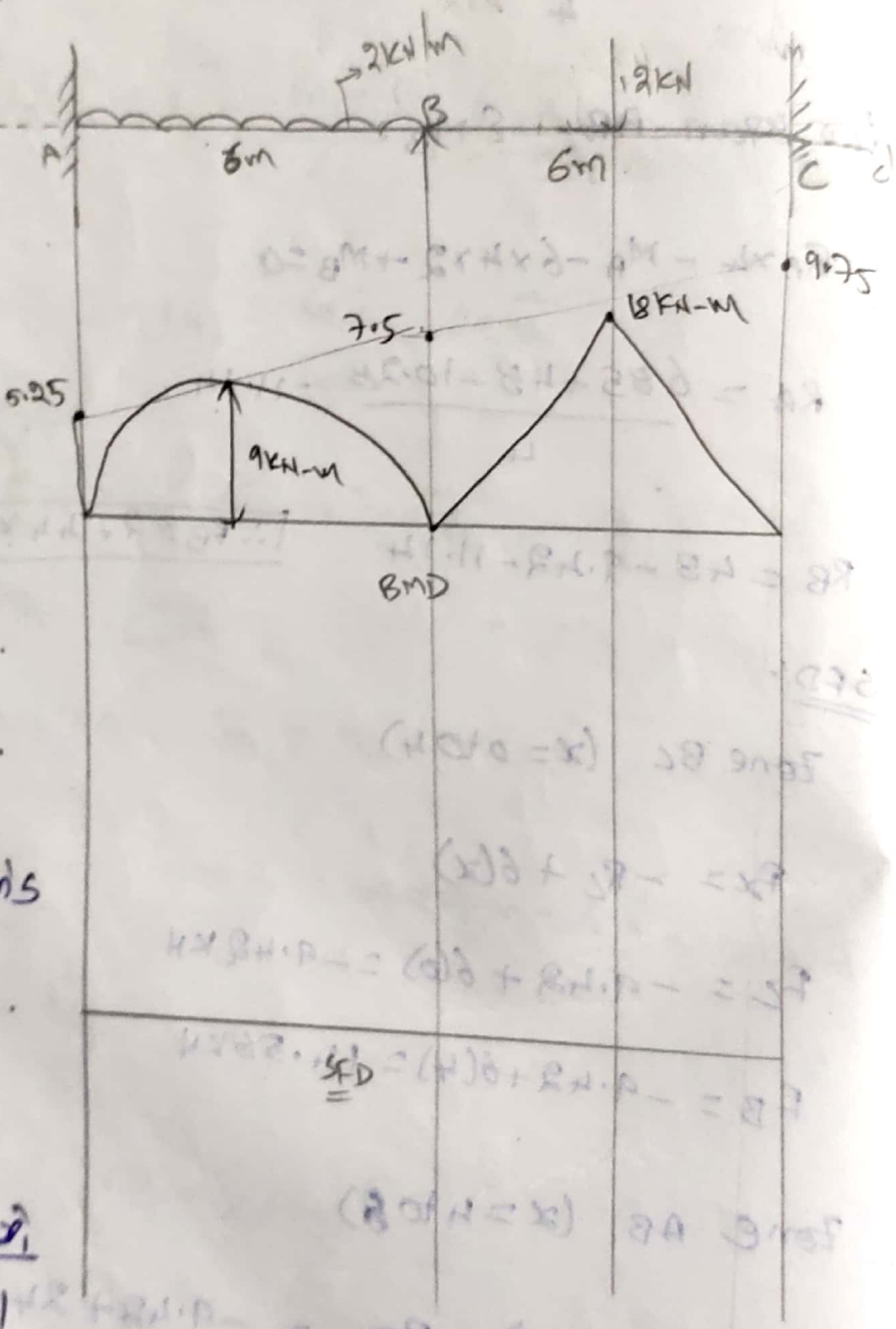
4) A continuous beam ABC having AB length 6m, BC length 6m. A UDL of 2 KN/m over the span AB and a point load of 12 KN. at middle of BC. both ends are fixed ends. Calculate support reactions, moments and draw SFD & BMD.



let us assume a fictious span of Aa' & Cc'.

length of Aa' = l_0 &

moment at a' = 0.

length of Cc' = l_0'.

moment at c' = 0.

Apply the clapeyrons theorem for AAB.

$M_0 l_0 + 2M_A(l_0 + l_1) + M_B(l_1) = \frac{6a_0\bar{x}_0}{l_0} + \frac{6a_1\bar{x}_1}{l_1}$

$2M_A(0 + l_1) + M_B(l_1) = \frac{6a_1\bar{x}_1}{l_1}$

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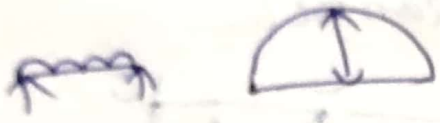
$2\times 6M_A + 6M_B = \frac{6a_1\bar{x}_1}{6}$

$12M_A + 6M_B = a_1\bar{x}_1 \rightarrow (1)$

a_1 = area of BMD due to load 2 KN/m on AB.



$\text{Max B.M} = \frac{wl^2}{8} = \frac{6\times 6^2}{8} = 9\ \text{KN-m}$

$a_1 = \frac{2}{3}\times 6\times 9 = 36 \quad ; \quad \bar{x}_1 = \frac{6}{2} = 3\text{m}.$

$12M_A + 6M_B = 36\times 3$

$12M_A + 6M_B = 108 \rightarrow (2).$

b) Apply clapeyron's theorem for span ABC.

$M_A l_1 + 2M_B(l_1+l_2) + M_C l_2 = \frac{6a_1\bar{x}_1}{l_1} + \frac{6a_2\bar{x}_2}{l_2}$

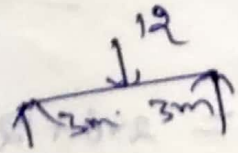
$M_A(6) + 2M_B(6+6) + (6)M_C = \frac{6a_1\bar{x}_1}{6} + \frac{6a_2\bar{x}_2}{6}$

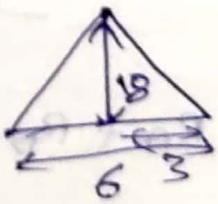
$6M_A + 24M_B + 6M_C = a_1\bar{x}_1 + a_2\bar{x}_2 \rightarrow (3).$

$a_1 = \frac{2}{3}\times 6\times 9 = 36.$



$\bar{x}_1 = 3\text{m}.$





$a_2 = \frac{1}{2}\times 18\times 6$

$a_2 = 54.$

$\bar{x}_2 = \frac{6}{2} = 3\text{m}.$

$\text{B.M} = \frac{wl}{4} = \frac{12\times 6}{4} = 18\ \text{KN-m}.$

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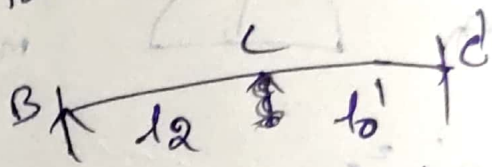
$6M_A + 24M_B + 6M_C = 36 \times 3 + 54$

$6M_A + 24M_B + 6M_C = 270 \rightarrow (4)$

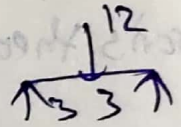
c) Apply clapeyron's theorem for span BCC'



$M_B(l_2) + 2M_C(l_2 + l_0') + M_C'(l_0') = \frac{6a_2\bar{x}_2}{l_2} + \frac{6a_0'\bar{x}_0}{l_0'}$

$M_B(6) + 2M_C(6+0) + 0 = \frac{6a_2\bar{x}_2}{6} + 0$

$6M_B + 12M_C = a_2\bar{x}_2 \rightarrow (5)$



$\frac{wl}{4} = \frac{12 \times 6}{4} = 18 kN$

B.M = ↑18

$a_2 = \frac{1}{2} \times 18 \times 6 = 54$

$\bar{x}_2 = 6/2 = 3m.$

$6M_B + 12M_C = 162 \rightarrow (6)$

Solve, (2), (4) & (6)

$M_A = 5.25 kN.m$, $M_B = 7.5 kN.m$, $M_C = 9.75 kN.m$.

$R_A + R_B + R_C = 2 \times 6 + 12 = 24$

for span BC, ΣM_B.

$R_C \times 6 - 2 \times 3 + M_B = 0$ $\quad R_C = \frac{36 - 7.5}{6} = 4.75 kN.$

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for span AB, ΣM_B.

$$R_A \times 6 - 2 \times 6 \times \frac{6}{2} - M_A + R_B = 0$$

$$6R_A - 36 - 5.25 + 7.5 = 0, \quad \boxed{R_A = 5.625\ KN}$$

$$\therefore R_B = 24 - 5.625 - 4.75, \quad \boxed{\therefore R_B = 13.625\ KN}$$

SFD:

Zone BC. ($x = 0$ to 6)

$$F_x = -R_C + 12. = -4.75 + 12 = 7.25 KN$$

$F_B = 7.25 KN$, $F_C = 7.25 KN$.

Zone AB ($x = 6$ to 12)

$$F_x = -R_C + 12 + 2x - R_B. = -4.75 + 12 + 2x - 13.63.$$

$$F_B = -6.38 + 2(6) = 5.62 KN.$$

$$F_A = -6.38 + 2(12) = 17.62 KN.$$

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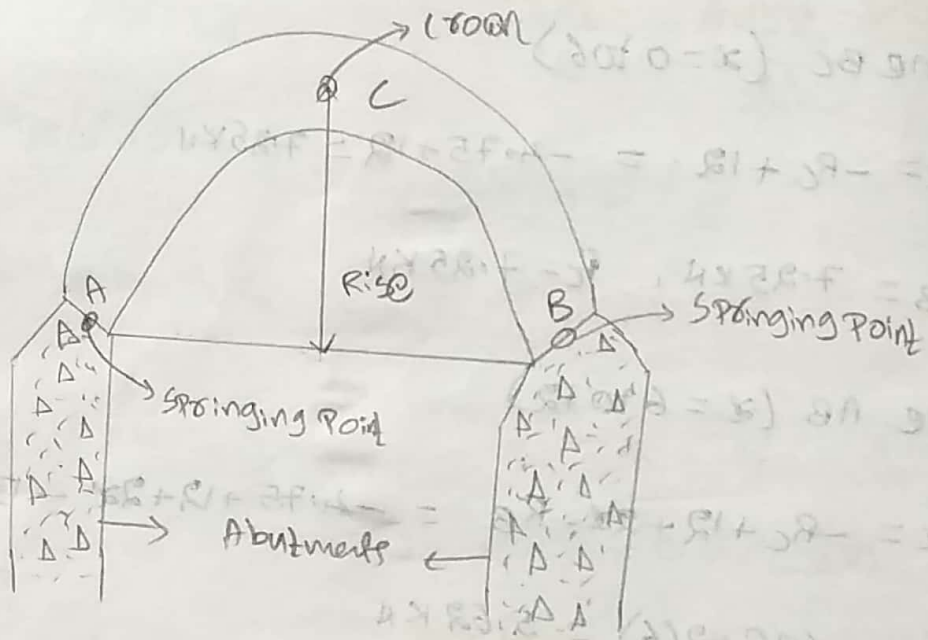
23/01/2020

Module-III

Girder → steel beam

ARCHES.

Arch:- An arch can be defined as curved beam subjected to transverse loads and other loads as well as the horizontal thrust at the supports.



* Arches transfers the loads to the abutments at springing points.

* Hinges may be provided at springing points.

* Because of the curved nature they subjected to horizontal force.

* At any section in the arch will be subjected to normal thrust, Radial thrust and Bending moment. but B.M considerably less.

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Types of Arches:-

There are 3 types of Arches. They are:

1) Three-Hinged arch.

2) Two-Hinged arch

3) Hingless arch or Fixed end arch.

1) Three-Hinged arch:-

2-hinges are provided at springing point and 1-hinge provided at crown.



2) Two-Hinged arch:-



two hinges are provided at springing points.

3) Hingless arch or Fixed end arch:-



Three-Hinged arch:-

There are 2 types of Three-hinged arches.

1) Parabolic 3-hinged arch.

2) Circular 3-hinged arch.

1) Parabolic 3-hinged arch:-

let us consider a section at "D".

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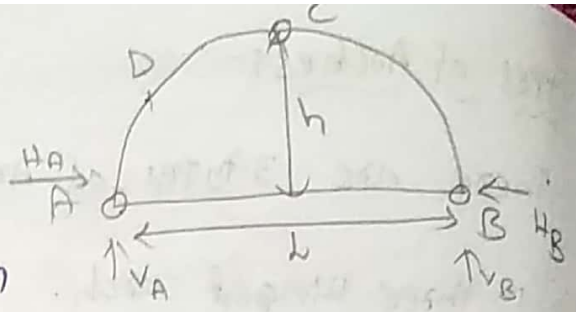
Consider the equilibrium of Part AD. The part AD is in equilibrium under the action of following forces.

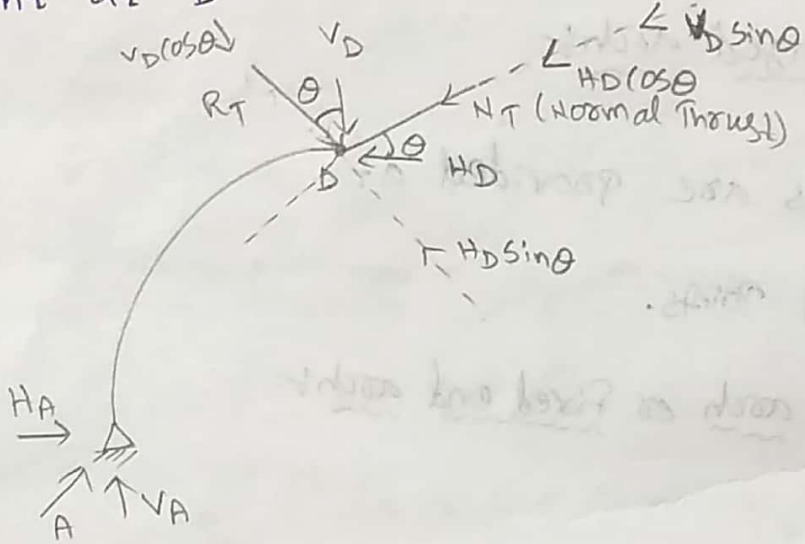


1) Reaction of H_A and V_A at A.

2) External loads between A and D.

3) Reacting forces H_D and V_D at D.

4) Moment at D.



Normal thrust $N_T = H_D\cos\theta + V_D\sin\theta$.

Radial thrust. $R_T = H_D\sin\theta - V_D\cos\theta$.

Equation of Parabola.

$$y = \frac{4hx}{L^2}(L-x).$$

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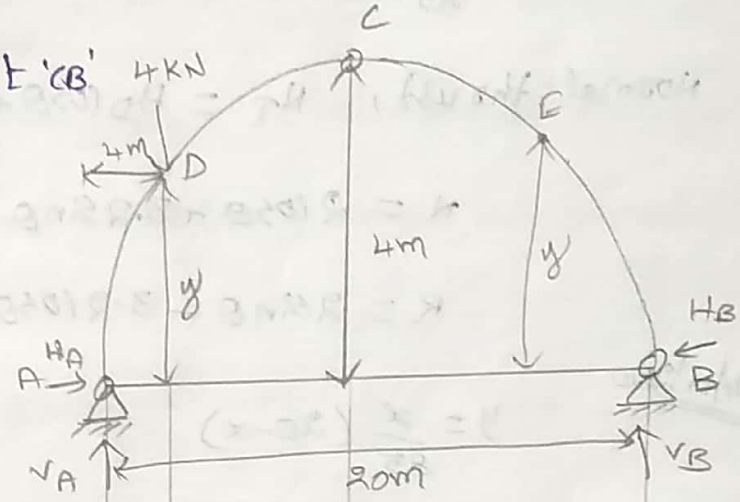
1) A 3-hinged parabolic arch of 20m span and 4m rise subjected to a point load of 4KN. at 4m from left support. Calculate i) reactions ii) Normal thrust, R_T & iii) calculate underload max +ve B.M & max -ve B.M.



Taking moment about 'B'

$V_A \times 20 - 4 \times 16 = 0$

$V_A = \frac{4 \times 16}{20}$

$V_A = 3.2KN.$

$V_A + V_B = 4$

$V_B = 4 - 3.2$; $V_B = 0.8KN.$

Taking moment about 'C' (Right Portion).

$V_B \times 10 - H_B \times 4 = 0$

$0.8 \times 10 = H_B \times 4$, $\therefore H_B = \frac{8}{4} = 2KN.$

$\therefore H_A = H_B = 2KN.$

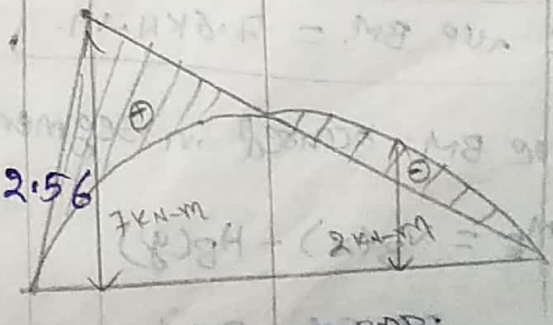
Eqn of Parabola, $y = \frac{4hx}{L^2}(L-x)$.

$y = \frac{4 \times 4 \times 4}{20^2}(20-4) = 2.56m$

BM under the load.

$M_D = V_A \times 4 - H_A \times 2.56$

$M_D = 7.68 KN-m.$



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$$y = \frac{4hx}{L^2}(L-x)$$

$$= \frac{4\times 4x}{20^2}(20-x)$$

$$y = \frac{x}{25}(20-x).$$

Normal thrust, $N_T = H_D\cos\theta + V_D\sin\theta$.

$$N = 2\cos\theta + 3.2\sin\theta.$$

$$R = 2\sin\theta - 3.2\cos\theta.$$

24/01/2020

$$y = \frac{x}{25}(20-x)$$

$$\frac{dy}{dx} = \tan\theta = \frac{20}{25} - \frac{2x}{25}$$

at $x = 4m$, $\frac{dy}{dx} = \tan\theta = \frac{20}{25} - \frac{8}{25} = 0.48$

$$\therefore \theta = \tan^{-1}(0.48) = 25.64.$$

$N = 2\cos(25.64) + (4-3.2)\sin(25.64) = 2.14$ KN

$R = 2\sin(25.64) - (4-3.2)\cos(25.64) = 0.144$ KN.

Max +ve BM.

Max +ve BM = 7.6 KN-m.

Max -ve BM occured in segment of CB.

$$M_E = V_B(x) - H_B(y)$$

$$= 0.8x - 2y$$

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$M_E = 0.8x - 2\left[\frac{x}{25}(20-x)\right] = 0.8x - \frac{40x}{25} + \frac{2x^2}{25}$.

$M_E = -0.8x + 0.08x^2 \longrightarrow (1)$

$\frac{dM_E}{dx} = 0 \quad \therefore -0.8 + 2\times0.08x = 0$

$x = \frac{0.8}{2\times0.08} = 5m$.

sub'tg $x=5m$ in $M_E = -0.8x + 0.08x^2$

$M_E = -0.8\times5 + 0.08(25)$

$\therefore M_E = -2KN\text{-}m$.

$\therefore$ Max -ve BM $= -2KN\text{-}m$.

2) A Parabolic arch Hinged at springing points and crown. has a span of 20m. The central rise of arch is 4m. It is loaded a UDL of 2KN/m. on left 8m length. i) Calculate the direction and magnitude of reactions at springing points. ii) B.M, N_T, R_T at 4m & 15m from the left. iii) Max +ve B.M and max -ve B.M.

$V_A + V_B = 2\times8 = 16$

Taking moment about 'B'.

$V_A\times20 - 2\times8\times16 = 0$

$V_A = \frac{2\times8\times16}{20} = 12.8KN$.

$V_B = 16 - 12.8 = 3.2KN$.

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Taking moment about C (Right Portion).

$V_B \times 10 - H_B \times 4 = 0$

$\frac{3.2 \times 10}{4} = H_B$.

$H_B = 8KN$.

$\therefore H_A = H_B = 8KN$.

Reaction at A.

$R_A = \sqrt{V_A^2 + H_A^2}$

$R_A = \sqrt{12.8^2 + 8^2}$

$R_A = 15.09KN$.

$\tan\theta_A = \frac{V_A}{H_A} = \frac{12.8}{8}$

$\theta_A = 57.9°$

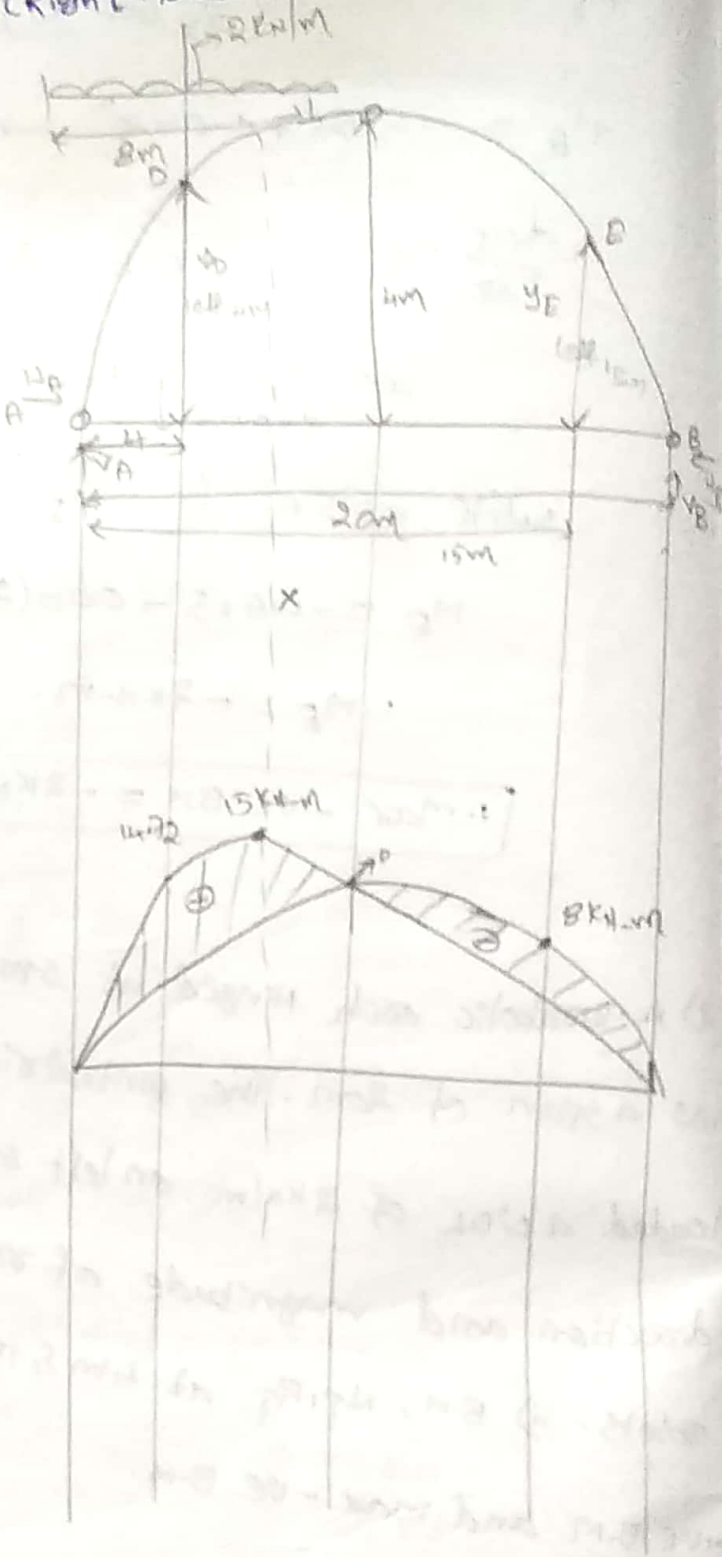
Reaction at B

$R_B = \sqrt{V_B^2 + H_B^2}$

$R_B = 8.61KN$.

$\tan\theta_B = \frac{V_B}{H_B} = \frac{3.2}{8}$

$\therefore \theta_B = 21.80°$



let us assume 'D' at 4m from left support.

$y = \frac{4hx}{L^2}(L-x)$

$y = \frac{4 \times 4 \times 4}{20^2}(20-4) = 2.56m$.

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Moment at "D".

$M_D = V_A \times 4 - H_A \times 2.56 - 2 \times 4 \times 2$

$M_D = 14.72 \text{ KN-m}$ ①

$y = \frac{4hx}{l^2}(l-x) = \frac{4 \times 4 \times x}{20^2}(20-x)$,

$y = \frac{x}{25}(20-x)$.

$\frac{dy}{dx} = \tan\theta = \frac{20}{25} - \frac{2x}{25}$

at $x = 4$, $\tan\theta = \frac{20}{25} - \frac{2 \times 4}{25} = 0.48$

$\theta = \tan^{-1}(0.48) = 25.64$.

$V_D = 12.8 - 8 = 4.8$ ② $V_A - 2 \times 4$

Normal thrust at 4m

$N = H_D \cos\theta + V_D \sin\theta$

$N = 8\cos(25.64) + 4.8\sin(25.64) = 9.28 \text{ KN}$.

Radial thrust at 4m.

$R = H_D \sin\theta - 4.8\cos\theta$.

$R = 8\sin(25.64) - 4.8\cos(25.64) = -0.8 \text{ KN}$.

let us assume 'E' at 15m from left support.

$y = \frac{4hx}{l^2}(l-x)$. $y = \frac{4 \times 4 \times 15}{20^2}(20-15) = 3m$.

Moment at 'E'.

$M_E = V_B \times 5 - H_B \times 3$ (y). $M_E = 3.2 \times 5 - 8 \times 3$

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$M_E = -11.2\ KN-m - 8\ KN-m$ ②

$y = \frac{4hx}{L^2}(L-x) = \frac{4\times 8\times x}{20^2}(20-x)$

$y = \frac{16x}{20^2}(20-x) = \frac{16x}{20^2}(20-x)$

$\frac{dy}{dx} = \tan\theta = \frac{16\times 20}{20^2} - \frac{16\times 2x}{20^2} = \frac{6}{5} + \frac{6x}{20}$

at $x = 15m$, $\tan\theta = \frac{6\times 15}{20} = 1.5\ \frac{16\times 20}{20^2} - \frac{16\times 2\times 3}{20^2} = 0.4$

$\theta = +21.80$

Normal thrust at 15m.

$N = H_E\cos\theta + V_E\sin\theta$.

$N = 8\cos(-56.30^{21.80}) + 3.2\sin(-56.30^{21.80})$

$N = 8.6\ KN$.

Radial thrust at 15m.

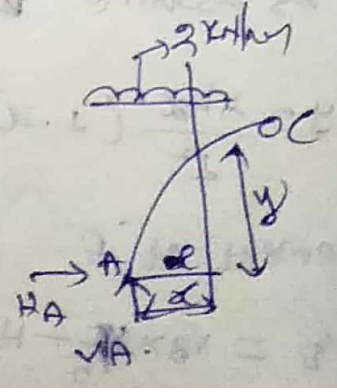
$R = H_E\sin\theta - V_E\cos\theta$

$R = 8\sin(-56.30^{21.80}) - 3.2\cos(-56.30^{21.80})$

$\therefore R = 0$

Take a section in segment AC.

$M_x = V_A \times x - H_A(y) - 2\frac{x^2}{2}$.



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$$M_x = 12.8x - 8y - x^2$$

$$M_x = 12.8x - 8\,\frac{4hx}{L^2}(L-x) - x^2$$

$$= 12.8x - \frac{8\times4\times4\times x}{20^2}(20-x) - x^2$$

$$= 12.8x - \frac{8\times4\times4\times20x}{20^2} + \frac{8\times4\times4\times x^2}{20^2} - x^2$$

$$= 12.8x - 6.4x + 0.32x^2 - x^2$$

$$M_x = 6.4x - 0.68x^2$$

$$\frac{dM_x}{dx} = 0 \Rightarrow 6.4 - 0.68(2x) = 0$$

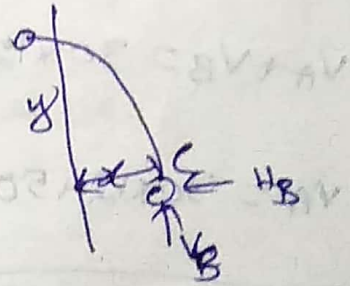
$$x = \frac{6.4}{2\times0.68} = 4.70\text{m}$$

$$\therefore x = 4.70\text{m}$$

$$\text{Max +ve BM} = 6.4\times4.70 - 0.68\times(4.70)^2$$

$$M_x = 15.05\,\text{KN-m} \quad (1)$$

Negative BM in segment BC.



$$M_x = V_B(x) - H_B(y)$$

$$M_x = 3.2x - 8(y)$$

$$M_x = 3.2x - 8\left(\frac{4hx}{L^2}(L-x)\right)$$

$$M_x = 3.2x - \frac{8\times4\times4x}{20^2}(20-x)$$

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$M_x = -3.2x + 0.32x^2$

$\frac{dM_x}{dx} = 0 \Rightarrow -3.2 + 0.32 \times 2x = 0 \quad x = \frac{3.2}{2 \times 0.32}$ $\boxed{x = 5m}$

Max -ve BM $= -3.2(5) + 0.32(5)^2 = -8 KN\text{-}m$

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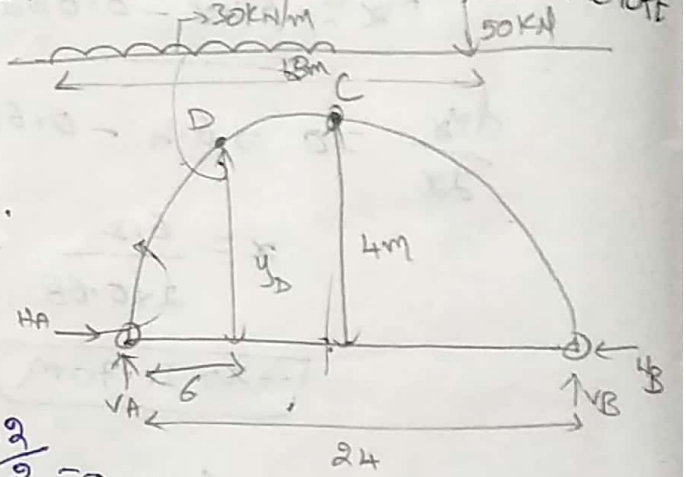
3) A 3 hinged arch of span 24m and central rise 4m carries a load of 50KN at 18m from left support and a UDL of 30KN/m over left half portion. Calculate support reactions ii) Moment, Normal thrust, radial shear a 6m from the left support.



Take a moment about A.

$\Sigma M_A = 0.$

$V_B \times 24 - 50 \times 18 - 30 \times 12 \times \frac{12}{2} = 0$

$24 V_B = 30 \times 12 \times 6 + 50 \times 18$

$\therefore V_B = 127.5 KN$

from left.

$[H_B \times 4 - V_A \times 12 + 30 \times 12 \times 6 = 0]$

$V_A + V_B = 30 \times 12 + 50$

$V_A = 30 \times 12 + 50 - 127.5$

$\therefore V_A = 282.5 KN$

Take a moment about C (from right side)

$V_B \times 12 - H_B \times 4 - 50 \times 6 = 0$

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$\therefore H_B = 307.5\ KN$

$\boxed{\therefore H_A = H_B = 307.5\ KN}$

let us assume "D" at 6m from left support.

$$y = \frac{4hx}{L^2}(L-x)$$

$$= \frac{4\times4\times6}{24^2}(24-6), \quad \boxed{\therefore y = 3m}$$

$$M_D = V_A(6) - H_A(3) - 30\times6\times3$$

$\boxed{\therefore M_D = 232.5\ KN\text{-}m}$

$$y = \frac{4hx}{L^2}(L-x) = \frac{4\times4x}{24^2}(24-x)$$

$$\frac{dy}{dx} = \tan\theta = \frac{16}{24^2}\times24 - \frac{16}{24^2}\times2x$$

$$\text{at } x=6, \ \tan\theta = \frac{16\times24}{24^2} - \frac{16\times2\times6}{24^2} = 0.33$$

$\boxed{\therefore \theta = 18.26}$

$$V_D = V_A - 30\times6 = 282.5 - 180 = 102.5\ KN.$$

$$N_T = H_D\cos\theta + V_D\sin\theta$$

$$= 307.5\cos(18.26) + 102.5\sin(18.26)$$

$\boxed{\therefore N_T = 324.1\ KN.}$

$$R_{Thor} = H_D\sin\theta - V_D\cos\theta$$

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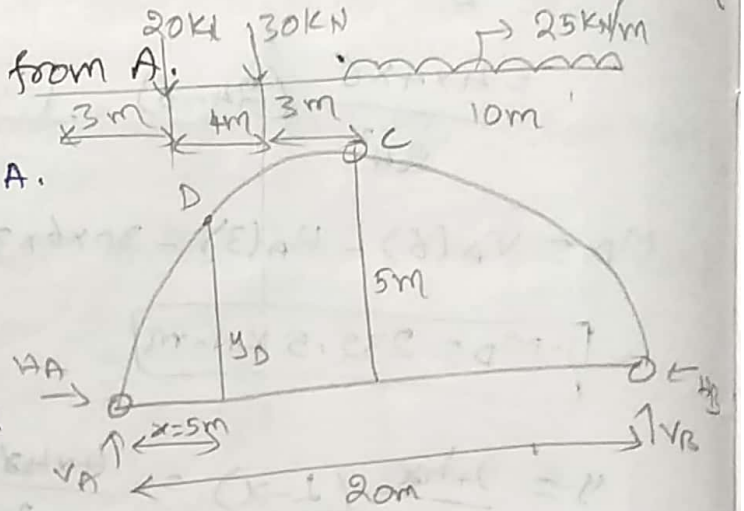
$= 307.5 \cos(18.26) - 102.5 \sin(18.26)$

$\therefore R_5 = 0.$

4) 3 hinged Parabolic arch subject to loads as shown in fig. Determine i) support reactions, ii) B.M, Normal thrust, radial ~~thrust~~ shear at 5m from A.



Take a moment about A.

$\Sigma M_A = 0.$

$V_B \times 20 - 25 \times 10 \times \frac{10}{2} - 30 \times 7 - 20 \times 3 = 0$

$V_B = 76\ KN$ $\quad \therefore V_A + V_B = 25 \times 10 + 30 * 20 = 300$

$\therefore V_A = 224\ KN.$

Take a moment about C from right side.

$V_B \times 10 - H_B \times 5 - 25 \times 10 \times \frac{10}{2} = 0$

$760 - 5H_B - 1250 = 0 \quad \therefore H_B = +98\ KN$

$\therefore H_A = H_B = 98\ KN$

let us assume "D" at 5m from A.

$$y = \frac{4hx}{L^2}(L-x)$$

$$= \frac{4 \times 5 \times 5}{20^2}(20-5)$$

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$\therefore y_D = 3.75\,m$

$M_D = V_A \times 5 - H_A \times y_D - 20 \times 3$

$M_D = 224 \times 5 - 98 \times 3.75 - 60$

$\therefore M_D = 692.5\ KNm$

$y = \frac{4hx}{l^2}(l-x) = \frac{4 \times 5 \times x}{20^2}(20-x)$

$y = \frac{20x}{20^2} - \frac{20x^2}{20^2} = x - \frac{x^2}{20}$

$\frac{dy}{dx} = \tan\theta = 1 - \frac{2x}{20}$

at $x = 5m$, $\tan\theta = 1 - \frac{2 \times 5}{20} = 1 - \frac{1}{2} = \frac{1}{2} = 0.5$

$\theta = \tan^{-1}(0.5)$

$\therefore \theta = 26.56$

$V_D = V_A - 20 * 3 = 224 - 60 = 204\ KN.$

$\therefore V_D = 204\ KN$

$N_T = H_D \cos\theta + V_D \sin\theta$

$N_T = 98\cos(26.56) + 204\sin(26.56)$

$\therefore N_T = 178.987\ KN$

$R_S = H_D \sin\theta - V_D \cos\theta = 98\sin(26.56) - 204\cos(26.56)$

$\therefore R_S = -138.87$ $\therefore R_S = -138.65\ KN$

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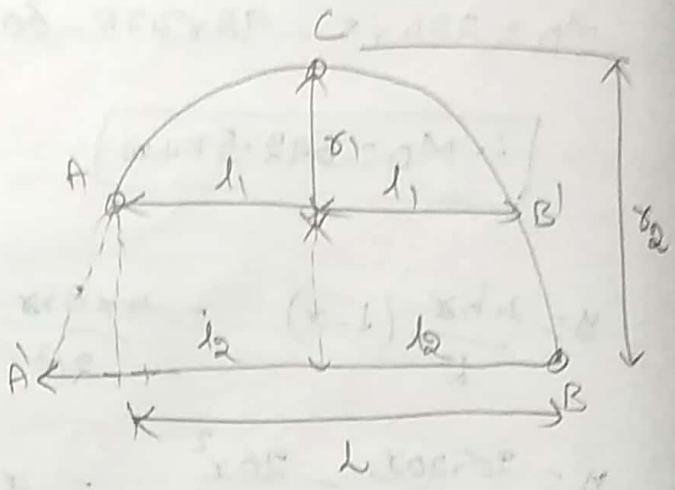
Settlement of supports at 3 hinged arch.

relation blw l_1, l_2 and r_1, r_2 is

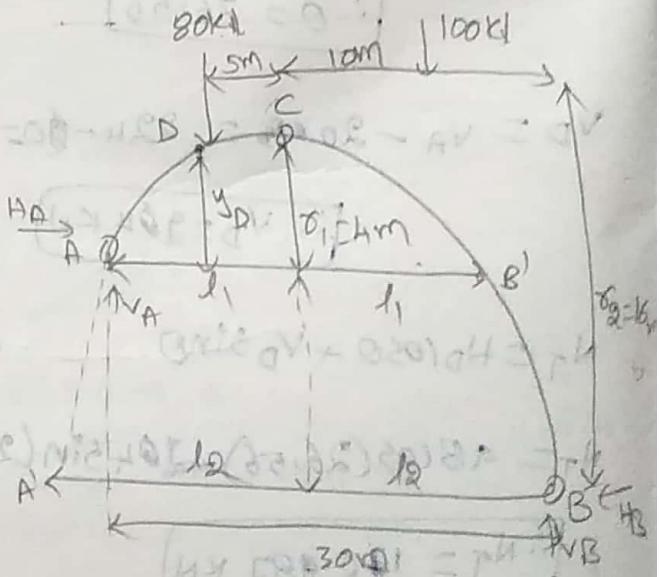
$$\frac{l_1}{l_2} = \sqrt{\frac{r_1}{r_2}}$$

(derive)



$L = l_1 + l_2$.

1) A 3hinged Parabolic arch of 30m span having supports at a depth of 4m and 16m below the crown. The arch covers a point load of 80 KN. at adistance of 5m from left of C. and 100KN at 10m to the right of C. Calculate support reactions, Normal thrust and radial shear under the load.



$L = l_1 + l_2$

$$\frac{l_1}{l_2} = \sqrt{\frac{r_1}{r_2}}$$

$$\frac{l_1}{L - l_1} = \sqrt{\frac{4}{16}} = \frac{1}{2}$$

$2l_1 = L - l_1$, $3l_1 = 30$

$\therefore l_1 = 10m$, $l_2 = 20m$

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Take amoment about C (left)

$V_A \times l_1 - H_A \times 4 - B0 \times 5 = 0$,

$10V_A - 4H_A - 400 = 0$, $2.5V_A - H_A - 100 = 0 \rightarrow (1)$

Take a moment about 'C' (Right)

$V_B \times l_2 - H_B \times 16 - 100 \times 10 = 0$.

$20V_B - 16H_B - 1000 = 0 \rightarrow (2)$.

$H_A = H_B$, $V_A + V_B = 180$.

$V_B = 180 - V_A$. sub in (2).

$20(180 - V_A) - 16H_A - 1000 = 0$

$3600 - 20V_A - 16H_A - 1000 = 0$

$-20V_A - 16H_A + 2600 = 0 \rightarrow (3)$

Solve (1) & (3)

$\therefore V_A = +70\ KN$

$H_A = +75\ KN$

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at Point D.

$$y_D = \frac{4 f_1 x}{(2 \times l_1)^2}(2l - x)$$

at $x = 5m$.

$$y_D = \frac{4 \times 4 \times 5}{(2 \times 10)^2}(20 - 5)$$

$y_D = 3m$.

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$$y_D = \frac{4x4x}{20^2}(20-x)$$

$$y_D = \frac{16\times 20x}{20^2} - \frac{16\times 2x^2}{20^2}$$

$$\frac{dy_D}{dx} = \tan\theta = \frac{16\times 20}{20^2} - \frac{16\times 2x}{20^2}$$

at $x=5$, $\tan\theta = \frac{16\times 20}{20^2} - \frac{16\times 2\times 5}{20^2}$

$\therefore \theta = 21.80^\circ$

$M_D = V_A \times 5 - H_A \times y_D$

$= 70\times 5 - 75\times 3$.

$M_D = 125\,KN\text{-}m$.

Normal thrust at D,

$V_D = 80 - V_A = 80 - 70 = 10$

$N_T = H_D\cos\theta + V_D\sin\theta$.

$= 75\cos(21.80) + 10\sin(21.80)$

$N_T = 73.35\,KN$.

Radial shear at D,

$R_S = H_D\sin\theta - V_D\cos\theta$

$= 75\sin(21.80) - 10\cos(21.80)$

$= 18.56\,KN$.

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calculate moment at E:-

$$y_E = \frac{4 h_2 x}{(2 \times l_2)^2} (40 - x)$$

at $x = 10m$, $= \frac{4 \times 16 \times 10}{(40)^2} (40 - 10)$

$$\therefore y_E = 12m$$

$$y_E = \frac{4 \times 16 x}{40^2} (40 - x) = \frac{4 \times 16 \times 40 x}{40^2} - \frac{64 x^2}{40^2}$$

$$\frac{dy_E}{dx} = \tan\theta = \frac{4 \times 16 \times 40}{40^2} - \frac{64 \times 2x}{40^2}$$

$$\tan\theta = \frac{64 \times 1}{40} - \frac{64 \times 2 \times 10}{40^2} = 0.8$$

$$\therefore \theta = \tan^{-1}(0.8)$$

$$\therefore \theta = 38.65^\circ$$

$$M_E = V_B \times 16 - H_B \times 12$$

$$= 110 \times 16 - 75 \times 12$$

$$\therefore M_E = 860 \text{ KN-m}$$

Normal thrust at E,

$$V_E = 110 - 100 = 10.$$

$$N_T = H_E \cos\theta + V_E \sin\theta.$$

$$= 75 \cos(38.65) + 10 \sin(38.65).$$

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$\therefore N_T = 64.84$ KN

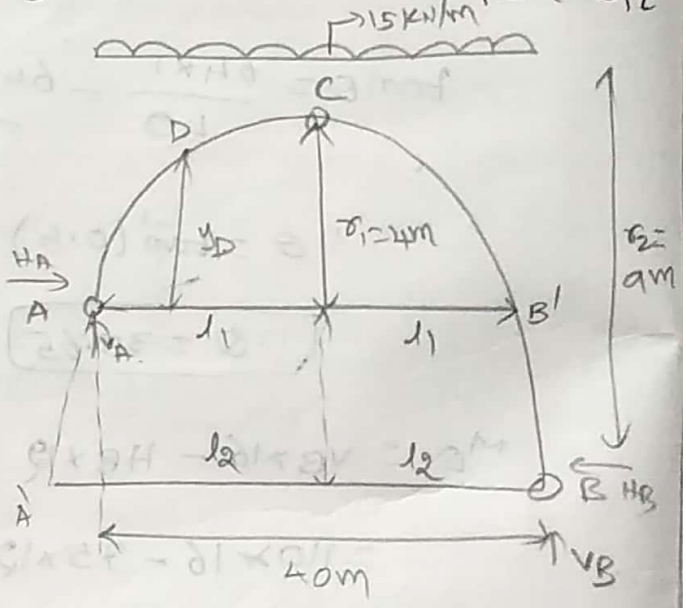
radial shear at E,

$R_S = H_E \sin\theta - V_E \cos\theta$

$= 75\sin(38.65) - 100\cos(38.65)$

$\therefore R_S = 38.99$ KN

2) A 3hinged Parabolic arch of 40m span has abutments of unequal level heights Point of the arch is 4m above left support and 9m above the right support. The arch is subjected to an UDL of 15KN/m. over entire span. Calculate i) support reactions, ii) N_T, R_S and moment at 8m from left support.



$L = l_1 + l_2$.

$\frac{l_1}{l_2} = \sqrt{\frac{y_1}{y_2}}$

$\frac{l_1}{l_2} = \sqrt{\frac{4}{9}} = \frac{2}{3}$

$3l_1 = 2l_2 = 2(L - l_1)$

$3l_1 = 2L - 2l_1$, $5l_1 = 2L = 2\times40 = 80$

$\therefore l_1 = 16$m , $l_2 = 24$m

Take a moment about C (left)

$V_A \times l_1 - H_A \times 4 - 15 \times 16 \times \frac{16}{2} = 0$

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$$16V_A - 4H_A = 1920 \longrightarrow (1)$$

Take a moment about C (Right side)

$$V_B \times 12 - H_B \times 9 - 15 \times 24 \times \frac{24}{2} = 0$$

$$24V_B - 9H_B = 4320 \rightarrow (2)$$

$H_A = H_B$, $V_A + V_B = 15 \times 40 = 600$, subtitle in (2).

$$24(600 - V_A) - 9H_A = 4320$$

$$-24V_A - 9H_A = -10080 \rightarrow (3)$$

solve (1) & (3).

$\therefore V_A = 240\ KN$, $H_A = 480\ KN$, $V_B = 360\ KN$

at Point D.

$$y_D = \frac{48,x}{(2\times12)^2}(32-x)$$

at $x = 8$.

$$y_D = \frac{4\times4\times8}{32^2}(32-8)$$

$\therefore y_D = 3m$.

$$M_D = V_A \times 8 - H_A \times 3 - 15 \times 8 \times 4$$

$$= 240 \times 8 - 480 \times 3 - 60 \times 8$$

$\therefore M_D = 0$

$$y_D = \frac{4\times4x}{32^2}(32-x)$$

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$$\frac{dy_D}{dx} = \tan\theta = \frac{16\times32}{32^2} - \frac{16\times2x}{32^2}$$

at $x=8m$, $\tan\theta = \frac{16}{32} - \frac{16\times2\times8}{32^2} = 0.25$

$$\therefore \theta = 14.04$$

Normal thrust, N_T at D.

$$V_D = V_A - 15\times8 = 240 - 15\times8$$

$$V_D = 120\,KN$$

$$N_T = H_D\cos\theta + V_D\sin\theta$$

$$N_T = 480\cos(14.04) + 120\sin(14.04)$$

$$\therefore N_T = 494.7\,KN$$

$$R_S = H_D\sin\theta - V_D\cos\theta$$

$$= 480\sin(14.04) - 120\cos(14.04)$$

$$\therefore R_S = 0.03 = 0.$$

3) A 3-hinged parabolic arch of span 25m and rise 4m carries a UDL of 25 KN/m on the whole span and a point load of 100 KN. at a distance of 15m from right end. Determine a) support reactions, b) N_T, R_S at quarter span.

$$V_A + V_B = 100 + 25\times25. = 725.$$

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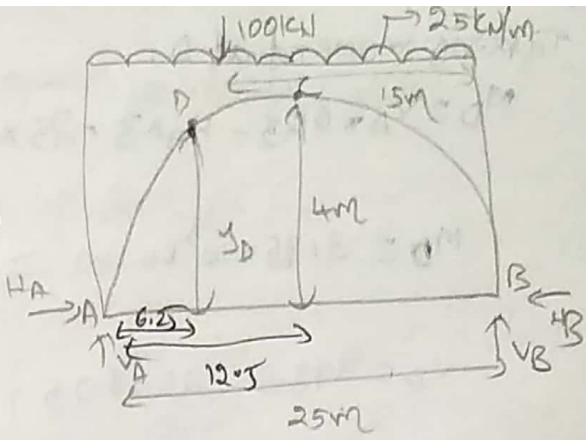

$\Sigma M_A = 0$.

$V_B \times 25 - 100 \times 10 - 25 \times 25 \times \frac{25}{2} = 0$

$V_B = 352.5$ KN

$V_A = 372.5$ KN

$\Sigma M_C = 0$ (from right)

$V_B \times 12.5 - H_B \times 4 - 25 \times 12.5 \times \frac{12.5}{2} = 0$

$352.5 \times 12.5 - 4H_B - 1953.12 = 0$

$\therefore H_B = 613.28$

$\therefore H_A = H_B = 613.28$ KN.

$y_D = \frac{4hx}{l^2}(l-x)$

$= \frac{4 \times 4 \times 6.25}{25^2}(25 - 6.25)$

$\therefore y_D = 3$m.

$y_D = \frac{4 \times 4 \times x}{25^2}(25-x)$

$y_D = \frac{16x}{25} - \frac{16x^2}{25}$

$\frac{dy_D}{dx} = \tan\theta = \frac{16}{25} - \frac{16 \times 2x}{25}$

at $x = 6.25$.

$\tan\theta = 0.32$.

$\therefore \theta = 17.7$

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Taking moment at D,

$$M_D = V_A \times 6.25 - H_A \times 3 - 25 \times 6.25 \times \frac{6.25}{2}$$

$$M_D = 3.75 \times 10^{-3}\ KN\text{-}m \simeq 0$$

$$V_D = 372.5 - 25 \times 6.25 = 216.25\ KN$$

$$N_T = H_D \cos\theta + V_D \sin\theta$$

$$N_T = 613.28 \cos(17.7) + 216.25 \sin(17.7)$$

$$\therefore N_T = 650\ KN$$

$$R_S = H_D \sin\theta - V_D \cos\theta$$

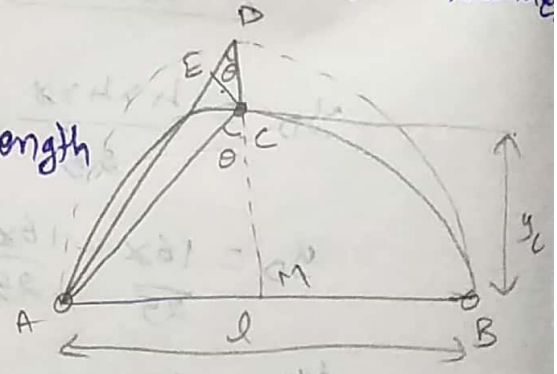
$$= 613.28 \sin(17.7) - 216.25 \cos(17.7)$$

$$\therefore R_S = 19.10\ KN$$

31/01/2020

Temperature effect on 3-hinged arches :-

let consider a 3-hinged arch ABC of span "l" and central rise y_c and are subjected to an increase of temperature "t".



As we increase the temp the length of arch increased. The end hinges A, B doesn't go any displacement as they are fixed in abutments, but the crown "c" changes from C to D.

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Assume arc length of AC, AD = chord length of AC, AD.

change in chord = ED = ACαt → (1)

from ΔEDC



$\cos\theta = \frac{ED}{DC}$

$\sec\theta = \frac{DC}{ED}$

$DC = ED\sec\theta$ → (2) $DC = AC\alpha t\sec\theta$ → (3)

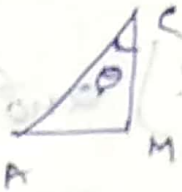
α → The strain per unit temp.

$\alpha = \frac{e}{t} = \frac{\delta l/l}{t}$

$\alpha = \frac{\delta l}{lt}$

$\delta l = \alpha l t$

from ΔACM,



∠ACM = ∠EDC

$\cos\theta = \frac{CM}{AC}$, $\sec\theta = \frac{AC}{CM}$ subtitute in (3) eqn.

there is deflation in increasing temp.

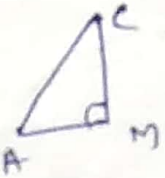
$\frac{\partial\theta}{\partial t} = l\alpha t$

$H = H_0 + H_1$

$\therefore DC = AC\alpha t\frac{AC}{CM}$

$\therefore DC = \frac{AC^2\alpha t}{CM}$ → (4)

from ΔACM.



$AC^2 = AM^2 + CM^2$

$AC^2 = AM^2 + (CM - DC)^2$

y_c^2

$AC^2 = \left(\frac{l}{2}\right)^2 + y_c^2 = \frac{l^2}{4} + y_c^2$ → subtitute in (4).

$\alpha = 11.2\times10^{-6}/°C$

coefficient of thermal expansion

$\alpha_{steel} = \alpha_{concrete}$

$\alpha_{Al} = 17\times10^{-6}$

If there is no temp.

$\frac{\partial\theta}{\partial t} = 0$

$DC = \frac{\left(\frac{l^2}{4} + y_c^2\right)\alpha t}{y_c} = \alpha t\left(\frac{l^2 + 4y_c^2}{4y_c}\right)$

$\therefore DC = \left(\frac{l^2 + 4y_c^2}{4y_c}\right)\alpha t$

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1) A 3 hinged parabolic arch of span 30m of central rise 5m find the rise of arch if the temperature rise 30°C. and the coefficient of thermal expansion $12\times10^{-6}/°C$.

G.T: $l=30m$, $y_c=5m$, $\alpha=12\times10^{-6}/°C$, $t=30°C$.

$$D_c=\left(\frac{l^2+4y_c^2}{4y_c}\right)\alpha t$$

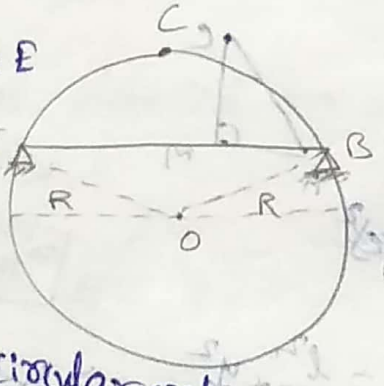
$$D_c=\left(\frac{30^2+4\times23}{4\times5}\right)12\times10^{-6}\times30$$

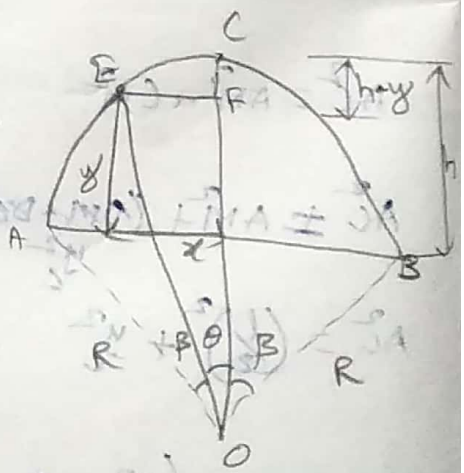
$$\therefore D_c=0.018\,m$$

$$\therefore D_c=18mm$$

Circular Arch:-

Let assume a point E on arch at a distance of "x" from D.





Consider ACB is circular arch.

ΔOEF,

$$OE^2=EF^2+OF^2$$

$$R^2=x^2+(OC-CF)^2$$

$$R^2=x^2+(R-(h-y))^2$$

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$$R^2 = x^2 + (R-h+y)^2 \rightarrow (1)$$

from ΔAOD,

$$OA^2 = AD^2 + OD^2$$

$$R^2 = \left(\frac{l}{2}\right)^2 + (R-h)^2 = \frac{l^2}{4} + R^2 + h^2 - 2Rh$$

$$2Rh = \frac{l^2}{4} + h^2$$

$$R = \frac{l^2}{8h} + \frac{h^2}{2h} = \frac{l^2}{8h} + \frac{h}{2}$$

$$\therefore R = \frac{l^2}{8h} + \frac{h}{2}$$

(1) $\Rightarrow$

$$R^2 - x^2 = (R-h+y)^2$$

$$\sqrt{R^2-x^2} = R-h+y$$

$$y = (\sqrt{R^2-x^2}) - R + h$$

$$\left[\because d(\sqrt{x^n}) = \frac{nx^{n-1}}{2\sqrt{x}}\right]$$

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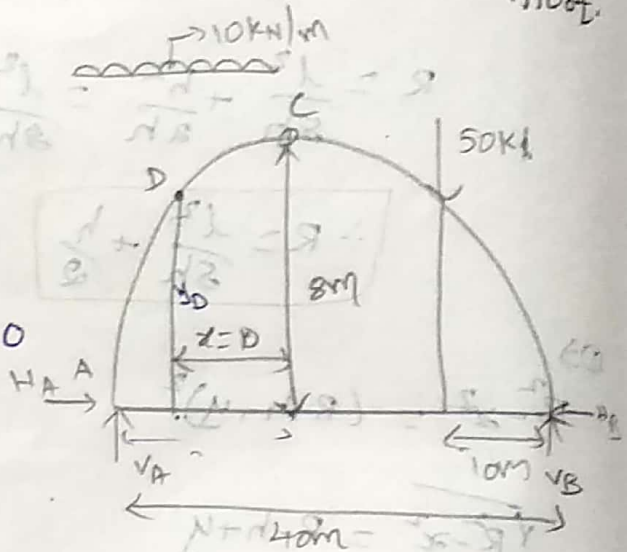
03/02/2020.

1) A 3-hinged circular arch of span 40m and rise 8m carries a UDL of 10KN/m over left Path of the arch and 50KN at a distance of 10m from the right calculate: i) support reactions ii) Moment, iii) N_T, R_S at 10m from left support.



Take a moment about A:

$\Sigma M_A = 0$.

$V_B \times 40 - 50 \times 30 - 10 \times 20 \times \frac{20}{2} = 0$

$40V_B = 3500$

$\boxed{V_B = 87.5}$

$V_A + V_B = 50 + 10 \times 20 = 50 + 200$

$V_A + V_B = 250$

$\boxed{\therefore V_A = 162.5}$

Taking moment about C (Right)

$\Sigma M_C = 0$

$V_B \times 20 - H_B \times 8 - 50 \times 10 = 0$

$8H_B = 20 \times 87.5 - 500$

$\boxed{\therefore H_B = 156.25}$ KN. $= H_A = 156.25$ KN.

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$$y_D = \sqrt{R^2 - x^2} - R + h \rightarrow (1)$$

$$y_D = \sqrt{R^2 - x^2} - (R - h) \rightarrow (2)$$

$$R = \frac{l^2}{8h} + \frac{h}{2} = \frac{40^2}{8 \times 8} + \frac{8}{2}$$

$$\therefore R = 29\text{m}$$

$$y_D = \sqrt{29^2 - 10^2} - 29 + 8$$

$$\therefore y_D = 6.22\text{m}$$

$$M_D = V_A \times 10 - H_A \times 6.22 - 10 \times 10 \times 5$$

$$M_D = 162.5 \times 10 - 156.28 \times 6.22 - 10 \times 50$$

$$\therefore M_D = 153.125\ \text{kN-m}$$

$$V_D = V_A - 10 \times 10$$

$$V_D = 62.5\ \text{kN}$$

$$\tan\theta = \frac{x}{\sqrt{R^2 - x^2}} = \frac{10}{\sqrt{29^2 - 10^2}}$$

$$\tan\theta = 0.3576. \quad \therefore \theta = 20.18$$

$$N_T = H_D \cos\theta + V_D \sin\theta$$

$$= 156.25 \cos(20.18) + 62.5 \sin(20.18)$$

$$\therefore N_T = 168.17\ \text{kN}$$

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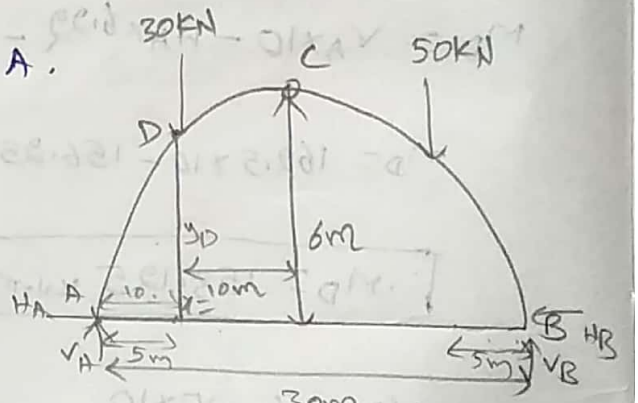
$R_S = H_D \sin\theta - V_D \cos\theta$.

$= 156.25 \sin(20.18) - 62.5 \cos(20.18)$

$\therefore R_S = -4.76\ KN$

*) A 3 hinged circular Arch of span 30m and central rise 6m. carries 2 Point loads 30KN & 50KN. at a distance of 5m from both ends respectively. Calculate i) support reactions. ii) Moment, N_T, R_S under the loads.

Taking moment about Point A.



$\Sigma M_A = 0$.

$V_B \times 30 - 50 \times 25 - 30 \times 5 = 0$

$V_B = 46.66\ KN$

$V_A + V_B = 30 + 50 = 80$.

$V_A = 80 - 46.66$

$\therefore V_A = 33.34\ KN$

Taking moment about C (right)

$V_B \times 15 - 50 \times 10 - H_B \times 6 = 0$

$46.66 \times 15 - 500 = 6 H_B$

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$\boxed{\therefore H_B = 33.32\ KN}$ $\boxed{H_A = H_B = 33.32\ KN}$

$$y_D = \sqrt{R^2 - x^2} - R + h \rightarrow ①$$

$$R = \frac{l^2}{8h} + \frac{h}{2}$$

$$= \frac{30^2}{8 \times 6} + \frac{6}{2}$$

$$\boxed{R = 21.75\ m}$$

$$y_D = \sqrt{21.75^2 - 10^2} - 21.75 + 6$$

$$\boxed{y_D = 3.56\ m}$$

$\boxed{H = \frac{W}{\pi}}$

$$M_D = V_A \times 5 - H_A \times 3.56 - \cancel{30 \times 5}$$ 30 (bear zero)

2-hinged

$$\boxed{M_D = 48.08\ KN}$$

$\int y^2 ds \rightarrow$ circular

$\hookrightarrow \frac{\pi r^3}{2}$

$$V_D = V_A - 30$$

$$V_D = 33.34 - 30 \quad \boxed{V_D = 3.34\ KN}$$

$$\tan\theta = \frac{x}{\sqrt{R^2 - x^2}}$$

$$\tan\theta = \frac{10}{\sqrt{21.75^2 - 100}} = 0.517$$

$$\boxed{\theta = 27.33}$$

$$N_T = H_D \cos\theta + V_D \sin\theta = 33.32 \times \cos(27.33) + 3.34 \sin(27.33)$$

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$\therefore N_T = 31.13\ KN$

$R_S = H_D \sin\theta - V_D \cos\theta$

$= 33.32 \sin(27.33) - 3.34 \cos(27.33)$

$\therefore R_S = 12.33\ KN.$

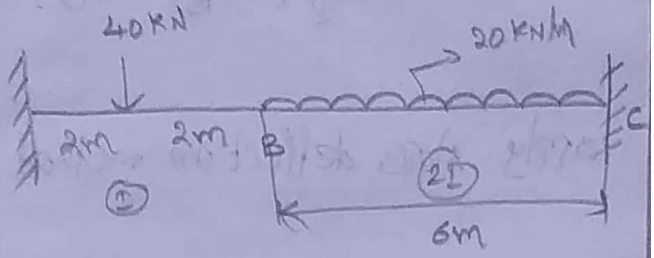
$M_E =$

07/02/2020

IV. Module

Slope deflection Method.

1) Analyse the 2 span continuous beam as shown in fig. by using slope deflection method and also draw SFD & BMD.



Step-1:

Fixed end moments.

for span AB,

$$M^f_{AB} = \frac{-wl}{8} = \frac{-40\times4}{8} = -20\,KN\text{-}m.$$

$$M^f_{BA} = \frac{wl}{8} = \frac{40\times4}{8} = 20\,KN\text{-}m.$$

for span BC.

$$M^f_{BC} = \frac{-wl^2}{12} = \frac{-20\times6^2}{12} = -60\,KN\text{-}m$$

$$M^f_{CB} = \frac{+wl^2}{12} = \frac{+20\times6^2}{12} = 60\,KN\text{-}m.$$

Step-2:

calculating slope deflection equation.

$$M_{AB} = \frac{2EI}{l}\left[2\theta_A + \theta_B - \frac{3\delta}{l}\right] + M^f_{AB}$$

[$\because \theta_A = \theta_B = 0$ at fixed ends, $\delta = 0$ (sinking support not mentioned)]

$$M_{AB} = \frac{2EI}{4}\left[2(0) + \theta_B - \frac{3\delta}{l}\right] + (-20)$$

$$M_{AB} = \frac{EI}{2}(\theta_B) - 20 \rightarrow (1)$$

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$$M_{BA} = \frac{2EI}{l_1}\left(2\theta_B + \theta_A - \frac{3\delta}{l}\right) + MF_{BA}$$

$$M_{BA} = \frac{2EI}{4}\left(2\theta_B + \theta_A - \frac{3\delta}{l}\right) + 20$$

$$M_{BA} = \frac{EI}{2}(2\theta_B) + 20 = EI(\theta_B) + 20 \rightarrow (2)$$

apply slope deflection method for span BC.

$$M_{BC} = \frac{2EI}{l_2}\left(2\theta_B + \theta_C - \frac{3\delta}{l}\right) + MF_{BC}$$

$$M_{BC} = \frac{2E(2I)}{6_3}\left(2\theta_B + \theta_C - \frac{3\delta}{l}\right) + (-60)$$

$$M_{BC} = \frac{4EI}{3}(\theta_B) - 60 \rightarrow (3)$$

$$M_{CB} = \frac{2EI}{l_2}\left(2\theta_C + \theta_B - \frac{3\delta}{l}\right) + MF_{CB}$$

$$M_{CB} = \frac{4EI}{6}\left(2\theta_C + \theta_B - \frac{3\delta}{l}\right) + 60$$

$$\therefore M_{CB} = \frac{2EI}{3}(\theta_B) + 60 \rightarrow (4)$$

$$M_{BA} + B_{BC} = EI(\theta_B) + 20 + \frac{4EI}{3}(\theta_B) - 60 = 0$$

$$= \frac{4EI(\theta_B) + 3EI(\theta_B)}{3} - 40 = 0$$

$$= 7EI\theta_B/3 - 40 = 0$$

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$$\frac{7EI(\theta_B)}{3} = 40 \quad , \quad 7EI(\theta_B) = 120.$$

$$EI.\theta_B = \frac{120}{7} = 17.14$$

$$\therefore \theta_B = \frac{17.14}{EI}$$

Subst θ_B in ①, ②, ③ & ④

① $\Rightarrow M_{AB} = \frac{EI}{2}(\theta_B) - 20$

$$M_{AB} = \frac{EI}{2} \times \frac{17.14}{EI} - 20$$

$$\therefore M_{AB} = -11.43 \text{ KN-m}$$

② $\Rightarrow M_{BA} = EI\left(\frac{17.14}{EI}\right) + 20$

$$\therefore M_{BA} = 37.14 \text{ KN-m.}$$

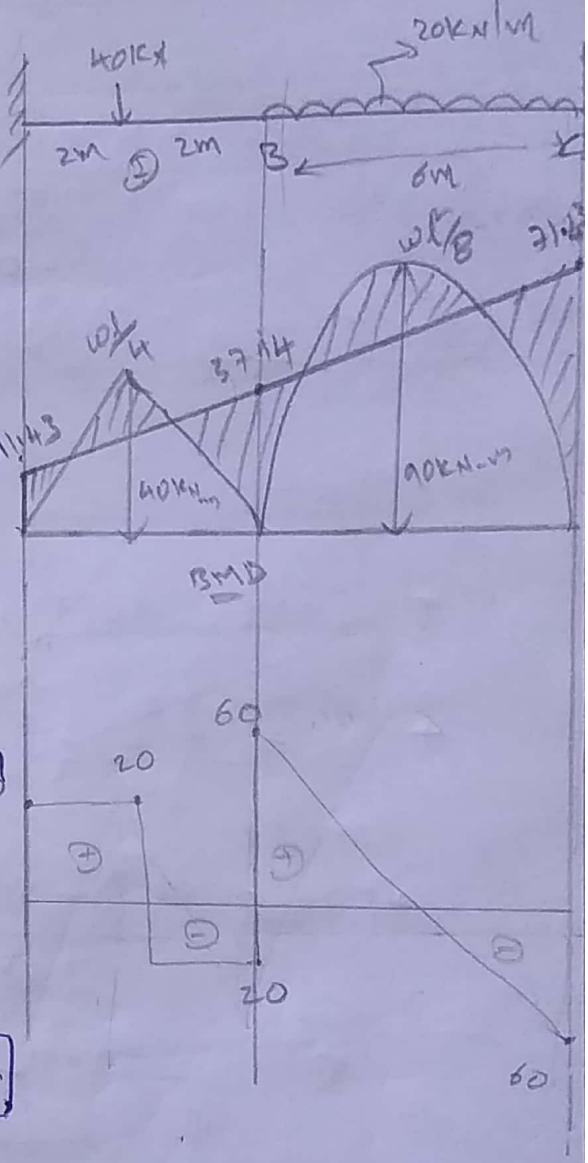
③ $\Rightarrow M_{BC} = \frac{4EI}{3}(\theta_B) - 60$

$$M_{BC} = \frac{4EI}{3} \times \frac{17.14}{EI} - 60$$

$$\therefore M_{BC} = -37.14 \text{ KN-m.}$$

④ $\Rightarrow M_{CB} = \frac{2EI}{3}\left(\frac{17.14}{EI}\right) + 60$

$$\therefore M_{CB} = 71.43 \text{ KN-m}$$



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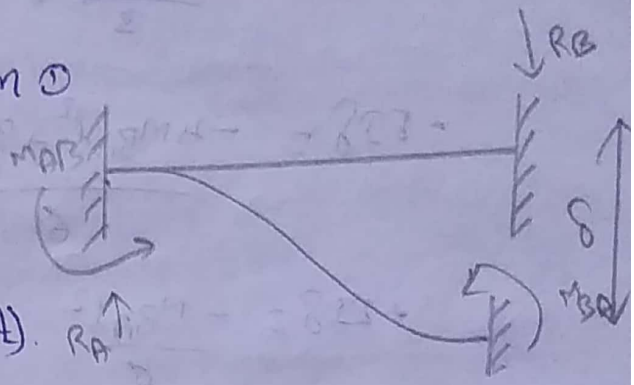
12/02/2020

Moment due to sinking of supports :-

let us consider a fixed beam of span AB of length 'l' and support B will sinking an amount of δ.

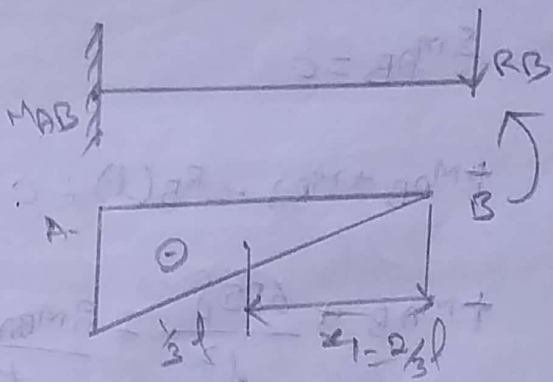


from moment area theorem ①

$$\theta_A - \theta_B = \frac{A}{EI}$$

$\theta_A = \theta_B = 0$ (fixed support)

$$0 = \frac{A_1}{EI} + \frac{A_2}{EI}$$



$$0 = \frac{-(\frac{1}{2} \times R_B l \times l)}{EI} + \frac{M_{BA} l}{EI}$$

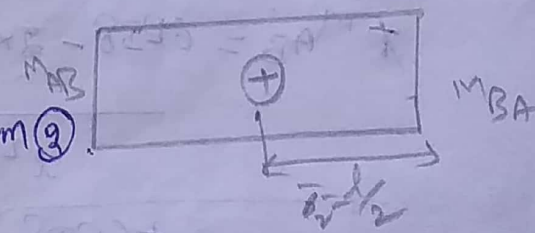
$$-\frac{1}{2} R_B l^2 + M_{BA} l = 0$$

$$+\frac{1}{2} R_B l^2 = +M_{BA} l$$

$$\therefore R_B = \frac{2M_{BA}}{l}$$



from moment area theorem ②



$$y_B = \frac{A\bar{x}}{EI}$$

$$y_B = \frac{A_1 \bar{x}_1}{EI} + \frac{A_2 \bar{x}_2}{EI}$$

sinking support at downwards

$$-\delta = \frac{(-\frac{1}{2} \times R_B l \times l \times \frac{2}{3} l)}{EI} + \frac{(M_{BA} l \times \frac{l}{2})}{EI}$$

$$-EI\delta = -\frac{R_B l^3}{3} + \frac{M_{BA} l^2}{2}$$

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$$-EI\delta = \frac{-2M_{BA}l^3}{3l} + \frac{M_{BA}l^2}{2}$$

$$-EI\delta = \frac{-2M_{BA}l^2}{3} + \frac{M_{BA}l^2}{2}$$

$$-EI\delta = \frac{-4M_{BA}l^2 + 3M_{BA}l^2}{6}$$

$$+EI\delta = \frac{+M_{BA}l^2}{6}$$

$$\therefore M_{BA} = \frac{6EI\delta}{l^2}$$

$$\Sigma M_{AB} = 0$$

$$+M_{AB} + M_{BA} - R_B(l) = 0$$

$$+M_{AB} + \frac{6EI\delta}{l^2} - \frac{2M_{BA}}{l} \times l = 0$$

$$+M_{AB} = \frac{6EI\delta - 2M_{BA}l^2}{l^2}$$

$$+M_{AB} = \frac{6EI\delta - 2\times\frac{6EI\delta}{l^2}\times l^2}{l^2} = +\frac{6EI\delta}{l^2}$$

$$\therefore M_{AB} = +\frac{6EI\delta}{l^2}$$

Moment due to rotation of support:-

Consider a beam of span AB of length "l" and subject to a moment M_{AB} at "A".

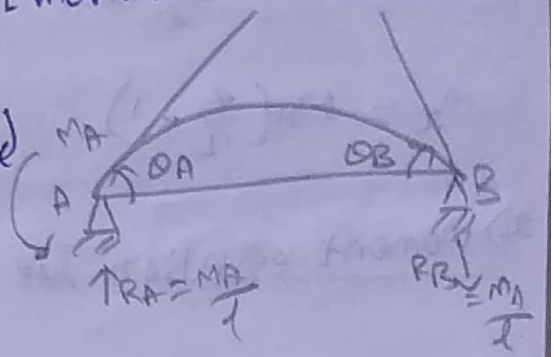
Due to moment at A the supports A, B will rotates by an

amount of θ_A, θ_B.

θ_A and θ_B can be calculated by unit load method.

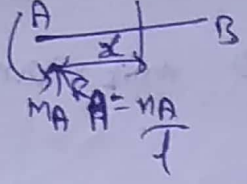
1) Moment equation due to M_A.

2) Moment equation due to unit moment.



1) Moment due to moment at A(M_x)

$$M_x = -M_A + \frac{M_A}{L} \cdot x$$

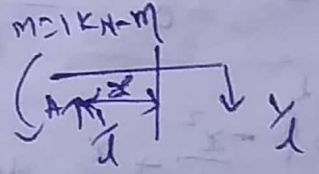


$$M_x = M_A\left(\frac{x}{L} - 1\right)$$

2) Moment equation due to unit moment.

$$M_x = -1 + \frac{1}{L}(x)$$

$$M_x = \left(\frac{x}{L} - 1\right)$$



$$\theta_A = \int_0^L \frac{M_x m_x}{EI}\, dx$$

$$\theta_A = \int_0^L \frac{M_A\left(\frac{x}{L} - 1\right)\left(\frac{x}{L} - 1\right)}{EI}\, dx = \frac{M_A}{EI}\int_0^L \frac{x^2}{L^2} + 1 - \frac{2x}{L}\, dx$$

$$\theta_A = \frac{M_A}{EI}\left[\frac{x^3}{3L^2} + x - \frac{2x^2}{2L}\right]_0^L = \frac{M_A}{EI}\left[\frac{L^3}{3L^2} + L - \frac{2L^2}{2L}\right]$$

$$\therefore \theta_A = \frac{M_A L}{3EI}$$

$$\therefore M_A = \frac{3EI\theta_A}{L}$$

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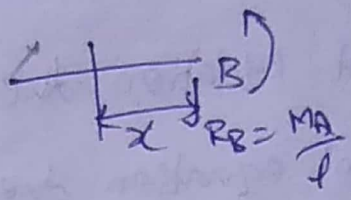
Calculation of θ_B:-

1) Moment due to moment at B (M_x)

$M_x = -M_A + R_A \times x$

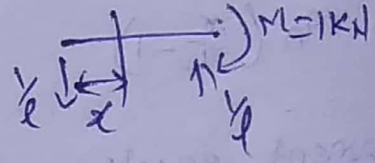


$M_x = -M_A + \frac{M_A}{l} \times x$

$M_x = M_A\left(\frac{x}{l} - 1\right)$

2) Moment equation due to unit moment at B.

$m_x = -\frac{x}{l}$



$$\theta_B = \int_0^l \frac{M_x \cdot m_x}{EI}\,dx = \int_0^l \frac{M_A\left(\frac{x}{l}-1\right)\left(-\frac{x}{l}\right)}{EI}\,dx$$

$$\theta_B = \frac{M_A}{EI}\int_0^l \frac{-x^2}{l^2} + \frac{x}{l}\,dx$$

$$\theta_B = \frac{M_A}{EI}\left[-\frac{x^3}{3l^2} + \frac{x^2}{2l}\right]_0^l = \frac{M_A}{EI}\left[-\frac{l^3}{3l^2} + \frac{l^2}{2l}\right]$$

$$\theta_B = \frac{M_A}{EI}\left[-\frac{l}{3} + \frac{l}{2}\right] = \frac{M_A}{EI}\left(\frac{l}{6}\right)$$

$$\theta_B = \frac{M_A l}{6EI}$$

$$\therefore M_A = \frac{6EI\theta_B}{l}$$

$$M_A = 1 \cdot \frac{3EI\theta_B}{l}$$

$$M_A = 2 \cdot M_A$$

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13/02/2020.

Slope-deflection Method:- IMP ***.

This method is used to findout the slope and deflection for the static Indeterminent beams and regid frames.

This method was developed by "G.A.Maney." in 1915.

Assumptions:-

1) All the joints of frame are regid.

2) Whenever the beams and frames are deflected the regid joints are considered to rotate as a whole.

3) The deflection and rotation due to shear stresses & axial stresses are neglected.

Slope-deflection Equation:-

let us consider a continuous beam as shown in figure. In that consider an Intermediate span of AB of length 'L'. we need to develop an equation which includes moments, rotations, deflections at A and B.

Treat AB as a fixed beam, the fixed end moments at A&B are M^f_{AB}, M^f_{BA}. The rotation at A, B are $\theta_A = 0$

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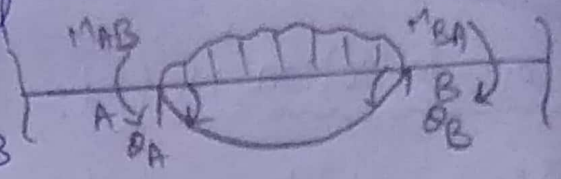
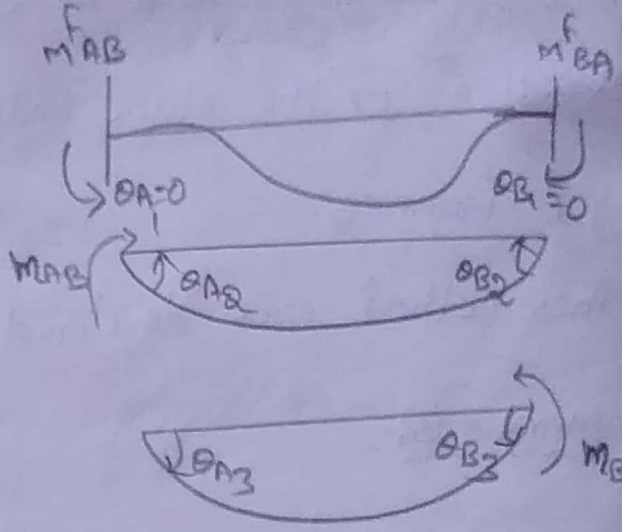
and $\theta_B = 0$. Due to external loadings the moments at A,B are m_{AB}, m_{BA}.





$$\theta_A = \theta_{A1} + \theta_{A2} + \theta_{A3}$$

$$\theta_A = 0 + \theta_{A2} + \theta_{A3}$$

$$\therefore \theta_A = \theta_{A2} + \theta_{A3} \rightarrow (1)$$

$$\theta_B = -(\theta_{B1} + \theta_{B2} + \theta_{B3})$$

$$\theta_B = -(0 + \theta_{B2} + \theta_{B3})$$

$$\therefore \theta_B = -(\theta_{B2} + \theta_{B3}) \rightarrow (2)$$

$$\theta_{A2} = \frac{m_{AB} l}{3EI}, \quad \theta_{B2} = \frac{m_{AB} l}{6EI}$$

$$\theta_{B3} = \frac{m_{BA} l}{3EI}, \quad \theta_{A3} = \frac{m_{BA} l}{6EI}$$

$$-m_{AB} = -m^f_{AB} + m_{AB}$$

$$m_{AB} = m^f_{AB} - m_{AB} \rightarrow (3)$$

$$m_{BA} = m^f_{BA} - m_{BA} \rightarrow (4)$$

Subst. $\theta_{A2}, \theta_{A3}, \theta_{B2}, \theta_{B3}$ in eqn (1) & (2).

$$\theta_A = \frac{m_{AB} l}{3EI} + \frac{m_{BA} l}{6EI} = \frac{l}{6EI}(2m_{AB} + m_{BA}).$$

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$$\theta_B = -\left[\frac{M_{AB}l}{6EI} + \frac{M_{BA}l}{3EI}\right] = \frac{-l}{6EI}\left[M_{AB} + 2M_{BA}\right].$$

$$\theta_A - \theta_B = \frac{l}{6EI}(2M_{AB} + M_{BA}) + \frac{l}{6EI}\left[M_{AB} + 2M_{BA}\right]$$

$$\theta_A - \theta_B = \frac{l}{6EI}\left[2M_{AB} + M_{BA} + M_{AB} + 2M_{BA}\right].$$

$$\therefore \theta_A - \theta_B = \frac{l}{6EI}\left[3M_{AB} + 3M_{BA}\right]$$

$$\theta_A - \theta_B = \frac{l}{2EI}\left[M_{AB} + M_{BA}\right]. \rightarrow (5)$$

$$M_{AB} + M_{BA} = \frac{(\theta_A - \theta_B)2EI}{l}$$

$$M_{BA} = \frac{(\theta_A - \theta_B)2EI}{l} - M_{AB} \cdot \text{ sub in } \theta_A.$$

$$\theta_A = \frac{l}{6EI}\left[2M_{AB} + \frac{(\theta_A - \theta_B)2EI}{l} - M_{AB}\right]$$

$$\theta_A = \frac{l}{6EI}\left[\frac{2lM_{AB} + (\theta_A - \theta_B)2EI - lM_{AB}}{l}\right]$$

$$\theta_A = \frac{lM_{AB} + (\theta_A - \theta_B)2EI}{6EI}$$

$$\theta_A = \frac{M_{AB}l}{6EI} + \frac{(\theta_A - \theta_B)}{3}$$

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$$\theta_A - \frac{(\theta_A - \theta_B)}{3} = \frac{m_{AB} l}{6EI}$$

$$\frac{2\theta_A + \theta_B}{3} = \frac{m_{AB} l}{6EI}, \quad 2\theta_A + \theta_B = \frac{m_{AB} l}{2EI}.$$

$$\therefore m_{AB} = -\frac{(2\theta_A + \theta_B) 2EI}{l}$$

$$(5) \Rightarrow \theta_A - \theta_B = \frac{l}{2EI} [m_{AB} + m_{BA}]$$

$$m_{AB} + m_{BA} = \frac{(\theta_A - \theta_B) 2EI}{l}$$

$$m_{AB} = \frac{(\theta_A - \theta_B) 2EI}{l} - m_{BA}, \text{ subtitle in } \theta_A.$$

$$\theta_A = \frac{l}{6EI} [2m_{AB} + m_{BA}]$$

$$\theta_A = \frac{l}{6EI} \left[2 \left[\frac{(\theta_A - \theta_B) 2EI}{l} - m_{BA} \right] + m_{BA} \right]$$

$$\theta_A = \frac{l}{6EI} \left[\frac{4EI(\theta_A - \theta_B)}{l} - m_{BA} \right]$$

$$\theta_A = \frac{l}{6EI} \left[\frac{4EI(\theta_A - \theta_B) - l m_{BA}}{l} \right]$$

$$\theta_A = \frac{2(\theta_A - \theta_B)}{3} - \frac{m_{BA} l}{6EI}$$

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$$\theta_A - \frac{2}{3}(\theta_A - \theta_B) = -\frac{m_{BA} l}{6EI}$$

$$\frac{1}{3}\theta_A + \frac{2\theta_B}{3} = -\frac{m_{BA} l}{6EI}$$

$$\frac{\theta_A + 2\theta_B}{3} = -\frac{m_{BA} l}{6EI}$$

$$\theta_A + 2\theta_B = -\frac{m_{BA} l}{2EI}$$

$$\therefore m_{BA} = -\frac{2EI}{l}(\theta_A + 2\theta_B).$$

③ $\Rightarrow M_{AB} = M^f_{AB} - m_{AB}$, subtitle m_{AB} in ③ eqn.

$$\therefore M_{AB} = M^f_{AB} + \frac{2EI(2\theta_A + \theta_B)}{l}$$

④ $M_{BA} = M^f_{BA} - m_{BA}$, subtitle m_{BA} in ④ eqn.

$$\therefore M_{BA} = M^f_{BA} + \frac{2EI}{l}(\theta_A + 2\theta_B).$$

If consider an deflection.

$$\therefore M_{AB} = M^f_{AB} + \frac{2EI}{l}\left(2\theta_A + \theta_B - \frac{3\delta}{l}\right).$$

$$\therefore M_{BA} = M^f_{BA} + \frac{2EI}{l}\left(\theta_A + 2\theta_B - \frac{3\delta}{l}\right).$$

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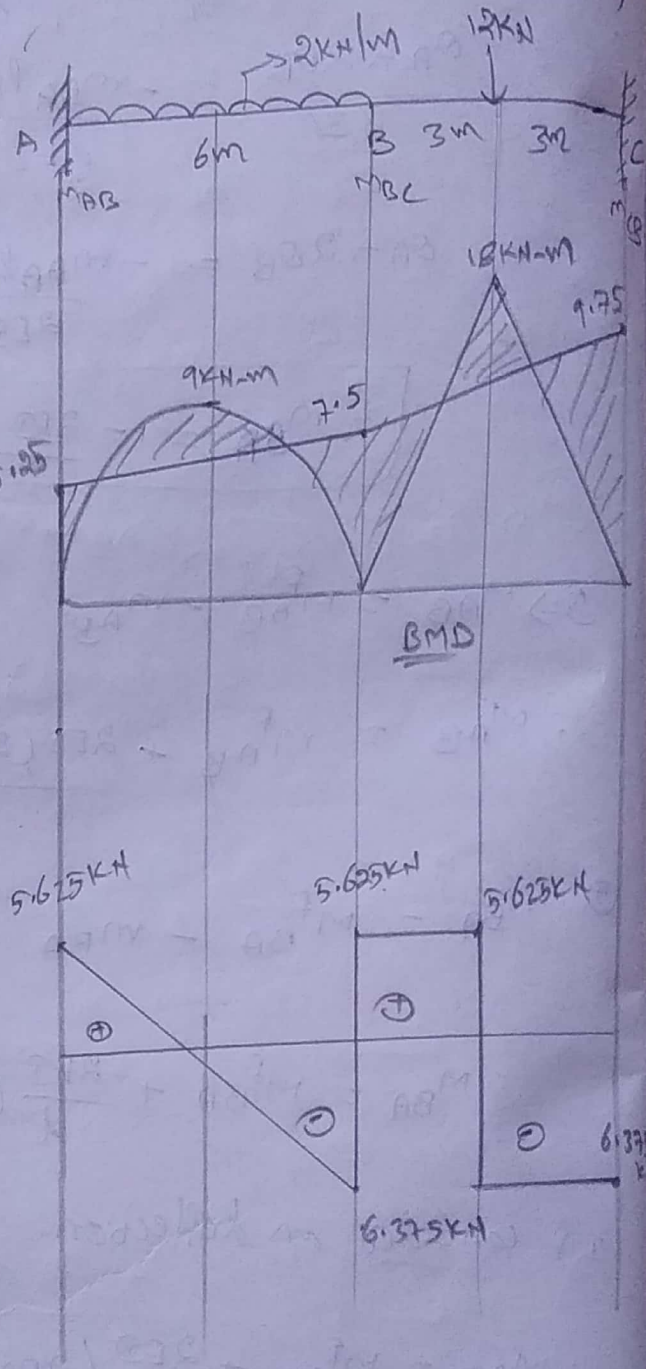
14/02/2020

1) Analyse the two span continuous beam as shown in fig. and calculate, find moments & reactions at supports. and also draw SFD & BMD



Given,

$l_1 = 6m$, $w_1 = 2KN/m$,

$\theta_A = \theta_C = 0$.

$l_2 = 6m$, $w_2 = 12KN$, $\delta = 0$

Step:1 fixed end moments

for span AB:-

$$M^F_{AB} = \frac{-w_1 l_1^2}{12} = \frac{-2 \times 6^2}{12}$$

$$M^F_{AB} = -6KN\text{-}m.$$

$$M^F_{BA} = \frac{+w_1 l_1^2}{12} = \frac{+2 \times 6^2}{12}$$

$$M^F_{BA} = +6KN\text{-}m.$$

for span BC:-

$$M^F_{BC} = \frac{-w_2 l_2}{8} = \frac{-12 \times 6}{8} = -9KN\text{-}m$$

$$M^F_{CB} = \frac{+w_2 l_2}{8} = \frac{+12 \times 6}{8} = +9KN\text{-}m.$$

Step-2:- slope deflection equation.

for span AB.

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$$M_{AB} = M^f_{AB} + \frac{2EI}{l_1}\left(2\theta_A + \theta_B - \frac{3\delta}{l_1}\right)$$

$$M_{AB} = -6 + \frac{2EI}{6}(2(0) + \theta_B - 3(0))$$

$$M_{AB} = -6 + \frac{EI}{3}(\theta_B) \longrightarrow (1)$$

$$M_{BA} = M^f_{BA} + \frac{2EI}{l_2}\left(2\theta_B + \theta_A - \frac{3\delta}{l_2}\right)$$

$$M_{BA} = +6 + \frac{2EI}{6}(2\theta_B + 0 - 3(0))$$

$$M_{BA} = +6 + \frac{2EI}{3}\theta_B \longrightarrow (2)$$

For span BC:-

$$M_{BC} = M^f_{BC} + \frac{2EI}{l_2}\left(2\theta_B + \theta_C - \frac{3\delta}{l}\right)$$

$$M_{BC} = -9 + \frac{2EI}{6}(2\theta_B + 0 - 3(0))$$

$$M_{BC} = -9 + \frac{2}{3}EI\,\theta_B \longrightarrow (3)$$

$$M_{CB} = M^f_{CB} + \frac{2EI}{l_2}\left(\theta_B + 2\theta_C - \frac{3\delta}{l_2}\right)$$

$$M_{CB} = +9 + \frac{2EI}{6}(\theta_B + 2(0) - 3(0))$$

$$M_{CB} = 9 + \frac{EI}{3}(\theta_B) \longrightarrow (4)$$

Step 3:- Equilibrium equations.

$\sum M_B = 0$.

$$M_{BA} + M_{BC} = 0.$$

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$$6 + \frac{2EI}{3}\theta_B - 9 + \frac{2}{3}EI\theta_B = 0$$

$$\frac{4}{3}EI\theta_B = 3, \qquad EI\theta_B = \frac{9}{4}$$

$$\therefore EI\theta_B = 2.25$$

Subst $EI\theta_B = 2.25$ in eq. ① ② & ③ & ④.

$$M_{AB} = -6 + \frac{1}{3}EI\theta_B \quad = -6 + \frac{1}{3}(2.25)$$

$$\therefore M_{AB} = -5.25\ KN\text{-}m$$

$$M_{BA} = +6 + \frac{2}{3}EI\theta_B \quad = +6 + \frac{2}{3}(2.25)$$

$$\therefore M_{BA} = 7.5\ KN\text{-}m$$

$$M_{BC} = -9 + \frac{2}{3}(2.25) \qquad \therefore M_{BC} = -7.5\ KN\text{-}m.$$

$$M_{CB} = +9 + \frac{EI}{3}\theta_B \quad = +9 + \frac{2.25}{3}$$

$$\therefore M_{CB} = 9.75\ KN\text{-}m$$

STEP-4:

Max. B.M

max B.M for $AB = \frac{wl^2}{8} = \frac{2 \times 6^2}{8} = 9\ KN\text{-}m$

max BM for $BC = \frac{wl}{4} = \frac{12 \times 6}{4} = 18\ KN\text{-}m.$

STEP-5:

SFD:

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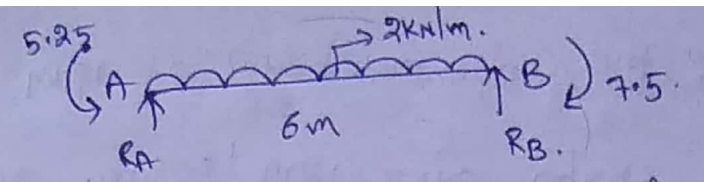
For Zone AB :-

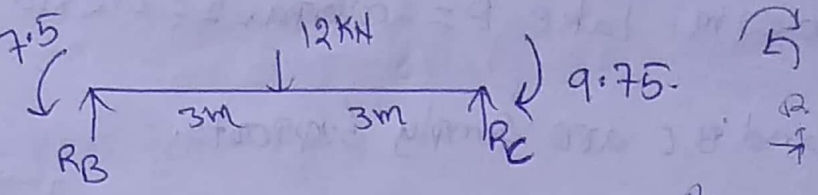


$\Sigma M_B = 0$.

$R_A \times 6 + 7.5 = 2 \times 6 \times \frac{6}{2} + 5.25$

$6R_A = 33.75$. $\therefore R_A = 5.625$ KN

for span BC.



$\Sigma M_B = 0$.

$R_C \times 6 + 7.5 = 12 \times 3 + 9.75$

$6R_C = 38.25$. $\therefore R_C = 6.375$ KN.

$R_A + R_B + R_C = 12 + 2 \times 6 = 12 + 12 = 24$.

$R_B = 24 - 6.375 - 5.625$.

$\therefore R_B = 12$ KN

$R_C = -6.375$ KN.

from that R_C, 12KN, $-6.375 + 12 = 5.625$ KN.

$5.625 - R_B = 5.625 - 12 = -6.375$ KN.

$2 \times 6 - 6.375 = 5.625$ KN $= R_A$.

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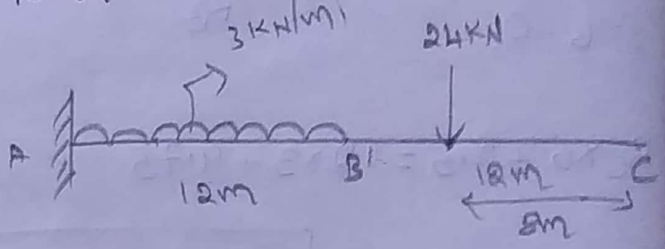
2) Analyse the continuous beam of span AB as 12m. subjected to 3KN/m over 12m length. and BC having 12m subjected to a point load of 24KN. at a distance of 8m from C. and support B sinks by an amount of 0.03m. take E = 200GPa, $I = 0.2 \times 10^{-3} m^4$. support A is fixed and B, C are simply supports.



G.T:

$l_1 = 12m$, $w_1 = 3KN/m$.

$\theta_A = 0$, $\theta_B = ?$

$l_2 = 12m$, $w_2 = 24KN$,

$\delta = 0.03m$, $E = 200GPa$.

$I = 0.2 \times 10^{-3} m^4$.

E: 200GPa

$= 200 \times 10^9 N/m^2$

$= 200 \times 10^6 \times (10^3) N/mm^2$

$= 200 \times 10^6 KN/mm^2$

$= 2 \times 10^8 KN/m^2$

Step:1

fixed end moments.

for span AB:

$$M^f_{AB} = -\frac{w_1 l_1^2}{12} = -\frac{3 \times 12^2}{12}$$

$$M^f_{AB} = -36 KN\text{-}m$$

$$M^f_{BA} = +\frac{w_1 l_1^2}{12} = +\frac{3 \times 12^2}{12}$$

$$\therefore M^f_{BA} = +36 KN\text{-}m.$$

for span BC:

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$$M^f_{BC} = \frac{-wab^2}{l^2} = \frac{-24 \times 4 \times 8^2}{12^2} = -42.66 \text{ KN-m}$$

$$M^f_{CB} = \frac{+wa^2b}{l^2} = \frac{+24 \times 4^2 \times 8}{12^2} = +21.33 \text{ KN-m}$$

Step 2:- Slope deflection equation

for span AB :-

$$M_{AB} = M^f_{AB} + \frac{2EI}{l_1}\left(2\theta_A + \theta_B - \frac{3\delta}{l_1}\right)$$

$$M_{AB} = -36 + \frac{2EI}{12}\left(2\theta_A + \theta_B - \frac{3(0.03)}{12}\right)$$

$$M_{AB} = -36 + \frac{EI}{6}\left(\theta_B - \frac{3}{400}\right)$$

$$M_{AB} = -36 + \frac{EI\theta_B}{6} - \frac{EI}{800} \rightarrow (1)$$

$$M_{BA} = M^f_{BA} + \frac{2EI}{l_1}\left(2\theta_B + \theta_A - \frac{3\delta}{l_1}\right)$$

$$M_{BA} = +36 + \frac{2EI}{12}\left(2\theta_B + 0 - \frac{3(0.03)}{12}\right)$$

$$M_{BA} = 36 + \frac{EI\theta_B}{3} - \frac{EI(0.03)}{24} \rightarrow (2)$$

for span BC :-

$$M_{BC} = M^f_{BC} + \frac{2EI}{l_2}\left(2\theta_B + \theta_C - \frac{3\delta}{l_2}\right)$$

$$M_{BC} = -42.66 + \frac{2EI}{12}\left(2\theta_B + \theta_C - \frac{3(-0.03)}{12}\right)$$

$$M_{BC} = -42.66 + \frac{EI}{6}\left(2\theta_B + \theta_C + \frac{0.03}{4}\right) \rightarrow (3)$$

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$$M_{CB} = M^F_{CB} + \frac{2EI}{12}\left(2\theta_C + \theta_B - \frac{3\delta}{12}\right)$$

$$M_{CB} = 42.66 + \frac{2EI}{12}\left(2\theta_C + \theta_B - \frac{3(0.03)}{12}\right)$$

$$M_{CB} = 42.66 + \frac{EI}{6}\left(2\theta_C + \theta_B + \frac{0.03}{4}\right) \rightarrow (4)$$

Step:3 Equilibrium equations.

$\Sigma M_B = 0$.

$M_{BA} + M_{BC} = 0$

$$36 + \frac{EI\theta_B}{3} - \frac{EI(0.03)}{24} + (-42.66) + \frac{EI\theta_B}{3} + \frac{EI\theta_C}{6} + \frac{EI(0.03)}{24} = 0$$

$$-6.66 + \frac{2EI\theta_B}{3} + \frac{EI}{400} + \frac{EI\theta_C}{6} = 0$$

$$-6.66 + \frac{(2\times 200\times 10^{+6}\times 0.2\times 10^{-3})\theta_B}{3} - \frac{200\times 10^{+6}\times 0.2\times 10^{-3}}{400} + \frac{(200\times 10^{+6}\times 0.2\times 10^{-6})\theta_C}{6} = 0$$

$$2.666\times 10^{+4}\,\theta_B + 6.66\times 10^{-5}\,\theta_C = 6.66 \rightarrow (5)$$

$$26666.66\,\theta_B + 6666.66\,\theta_C = 6.66. \rightarrow (5)$$

$\Sigma M_C = 0$.

$M_{CB} = 0$.

$$M_{CB} = \frac{EI\theta_C}{3} + \frac{EI\theta_B}{6} + \frac{200\times 10^{-9}\times 0.2\times 10^{-3}\times 0.03}{24} + 42.66 = 0$$

$$\frac{200\times 10^{+6}\times 0.2\times 10^{-3}\,\theta_C}{3} + \frac{200\times 10^{+6}\times 0.2\times 10^{-3}\,\theta_B}{6} + 42.66 = 0$$

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$1333.33\theta_C + 6666.66\theta_B = -42.66 \rightarrow (6)$

solve (5) & (6).

$\theta_C = -6.25 \times 10^{-3}$.

$\theta_B = 1.81 \times 10^{-3}$

Step: 4:

final moments.

subst. θ_B and θ_C in M_{AB}.

$$M_{AB} = -36 + \frac{EI\theta_B}{6} - \frac{EI}{800}$$

$$= -36 + \frac{2\times10^8 \times 0.2\times10^{-3} \times 1.81\times10^{-3}}{6} - \frac{2\times10^8 \times 0.2\times10^{-3}}{800}$$

$$M_{AB} = -73.9 \simeq -74 \text{ KN-m}.$$

$$M_{BA} = +36 + \frac{EI}{6}\left(2\theta_B - \frac{3(0.03)}{12}\right)$$

$$= 36 + \frac{2\times10^8 \times 0.2\times10^{-3}}{6}\left(2\times1.81\times10^{-3} - \frac{3}{400}\right)$$

$$M_{BA} = 10.1 \text{ KN-m}.$$

$$M_{BC} = -42.6 + \frac{EI}{6}\left(2\theta_B + \theta_C + \frac{0.03}{4}\right)$$

$$\boxed{M_{BC} = -10.1 \text{ KN-m}}$$

$$M_{CB} = 42.6 + \frac{EI}{6}\left(2\theta_C + \theta_B + \frac{0.03}{4}\right)$$

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$\therefore M_{CB} = 0$

Step 5:

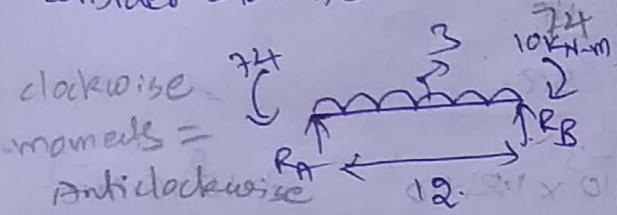
Max BM.

For span AB $= \frac{wl^2}{8} = \frac{3 \times 12^2}{8} = 54$ KN-m.

For span BC $= \frac{wab}{l} = \frac{24 \times 4 \times 8}{12} = 64$ KN-m.

Step-6: SFD.

Consider span AB.

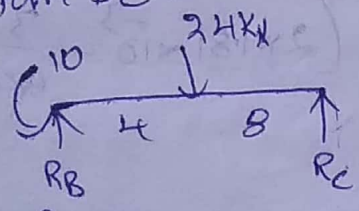


$R_A \times 12 + 10 = 3 \times 12 \times \frac{12}{2} + 74$

$R_A = \frac{74 + 3 \times 12 \times 6 - 10}{12}$

$R_A = 23.33$ KN.

Consider span BC.

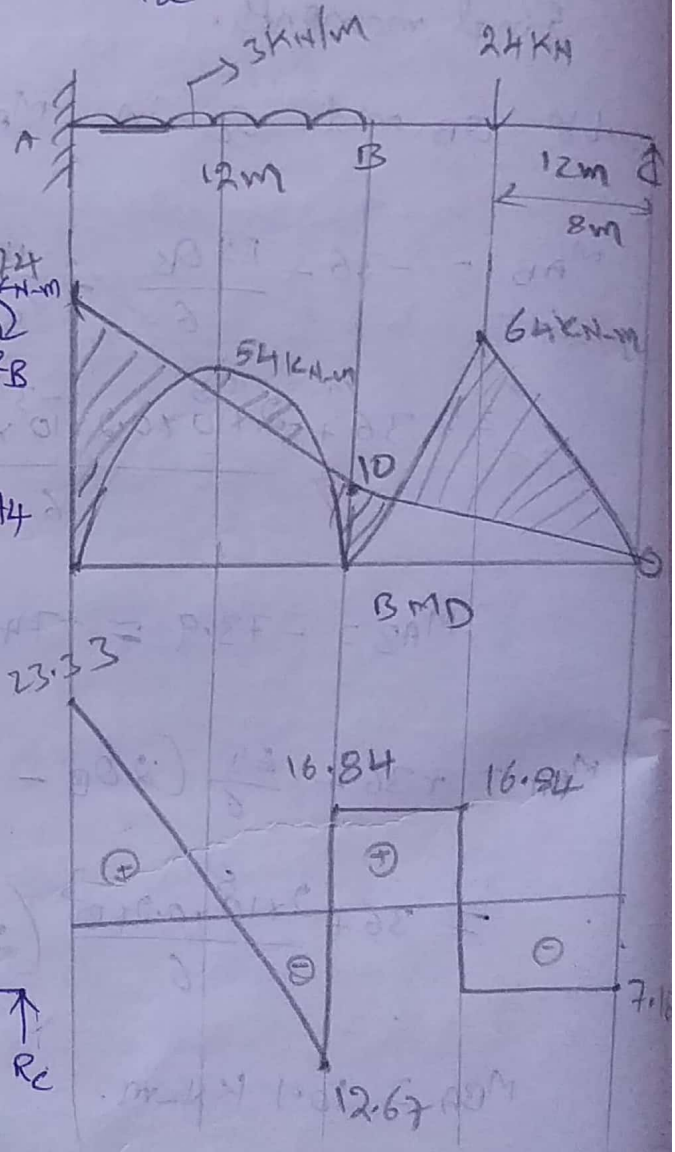


$R_C \times 12 + 10 = 24 \times 4$

$\therefore R_C = 7.16$ KN

$R_A + R_B + R_C = 24 + 3 \times 12 = 24 + 36$

$\therefore R_B = 29.51$ KN



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$R_C = 7.16$ KN, direction should be take as -7.16 KN.

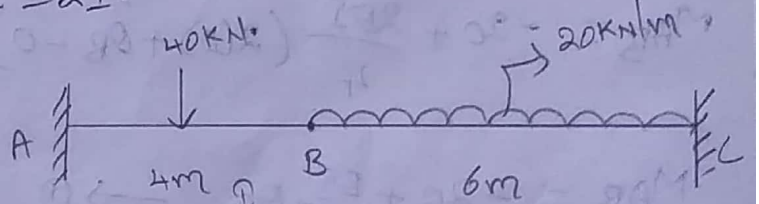
at point load, $+24$ KN $\Rightarrow$ $+24 - 7.16 = 16.84$ KN.

from 16.84 KN no load. at B also same 16.84 from B,,

there is an $R_B = 29.51$ KN, $-29.51 + 16.84 = -12.67$ KN.

$\Rightarrow -12.67$ an UDL $\rightarrow -12.67 + 3 \times 12 = 23.33$ (this shall be same as R_A).

3) Analyse the continuous beam by slope deflection method as shown in figure. by slope deflection method. span AB = 4m, subjected to a point load of 40KN at centre. and span BC of length 6m. having a UDL of 20KN/m over 6m. Both A & C are fixed supports and B is simply support. for AB, $I = I$, BC span, $I = 2I$.



Given,

$l_1 = 4$m, $w_1 = 40$KN.

$\theta_A = \theta_C = 0$.

$l_2 = 6$m, $w_2 = 20$KN/m, $\delta = 0$

Step: 1:

fixed end moments

for span AB:-

$$M^f_{AB} = \frac{-w_1 l_1}{8} = \frac{-40 \times 4}{8} = -20 \text{ KN-m}$$

$$M^f_{BA} = \frac{w_1 l_1}{8} = \frac{40 \times 4}{8} = +20 \text{ KN-m}$$

For span BC:

$$M^f_{BC} = \frac{-w_2 l_2^2}{12} = \frac{-20 \times 36}{12} = -60 \text{ KN-m}$$

$$M^f_{CB} = \frac{+w_2 l_2^2}{12} = +\frac{20 \times 36}{12} = +60 \text{ KN-m}.$$

Step-2:

Slope deflection equation.

For span AB.

$$M_{AB} = M^f_{AB} + \frac{2EI}{l_1}\left(2\theta_A + \theta_B - \frac{3\delta}{l_1}\right)$$

$$M_{AB} = -20 + \frac{2EI}{4}(2(0) + \theta_B - 0)$$

$$M_{AB} = -20 + \frac{EI}{2}\theta_B \longrightarrow (1)$$

$$M_{BA} = M^f_{BA} + \frac{2EI}{l_1}\left(2\theta_B + \theta_A - \frac{3\delta}{l_1}\right)$$

$$M_{BA} = 20 + \frac{2EI}{4}(2\theta_B)$$

$$M_{BA} = 20 + EI\cdot\theta_B \longrightarrow (2)$$

For span BC.

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$$M_{BC} = M^f_{BC} + \frac{2EI}{l_2}\left(2\theta_B + \theta_C - \frac{3\delta}{l_2}\right)$$

$$M_{BC} = -60 + \frac{2E(2I)}{6}(2\theta_B + 0 - 0)$$

$$M_{BC} = -60 + \frac{4EI}{3}\theta_B \longrightarrow (3)$$

$$M_{CB} = M^f_{CB} + \frac{2EI}{l_2}\left(2\theta_C + \theta_B - \frac{3\delta}{l_2}\right)$$

$$M_{CB} = 60 + \frac{2E(2I)}{6}(2(0) + \theta_B - 0)$$

$$M_{CB} = 60 + \frac{2EI}{3}\theta_B \longrightarrow (4)$$

STEP: 3.

Equilibrium equations.

$$\Sigma M_B = 0, \quad M_{BA} + M_{BC} = 0$$

$$20 + EI\cdot\theta_B + (-60) + \frac{4EI}{3}\theta_B = 0$$

$$-40 + \frac{7EI\cdot\theta_B}{3} = 0$$

$$7EI\cdot\theta_B = 120$$

$$\therefore EI\theta_B = 17.14$$

Subst $EI\theta_B = 17.14$ in (1), (2), (3) & (5) eqn.

$$M_{AB} = -20 + \frac{EI\theta_B}{2} = -20 + \frac{17.14}{2} = -11.43 \text{ KN-m}$$

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$M_{BA} = 20 + EI\theta_B = 20 + 17.14$

$\therefore M_{BA} = 37.14$ KN-m.

$M_{BC} = -60 + \frac{4}{3} \cdot EI\theta_B = -60 + \frac{4}{3}(17.14)$

$\therefore M_{BC} = -37.14$ KN-m

$M_{CB} = 60 + \frac{2}{3} \cdot EI\theta_B$

$M_{CB} = 60 + \frac{2}{3} \times 17.14$ $\therefore M_{CB} = 71.43$ KN-m

Step-4:

Max B.M

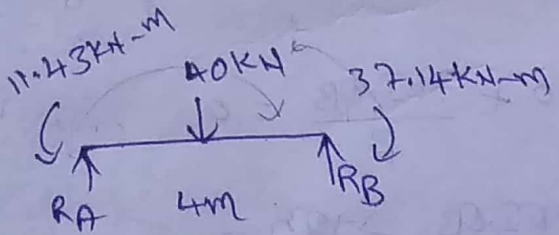
max BM for span AB $= \frac{wl}{4} = \frac{40 \times 4}{4} = 40$ KN-m

max BM for span BC $= \frac{wl^2}{8} = \frac{20 \times 36}{8} = 90$ KN-m.

Step-5:

SFD

for span AB.



$R_A \times 4 + 37.14 = 11.43 + 40 \times 2$

$4R_A + 37.14 = 11.43 + 80$

$R_A = 13.57$ KN.

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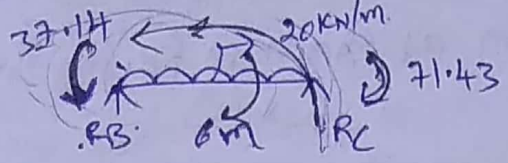
for span BC.



$R_C \times 6 + 37.14 = 20 \times 6 \times \frac{6}{2} + 71.43$

$6R_C + 37.14 = 20 \times 6 \times 3 + 71.43$.

$6R_C = 394.29$, $\therefore R_C = 65.715$ KN.

$R_A + R_B + R_C = 40 + 20 \times 6 = 40 + 120 = 160$.

$\therefore R_B = 160 - 65.715 - 13.57$.

$\therefore R_B = 80.715$ KN.

$R_C = -65.715$ KN.

load of 20 KN/m is acting

$-65.715 + 20 * 6 = 54.285$ KN

$54.285 - R_B$

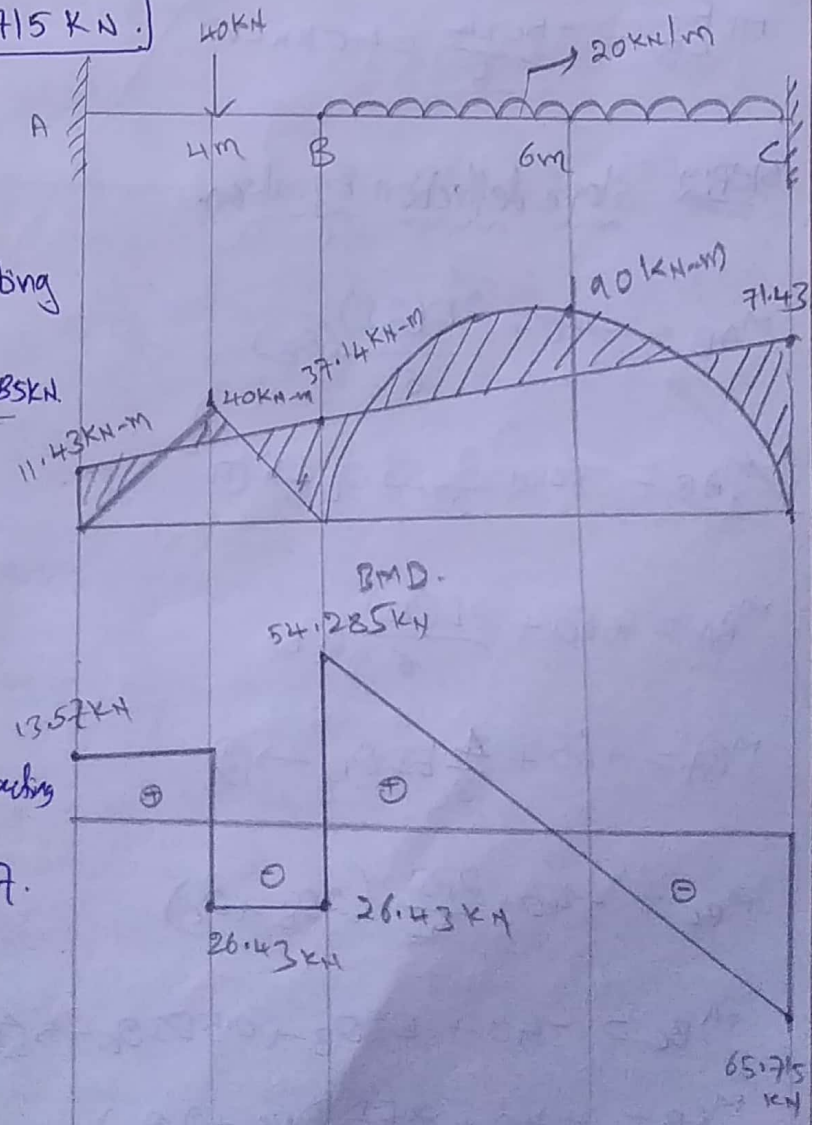
$= 54.285 - 80.715$

$= -26.43$ KN.

The point load 40 KN acting

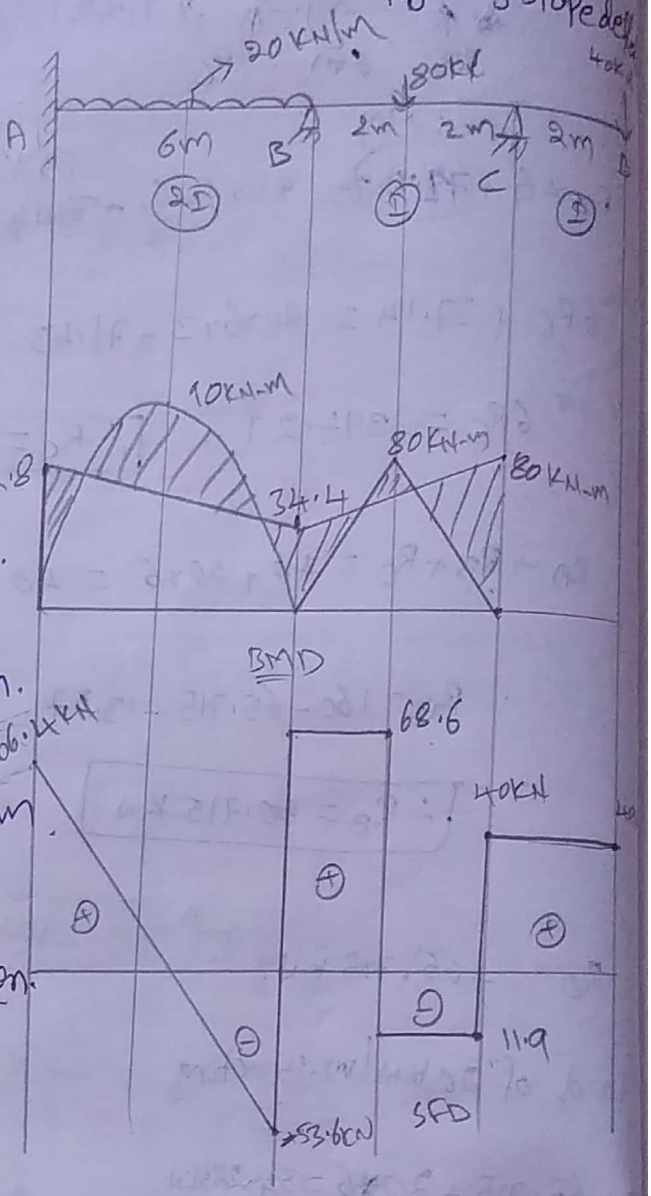
$-26.43 + 40 = 13.57$.

$= 13.57$ KN.



19/02/2020

4) Analyse the continuous beam as shown in fig. by slope deflection method.



Step-1: Fixed end moments

$$M^F_{AB} = \frac{-20\times6^2}{12} = -60\,KN\text{-}m.$$

$$M^F_{BA} = \frac{+20\times6^2}{12} = +60\,KN\text{-}m.$$

$$M^F_{BC} = \frac{-80\times4}{8} = -40\,KN\text{-}m.$$

$$M^F_{CB} = \frac{+80\times4}{8} = +40\,KN\text{-}m$$

Step:2: Slope deflection Equation.

$$M_{AB} = -60 + \frac{2E(2I)}{6}(\theta_B)$$

$$M_{AB} = -60 + \frac{2}{3}EI\theta_B \rightarrow (1)$$

$$M_{BA} = +60 + \frac{2E(2I)}{6}(2\theta_B)$$

$$M_{BA} = +60 + \frac{4}{3}EI\theta_B \rightarrow (2)$$

$$M_{BC} = -40 + \frac{2EI}{4}(2\theta_B + \theta_C)$$

$$M_{BC} = -40 + EI\theta_B + 0.5EI\theta_C \rightarrow (3)$$

$$M_{CB} = +40 + \frac{2EI}{4}(\theta_B + 2\theta_C).$$

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$M_{CB} = 40 + 0.5EI\theta_B + EI\theta_C \longrightarrow (4)$

$M_{CD} = -40 \times 2 = -80.$

Step:3: Equilibrium equations.

$\Sigma M_B = 0, \quad M_{BA} + M_{BC} = 0.$

$60 + \frac{4}{3}EI\theta_B - 40 + EI\theta_B + 0.5EI\theta_C = 0.$

$\frac{7}{3}EI\theta_B + 0.5EI\theta_C = -20 \longrightarrow (5).$

$\Sigma M_C = 0, \quad M_{CB} + M_{CD} = 0.$

$40 + 0.5EI\theta_B + EI\theta_C - 80 = 0.$

$0.5EI\theta_B + EI\theta_C = +40 \longrightarrow (6)$

Solve (5) & (6).

$\therefore EI\theta_B = -19.2, \quad EI\theta_C = 49.6.$

Step:4: Final moments.

sub'ing $EI\theta_B$ & $EI\theta_C$ in (1), (2), (3) & (4).

$M_{AB} = -60 + \frac{2}{3}(-19.2) = -72.8 \text{ KN-m}$

$M_{BA} = +60 + \frac{4}{3}(-19.2) = 34.4 \text{ KN-m}$

$M_{BC} = -40 - 19.2 + 0.5(49.6) = -34.4 \text{ KN-m}.$

$M_{CB} = +40 + 0.5(-19.2) + 49.6 = 80 \text{ KN-m}.$

$M_{CD} = -80 \text{ KN-m}.$

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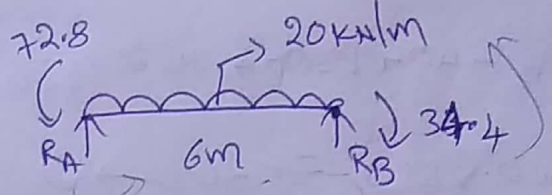
Step-5: Max BM:-

for span AB $= \frac{wl^2}{8} = \frac{20 \times 6^2}{8} = 90 \text{ KN-m}$

for span BC $= \frac{wl}{4} = \frac{80 \times 4}{4} = 80 \text{ KN-m}$

Step-6:- SFD.

Consider span AB.

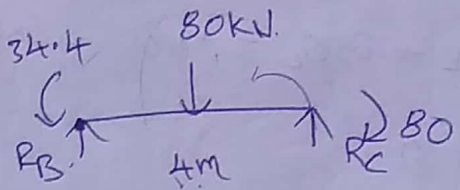


$\Sigma M_B = 0$.

$R_A \times 6 + 33.4 - 20 \times 6 \times \frac{6}{2} - 72.8 = 0$

$\boxed{R_A = 66.4 \text{ KN}}$

consider span BC.



$\Sigma M_B = 0$

$R_C \times 4 + 34.4 = 80 + 80 \times 2$

$\boxed{R_C = 51.4 \text{ KN}}$

$R_A + R_B + R_C = 240$

$R_B = 240 - 51.4 - 66.4$, $\boxed{\therefore R_B = 122.2 \text{ KN}}$

$R_C = +51.4 \text{ KN}$, at D $= 40 \text{ KN}$.

$20 \times 6 - 53.6 = 66.4$

for SFD downward ve +ve upward ve -ve

$40 - 51.4 = -11.9$, $-11.9 + 80 = 68.6$

$68.6 - 122.2 = -53.6 \text{ KN}$, $-53.6 + 20 \times 6 = 66.4 \text{ KN} = R_A$.

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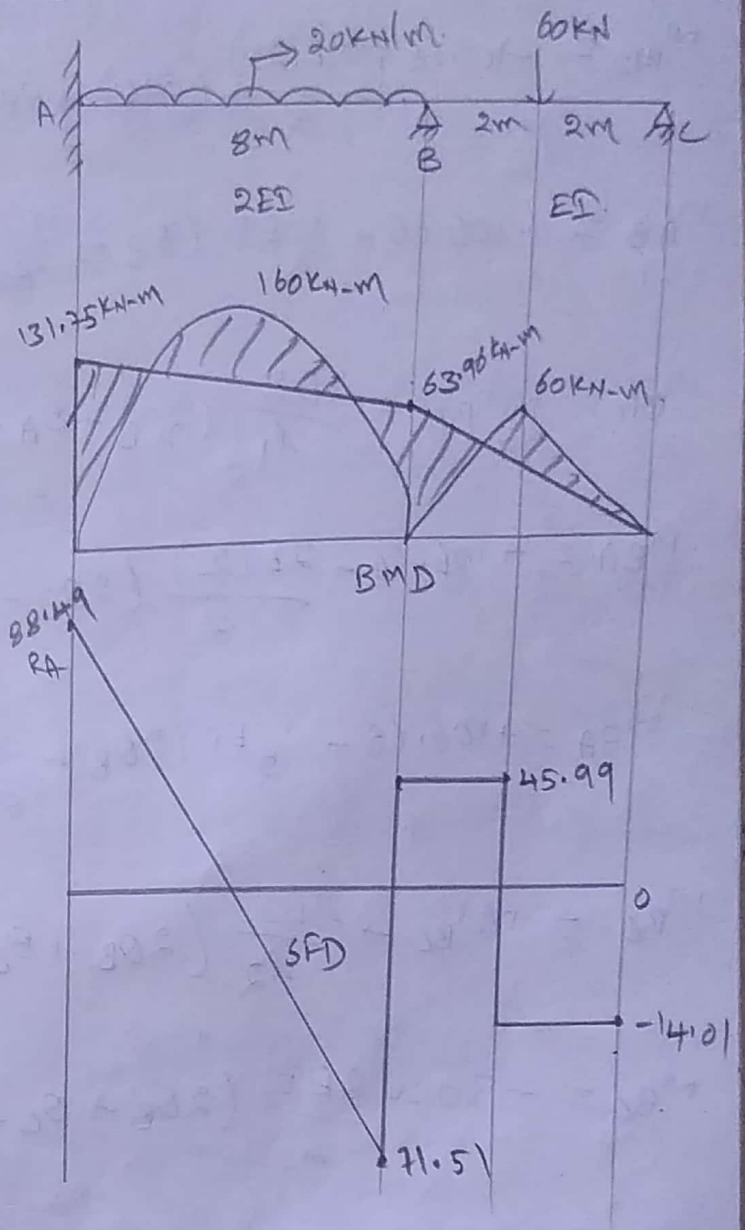
5) Analyse continuous beam as shown in fig by slope deflection method. If support B sinks by an amount of 10mm take EI is = 4000 KN-m².



step:1

fixed end moments:-

for span AB:-

$$M^f_{AB} = \frac{-w_1 l_1^2}{12} = -\frac{20 \times 64}{12}$$

$$M^f_{AB} = -106.66 \text{ KN-m}$$

$$M^f_{BA} = \frac{+w_1 l_1^2}{12} = +\frac{20 \times 64}{12}$$

$$M^f_{BA} = +106.66 \text{ KN-m}.$$

for span BC:-

~~$M^f_{BC} = \frac{-w_2 l_2^2}{12} - 60$~~

$$M^f_{BC} = \frac{-w_2 l_2}{8} = -\frac{60 \times 4}{8}$$

$$M^f_{BC} = -30 \text{ KN-m}.$$

$$M^f_{CB} = \frac{+w_2 l_2}{8} = +\frac{60 \times 4}{8}$$

$$M^f_{CB} = +30 \text{ KN-m}.$$

$\delta = 10\text{mm} \Rightarrow 0.01\text{m}$, $EI = 4000 \text{ KN-m}^2$.

step:2: slope deflection equation.

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$$M_{AB} = Mf_{AB} + \frac{2EI}{l_1}\left(2\theta_A + \theta_B - \frac{3\delta}{l_1}\right)$$

$$M_{AB} = -106.66 + \frac{2E(2I)}{8}\left(2(0) + \theta_B - \frac{3(0.01)}{8}\right)$$

$$M_{AB} = -106.66 + \frac{1}{2}EI\left(\theta_B - \frac{3}{800}\right) \rightarrow (1)$$

$$M_{BA} = Mf_{BA} + \frac{2EI}{l_1}\left(2\theta_B + \theta_A - \frac{3\delta}{l_1}\right)$$

$$M_{BA} = +106.66 + \frac{2E(2I)}{8}\left(2\theta_B - \frac{3(0.01)}{8}\right)$$

$$M_{BA} = +106.66 + \frac{1}{2}EI\left(2\theta_B - \frac{3}{800}\right) \rightarrow (2).$$

$$M_{BC} = Mf_{BC} + \frac{2EI}{l_2}\left(2\theta_B + \theta_C - \frac{3\delta}{l_2}\right)$$

$$M_{BC} = -30 + \frac{2EI}{4}\left(2\theta_B + \theta_C - \frac{3(-0.01)}{4}\right)$$

$$M_{BC} = -30 + \frac{1}{2}EI\left(2\theta_B + \theta_C + \frac{3}{400}\right) \rightarrow (3)$$

$$M_{CB} = Mf_{CB} + \frac{2EI}{l_2}\left(2\theta_C + \theta_B - \frac{3\delta}{l_2}\right)$$

$$M_{CB} = 30 + \frac{2EI}{4}\left(2\theta_C + \theta_B - \frac{3(-0.01)}{4}\right)$$

$$M_{CB} = 30 + \frac{1}{2}EI(2\theta_C) + \frac{1}{2}EI\theta_B + \frac{6EI}{1600}$$

$$M_{CB} = 30 + \frac{6\times4000}{1600} + EI\theta_C + 0.5EI\theta_B \rightarrow (4).$$

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Step:- 3. Equilibrium equation.

$\Sigma M_B = 0$; $M_{BA} + M_{BC} = 0$

$$106.66 + EI\theta_B - \frac{3}{1600}EI - 30 + EI\theta_B + 0.5EI\theta_C + \frac{3}{800}EI = 0$$

$$76.66 + 2EI\theta_B + 0.5EI\theta_C + \frac{3}{1600}EI = 0$$

$$2EI\theta_B + 0.5EI\theta_C + 76.66 + 7.5 = 0$$

$$2EI\theta_B + 0.5EI\theta_C = -84.16 \rightarrow (5)$$

$\Sigma M_C = 0$, $M_{CB} = 0$.

$$30 + 15 + EI\theta_C + 0.5EI\theta_B = 0.$$

$$0.5EI\theta_B + EI\theta_C = -45 \longrightarrow (6)$$

solve (5) & (6).

$EI\theta_B = -35.23$, $EI\theta_C = -27.38$.

$\therefore \theta_B = -8.80 \times 10^{-3}$; $\theta_C = -6.845 \times 10^{-3}$.

Step 4:- final moments.

Sub'te θ_B and θ_C in (1), (2), (3) & (4).

$$M_{AB} = -106.66 + \frac{4000}{2}\left(-8.80 \times 10^{-3} - \frac{3}{800}\right)$$

$\therefore M_{AB} = -131.75$ KN-m.

$M_{BA} = 63.96$ KN-m.

$$M_{BC} = -30 + 2000\left(2 \times -8.80 \times 10^{-3} - 6.845 \times 10^{-3} + \frac{3}{400}\right).$$

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$\therefore M_{BC} = -63.96\ KN\text{-}m.$

$$M_{CB} = +30 + \frac{6\times4000}{1600} + 4000\times(-6.845\times10^{-3}) + 0.5(4000)\times(-8.80\times10^{-3})$$

$\therefore M_{CB} = 0.02$

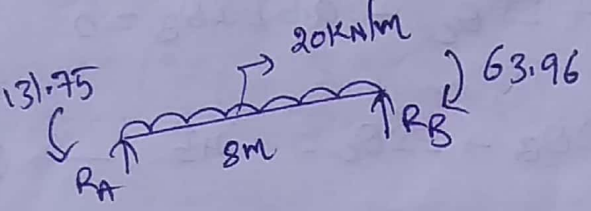
STEP-5: Max B.M.

for span AB $= \frac{wl^2}{8} = \frac{20\times64}{8} = 160\ KN\text{-}m.$

for span BC $= \frac{wl}{4} = \frac{60\times4}{4} = 60\ KN\text{-}m.$

STEP-6: SFD:

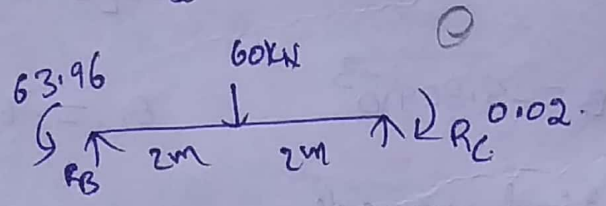
Consider span AB.



$\Sigma M_B = 0.$

$R_A \times 8 + 63.96 = 131.75 + 20\times8\times\frac{8}{2}.$

$R_A = 88.47\ KN$



consider span BC.

$\Sigma M_B = 0$

$R_C \times 4 + 63.96 = 0.02 + 60\times2$

$\therefore R_C = 14.01\ KN$ $\quad \therefore R_A + R_B + R_C = 60 + 20\times8 = 220$

$\therefore R_B = 117.5\ KN.$

$R_C = -14.01$, load 60KN, $-14.01 + 60 = 45.99.$

from 45.99 no load, R_B, $\therefore 45.99 - R_B = -71.51.$

from this UDL load, $-71.51 + 20\times8 = 88.49\ KN.$

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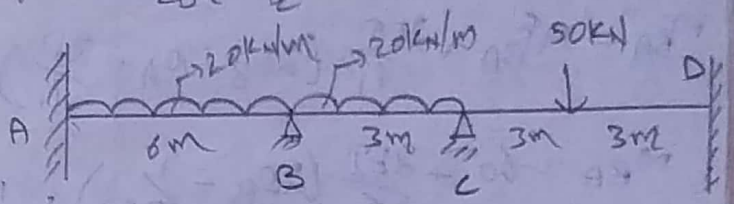
6) Analyse continuous beam as shown in fig by slope deflection method. If the support B sinks by an amount of 10mm. take $E = 2\times10^3\ N/mm^2$, $I = 16\times10^7\ mm^4$.

M_{AB}, M_{BA}, M_{BC} & $M_{CB} \rightarrow \delta \neq 0$; M_{CD} & $M_{DC} = \delta = 0$.



G.T:

$\delta = 10mm = 0.01\ m$.

$E = 2\times10^3\ N/mm^2 = 2\times10^3 \times \frac{10^{-3}\ KN}{10^{-6}\ m^2} = 2\times10^8\ KN/m^2$.

$I = 16\times10^7\ mm^4 = 16\times10^7 \times 10^{-12}\ m^4 = 16\times10^{-5}\ m^4$.

step:1: fixed end moments:- $\theta_A = \theta_D = 0$.

for span AB.

$M^f_{AB} = \frac{-wl^2}{12} = \frac{-20\times36}{12} = -60\ KN\text{-}m$

$M^f_{BA} = \frac{+wl^2}{12} = \frac{+20\times36}{12} = +60\ KN\text{-}m$

for span BC.

$M^f_{BC} = \frac{-wl^2}{12} = \frac{-20\times9}{12} = -15\ KN\text{-}m$

$M^f_{CB} = \frac{+wl^2}{12} = \frac{+20\times9}{12} = +15\ KN\text{-}m$

for span CD:-

$M^f_{CD} = \frac{-wl}{8} = \frac{-50\times6}{8} = -37.5\ KN\text{-}m$

$M^f_{DC} = \frac{+wl}{8} = \frac{+50\times60}{8} = +37.5\ KN\text{-}m$

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Step:2: slope deflection equation:-

$$M_{AB} = M^f_{AB} + \frac{2EI}{l_1}\left(2\theta_A + \theta_B - \frac{3\delta}{l_1}\right)$$

$$M_{AB} = -60 + \frac{2EI}{6}\left(0 + \theta_B - \frac{3(0.01)}{6}\right) = -60 + \frac{EI}{3}\left(\theta_B - \frac{1}{200}\right) \rightarrow 0$$

$$M_{BA} = M^f_{BA} + \frac{2EI}{l_1}\left(2\theta_B + \theta_A - \frac{3\delta}{l_1}\right)$$

$$M_{BA} = 60 + \frac{2EI}{6}\left(2\theta_B - \frac{1}{200}\right). \quad M_{BA} = 60 + \frac{EI}{3}\left(2\theta_B - \frac{1}{200}\right) \rightarrow (2)$$

$$M_{BC} = M^f_{BC} + \frac{2EI}{l_2}\left(2\theta_B + \theta_C - \frac{3\delta}{l_2}\right)$$

$$M_{BC} = -15 + \frac{2EI}{3}\left(2\theta_B + \theta_C - \frac{3(-0.01)}{3}\right)$$

$$M_{BC} = -15 + \frac{2EI}{3}\left(2\theta_B + \theta_C + \frac{1}{100}\right) \rightarrow (3)$$

$$M_{CB} = M^f_{CB} + \frac{2EI}{l_2}\left(2\theta_C + \theta_B - \frac{3\delta}{l_2}\right)$$

$$\therefore M_{CB} = 15 + \frac{2EI}{3}\left(2\theta_C + \theta_B + \frac{1}{100}\right) \rightarrow (4)$$

$$M_{CD} = M^f_{CD} + \frac{2EI}{l_3}\left(2\theta_C + \theta_D - \frac{3\delta}{l_3}\right)$$

$$M_{CD} = -37.5 + \frac{2EI}{6}\left(2\theta_C + 0 - \frac{3(0)}{6}\right)$$

$$M_{CD} = -37.5 + \frac{EI}{3}(2\theta_C) \rightarrow (5)$$

$$M_{DC} = M^f_{DC} + \frac{2EI}{l_3}\left(2\theta_D + \theta_C - \frac{3\delta}{l_3}\right)$$

$$\therefore M_{DC} = 37.5 + \frac{EI}{3}(\theta_C) \rightarrow (6)$$

Step:3: Equilibrium equation.

$\Sigma M_B = 0$. $M_{BA} + M_{BC} = 0$.

$\Sigma M_C = 0$

$M_{CB} + M_{CD} = 0$.

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$$60+\frac{2}{3}EI\theta_B-\frac{1}{600}EI+(-15)+\frac{4}{3}EI\theta_B+\frac{2}{3}EI\theta_C+\frac{2}{300}EI=0$$

$$45+2EI\theta_B+\frac{2}{3}EI\theta_C+\frac{1}{200}\times 2\times10^{8}\times16\times10^{-5}=0$$

$$205+64000\,\theta_B+21333.33\,\theta_C=0 \rightarrow (7)$$

$$15+\frac{4}{3}\times EI\theta_C+\frac{2}{3}EI\theta_B+\frac{2}{300}EI+(-37.5)+\frac{2}{3}EI\theta_C=0$$

$$-22.5+2EI\theta_C+\frac{2}{3}EI\theta_B+213.33=0$$

$$64000\,\theta_C+21333.33\,\theta_B=-190.83 \rightarrow (8)$$

solve (7) & (8).

$$\therefore \theta_B=-2.48\times10^{-3},\quad \theta_C=-2.15\times10^{-3}$$

step:4. Final moments:-

sub'te θ_B and θ_C in (1), (2), (3), (4), (5) and (6) equations.

$$M_{AB}=-60+\frac{2\times10^{8}\times16\times10^{-5}}{3}\left(-2.48\times10^{-3}-\frac{1}{200}\right).$$

$$\therefore M_{AB}=-139.78\ KN\text{-}m.$$

$$M_{BA}=60+\frac{2\times10^{8}\times16\times10^{-5}}{3}\left(2\times-2.48\times10^{-3}-\frac{1}{200}\right).$$

$$\therefore M_{BA}=-46.24\ KN\text{-}m.$$

$$M_{BC}=-15+\frac{64000}{3}\left(2\times-2.48\times10^{-3}-2.15\times10^{-3}+\frac{1}{100}\right)$$

$$\therefore M_{BC}=46.65\ KN\text{-}m.$$

$$M_{CB}=15+\frac{64000}{3}\left(2\times-2.15\times10^{-3}+(-2.48\times10^{-3})+\frac{1}{100}\right)$$

$$\therefore M_{CB}=83.69\ KN\text{-}m.$$

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$M_{CD} = -37.5 + \frac{32000}{3} \times 2(-2.15\times10^{-3})$, $\therefore M_{CD} = -83.36$ KN-m

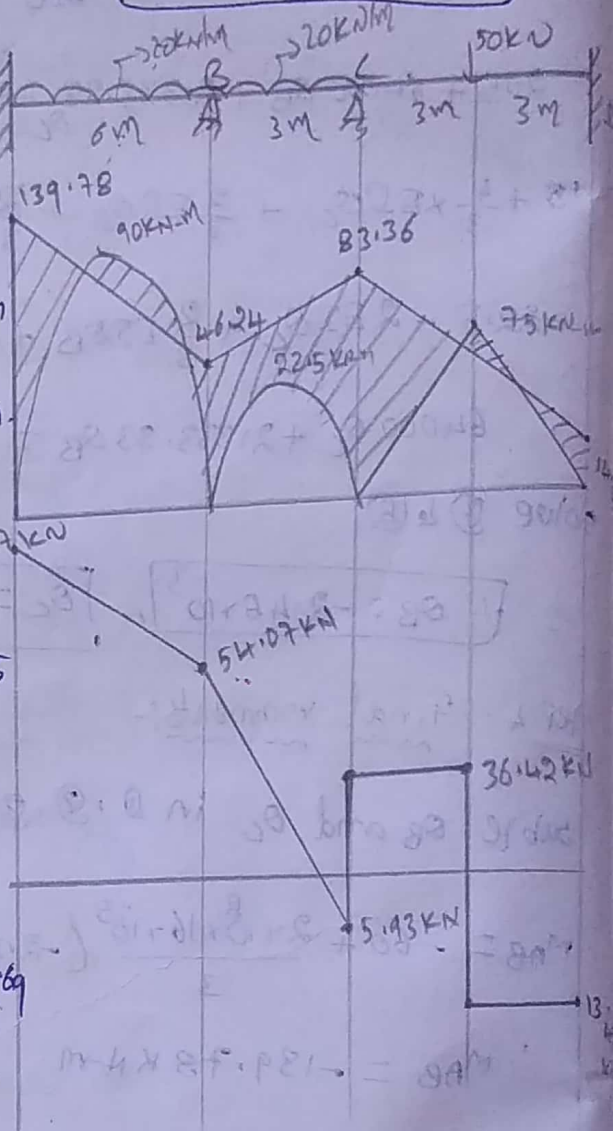
$M_{DC} = 37.5 + \frac{32000}{3}(-2.15\times10^{-3})$, $\therefore M_{DC} = 14.56$ KN-m

Step:5 Max B.M:

for span AB $= \frac{wl^2}{8} = \frac{20\times36}{8} = 90$ KN-m

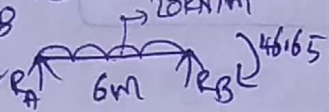
for span BC $= \frac{wl^2}{8} = \frac{20\times9}{8} = 22.5$ KN-m

for span CD $= \frac{wl}{4} = \frac{50\times6}{4} = 75$ KN-m



Step:6 SFD:

Consider span AB.

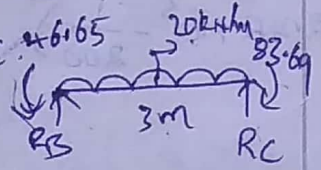


$\Sigma M_B = 0$.

$R_A \times 6 + 46.65 = 20\times6\times\frac{6}{2} + 139.78$

$\therefore R_A = 75.52$ KN

Consider span BC



$\Sigma M_B = 0$

$R_C \times 3 + 46.65 = 20\times3\times\frac{3}{2} + 83.69$

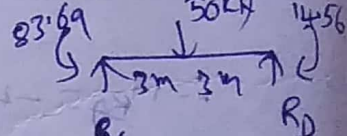
$\therefore R_C = 42.35$ KN.

$R_A + R_B + R_C + R_D = 20\times6 + 20\times3 + 50$

$= 230$.

$\therefore R_B = 98.65$ KN.

Consider span CD.



$\Sigma M_C = 0$.

$R_D \times 6 + 83.69 = 50\times3 + 14.56$

$\therefore R_D = 13.148$ KN

$R_D = -13.148$ KN, $-13.148 + 50 = 36.42$ KN, $36.42 - R_C$

$36.42 - 42.35 = -5.93$, $-5.93 + 20\times3 = 54.07$

$54.07 + 20 = 74.07$ KN.

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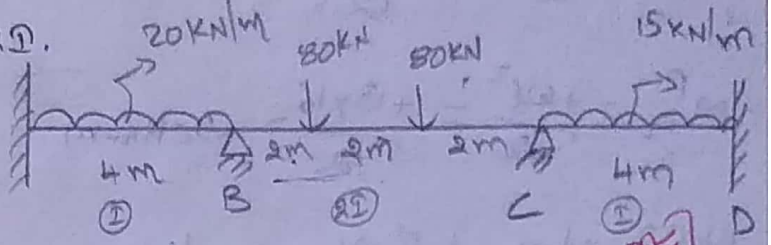
7) Analyse continuous beam as shown in fig. by slope deflection method.

$2I_{AB} = I_{BC} = 2I_{CD} = 2I$.



S.T.

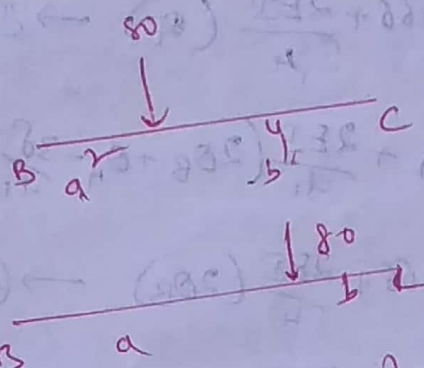
$2I_{AB} = I_{BC} = 2I_{CD} = 2I$.

$2I_{AB} = 2I$

$\therefore I_{AB} = I$

$I_{BC} = 2I$, $2I_{CD} = 2I$

$\therefore I_{CD} = I$



Step 1: Fixed end moments:-

For span AB.

$M^F_{AB} = \frac{-wl^2}{12} = \frac{-20\times16}{12} =$

$M^F_{AB} = -26.66\ KN\text{-}m$

$M^F_{BA} = \frac{+wl^2}{12} = \frac{+20\times16}{12}$

$M^F_{BA} = +26.66\ KN\text{-}m$

For span BC.

$M^F_{BC} = -\left[\frac{wab^2}{l^2} + \frac{wab^2}{l^2}\right] = -\left[\frac{80\times2\times4^2}{36} + \frac{80\times4\times2^2}{36}\right]$

$\therefore M^F_{BC} = -106.66\ KN\text{-}m$

$M^F_{CB} = +\left[\frac{wa^2b}{l^2} + \frac{wa^2b}{l^2}\right]$

$\therefore M^F_{CB} = +106.66\ KN\text{-}m$

~~$M^F_{BC} = -\left[\frac{wab^2}{8} + \frac{wab^2}{8}\right] = \left[\frac{80\times2\times4^2}{8} + \frac{80\times4\times2^2}{8}\right]$~~

~~$\therefore M^F_{BC} = -480\ KN\text{-}m$~~

~~$M^F_{CB} = +\left[\frac{wab^2}{8} + \frac{wab^2}{8}\right] = \left[\frac{80\times2\times4^2}{8} + \frac{80\times4\times2^2}{8}\right]$~~

~~$\therefore M^F_{CB} = +480\ KN\text{-}m$~~

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For span CD:

$$M^F_{CD} = \frac{-wl^2}{12} = -\frac{15\times16}{12} = -20\ KN\text{-}m$$

$$M^F_{DC} = \frac{+wl^2}{12} = +\frac{15\times16}{12} = +20\ KN\text{-}m.$$

Step.2: slope deflection equation.

$$M_{AB} = M^F_{AB} + \frac{2EI}{l_1}\left(2\theta_A + \theta_B - \frac{3\delta}{l_1}\right)$$

$$M_{AB} = -26.66 + \frac{2EI}{4}(\theta_B) \rightarrow (1)$$

$$M_{BA} = M^F_{BA} + \frac{2EI}{l_1}\left(2\theta_B + \theta_A - \frac{3\delta}{l_1}\right)$$

$$M_{BA} = +26.66 + \frac{2EI}{4}(2\theta_B) \rightarrow (2).$$

$$M_{BC} = M^F_{BC} + \frac{2EI}{12}\left(2\theta_B + \theta_C - \frac{3\delta}{12}\right)$$

$$M_{BC} = -106.66 + \frac{2E(2I)}{6}(2\theta_B + \theta_C)$$

$$M_{BC} = -106.66 + \frac{2}{3}EI(2\theta_B + \theta_C) \rightarrow (3).$$

$$M_{CB} = M^F_{CB} + \frac{2EI}{12}\left(2\theta_C + \theta_B - \frac{3\delta}{12}\right)$$

$$M_{CB} = +106.66 + \frac{2E(2I)}{6}(2\theta_C + \theta_B) \rightarrow (4).$$

$$M_{CB} = 106.66 + \frac{2}{3}EI(2\theta_C + \theta_B) \rightarrow (4).$$

$$M_{CD} = M^F_{CD} + \frac{2EI}{13}\left(2\theta_C + \theta_D - \frac{3\delta}{13}\right)$$

$$M_{CD} = -20 + \frac{2EI}{4}(2\theta_C) \rightarrow (5)$$

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$$M_{DC} = M^f_{DC} + \frac{2EI}{l_3}\left[2\theta_D + \theta_C - \frac{3\delta}{l_3}\right]$$

$$M_{DC} = 20 + \frac{2EI}{4}[\theta_C] \rightarrow (6).$$

Step:3 Equilibrium equation.

$\Sigma M_B = 0$.

$$M_{BA} + M_{BC} = 0$$

$$26.66 + EI\theta_B - 106.66 + \frac{2}{3}EI(2\theta_B + \theta_C) = 0.$$

$$-80 + \frac{7}{3}EI\theta_B + \frac{2}{3}EI\theta_C = 0. \rightarrow (7)$$

$\Sigma M_C = 0$

$$M_{CB} + M_{CD} = 0.$$

$$106.66 + \frac{2}{3}EI(2\theta_C + \theta_B) - 20 + EI\theta_C = 0.$$

$$86.66 + \frac{7}{3}EI\theta_C + \frac{2}{3}EI\theta_B = 0 \rightarrow (8)$$

solve (7) & (8). $\frac{7}{3} = 2.33$, $\frac{2}{3} = 0.66$

$\therefore EI\theta_B = 48.78$, $EI\theta_C = -51.01$

Step:4: Final moments:-

Sub'te $EI\theta_B$ and $EI\theta_C$ in eqn. (1), (2), (3), (4) (5) and (6).

$$M_{AB} = -26.66 + \frac{1}{2}EI\theta_B$$

$$\therefore M_{AB} = -26.66 + \frac{1}{2}(48.78)$$

$$\therefore M_{AB} = -2.27 \text{ KN-m}.$$

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$M_{BA} = 26.66 + EI\theta_B$.

$M_{BA} = 26.66 + 48.78$, $\therefore M_{BA} = 75.44$ KN-m

$M_{BC} = -106.66 + \frac{4}{3} EI\theta_B + \frac{2}{3} EI\theta_C$.

$M_{BC} = -106.66 + \frac{4}{3}(48.78) + \frac{2}{3}(-51.01)$

$\therefore M_{BC} = -75.62$ KN-m.

$M_{CB} = +106.66 + \frac{4}{3} EI\theta_C + \frac{2}{3} EI\theta_B$.

$M_{CB} = +106.66 + \frac{4}{3}(-51.01) + \frac{2}{3}(48.78)$

$\therefore M_{CB} = 71.16$ KN-m.

$M_{CD} = -20 + EI\theta_C = -20 - 51.01$

$\therefore M_{CD} = -71.01$ KN-m.

$M_{DC} = 20 + \frac{1}{2} EI\theta_C = 20 + \frac{1}{2}(-51.01)$

$\therefore M_{DC} = -5.505$ KN-m

Step:-5: Max B.M:-

for span AB $= \frac{wl^2}{8} = \frac{20 \times 16}{8} = 40$ KN-m.

for span BC $= \frac{wab}{l} + \frac{wab}{l} =$

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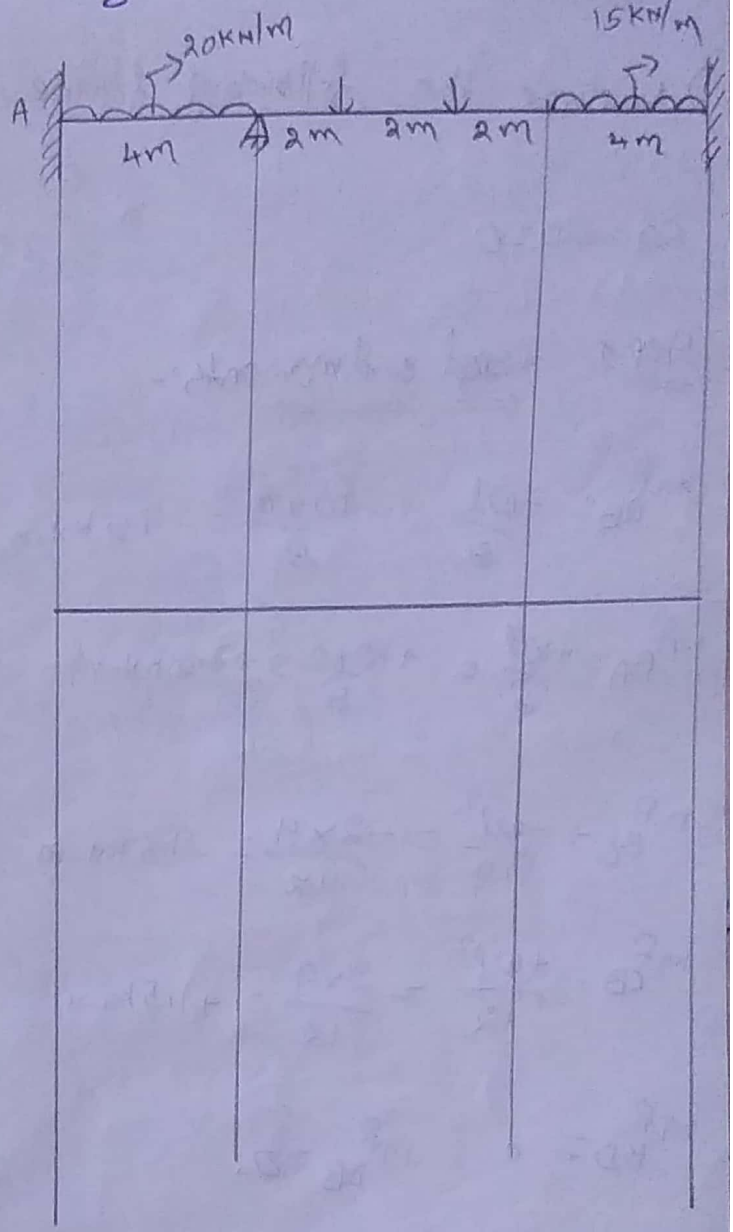
for span CD $= \frac{wl^2}{8} = \frac{15 \times 16}{8} = 30$ KN-m.

step: 6: SFD:-



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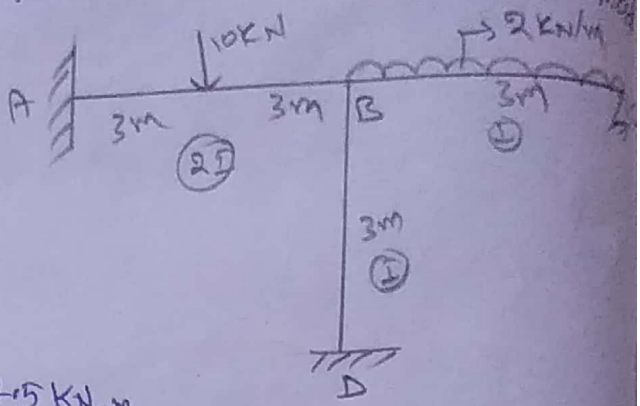
Analysis of frames by slope deflection Method.

1) Analyse the following frame by slope deflection method.



$\theta_A = \theta_D = 0$.

Step-1: Fixed end moments:-

$$M^F_{AB} = \frac{-wl}{8} = \frac{-10 \times 6}{8} = -7.5 \text{ KN-m}$$

$$M^F_{BA} = \frac{+wl}{8} = \frac{+10 \times 6}{8} = +7.5 \text{ KN-m}.$$

$$M^F_{BC} = \frac{-wl^2}{12} = \frac{-2 \times 9}{12} = -1.5 \text{ KN-m}$$

$$M^F_{CB} = \frac{+wl^2}{12} = \frac{2 \times 9}{12} = +1.5 \text{ KN-m}$$

$$M^F_{BD} = 0 \;,\; M^F_{DB} = 0.$$

Step:2: slope deflection equation.

$$M_{AB} = M^F_{AB} + \frac{2EI}{l_1}\left(2\theta_A + \theta_B - \frac{3\delta}{l_1}\right)$$

$$M_{AB} = -7.5 + \frac{4EI}{6}(\theta_B) \rightarrow (1)$$

$$M_{BA} = M^F_{BA} + \frac{2EI}{l_1}\left(2\theta_B + \theta_A - \frac{3\delta}{l_1}\right)$$

$$M_{BA} = +7.5 + \frac{4EI}{6}(2\theta_B) \rightarrow (2)$$

$$M_{BC} = M^F_{BC} + \frac{2EI}{l_2}\left(2\theta_B + \theta_C - \frac{3\delta}{l_2}\right)$$

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$$M_{BC} = -1.5 + \frac{2EI}{3}(2\theta_B + \theta_C) \rightarrow (3)$$

$$M_{CB} = M^f_{CB} + \frac{2EI}{l_2}(2\theta_C + \theta_B - \frac{3\delta}{l_2})$$

$$M_{CB} = 1.5 + \frac{2EI}{3}(2\theta_C + \theta_B) \rightarrow (4)$$

$$M_{BD} = M^f_{BD} + \frac{2EI}{l_3}(2\theta_B + \theta_D - \frac{3\delta}{l_3})$$

$$M_{BD} = 0 + \frac{2EI}{3}(2\theta_B) \rightarrow (5)$$

$$M_{DB} = M^f_{DB} + \frac{2EI}{l_3}(2\theta_D + \theta_B - \frac{3\delta}{l_3})$$

$$M_{DB} = 0 + \frac{2EI}{3}(\theta_B) \rightarrow (6)$$

STEP:3. Equilibrium equations.

$\Sigma M_B = 0$.

$$M_{BA} + M_{BC} + M_{BD} = 0$$

$$7.5 + \frac{4EI\theta_B}{3} - 1.5 + \frac{4EI\theta_B}{3} + \frac{2EI\theta_C}{3} + \frac{4EI\theta_B}{3} = 0$$

$$\frac{12}{3}EI\theta_B + \frac{2}{3}EI\theta_C = -6$$

$$12EI\theta_B + 2EI\theta_C = -18 \rightarrow (7)$$

$\Sigma M_C = 0$.

$$M_{CB} = 1.5 + \frac{4EI\theta_C}{3} + \frac{2EI\theta_B}{3} = 0$$

$$4EI\theta_C + 2EI\theta_B = -4.5 \rightarrow (8)$$

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solve ⑦ & ⑧.

$\therefore EI\theta_B = -1.43$, $EI\theta_C = -0.40$.

Step 4: final moments.

Sub $EI\theta_B$ and $EI\theta_C$ in ①, ②, ③, ④, ⑤ & ⑥ equations.

$M_{AB} = -7.5 + \frac{4}{6} EI\theta_B = -7.5 + \frac{4}{6}(-1.43)$.

$\therefore M_{AB} = -8.45$ KN-m

$M_{BA} = 7.5 + \frac{4}{6} EI.2\theta_B = 7.5 + \frac{4}{3}(-1.43)$

$\therefore M_{BA} = 5.59$ KN-m

$M_{BC} = -1.5 + \frac{4}{3} EI\theta_B + \frac{2}{3} EI\theta_C$.

$M_{BC} = -1.5 + \frac{4}{3}(-1.43) + \frac{2}{3}(-0.40)$.

$\therefore M_{BC} = -3.67$ KN-m.

$M_{CB} = 1.5 + \frac{4}{3} EI\theta_C + \frac{2}{3} EI\theta_B$

$M_{CB} = 1.5 + \frac{4}{3}(-0.40) + \frac{2}{3}(-1.43)$.

$\therefore M_{CB} = 0.01$ KN-m.

$M_{BD} = \frac{4}{3} EI\theta_B = \frac{4}{3}(-1.43) \Rightarrow \therefore M_{BD} = -1.91$ KN-m.

$M_{DB} = \frac{2}{3} EI\theta_B = \frac{2}{3}(-1.43)$. $\therefore M_{DB} = -0.95$ KN-m

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Step:5 Max B.M.

for span AB = $\frac{wl}{4} = \frac{10 \times 6}{4} = 15$ KN-m.

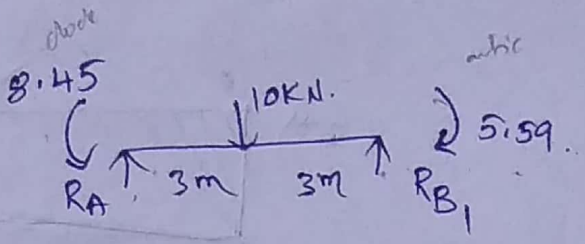
for span BC = $\frac{wl^2}{8} = \frac{2 \times 9}{8} = 2.25$ KN-m.

for span BD = 0.

26/04/2020

Step:6: SFD.

Consider span AB.



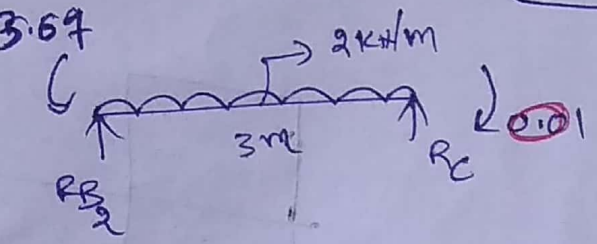
$\Sigma M_B = 0$.

$R_A \times 6 + 5.59 = 10 \times 3 + 8.45$.

$\therefore R_A = 5.47$ KN

$R_{B_1} = 10 - 5.47$; $\therefore R_{B_1} = 4.53$ KN

Consider span BC.

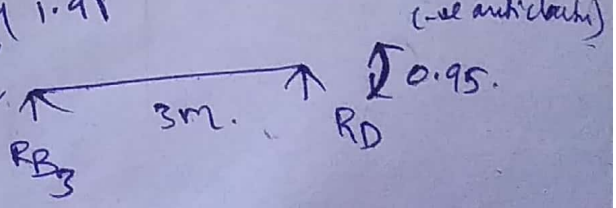


$\Sigma M_B = 0$.

$R_C \times 3 + 3.69 = 2 \times 3 \times \frac{3}{2} + 0.01$

$\therefore R_C = 1.77$ KN ; $R_{B_2} = 6 - 1.77 = 4.29$ KN.

Consider span BD.



$\Sigma M_B = 0$.

$R_D \times 3 + 5.59 = 0.95$; $R_D \times 3 = 0.95 + 1.91$

$\therefore R_D = 0.95$ KN ; R_{B_3} = load - total dist

R_D & R_{B_3} couples on couple. $\therefore R_D = R_{B_3}$

$R_{B_3} = -0.95$ KN.

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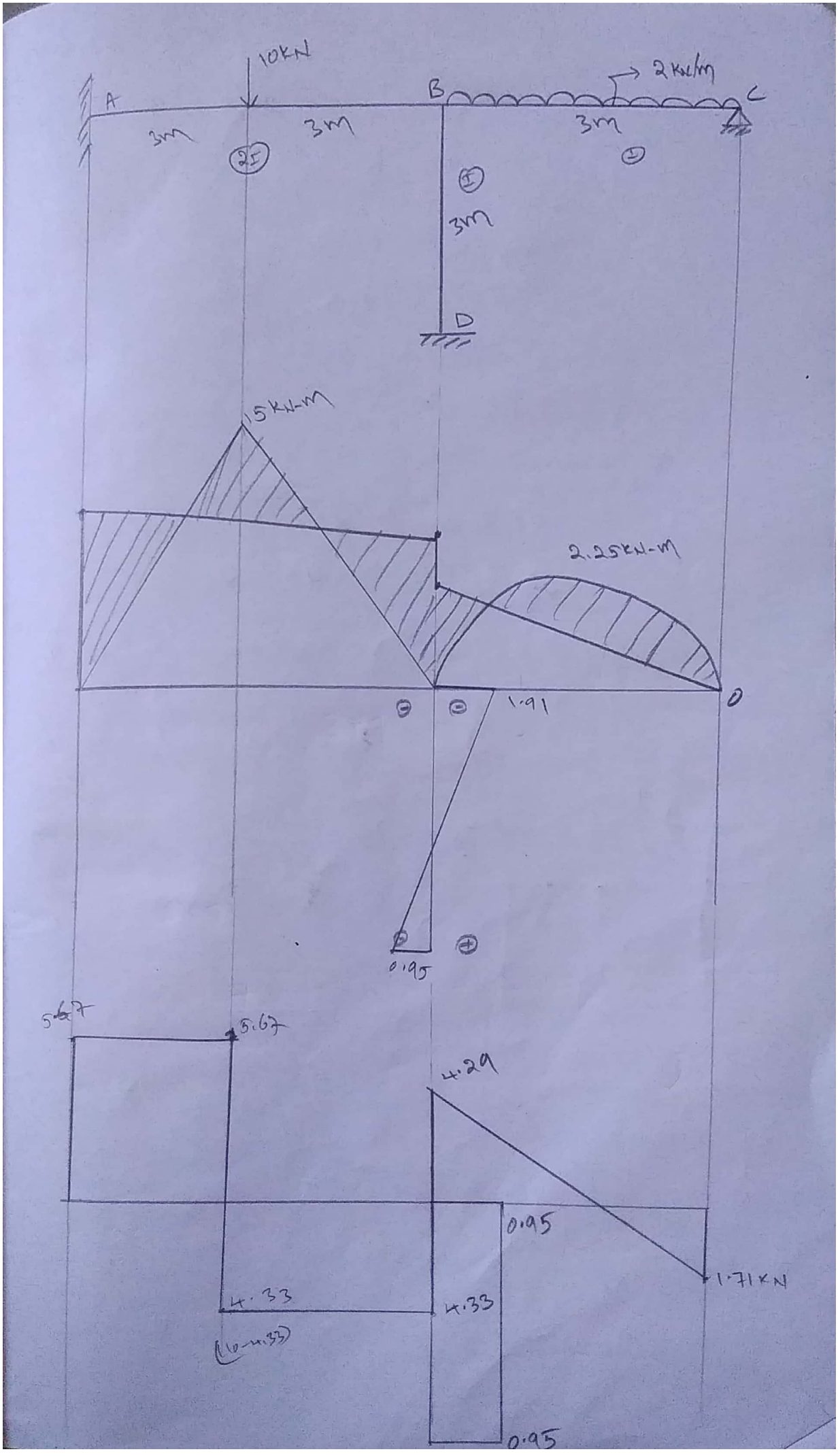

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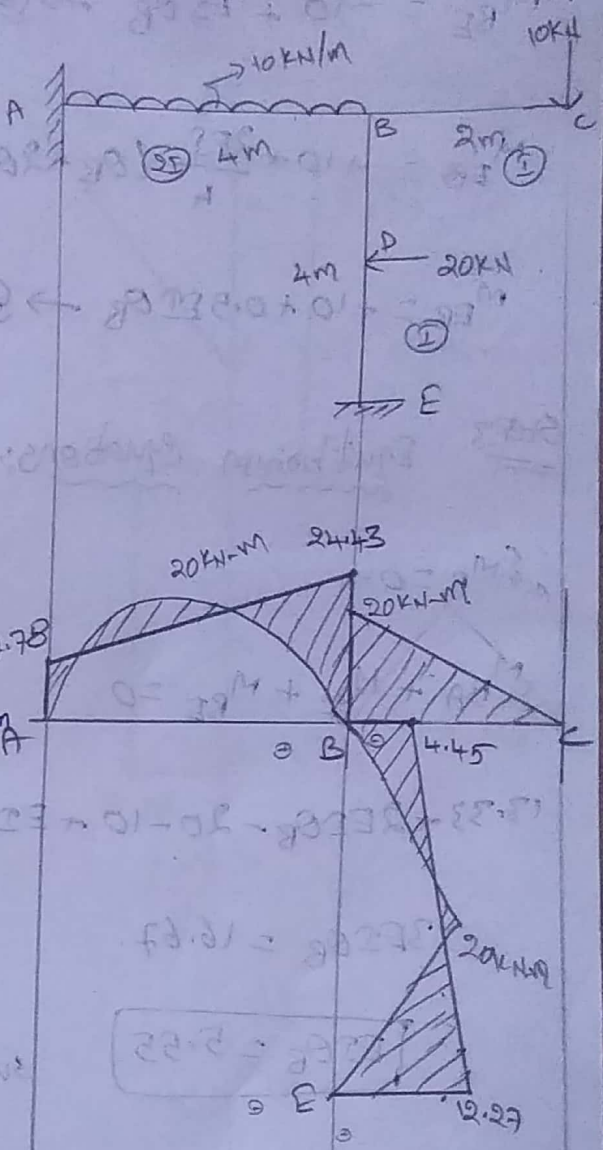
IV - Module

2) Analyse the following frame by slope deflection method.



Step: 1:

Fixed end Moments:-

$$M^f_{AB} = -\frac{wl^2}{12} = -\frac{10 \times 16}{12}$$

$$M^f_{AB} = -13.33 \text{ KN-m}.$$

$$M^f_{BA} = +\frac{10 \times 16}{12} = +13.33 \text{ KN-m}.$$

$$M^f_{BE} = -\frac{wl}{8} = -\frac{20 \times 4}{8} = -10 \text{ KN-m}$$

$$M^f_{EB} = +\frac{wl}{8} = +10 \text{ KN-m}.$$

$$\theta_A = \theta_E = 0.$$

Step: 2: Slope deflection equation.

$$M_{AB} = -13.33 + \frac{2E(2I)}{4}\left(2\theta_A + \theta_B - \frac{3\delta}{l_1}\right)$$

$$M_{AB} = -13.33 + EI\theta_B \rightarrow (1)$$

$$M_{BA} = +13.33 + \frac{2E(2I)}{4}\left(2\theta_B + \theta_A - \frac{3\delta}{l_1}\right)$$

$$M_{BA} = 13.33 + 2EI\theta_B \rightarrow (2)$$

$$M_{BC} = -20.$$

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$$M_{BE} = -10 + \frac{2EI}{4}\left(2\theta_B + \theta_E - \frac{3\delta}{l_2}\right)$$

$$M_{BE} = -10 + EI\theta_B \rightarrow (3)$$

$$M_{EB} = +10 + \frac{2EI}{4}\left(\theta_B + 2\theta_E - \frac{3\delta}{l_2}\right)$$

$$M_{EB} = +10 + 0.5EI\theta_B \rightarrow (4)$$

Step:3: Equilibrium equations:-

$$\Sigma M_B = 0.$$

$$M_{BA} + M_{BC} + M_{BE} = 0.$$

$$13.33 + 2EI\theta_B - 20 - 10 + EI\theta_B = 0$$

$$3EI\theta_B = 16.67.$$

$$\boxed{EI\theta_B = 5.55}$$ sub $EI\theta_B$ in (1), (2), (3) & (4)

Step:4 Final moments:-

$$M_{AB} = -13.33 + 5.55 = -7.78 \text{ KN-m}$$

$$M_{BA} = 13.33 + 2(5.55) = 24.43 \text{ KN-m}.$$

$$M_{BC} = -20. \text{ KN-m}$$

$$M_{BE} = -10 + 5.55 = -4.45 \text{ KN-m}.$$

$$M_{EB} = +10 + 0.5(5.55) = 12.77 \text{ KN-m}.$$

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Step:5. Max B.M:-

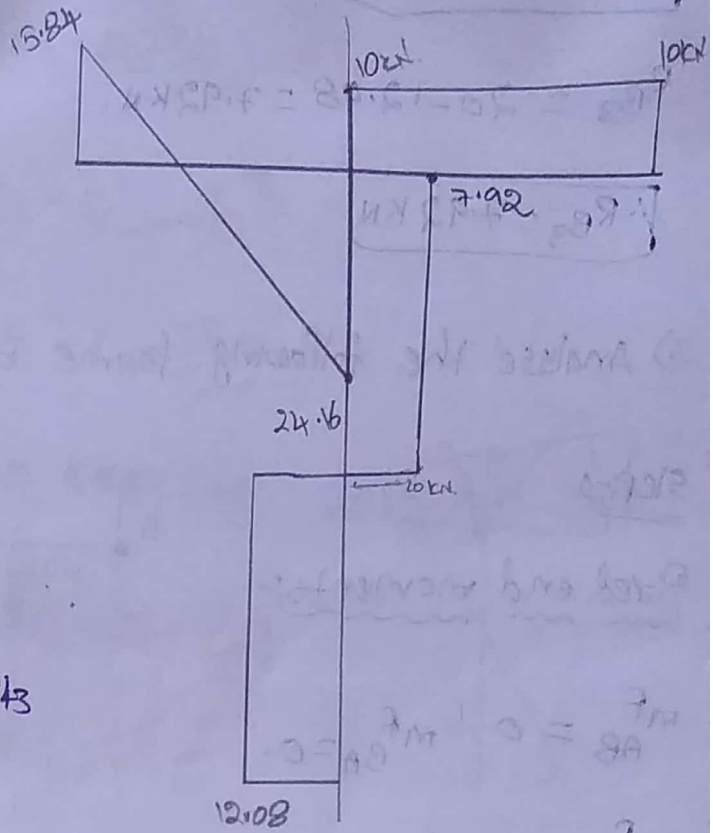
for span AB $= \frac{10 \times 4^2}{8}$

$= 20$ KN-m.

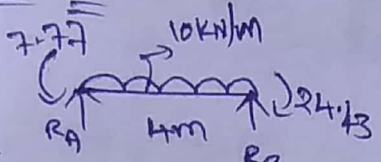
for span BE $= \frac{20 \times 4}{4}$

$= 20$ KN-m.

Step:6. SFD.



for span AB.



$\Sigma M_B = 0$.

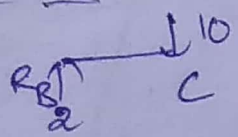
$R_A \times 4 + 24.43 = 10 \times 4 \times \frac{4}{2}$

$\therefore R_A = 15.84$ KN

$R_{B_1} = 40 - 15.84$

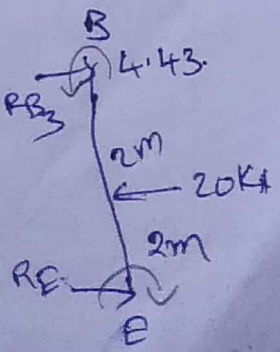
$R_{B_1} = 24.16$ KN.

for span BC:-



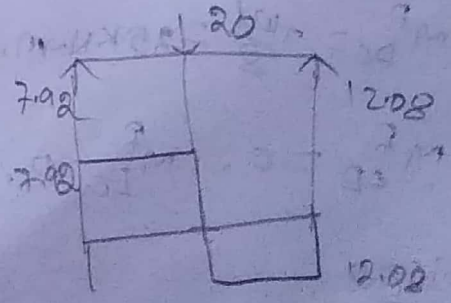
$R_{B_2} = 10$ KN.

for span BE:-



$\Sigma M_B = 0$.

for BC SFD



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$R_E \times 4 + 4.43 = 20 \times 2 + 12.77$.

$\boxed{R_E = 12.08\ KN.}$

$R_{B_3} = 20 - 12.08 = 7.92 KN.$

$\boxed{\therefore R_{B_3} = 7.92\ KN.}$

3) Analyse the following frame by slope deflection method.

Step:-1

Fixed end moments:-

$M^F_{AB} = 0.\quad M^F_{BA} = 0.$

$M^F_{BC} = \frac{-wl^2}{12} = \frac{-20 \times 9}{12}$

$M^F_{BC} = -15\ KN\text{-}m$

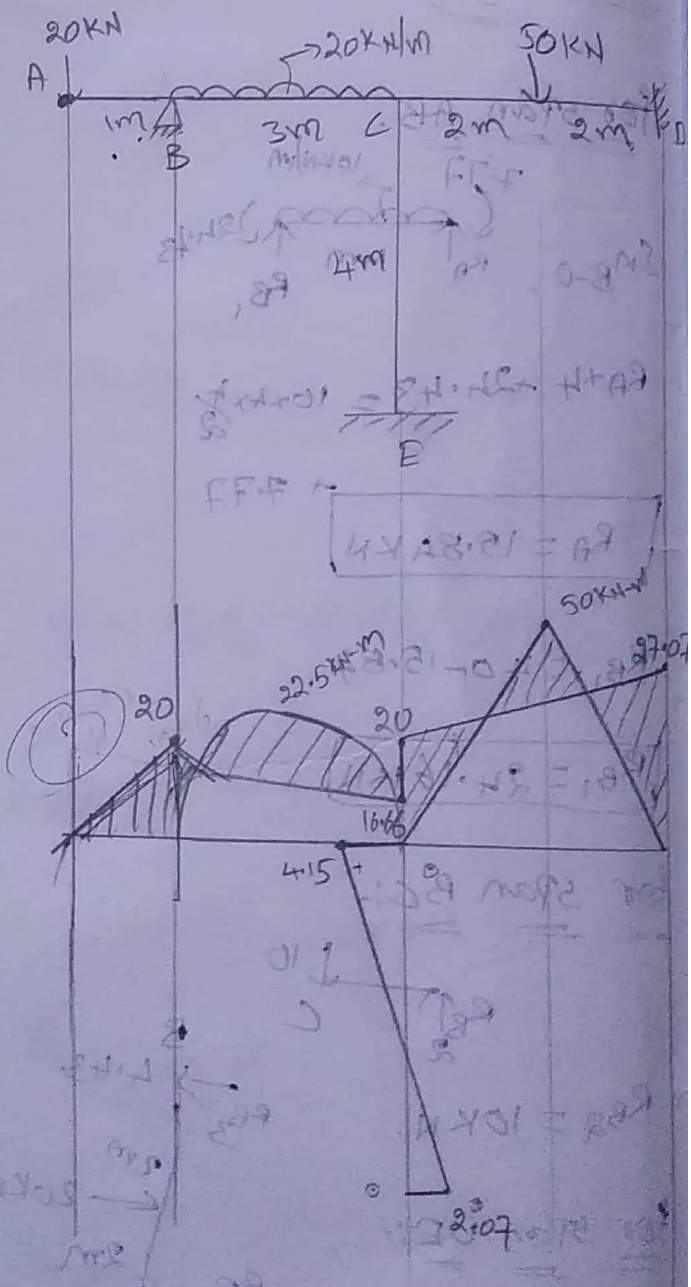
$M^F_{CB} = \frac{+wl^2}{12} = +15\ KN\text{-}m.$

$M^F_{CD} = \frac{-wl}{8} = \frac{-50 \times 4}{8}$

$M^F_{CD} = -25\ KN\text{-}m.$

$M^F_{DC} = \frac{+wl}{8} = 25\ KN\text{-}m.$

$M^F_{CE} = 0.\quad M^F_{EC} = 0.$



Step:2

Slope deflection equation:-

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$M_{AB} = -20 \times 1 = -20\ KN\text{-}m$

$M_{BA} = +20\ KN\text{-}m$

$M_{BC} = M^f_{BC} + \frac{2EI}{l}\left(2\theta_B + \theta_C - \frac{3\delta}{l}\right)$

$M_{BC} = -15 + \frac{2EI}{4}(2\theta_B + \theta_C) \rightarrow (1)$

$M_{CB} = M^f_{CB} + \frac{2EI}{l}\left(2\theta_C + \theta_B - \frac{3\delta}{l}\right)$

$M_{CB} = +15 + \frac{2EI}{3}(2\theta_C + \theta_B) \rightarrow (2)$



$M_{CD} = M^f_{CD} + \frac{2EI}{l}\left(2\theta_C + \theta_D - \frac{3\delta}{l}\right)$

$M_{CD} = -25 + \frac{2EI}{4}(2\theta_C) \rightarrow (3)$

$M_{DC} = M^f_{DC} + \frac{2EI}{l}\left(2\theta_D + \theta_C - \frac{3\delta}{l}\right)$

$M_{DC} = +25 + \frac{2EI}{4}(\theta_C) \rightarrow (4)$

$M_{CE} = 0 + \frac{2EI}{4}(2\theta_C + \theta_E)$ $M_{EC} = 0 + \frac{2EI}{4}(2\theta_E + \theta_C) = \frac{1}{2}EI\theta_C \rightarrow (6)$

Step:3 $= EI\theta_C \rightarrow (5)$

Equilibrium equation:

$\Sigma M_B = 0$

$M_{BA} + M_{BC} = 0$

$20 - 15 + \frac{4EI}{3}\theta_B + \frac{2}{3}EI\theta_C = 0$

$5 + \frac{4}{3}EI\theta_B + \frac{2}{3}EI\theta_C = 0 \rightarrow (7)$

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$\Sigma M_C = 0$.

$M_{CB} + M_{CD} + M_{CE} = 0$

$15 + \frac{4}{3}EI\theta_C + \frac{2}{3}EI\theta_B + (-25) + EI\theta_C + 0 = 0$

$+EI\theta_C$

$-10 + \frac{4}{3}EI\theta_C + \frac{2}{3}EI\theta_B + 2EI\theta_C = 0$

$\frac{2}{3}EI\theta_B + \frac{10}{3}EI\theta_C = 10 \rightarrow (6)$

solve (4) & (6):

$\therefore EI\theta_B = -5.80$

$EI\theta_C = 4.15$.

subst $EI\theta_B$ and $EI\theta_C$ in (1), (2), (3), (4), (5), (6).

Step:4: final moments:-

$M_{AB} = +20\,KN\text{-}m$, $M_{BA} = +20\,KN\text{-}m$

$M_{BC} = -15 + \frac{4}{3}EI\theta_B + \frac{2}{3}EI\theta_C$

$M_{BC} = -15 + \frac{4}{3}(-5.80) + \frac{2}{3}(4.15)$

$\therefore M_{BC} = -19.96\,KN\text{-}m.$

$M_{CB} = +15 + \frac{4}{3}EI\theta_C + \frac{2}{3}EI\theta_B = 15 + \frac{4}{3}(4.15) + \frac{2}{3}(-5.80)$

$\therefore M_{CB} = 16.66\,KN\text{-}m.$

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$M_{CD} = -25 + EI\theta_C = -25 + 4.15$

$\boxed{M_{CD} = -20.85\ KN\text{-}m}$

$M_{DC} = +25 + \frac{1}{2}EI\theta_C = 25 + \frac{1}{2}(4.15)$

$\boxed{M_{DC} = 27.07\ KN\text{-}m}$

$M_{CE} = EI\theta_C = 4.15\ KN\text{-}m.$

$M_{EC} = \frac{1}{2}(EI\theta_C) = \frac{1}{2}(4.15).$

$\boxed{M_{EC} = 2.07\ KN\text{-}m.}$

Step:5: Max B.M:

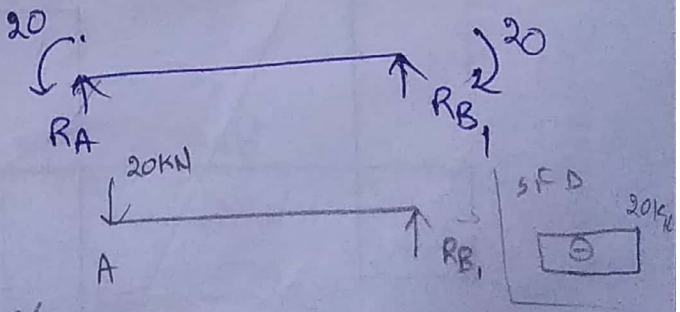
for span BC $= \frac{wl^2}{8} = \frac{20\times 9}{8} = 22.5\ KN\text{-}m.$

for span CD $= \frac{wl}{4} = \frac{50\times 4}{4} = 50\ KN\text{-}m$, for CE $= 0$.

Step:6. SFD.

Consider span AB:-

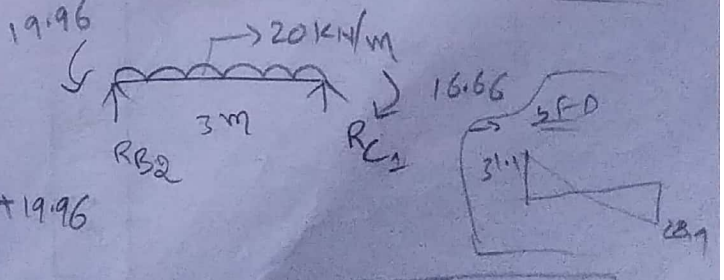
$R_B = 20\ KN.$



for span BC.

$\Sigma M_C = 0.$

$R_{B_2}\times 3 + 16.66 = 20\times 3\times\frac{3}{2} + 19.96$



$\boxed{R_{B_2} = 31.1\ KN}$, $R_{C_1} = 20\times 3 - 31.1$, $\boxed{R_{C_1} = 28.9\ KN}$

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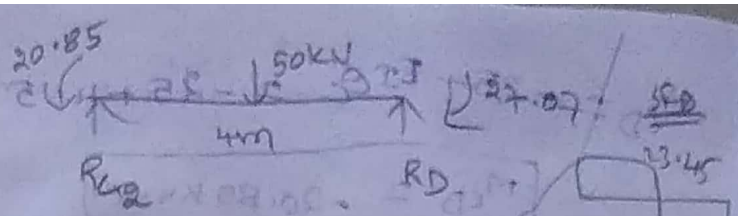
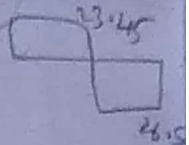
for span CD.

$\Sigma M_C = 0$

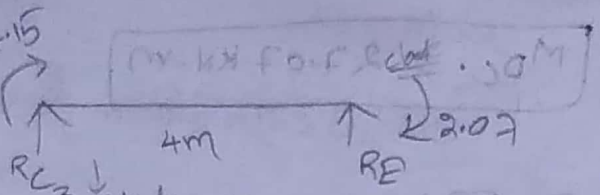




$R_D \times 4 + 20.85 = 50 \times 2 + 27.07$

$R_D = 26.55$ KN $\Rightarrow R_{C2} = 50 - 26.55 \Rightarrow$ **$R_{C2} = 23.45$ KN**

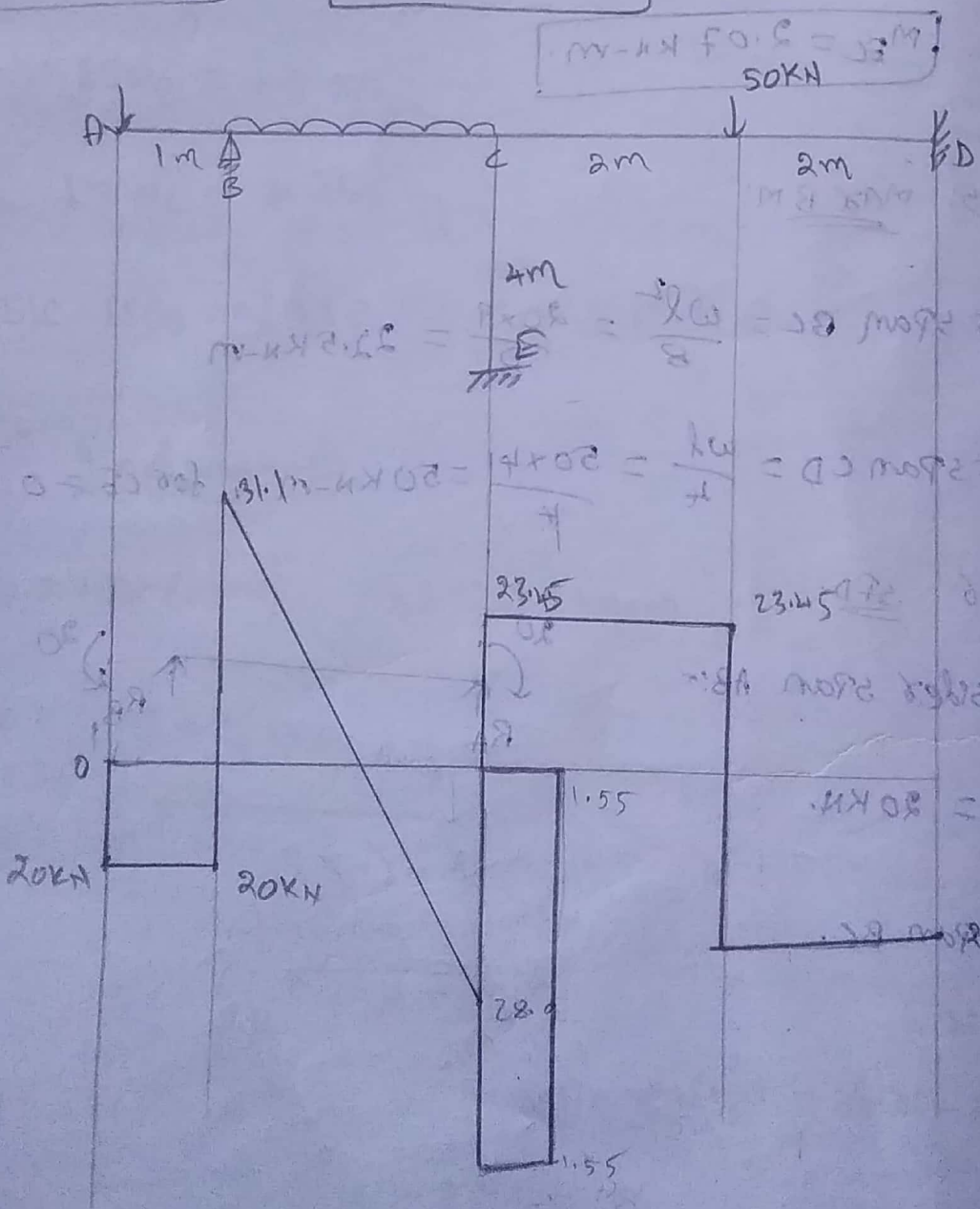
for span CE



$\Sigma M_C = 0$

$R_E \times 4 = 4.15 + 2.07 = 6.22$

$R_E = 1.55$ KN **$R_{C3} = -1.55$ KN**



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03/03/2020.

sway → Movement of joints.

frames with side sway:-

Reasons due to sway:-

1). Due to unsymmetrical loading.

2) Due to sinking of supports.

3). Due to unsymmetrical shape of column.

4) Due to different end conditions.

5) Due to non-uniform cross section of column.

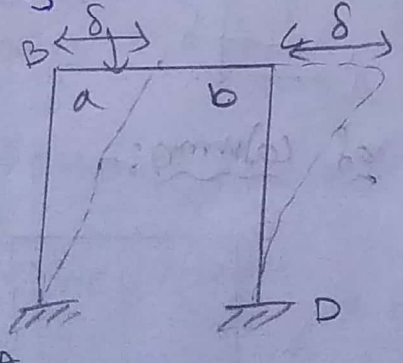
6) Due to horizontal loading on column.

7). Due to all the above.

*If a frame subjected to different loading conditions the members in the frames ex. subjected to sway

'sway means movement of joints.

1).

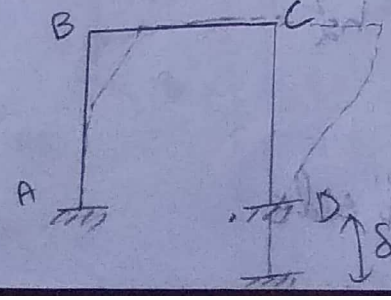


$$M^f_{BC} = \frac{Wab^2}{l^2}$$

$$M^f_{CB} = \frac{Wa^2b}{l^2}$$

$\therefore M^f_{CB} < M^f_{BC}$.

2).



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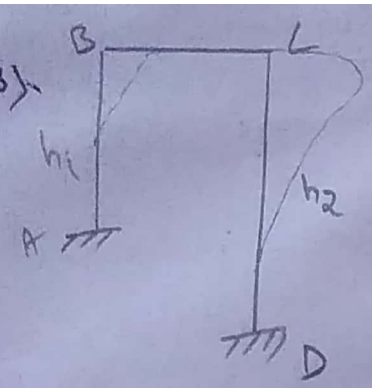
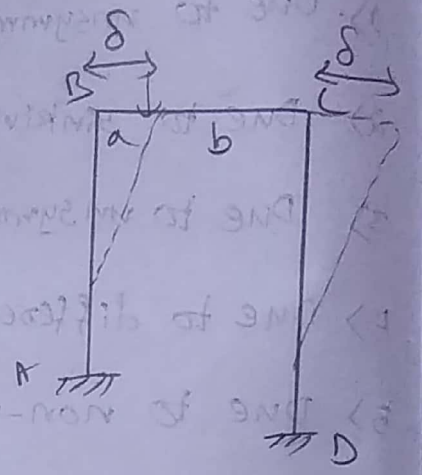
3).



1. Due to unsymmetrical loading:-



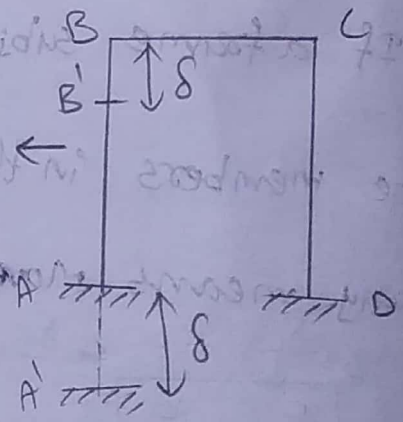
sway towards less movement

$$M^f_{BC} = \frac{wab^2}{l^2} \quad , \quad M^f_{CB} = \frac{wa^2b}{l^2}$$

$$M^f_{BC} > M^f_{CB}.$$

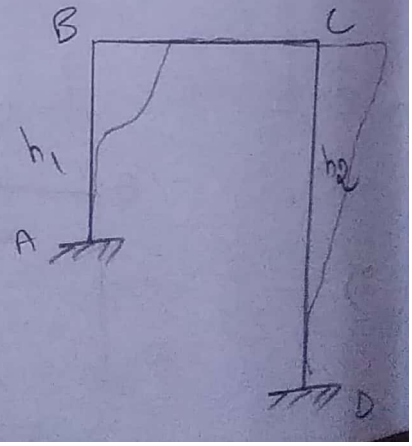
∴ sway towards right (having less moment).

2). Due to sinking of supports:-



If support A sinks by an amount of δ. then the sway towards left.

3) Due to unsymmetrical shape of column:-



$$M_B = \frac{6EI\delta}{h_1^2} \quad , \quad M_C = \frac{6EI\delta}{h_2^2}$$

$$M_C < M_B.$$

∴ sway towards right side.

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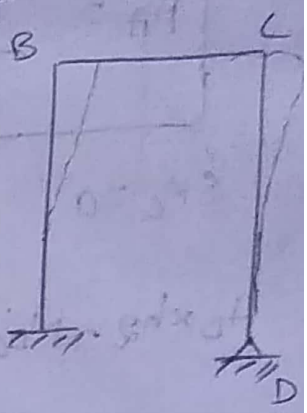
4) Due to different end conditions:-



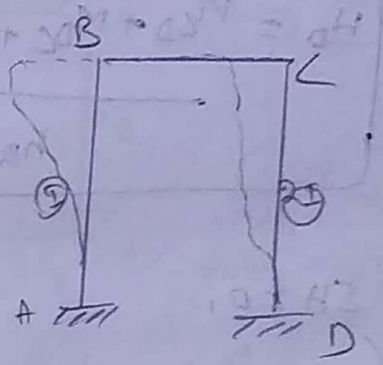
Sway towards the Hinge side.

5) Due to non-uniform c/s of column:-

$M_B = \frac{6E(I)\delta}{l^2}$ & $M_C = \frac{6E(2I)\delta}{l^2}$

$M_B < M_C$

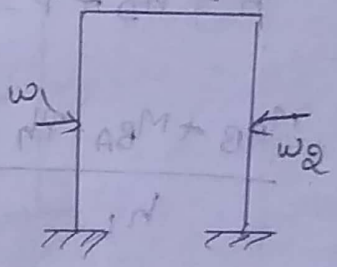
∴ Sway towards left side.



6) Due to horizontal loading on column:-

Sway towards Netforce direction.

$\omega_1 < \omega_2$.



Shear equation for different cases:-

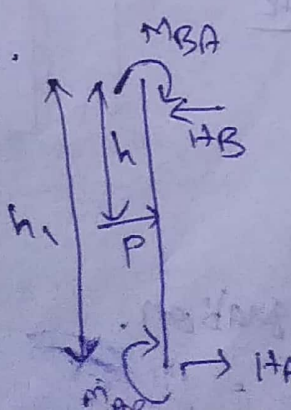
Shear equation:- Additional equilibrium equation is known as shear equation.

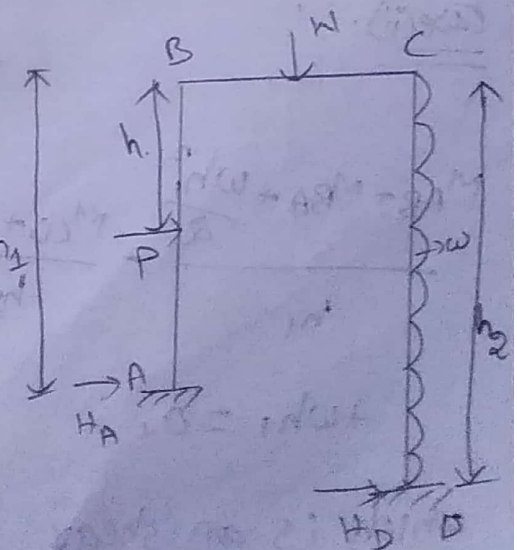
assume Posibve moment on the column.

$\Sigma M_B = 0$.

$H_A \times h_1 - M_{AB}$

$-M_{BA} + Ph = 0$





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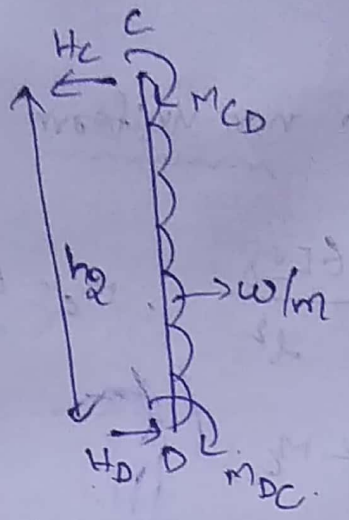
$$H_A = \frac{M_{AB} + M_{BA} - Ph}{h_1}$$

$\Sigma M_C = 0$



$$H_D \times h_2 - M_{DC} - M_{CD} - \frac{\omega h_2^2}{2} = 0$$

$$H_D = \frac{M_{CD} + M_{DC} + \frac{\omega h_2^2}{2}}{h_2}$$

$\Sigma H = 0,$

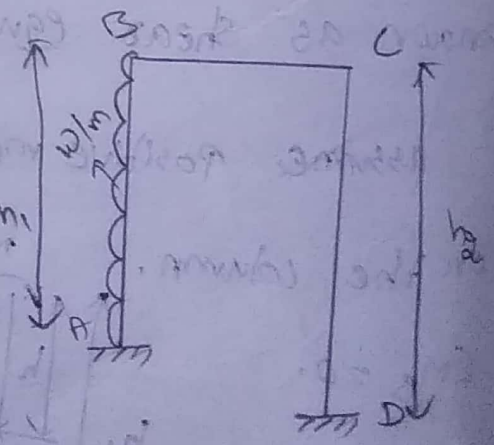
$$P + H_A + H_D - \omega h_2 = 0,$$

$$H_A + H_D + P - \omega h_2 = 0.$$

$$\frac{M_{AB} + M_{BA} - Ph}{h_1} + \frac{M_{CD} + M_{DC} + \frac{\omega h_2^2}{2}}{h_2} + P - \omega h_2 = 0,$$

which is an shear equation.

Case(ii).



$$\frac{M_{AB} + M_{BA} + \frac{\omega h_1^2}{2}}{h_1} + \frac{M_{CD} + M_{DC}}{h_2}$$

$$+ \omega h_1 = 0,$$

which is an shear equation.

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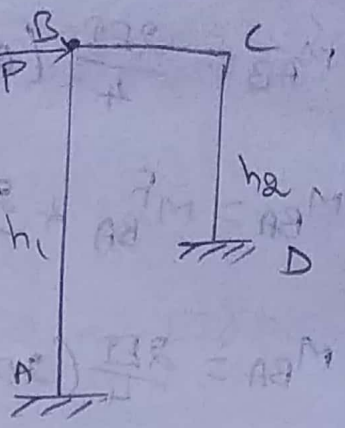
case iii):

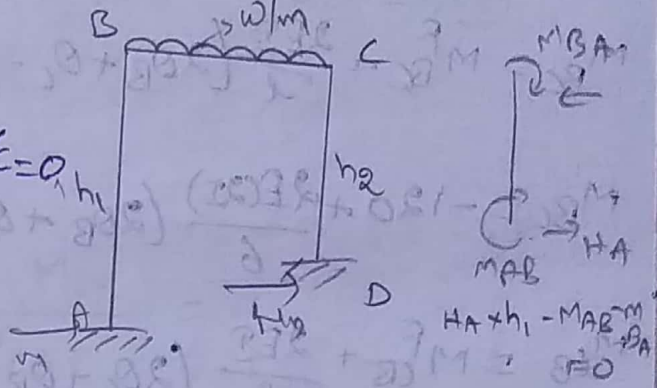
$$\frac{M_{AB}+M_{BA}}{h_1}+\frac{M_{CD}+M_{DC}}{h_2}+P=0.$$



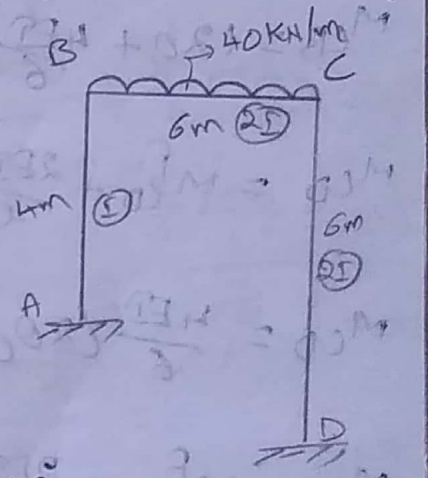
which is an shear equation.

case iv):

$$\frac{M_{AB}+M_{BA}}{h_1}+\frac{M_{CD}+M_{DC}}{h_2}=0$$



1). Analyse the following frame by slope deflection method.



Step:1 fixed end moments:-

$$M^f_{AB}=0, \quad M^f_{BA}=0.$$

$$M^f_{BC}=\frac{-wl^2}{12}=\frac{-40\times 36}{12}=-120\ KN\text{-}m$$

$$M^f_{CB}=\frac{+wl^2}{12}=+120\ KN\text{-}m.$$

$$M^f_{CD}=M^f_{DC}=0.$$

Step:2

slope deflection equations:-

$$M_{AB}=M^f_{AB}+\frac{2EI}{l}\left(2\theta_A+\theta_B-\frac{3\delta}{l}\right)$$

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$$M_{AB} = \frac{2EI}{4}\left(\theta_B - \frac{3\delta}{4}\right) \rightarrow (1)$$

$$M_{BA} = M^f_{BA} + \frac{2EI}{l}\left(2\theta_B + \theta_A - \frac{3\delta}{l}\right)$$

$$M_{BA} = \frac{2EI}{4}\left(2\theta_B - \frac{3\delta}{l}\right) \rightarrow (2)$$

$$M_{BC} = M^f_{BC} + \frac{2EI}{l}\left(2\theta_B + \theta_C - \frac{3\delta}{l}\right)$$

$$M_{BC} = -120 + \frac{2E(2I)}{6}\left(2\theta_B + \theta_C - \frac{3\delta}{l}\right)^{(\delta=0)} \rightarrow (3)$$

$$M_{CB} = M^f_{CB} + \frac{2EI}{l}\left(2\theta_C + \theta_B - \frac{3\delta}{l}\right)$$

$$M_{CB} = 120 + \frac{4EI}{6}\left(2\theta_C + \theta_B - \frac{3\delta}{l}\right)^{(\delta=0)} \rightarrow (4)$$

$$M_{CD} = M^f_{CD} + \frac{2EI}{l}\left(2\theta_C + \theta_D - \frac{3\delta}{l}\right)$$

$$M_{CD} = \frac{4EI}{6}\left(2\theta_C - \frac{3\delta}{6}\right) \rightarrow (5)$$

$$M_{DC} = M^f_{DC} + \frac{2EI}{l}\left(2\theta_D + \theta_C - \frac{3\delta}{l}\right)$$

$$M_{DC} = \frac{4EI}{6}\left(\theta_C - \frac{3\delta}{6}\right) \rightarrow (6)$$

Step:3.

Equilibrium equations:-

$$\Sigma M_B = 0 \quad ; \quad M_{BA} + M_{BC} = 0$$

$$EI\theta_B - \frac{3}{8}EI\delta - 120 + \frac{4}{3}EI\theta_B + \frac{2}{3}EI\theta_C = 0$$

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$$\frac{7}{3}EI\theta_B + \frac{2}{3}EI\theta_C - \frac{3}{8}EI\delta = 120. \rightarrow (7)$$

$2.33 \quad 0.66 \quad 0.375$

$\Sigma M_C = 0, \quad M_{CB} + M_{CD} = 0.$

$$120 + \frac{4}{3}EI\theta_C + \frac{2}{3}EI\theta_B + \frac{4}{3}EI\theta_C - \frac{1}{3}EI\delta = 0.$$

$$\frac{2}{3}EI\theta_B + \frac{8}{3}EI\theta_C - \frac{1}{3}EI\delta = -120 \rightarrow (8)$$

$2.66 \quad 0.33$

Step:4 Shear Equation:-

$$\frac{M_{AB} + M_{BA}}{l_{AB}} + \frac{M_{CD} + M_{DC}}{l_{CD}} = 0.$$

$$\frac{\frac{1}{2}EI\theta_B - \frac{3}{8}EI\delta + EI\theta_B - \frac{3}{8}EI\delta}{4} + \frac{\frac{4}{3}EI\theta_C - \frac{1}{3}EI\delta + \frac{2}{3}EI\theta_C - \frac{1}{3}EI\delta}{6} = 0$$

$$\frac{1.5EI\theta_B - \frac{3}{4}EI\delta}{4} + \frac{2EI\theta_C - \frac{2}{3}EI\delta}{6} = 0$$

$$\frac{3\left(1.5EI\theta_B - \frac{3}{4}EI\delta\right) + 2\left(2EI\theta_C - \frac{2}{3}EI\delta\right)}{12} = 0$$

$$4.5EI\theta_B - 2.25EI\delta + 4EI\theta_C - 1.33EI\delta = 0.$$

$$4.5EI\theta_B + 4EI\theta_C - 3.58EI\delta = 0 \rightarrow (9)$$

solve. (7), (8) and (9).

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$\therefore EI\theta_B = 72.44$, $EI\theta_C = -60.12$, $EI\delta = 23.85$.

Step 5. Final Moments:-

Sub the above values in ①, ②, ③, ④, ⑤ & ⑥.

$$M_{AB} = \frac{1}{2}EI\theta_B - \frac{3}{8}EI\delta = \frac{1}{2}(72.44) - \frac{3}{8}(23.85)$$

$$\boxed{M_{AB} = 27.27\ KN\text{-}m}$$

$$M_{BA} = EI\theta_B - \frac{3}{8}EI\delta = 72.44 - \frac{3}{8}(23.85)$$

$$\boxed{\therefore M_{BA} = 63.49\ KN\text{-}m}$$

$$M_{BC} = -120 + \frac{4}{3}EI\theta_B + \frac{2}{3}EI\theta_C = -120 + \frac{4(72.44)}{3} + \frac{2}{3}(-60.$$

$$\boxed{M_{BC} = -63.49\ KN\text{-}m}$$

$$M_{CB} = 120 + \frac{4}{3}EI\theta_C + \frac{2}{3}EI\theta_B = 120 + \frac{4(-60.12)}{3} + \frac{2(72.44)}{3}$$

$$\boxed{\therefore M_{CB} = 88.13\ KN\text{-}m}$$

$$M_{CD} = \frac{4}{3}EI\theta_C - \frac{1}{3}EI\delta = \frac{4}{3}(-60.12) - \frac{1}{3}(23.85)$$

$$\boxed{\therefore M_{CD} = -88.11\ KN\text{-}m}$$

$$M_{DC} = \frac{2}{3}EI\theta_C - \frac{1}{3}EI\delta = \frac{2}{3}(-60.12) - \frac{1}{3}(23.85)$$

$$\boxed{\therefore M_{DC} = -48.03\ KN\text{-}m}$$

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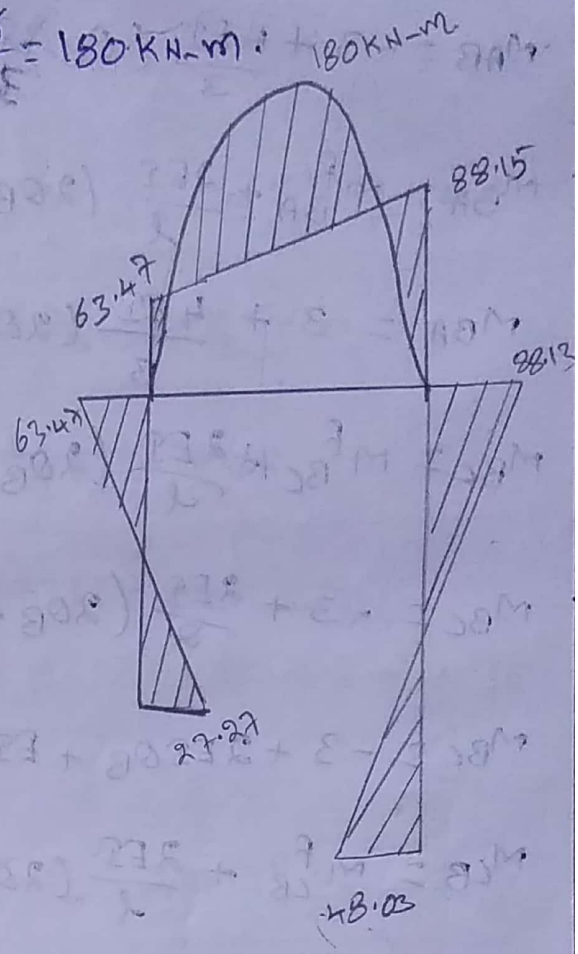
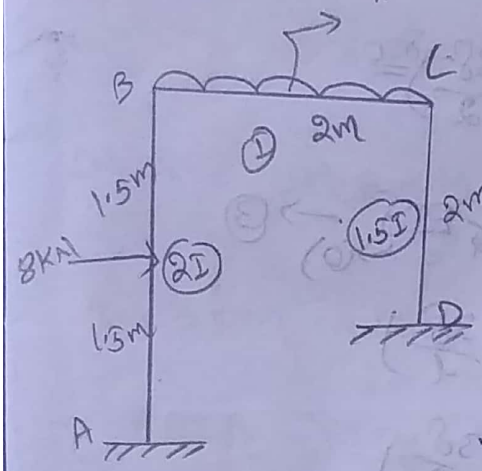
Step:6 Max B.M.

$$\text{Max B.M} = \frac{wl^2}{8} = \frac{40 \times 36}{8} = 180 \text{ KN-m}$$

BMD



2). Analyse the following frame by slope deflection equation.



Step:1 fixed end moments:-

$$M^f_{AB} = \frac{-wl}{8} = \frac{-8 \times 3}{8} = -3 \text{ KN-m}$$

$$M^f_{BA} = \frac{+wl}{8} = \frac{+8 \times 3}{8} = 3 \text{ KN-m}$$

$$M^f_{BC} = \frac{-wl^2}{12} = \frac{-9 \times 4}{12} = -3 \text{ KN-m}$$

$$M^f_{CB} = \frac{+wl^2}{12} = \frac{+9 \times 4}{12} = 3 \text{ KN-m}$$

$$M^f_{CD} = 0, \quad M^f_{DC} = 0.$$

Step:2 slope deflection equation:-

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$$M_{AB} = M^f_{AB} + \frac{2EI}{l}\left(2\theta_A + \theta_B - \frac{3\delta}{l}\right)$$

$$M_{AB} = -3 + \frac{4EI}{3}\left(\theta_B - \frac{3\delta}{3}\right) \rightarrow (1)$$

$$M_{BA} = M^f_{BA} + \frac{2EI}{l}\left(2\theta_B + \theta_A - \frac{3\delta}{l}\right)$$

$$M_{BA} = 3 + \frac{4EI}{3}\left(2\theta_B - \frac{3\delta}{3}\right) \rightarrow (2)$$

$$M_{BC} = M^f_{BC} + \frac{2EI}{l}\left(2\theta_B + \theta_C - \frac{3\delta}{l}\right)$$

$$M_{BC} = -3 + \frac{2EI}{2}\left(2\theta_B + \theta_C - \frac{3\delta}{2}\right) \quad \delta = 0$$

$$M_{BC} = -3 + 2EI\theta_B + EI\theta_C - \frac{3}{2}EI\delta \rightarrow (3)$$

$$M_{CB} = M^f_{CB} + \frac{2EI}{l}\left(2\theta_C + \theta_B - \frac{3\delta}{l}\right)$$

$$M_{CB} = 3 + \frac{2EI}{2}\left(2\theta_C + \theta_B - \frac{3\delta}{2}\right)$$

$$M_{CB} = 3 + 2EI\theta_C + EI\theta_B - \frac{3}{2}EI\delta \rightarrow (4)$$

$$M_{CD} = M^f_{CD} + \frac{2EI}{l}\left(2\theta_C + \theta_D - \frac{3\delta}{l}\right)$$

$$M_{CD} = \frac{2E(1.5I)}{2}\left(2\theta_C - \frac{3\delta}{2}\right)$$

$$M_{CD} = 3\frac{EI}{2}\left(2\theta_C - \frac{3\delta}{2}\right) = 3EI\theta_C - \frac{9}{4}EI\delta \rightarrow (5)$$

$$M_{DC} = M^f_{DC} + \frac{2EI}{l}\left(2\theta_D + \theta_C - \frac{3\delta}{l}\right)$$

$$M_{DC} = 0 + \frac{2E(1.5I)}{2}\left(\theta_C - \frac{3\delta}{2}\right)$$

$$\therefore M_{DC} = \frac{3}{2}EI\theta_C - \frac{9}{4}EI\delta \rightarrow (6)$$

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Step:3 Equilibrium equations:-

$\Sigma M_B = 0, \quad M_{BC} + M_{BA} = 0$

$3 + \frac{4}{3} EI(2\theta_B - \delta) - 3 + 2EI\theta_B + EI\theta_C = 0$

$\frac{8}{3} EI\theta_B - \frac{4}{3} EI\delta + 2EI\theta_B + EI\theta_C = 0$

$\frac{14}{3} EI\theta_B + EI\theta_C - \frac{4}{3} EI\delta = 0.$

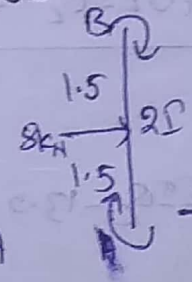
$4.666\, EI\theta_B + EI\theta_C - 1.33 EI\delta = 0 \rightarrow (7)$

$\Sigma M_C = 0, \quad M_{CB} + M_{CD} = 0$

$3 + 2EI\theta_C + EI\theta_B + 1.5 \times 2 EI\theta_C - 2.25 EI\delta = 0.$

$5EI\theta_C + EI\theta_B - 2.25 EI\delta = -3 \rightarrow (8)$

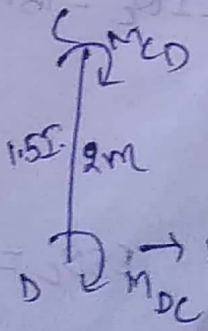
Step:4 shear equation:-



$\Sigma M_B = 0.$

$H_A \times 3 - M_{AB} - M_{BA} + 8 \times 1.5 = 0.$

$H_A = \frac{M_{AB} + M_{BA} - 8 \times 1.5}{3}.$



$\Sigma M_C = 0.$

$H_D \times 2 - M_{CD} - M_{DC} = 0$

$H_D = \frac{M_{CD} + M_{DC}}{2}.$

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$\Sigma H = 0$.

$$P + H_A + H_D = 0.$$

$$H_A + H_D + P = 0$$

$$\frac{M_{AB} + M_{BA} - 8 \times 1.5}{3} + \frac{M_{CD} + M_{DC}}{2} + 8 = 0$$

$$\frac{-3 + 1.33EI\theta_B - 1.33EI\delta + 3 + 1.33 \times 2EI\theta_B - 1.33EI\delta - 12}{3} +$$

$$\frac{3EI\theta_C - 2.25EI\delta + 1.5EI\theta_C - 2.25EI\delta}{2} + 8 = 0$$

$$\frac{3.99EI\theta_B - 2.66EI\delta - 12}{3} + \frac{4.5EI\theta_C - 4.5EI\delta}{2} + 8 = 0$$

$$\frac{2(3.99EI\theta_B - 2.66EI\delta - 12) + 3(4.5EI\theta_C - 4.5EI\delta) + 48}{6} = 0$$

$$7.98EI\theta_B - 5.32EI\delta - 24 + 13.5EI\theta_C - 13.5EI\delta + 48 = 0$$

$$7.98EI\theta_B + 13.5EI\theta_C - 18.82EI\delta + 24 = 0.$$

$$7.98EI\theta_B + 13.5EI\theta_C - 18.82EI\delta = -24 \rightarrow (9)$$

Solve (7) (8) & (9).

$\therefore EI\theta_B = 0.4139 \quad EI\theta_C = -0.044, \quad EI\delta = 1.419$

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Step:5 final moments:-

sub the above values in ① ② ③ ④ ⑤ & ⑥.

$$M_{AB} = -3 + \frac{4}{3}EI\theta_B - \frac{4}{3}EI\delta = -4.34\ KN\text{-}m.$$

$$M_{BA} = 3 + \frac{8}{3}EI\theta_B - \frac{4}{3}EI\delta = 2.217\ KN\text{-}m.$$

$$M_{BC} = -3 + 2EI\theta_B + EI\theta_C = -2.216\ KN\text{-}m.$$

$$M_{CB} = 3 + 2EI\theta_C + EI\theta_B = 3.325\ KN\text{-}m.$$

$$M_{CD} = \frac{6}{2}EI\theta_C - \frac{9}{4}EI\delta = -3.324\ KN\text{-}m.$$

$$M_{DC} = \frac{3}{2}EI\theta_C - \frac{9}{4}EI\delta = -3.258\ KN\text{-}m.$$

Step:6

Max B.M

Max B.M $= \frac{wl^2}{8}$

BC span.

$= \frac{9 \times 4}{8} = 4.5\ KN\text{-}m.$

max B.M for AB $= \frac{wl}{4}$

$\frac{8 \times 3}{4} = 6\ KN\text{-}m.$

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