MALLA REDDY ENGINEERING COLLEGE

MAISAMMAGUDA, KOMPALLY, SECUNDERABAD – 500100

DEPARTMENT OF CIVIL ENGINEERING

COURSE FILE

II B.TECH – II SEMESTER (A & C SECTIONS)

SUBJECT: HYDRAULICS AND HYDRAULIC MACHINERY (80111)

FACULTY NAME: Mr. SOHANG DEBNATH *(ASSISTANT PROFESSOR)*

MODULE I

→ One drawback of hydraulic machine is that any transmission of power results in some losses due to resistance of fluid flow through the piping.

* VELOCITY DISTRIBUTION :-

→ Velocity distribution shows the difference in intensity of resistance of fluid particles across the flow, due to cohesive and adhesive forces.

→ Not all fluid particles travel at the same velocity within a pipe.

→ The shape of the velocity curve (the velocity profile across any given section of the pipe) depends upon whether the flow is laminar or turbulent.

→ If the flow in a pipe is laminar, the velocity distribution at a cross section will be parabolic in shape with the maximum velocity at the center being about twice the average velocity in the pipe.

→ In case of a turbulent flow, a fairly flat velocity distribution exists across the section of pipe.

→ The velocity of the fluid in contact with the pipe wall is essentially zero and it increases further, as we move away from the wall along the fluid layers.

→ In an open channel flow, velocity distribution is non-uniform which means that velocity is different at different depths. Various factors such as channel slope, alignment, shape, roughness etc. plays key role in velocity distribution.

→ The ratio of the actual velocity to the theoritical velocity of a fluid jet is called the velocity distribution coefficient.

* ENERGY AND MOMENTUM CORRECTION FACTORS :→

→ Kinetic energy correction factor is defined as the ratio of the kinetic energy of the flow per second based on actual velocity across a section to the kinetic energy of the flow per second based on average velocity across the same section. It is denoted by α.

$$\alpha = \frac{\text{K.E. per sec based on actual velocity}}{\text{K.E. per sec based on average velocity}}$$

→ Momentum correction factor is defined as the ratio of momentum of the flow per second based on actual velocity to the momentum of the flow per second based on average velocity across a section. It is denoted by β.

$$\beta = \frac{\text{Momentum per second based on actual velocity}}{\text{Momentum per second based on average velocity}}$$

* TYPES OF CHANNELS :→

→ Natural Channels: It is one with irregular sections of varying shapes, developing in natural way.
Eg:- rivers, streams etc.

→ Artificial channels: It is the one built artificially for carrying water for various purposes.
Eg:- Canals.

→ Open Channel : A channel without any cover at the top.
Eg:- Canals, rivers, streams etc.

→ Covered channels: A channel having cover at the top.
Eg:- Partially filled conduits carrying water.

→ Prismatic channel: A channel with constant bed slope and cross-section along its depth.

Eg:- Man made open channels.

→ Comparision between Open Channel Flow and Pipe Flow:

ASPECT	OPEN CHANNEL	PIPE FLOW
Cause of flow	Gravity force (provided by sloping bottom)	Pipes run full and flow takes place under hydraulic pressure.
Cross-sectional shape	Open channels may have any shape, e.g: triangular, rectangular, trapezoidal, parabolic or circular etc.	Pipes are generally round in cross-section which is uniform along length.
Surface roughness	Varies with depth of flow	Varies with type of pipe material.
Piezometric head	$(z+h)$, where h is the depth of channel	$\left(z+\frac{P}{\rho g}\right.$ or $\left.z+\frac{P}{\gamma}\right)$, where P is the pressure in pipe
Velocity distribution	Maximum velocity occurs at a little distance below the water surface. The shape of the velocity profile is dependent on the channel roughness	The velocity distribution is symmetrical about the pipe axis. Maximum velocity occurs at the pipe center and the velocity at pipe walls is reduced to zero.

* SPECIFIC ENERGY AND SPECIFIC ENERGY CURVE :→

The total energy of flow in an open channel is given by $z+y+\frac{v^2}{2g}$

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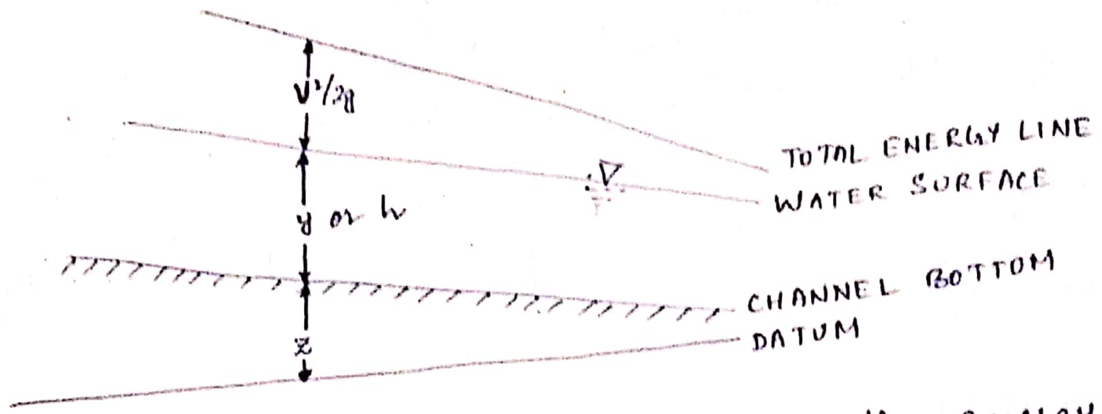


If the channel bottom is considered to be the datum, the energy of the flow is termed as specific energy of flow, given as :-

$$y + \frac{V^2}{2g} \quad \text{or} \quad h + \frac{V^2}{2g}$$

→ <u>Specific Energy Curve</u>:

It is a curve plotted between specific energy on the X axis and depth of flow on the Y axis.

Specific energy is given by:-

$$E = y + \frac{V^2}{2g} \quad \text{or} \quad h + \frac{V^2}{2g} = E_p + E_k$$

where,

$$E_p = y \text{ or } h \quad \text{and} \quad E_k = \frac{V^2}{2g}$$

we know, $Q = A \times V \Rightarrow V = \frac{Q}{A} = \frac{Q}{b \times h} = \frac{q}{h}$ $\left[\because q = \frac{Q}{b}\right]$

where, q is the discharge per unit width

$$\therefore E_k = \frac{q^2}{2gh^2}$$

$$\therefore E = E_p + E_k = h + \frac{q^2}{2gh^2} \quad \text{or} \quad y + \frac{q^2}{2gy^2}$$

Since the specific energy curve is plotted between E and y or h, therefore the entire equation of specific energy is converted in terms of depth (h or y).

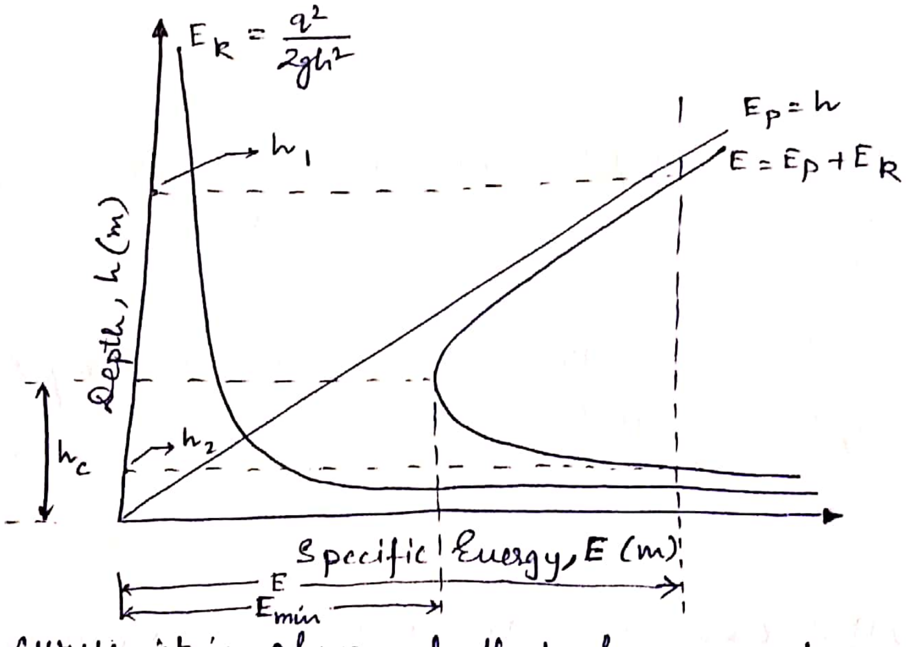
The equation $E_p = y$ or $E_p = h$ resembles the equation of a ~~straight line~~. linear equation.

In the equation $E_k = \frac{q^2}{2gh^2}$ or $E_k = \frac{q^2}{2gy^2}$

If $y = 0$, then $E_k = \infty$ and if $y = \infty$, then $E_k = 0$

With this reference, the curves for E_p and E_k is plotted as shown below.

The specific energy curve is $E = h + \frac{q^2}{2gh^2}$, which is a combination of the curves E_p and E_k.



From the curve, it is observed that for a value of specific energy at any point along the X axis, say E, we have 2 values of flow depths h_1 and h_2 as shown in the curve.

However, there is only one particular point on this curve or one particular value of the specific energy curve, corresponding to which only one particular value of flow depth can be obtained. That particular point corresponds to minimum specific energy (E_{min}), and corresponding to this value of E_{min}, the depth of flow obtained is termed as critical depth, termed as h_c.

→ The salient features of the specific energy curve are:

✓ The specific energy curve has 2 limbs, one of which is asymptotical (reaches out but never touches) to the ~~X axis~~ Ek and another which is asymptotical to the line $E_p = h$.

✓ Each value of specific energy corresponds to 2 different values of flow depth. This means that there are 2 different depths of flow that have the same specific energy.

✓ The minimum value of specific energy corresponds ~~to~~ to just one flow depth. This depth is known as critical depth.

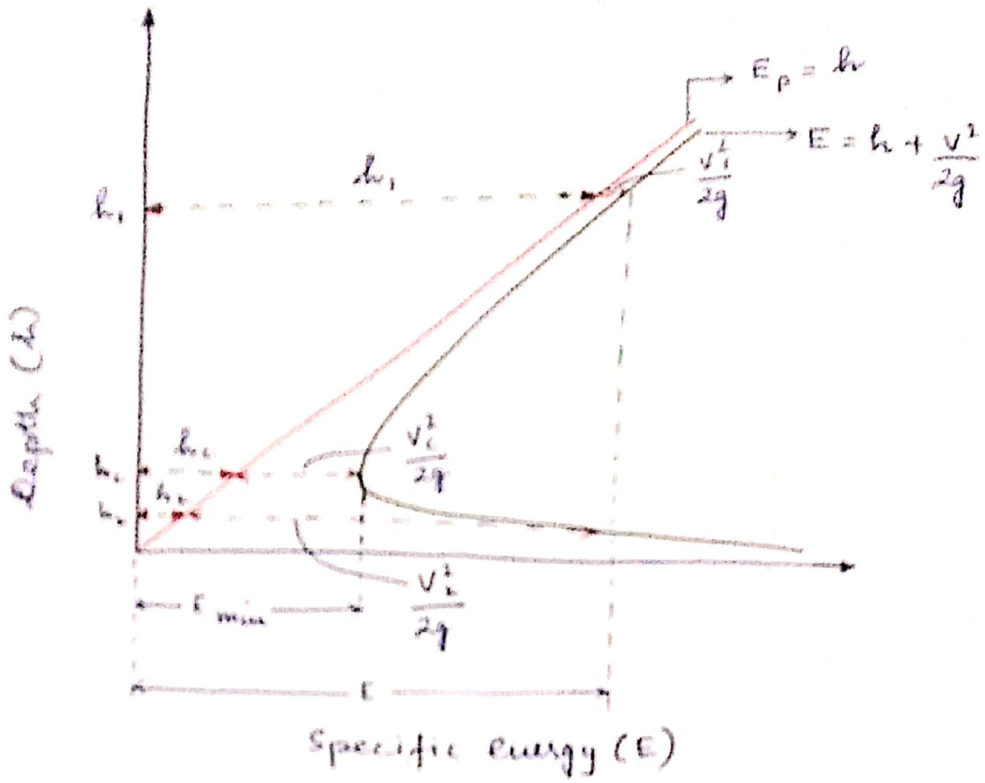
→ Alternate depths: The depths corresponding to the same specific energy.

From the specific energy curve, there are two depths h_1 and h_2 corresponding to E. So h_1 and h_2 are alternate depths.

From the curve, it is observed that at y_2, the value of y_2 is lesser ~~the~~ and the value of $\frac{V_2^2}{2g}$ is greater.

Similarly, at y_1, the value of y_1 is greater and the value of $\frac{V_1^2}{2g}$ is smaller.

Corresponding to critical depth h_c, the critical velocity is V_c



From this analysis, it can be stated that:-

$$h_1 > h_c > h_2 \quad \text{and} \quad V_2 > V_c > V_1$$

Thus, it is observed that there is a inverse relationship between critical depth h_c and critical velocity V_c.

The depth h_1 corresponds to depth greater than the critical depth and velocity less than critical velocity. This depth is termed as sub-critical depth.

Similarly, the depth h_2 corresponds to depth lesser than the critical depth and velocity greater than critical velocity. This depth is termed as super-critical depth.

→ Critical depth (h_c) : The depth of flow corresponding to the minimum specific energy (E_{min}).

We know that.

$$E = h + \frac{q^2}{2gh^2}$$

For specific energy to be minimum, $\frac{dE}{dh} = 0$

$$\frac{dE}{dh} = 1 - \frac{2q^2}{2gh^3} = 0$$

$$\Rightarrow 1 = \frac{q^2}{gh^3}$$

$$\Rightarrow h^3 = \frac{q^2}{g}.$$

$$\Rightarrow h = \left(\frac{q^2}{g}\right)^{1/3}$$

i.e, $$\boxed{h_c = \left(\frac{q^2}{g}\right)^{1/3}}$$

This is the expression for critical depth.

→ Critical Velocity (V_c): Velocity corresponding to the critical depth of flow (h_c)

We know that,

$$h_c = \left(\frac{q^2}{g}\right)^{1/3}$$

$$\Rightarrow h_c^3 = \frac{q^2}{g}.$$

$$\Rightarrow h_c^3 = \frac{Q^2}{b^2 g} \qquad \left[\because q = \frac{Q}{b}\right]$$

$$\Rightarrow h_c^3 = \frac{A^2 \times V^2}{b^2 g} \qquad [\because Q = A \times V]$$

$$\Rightarrow h_c^3 = \frac{b^2 \times h_c^2 \times V_c^2}{b^2 \times g} \qquad [\because A = b \times h]$$

$$\Rightarrow h_c^3 = \frac{h_c^2 V_c^2}{g}.$$

$$\Rightarrow h_c = \frac{V_c^2}{g}$$

$$\Rightarrow \boxed{V_c = \sqrt{g h_c}}$$

This is the expression for critical velocity.

→ **Minimum Specific Energy (E_{min}):**

We know that,

$$E = h + \frac{V^2}{2g}$$

So,

$$E_{min} = h_c + \frac{V_c^2}{2g}.$$

$$\Rightarrow E_{min} = h_c + \frac{(\sqrt{g h_c})^2}{2g}$$

$$\Rightarrow E_{min} = h_c + \frac{\cancel{g} h_c}{2\cancel{g}}$$

$$\Rightarrow E_{min} = h_c + \frac{h_c}{2} = \frac{3 h_c}{2}$$

$$\therefore \boxed{E_{min} = 1.5\, h_c}$$

This is the expression for minimum specific energy.

Here the energy is expressed in terms of metres

Problems:→

Q. Find the specific energy of the flowing water through a rectangular channel of width 5m, when the discharge is 10 m³/sec and the depth of water is 3m.

Solⁿ: Given:-

$h = 3m$

$Q = 10\ m^3/s$.

$b = 5m$

P.T.O

$$\therefore q = \frac{Q}{b} = \frac{10}{5} = 2\ m^2/s.$$

This is the discharge per unit width

We know,

$$E = h + \frac{q^2}{2gh^2}$$

$$= 3 + \frac{2^2}{2 \times 9.81 \times 3^2} = 3.0226\ m.$$

Q. Find the critical depth and critical velocity of the water flowing through a rectangular channel of width 5m, when discharge is 15 m^3/s.

Soln:- given,

$$Q = 15\ m^3/s$$

$$b = 5m$$

$$\therefore q = \frac{Q}{b} = \frac{15}{3} = 5\ m^2/s$$

We know, $h_c = \left(\frac{q^2}{g}\right)^{1/3}$

$$\therefore h_c = \left(\frac{3^2}{9.81}\right)^{1/3} = 0.972\ m$$

Also,

$$V_c = \sqrt{gh_c}$$

$$\therefore V_c = \sqrt{9.81 \times 0.972} = 3.088\ m/s.$$

Q. The specific energy of 5m wide rectangular channel is 4m. If the rate of flow through the channel is 20 m^3/s, then determine the alternate depths of flow.

Soln:- given,

$$Q = 20\ m^3/s$$

$$b = 5\ m$$

$$\therefore q = \frac{Q}{b} = \frac{20}{5} = 4\ m^2/s$$

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Also, $E = 4m$

We know,
$$E = h + \frac{q^2}{2gh^2}$$

$$\therefore h + \frac{q^2}{2gh^2} = 4$$

$$\Rightarrow h + \frac{4^2}{2 \times 9.81 \times h^2} = 4.$$

$$\Rightarrow 19.62 h^3 - 78.48 y^2 + 16 = 0.$$

Solving we get,
$$h = 3.95, -0.42 \text{ and } 0.48$$

$\therefore$ Alternate depths of flow = 3.93m and 0.48 m.

Q. The discharge of water through a rectangular channel of width 8m is 15 m³/s, when the depth of flow is 1.2m. Calculate, (a) Specific energy of the flowing water
(b) Critical depth and critical velocity
(c) Minimum specific energy.

Solⁿ:- Given,

$h = 1.2m.$

$Q = 15 m^3/s.$

$b = 8m$

$$\therefore q = \frac{Q}{b} = \frac{15}{8} = 1.875 \ m^2/s.$$

(a) $E = h + \frac{q^2}{2gh^2} = 1.2 + \frac{1.875^2}{2 \times 9.81 \times 1.2^2} = 1.324 \ m.$

(b) $h_c = \left(\frac{q^2}{g}\right)^{1/3} = \left(\frac{1.875^2}{9.81}\right)^{1/3} = 0.71 \ m$

(c) $E_{min} = ?$

We know, $V_c = \sqrt{g h_c}$

$\Rightarrow V_c = \sqrt{9.81 \times 0.71} = 2.639$ m/s

$\therefore E_{min} = \dfrac{3h_c}{2}$.

$= \dfrac{3 \times 0.71}{2} = 1.065$ m.

Froude Number (F_r):

- It is a dimensionless number that is significant in an open channel flow.
- It is given by: $F_r = \dfrac{V_c}{\sqrt{gh_c}}$

For critical flow, $V_c = \sqrt{gh_c}$

$\therefore$ For critical flow, the corresponding Froude number is 1.

For sub-critical flow, V_1 is lesser than V_c and h_1 is greater than h_c

$\therefore$ For sub-critical flow, $F_r = \dfrac{V_1}{\sqrt{gh_1}} < \dfrac{V_c}{\sqrt{gh_c}}$

hence, for sub-critical flow, froude number is less than 1
i.e, $F_r < 1$.

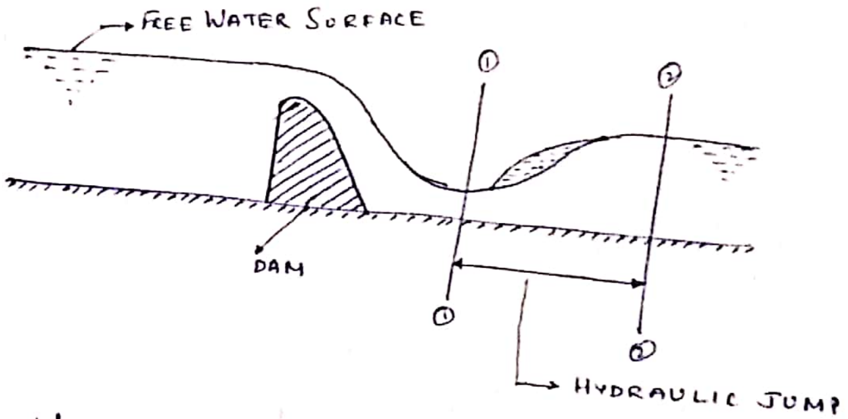
For super-critical flow, V_2 is greater than V_c and h_2 is lesser than h_c

$\therefore$ For super-critical flow, $F_r = \dfrac{V_2}{\sqrt{gh_2}} > \dfrac{V_c}{\sqrt{gh_c}}$

hence, for super-critical flow, froude number is greater than 1.
i.e, $F_r > 1$.

Thus, Froude number is a dimensionless number used to differentiate between critical, sub critical and super critical flow.

* HYDRAULIC JUMP :→



→ Let us consider a channel in which a dam has been built across the channel as a barrier in the flow.

→ Due to the construction of this dam, water will start to store up in the upstream direction of this dam.

→ The depth of water goes on increasing and a point will come when the depth of water (i.e, upstream of the dam) will become equal to the height of the crest of the dam.

→ Once the water depth reaches the crest height, it then starts to spill over the dam along its slope.

→ Initially, the water flow is sub-critical, i.e it has less velocity and the Froude number is less than 1 ($F_r < 1$).

→ When the water starts to spill over the dam, the flow gets accelerated.

→ Due to this sudden acceleration, there is a sudden increase in the velocity and the froude number also starts to increase. A time will come when the subcritical nature of the flow transforms into critical, and then further gets converted into super critical flow.

→ As shown in the figure, in section 1-1, the depth of the flow is less than the critical depth and the flow is super-critical in nature or shooting flow.

→ This shooting flow is generally unstable in nature and doesnot further travel on the downstream side.

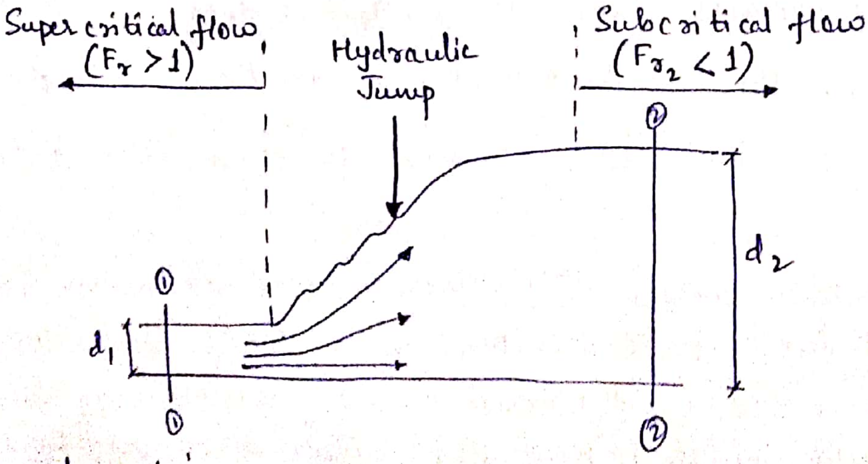
→ Thus, it itself gets converted into sub-critical or streaming or tranquil flow again and hence depth of water will again increase.

→ This rise in depth takes place over a very short length, i.e upto section 2-2 as shown in the figure.

→ This sudden increase in the depth of water on transformation of flow from supercritical to sub-critical between section 1-1 and 2-2 is called a hydraulic jump or a standing wave.

→ Thus hydraulic jump is defined as:
"The rise of water level, which takes place due to the transformation of the unstable shooting flow (super-critical) to the stable streaming flow (sub-critical flow)."

→ When hydraulic jump occurs, a loss of energy due to eddy formation and turbulence flow occurs.



From the above figure, Depth of Jump = $(d_2 - d_1)$

→ If water is going to travel with high velocity or higher amount of kinetic energy, then channel gets subjected to ~~knurling~~ scouring, which hampers the life of channel. Therefore hydraulic jump is advisible, since it can dissipate this high energy and prevent ~~knurling~~ scouring in channels.

The most typical cases for the location of hydraulic jump are:

1. Below control structures like weir, sluice, which are used in channel.
2. when any obstruction is found in a channel.
3. when a sharp change in the channel slope takes place.
4. At the toe of a spillway dam.

The various effects of Hydraulic Jump are listed below:

1. Actually the hydraulic jump usually acts as the energy dissipator. It clears the surplus energy of water.
2. Due to hydraulic jump, many noticable disturbances are created in the flowing water like eddies, reverse flow.
3. Usually when the hydraulic jump takes place, the considerable amount of air is trapped in the water. That air can be helpful in removing the wastes in the streams that are causing pollution.
4. Hydraulic jump also makes the work of different hydraulic structures effective like weirs, notches, flumes etc.
5. Hydraulic jump helps to dissipate energy in water flowing over hydraulic structures as dams, weirs etc. and prevents scouring in downstream.
6. Hydraulic jump usually maintains higher water level on the downstream side. This higher water level can be used for irrigation purposes.

7. Hydraulic jump reverses the flow of water. This phenomenon can be used to mix chemicals for water purification.

8. Hydraulic jump can be used to remove the air pockets from water supply and sewage lines to prevent the air locking.

Types of Hydraulic Jumps (based on Froude's number):

1. Undular Hydraulic Jump
   → Froude number: 1 to 1.7
   → Low Energy dissipation: about 5%

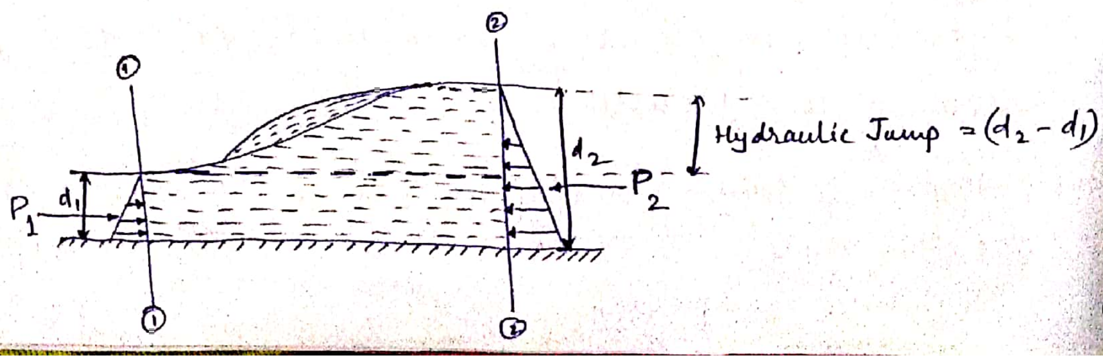
2. Weak Hydraulic Jump
   → Froude number: 1.7 to 3.
   → Energy dissipation: 5% to 15%

3. Oscillating Hydraulic Jump
   → Froude number: 2.5 to 4.5
   → Energy dissipation: 15% to 45%

4. Steady Hydraulic Jump
   → Froude number: 4.5 to 9
   → Energy dissipation: 45% to 70%

5. Steady Hydraulic Jump
   → Froude number: above 9
   → Energy dissipation: 75% to 85%

Expression for Depth of Hydraulic Jump:→



Before deriving an expression for the depth of hydraulic jump, the following assumptions are made:

1. The flow is uniform (throughout the length of the channel, the depth is constant) and pressure distribution is hydrostatic (pressure exerted by the liquid at rest) before and after the jump.
2. Losses due to friction on the surface of the bed of the channel are small and hence neglected.
3. The slope of the bed of the channel is small, so that the component of the weight of the fluid in the direction of flow is negligibly small.

Consider a hydraulic jump formed in a channel of horizontal bed as shown in the figure. Consider 2 sections, 1-1 and 2-2 before and after the hydraulic jump respectively.

Let d_1 = Depth of flow at section 1-1

d_2 = Depth of flow at section 2-2.

V_1 = Velocity of flow at section 1-1.

V_2 = Velocity of flow at section 2-2.

$\bar{Z}_1$ = Depth of centroid of area at section 1-1 below free surface.

$\bar{Z}_2$ = Depth of centroid of area at section 2-2 below free surface

A_1 = Area of cross-section at section 1-1

A_2 = Area of cross-section at section 2-2.

Consider unit width of the channel.

The forces acting on the mass of water between section 1-1 and 2-2 are:

1. Pressure force P_1 on section 1-1
2. Pressure force P_2 on section 2-2.
3. Frictional force on the floor of the channel.

As per the initial assumptions, the frictional force on the floor of the channel is assumed to be negligible.

P.T.O

Let q = discharge per unit width

$$= \frac{Q}{b} = \frac{A \times V}{b} = \frac{\cancel{b} \times d \times V}{\cancel{b}} = dV = Vd$$

As per continuity equation, $q = V_1 d_1 = V_2 d_2$ —— ii)

Now,

Pressure force P_1 on section 1-1 = Specific weight × height of rise of water in 1-1

$$= w \times h$$

$$= \rho g \times A_1 \bar{Z}_1$$

$$= \rho g \times b_1 \times d_1 \times \frac{d_1}{2}.$$

$$P_1 = \frac{\rho g d_1^2}{2} \quad [\because b_1 = 1 \text{, unit width}]$$

Similarly, pressure force P_2 on section 2-2 = $\rho g \times A_2 \bar{Z}_2$

$$= \rho g \times b_2 \times d_2 \times \frac{d_2}{2}$$

$$P_2 = \frac{\rho g d_2^2}{2} \quad [\because b_2 = 1]$$

From the figure, d_2 is greater than d_1, so P_2 is also greater than P_1

$\therefore$ Net force acting on the mass of water between sections 1-1 and 2-2 is : $P_2 - P_1$

$$= \frac{\rho g}{2}\left[d_2^2 - d_1^2\right] \quad \text{—— iii)}$$

But from momentum principle, the net force acting on a mass of fluid must be equal to the rate of change of momentum in the same section in the direction of force.

$\therefore$ Rate of change of momentum in the direction of force is equal to = mass of water per second × Change of velocity in direction of force

Contd....

Here, Mass of water per second = Density × discharge per unit width × width

$= \rho \times q \times b = \rho \times q \times 1 \ m^3/sec$

and, Change of velocity in the direction of force $= (V_1 - V_2)$

[∵ $Q = A \times V \Rightarrow V = \frac{Q}{A} \Rightarrow V \propto \frac{1}{A}$.

In section 1-1, Area is less, so velocity is more. Similarly in section 2-2, Area is more, so velocity is less. Hence $V_1 > V_2$]

∴ Rate of change of momentum in the direction of force $= \rho q (V_1 - V_2)$ —— (iii)

So, according to impulse momentum theorem or momentum principle, eqⁿ (ii) is equal to eqⁿ (iii)

$$\therefore \frac{\rho g}{2}(d_2^2 - d_1^2) = \rho q (V_1 - V_2)$$

From eqⁿ (i), $V_1 = \frac{q}{d_1}$ and $V_2 = \frac{q}{d_2}$

$$\therefore \frac{\rho g}{2}(d_2^2 - d_1^2) = \rho q \left(\frac{q}{d_1} - \frac{q}{d_2}\right)$$

$$\Rightarrow \frac{g}{2}(d_2 + d_1)(d_2 - d_1) = q^2\left(\frac{1}{d_1} - \frac{1}{d_2}\right)$$

$$\Rightarrow \frac{g}{2}(d_2 + d_1)(d_2 - d_1) = q^2\left(\frac{d_2 - d_1}{d_1 d_2}\right)$$

$$\Rightarrow \frac{g}{2}(d_2 + d_1) = \frac{q^2}{d_1 d_2}$$

$$\Rightarrow (d_2 + d_1) = \frac{2q^2}{g d_1 d_2} \quad \text{—— (iv)}$$

Multiplying both sides by d_2, we get:-

$$\Rightarrow d_2^2 + d_1 d_2 = \frac{2q^2}{g d_1}$$

$$\Rightarrow d_2^2 + d_1 d_2 - \frac{2q^2}{g d_1} = 0 \quad \text{—— (v)}$$

Equation (v) is in the form of a quadratic equation in terms of d_2 and hence its solution is :-

$$d_2 = \frac{-d_1 \pm \sqrt{d_1^2 - 4 \times 1 \times \left(\frac{-2q^2}{gd_1}\right)}}{2 \times 1} \qquad \left[\because ax^2 + bx + c = 0 \quad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\right]$$

$$d_2 = \frac{-d_1 \pm \sqrt{d_1^2 + \frac{8q^2}{gd_1}}}{2}$$

$$= \frac{-d_1}{2} \pm \sqrt{\frac{d_1^2}{4} + \frac{2q^2}{gd_1}}$$

The two roots of the equation are :-

$$\frac{-d_1}{2} - \sqrt{\frac{d_1^2}{4} + \frac{2q^2}{gd_1}} \quad \text{and} \quad \frac{-d_1}{2} + \sqrt{\frac{d_1^2}{4} + \frac{2q^2}{gd_1}}$$

First root is not possible as it gives negative depth. Hence

$$d_2 = -\frac{d_1}{2} + \sqrt{\frac{d_1^2}{4} + \frac{2q^2}{gd_1}}$$

$$d_2 = -\frac{d_1}{2} + \sqrt{\frac{d_1^2}{4} + \frac{2(V_1 d_1)^2}{gd_1}} \qquad [\because q_1 = V_1 d_1]$$

in terms of section 1-1.

$$\Rightarrow d_2 = -\frac{d_1}{2} + \sqrt{\frac{d_1^2}{4} + \frac{2V_1^2 d_1}{g}}$$

From the above equation, the value of d_1 and d_2 can be obtained.

and, Depth of Hydraulic Jump $= (d_2 - d_1)$ [from the figure].

Contd....

Expression for loss of Energy due to Hydraulic Jump:→

When hydraulic jump takes place, a loss of energy due to eddies formation and turbulence occurs. This loss of energy is equal to the difference of specific energies at section 1-1 and 2-2.

∴ Loss of energy due to hydraulic jump, $h_L = E_1 - E_2$

$$h_L = \left(d_1 + \frac{V_1^2}{2g}\right) - \left(d_2 + \frac{V_2^2}{2g}\right)$$

$$= \left(\frac{V_1^2}{2g} - \frac{V_2^2}{2g}\right) - (d_2 - d_1)$$

$$= \left(\frac{q^2}{2g\,d_1^2} - \frac{q^2}{2g\,d_2^2}\right) - (d_2 - d_1) \quad \left[\because V_1 = \frac{q}{d_1} \text{ and } V_2 = q/d_2\right]$$

*

From eqⁿ (iv),

$$(d_2 + d_1) = \frac{2q^2}{g d_1 d_2}.$$

$$\Rightarrow q^2 = g d_1 d_2 \frac{(d_2 + d_1)}{2}$$

*

$$h_L = \frac{q^2}{2g}\left[\frac{d_2^2 - d_1^2}{d_1^2 d_2^2}\right] - (d_2 - d_1) \text{ —— (vi)}$$

Substituting the value of q^2 in eqⁿ (vi), we get:-

$$h_L = g d_1 d_2 \frac{(d_2 + d_1)}{2} \times \frac{d_2^2 - d_1^2}{2g\,d_1^2 d_2^2} - (d_2 - d_1)$$

$$= \frac{(d_2 + d_1)(d_2^2 - d_1^2)}{4 d_1 d_2} - (d_2 - d_1)$$

$$= \frac{(d_2 + d_1)(d_2 + d_1)(d_2 - d_1)}{4 d_1 d_2} - (d_2 - d_1)$$

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$$= (d_2 - d_1)\left[\frac{(d_2+d_1)^2}{4d_1d_2} - 1\right]$$

$$= (d_2 - d_1)\left[\frac{d_2^2 + d_1^2 + 2d_1d_2}{4d_1d_2} - 1\right]$$

$$= (d_2 - d_1)\left[\frac{d_2^2 + d_1^2 + 2d_1d_2 - 4d_1d_2}{4d_1d_2}\right]$$

$$= (d_2 - d_1)\left[\frac{d_2^2 - 2d_1d_2 + d_1^2}{4d_1d_2}\right] = (d_2 - d_1)\frac{(d_2 - d_1)^2}{4d_1d_2}$$

$$\therefore h_L = \frac{[d_2 - d_1]^3}{4d_1d_2}$$

Expression for Depth of Hydraulic Jump in Terms of Upstream Froude Number:

Let V_1 = Velocity of flow on the upstream side

d_1 = Depth of flow on upstream side.

Then Froude number $(F_e)_1$ on the upstream side of the jump is given by:

$$(F_e)_1 = \sqrt{\frac{\text{Inertia Force}}{\text{Force due to gravity}}}$$

$$= \sqrt{\frac{\rho A V^2}{\text{Mass} \times \text{Accl}^n \text{ due to gravity}}}$$

$$= \sqrt{\frac{\rho A V^2}{\text{Density} \times \text{volume} \times g}}$$

$$= \sqrt{\frac{\rho \times d_1^2 \times V_1^2}{\rho \times d_1^3 \times g}}$$

$$(Fe)_1 = \sqrt{\frac{V_1^2}{gd_1}} = \frac{V_1}{\sqrt{gd_1}} \quad \text{—— (vii)}$$

Now, the depth of flow after the hydraulic jump is d_2 and it is given by:

$$d_2 = -\frac{d_1}{2} + \sqrt{\frac{d_1^2}{4} + \frac{2V_1^2 d_1}{g}}$$

$$= -\frac{d_1}{2} + \sqrt{\frac{d_1^2}{4} + \frac{2V_1^2 d_1}{g} \times \frac{4}{4} \times \frac{d_1}{d_1}}$$

$$= -\frac{d_1}{2} + \sqrt{\frac{d_1^2}{4}\left(1 + \frac{8V_1^2}{gd_1}\right)}$$

$$= -\frac{d_1}{2} + \frac{d_1}{2}\sqrt{1 + \frac{8V_1^2}{gd_1}} \quad \text{—— (viii)}$$

But, from eqⁿ (vii),

$$(Fe)_1 = \frac{V_1}{\sqrt{gd_1}} \Rightarrow (Fe)_1^2 = \frac{V_1^2}{gd_1}$$

Substituting the value of $(Fe)_1^2$ in eqⁿ (viii), we get:

$$d_2 = -\frac{d_1}{2} + \frac{d_1}{2}\sqrt{1 + 8(F_e)_1^2}$$

$$d_2 = \frac{d_1}{2}\sqrt{1 + 8(Fe)_1^2} - \frac{d_1}{2}$$

$$d_2 = \frac{d_1}{2}\left(\sqrt{1 + 8(Fe)_1^2} - 1\right)$$

Length of Hydraulic Jump:

This is defined as the length between the two sections where one section is taken before the hydraulic jump and the second section is taken immediately after the jump.

Problems:→

Q. The depth of flow of water, at a certain section of a ▭ lr channel of 4m wide, is 0.5 m. This discharge through the channel is 16 m³/sec. If a hydraulic jump takes place on the downstream side, find the depth of flow after the jump.

Solⁿ:- Given:

Width of channel, $b = 4 m$

Depth of flow before jump, $d_1 = 0.5 m$

Discharge, $Q = 16 m^3/sec$

∴ Discharge per unit width, $q = \frac{Q}{b} = \frac{16}{4} = 4 m^2/sec$

Let the depth of flow after jump be d_2

We know,

$$d_2 = \frac{-d_1}{2} + \sqrt{\frac{d_1^2}{4} + \frac{2q^2}{gd_1}}$$

$$d_2 = \frac{-0.5}{2} + \sqrt{\frac{0.5^2}{4} + \frac{2 \times 4^2}{9.81 \times 0.5}} = 2.316 m$$

Q. The depth of flow of water, at a certain section of a ▭ lr channel of 2m wide is 0.3 m. The discharge through the channel is 1.5 m³/sec. Determine whether a hydraulic jump will occur, and if so, find its height and loss of energy per kg of water?

Solⁿ:- Given,

Depth of flow, $d_1 = 0.3 m$

Width of channel, $b = 2m$

Discharge, $Q = 1.5 m^3/sec$

Discharge per unit width, $q = \frac{Q}{b} = \frac{1.5}{2.0} = 0.75 m^2/sec$

Hydraulic jump will occur if the depth of the flow on the upstream side is less than the critical depth on the upstream side

(OR)

if the Froude number in the upstream side is more than one.

We know,

Critical depth, $h_c = \left(\frac{q^2}{g}\right)^{1/3}$

$$h_c = \left(\frac{0.75^2}{9.81}\right)^{1/3} = 0.3859 \text{ m.}$$

But the depth of flow in the upstream side, i.e d_1 is 0.3 m
Thus, depth of flow on the upstream side is less than the critical depth on the upstream side. Hence hydraulic jump occurs.

We know,

Depth of flow after hydraulic jump. i.e d_2 is :-

$$d_2 = -\frac{d_1}{2} + \sqrt{\frac{d_1^2}{4} + \frac{2q^2}{gd_1}}$$

$$= -\frac{0.3}{2} + \sqrt{\frac{0.3^2}{4} + \frac{2 \times 0.75^2}{9.81 \times 0.3}}$$

$$d_2 = 0.4862 \text{ m}$$

$\therefore$ Height of hydraulic jump = $(d_2 - d_1) = 0.1862$ m.

Also,

Loss of energy per kg of water due to hydraulic jump is :-

$$h_L = \frac{(d_2 - d_1)^3}{4d_1d_2}$$

$$= \frac{(0.4862 - 0.30)^3}{4 \times 0.3 \times 0.4862}$$

$\therefore$ $h_L = 0.01106$ m-kg/kg

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Q. A sluice gate discharges water into a horizontal rectangular channel with a velocity of 10 m/sec and depth of flow of 1m. Determine the depth of flow after the jump and consequent loss in total head?

Solⁿ:- Given:

Velocity of flow before hydraulic jump, $V_1 = 10$ m/sec

Depth of flow before hydraulic jump, $d_1 = 1$ m.

Discharge per unit width, $q = V_1 d_1 = 10 \times 1 = 10\ m^2/sec$.

The depth of flow after jump is given by:-

$$d_2 = -\frac{d_1}{2} + \sqrt{\frac{d_1^2}{4} + \frac{2q^2}{g d_1}}$$

$$= -\frac{1}{2} + \sqrt{\frac{1^2}{4} + \frac{2 \times 10^2}{9.81 \times 1}}$$

$$\therefore d_2 = 4.043 \text{ m.}$$

And, Loss in total head is:-

$$h_L = \frac{(d_2 - d_1)^3}{4 d_1 d_2}$$

$$= \frac{(4.043 - 1)^3}{4 \times 1 \times 4.043}$$

$$\therefore h_L = 1.742 \text{ m.}$$

Q. A sluice gate discharges water into a horizontal rectangular channel with a velocity of 6m/sec and depth of flow is 0.4 m. The width of the channel is 8 m. Determine whether a hydraulic jump will occur, and if so, find the height and loss of energy per kg of water. Also determine the power lost in the hydraulic jump?

Sol^n:- Given,

Velocity of flow, $V_1 = 6$ m/sec.

Depth of flow, $d_1 = 0.4$ m

Width of channel, $b = 8$ m

$\therefore$ Discharge per unit width, $q = \dfrac{Q}{b} = \dfrac{A_1 \times V_1}{b}$

$$q = \frac{b \times d_1 \times V_1}{b} = V_1 \times d_1 = 6 \times 0.4 = 2.4 \text{ m}^2/\text{sec.}$$

We know,

Froude number on the upstream side is:

$$(Fe)_1 = \frac{V_1}{\sqrt{g d_1}} = \frac{6}{\sqrt{9.81 \times 0.4}} = 3.0289 \approx 3.029$$

As Froude number in the upstream side is more than 1, therefore the flow is a shooting flow in the upstream side. Shooting flow is an unstable flow and it will convert itself into streaming flow by raising its height and hence hydraulic jump will take place.

Let the depth of hydraulic jump $= d_2$

We know,

$$d_2 = \frac{d_1}{2}\left(\sqrt{1 + 8(Fe)_1^2} - 1\right)$$

$$= \frac{0.4}{2}\left(\sqrt{1 + 8\,(3.029)^2} - 1\right) = 1.525 \text{ m.}$$

$\therefore$ Height of hydraulic jump $= (d_2 - d_1) = 1.525 - 0.4 = 1.125$ m.

Also, Loss of energy per kg of water is:-

$$h_L = \frac{(d_2 - d_1)^3}{4 d_1 d_2}$$

$$= \frac{(1.525 - 0.4)^3}{4 \times 0.4 \times 1.525} = 10.5835 \text{ m-kg/kg of water}$$

P.T.O

And,

$$\text{Power lost in kW} = \frac{\rho g \times Q \times h_L}{1000}$$

$$P = \frac{\rho g \times A_1 \times V_1 \times h_L}{1000}$$

$$= \frac{\rho g \times b \times d_1 \times V_1 \times h_L}{1000}$$

$$= \frac{1000 \times 9.81 \times 8 \times 0.4 \times 6 \times 0.5835}{1000}$$

$\therefore P = 109.9$ kW.

Q. A hydraulic jump forms at the downstream end of spillway carrying 17.93 m³/sec discharge. If the depth before jump is 0.80 m, determine the depth after the jump and energy loss.

Soln:- Given, Discharge, $Q = 17.93\,m^3/sec$

Depth before jump, $d_1 = 0.8$ m

Taking ~~depth~~ width $b = 1$ m, we get:-

Discharge per unit width, $q = \frac{Q}{b} = \frac{17.93}{1} = 17.93\ m^2/s$

Let d_2 = Depth after jump and h_L = Loss of Energy

$$\therefore d_2 = -\frac{d_1}{2} + \sqrt{\frac{d_1^2}{4} + \frac{2q^2}{gd_1}} = \frac{-0.8}{2} + \sqrt{\frac{0.8^2}{4} + \frac{2 \times 17.93^2}{9.81 \times 0.8}}$$

$\therefore d_2 = 8.66$ m.

and, $$h_L = \frac{(d_2 - d_1)^3}{4d_1d_2} = \frac{(8.66 - 0.8)^3}{4 \times 0.8 \times 8.66} = 17.52\text{ m}$$

* FORMULAE USED TO DESIGN OPEN CHANNEL (OR) USED TO ANALYSE OPEN CHANNEL (OR) UNIFORM FLOW FORMULAE :→

Uniform flow :

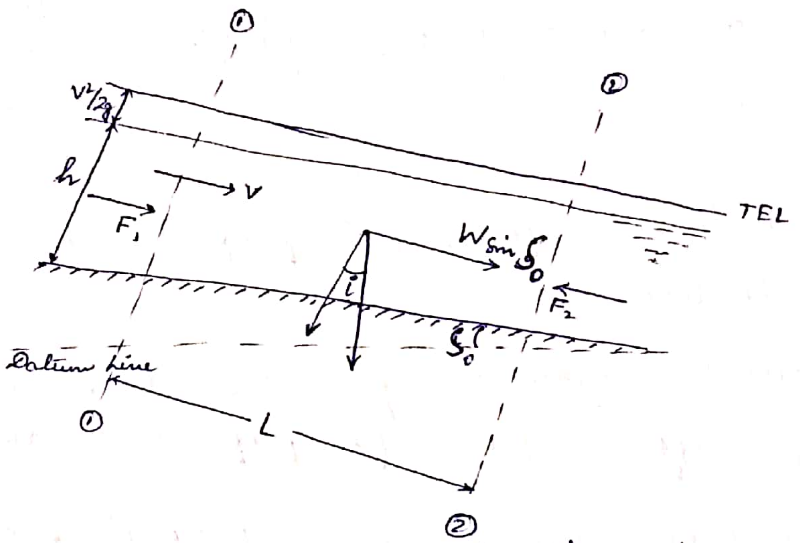
- ✓ A flow is said to be uniform if its properties remains constant with respect to distance.
- ✓ The term uniform flow in open channels is understood to mean steady uniform flow (flow properties remains const w.r.t time and length)
- ✓ The depth of flow remains constant at all sections in a uniform flow.
- ✓ Slope of channel bottom = Slope of water surface elevation = Slope of energy line.

Assumptions :

1. Flow is steady and uniform.
2. Channel bottom slope is small, i.e $\sin i = i$ or $\sin\theta = \theta$.
3. Channel is prismatic (artificial channel). Hence various channel properties like area of cross-section of channel, roughness of the channel surface will not change, remain constant.
4. Since velocity is constant, thus energy line will also remain constant. Thus Slope of energy line (S_f) = Slope of bottom of channel (S_0) = Slope of water surface elevation (S_w), as depth (y) is constant.

• Chezy's Formula (Developed in 1775) →

Consider a longitudinal section of an open channel in which the flow is steady and uniform, as shown in the figure. The forces acting on the free body of water between sections 1-1 and 2-2 in the direction of flow are as follows :



let L = Characteristic length of the channel.

A = Area of flow of water

S_0 = i or θ = Slope of the bed. (Angle of inclination of channel bottom with the horizontal)

V = Mean velocity of flow of water.

P = Wetted perimeter of the cross-section

f = Frictional resistance per unit velocity per unit area.

w = Specific weight of Water

∴ The weight of water between section 1-1 and 2-2 is:

$$W = \text{Specific weight of water} \times \text{volume of water}$$

$$= w \times A \times L$$

The component of weight of water along the direction of flow $= W \times \sin i = wAL \sin i$

The forces acting on the water between sections 1-1 and 2-2 are:

1. Pressure forces F_1 and F_2 acting on the two ends of the body at section 1-1 and 2-2 respectively. Since the depths of water at section 1-1 and 2-2 are the same, hence the pressure forces on these two sections are same, which act in opposite direction. Hence they cancel each other.

2. The component of weight of water in the direction of flow, which is $= W \sin\theta = wAL \sin i$.

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3. Frictional resistance acting against motion of water, offered by the sides of the channel which is given by:

$$F_R \propto AV^n$$

Experimentally, the value of n is found to be 2

$$\therefore F_R \propto AV^2$$

Here, A = wetted area = $P \times L$

$$\therefore F_R \propto P \times L \times V^2$$

$$\Rightarrow F_R = f\,PLV^2$$

where, f is a non-dimensional factor, called frictional coefficient, whose value depends upon the material and nature of flow surface.

As the flow is steady and uniform, it is neither accelerating nor decelerating; the liquid mass is in equilibrium and the frictional resistance acting against the motion of water is equal to the weight component of water in the direction of flow.

$$\therefore wAL\sin i = f\,PLV^2$$

$$\Rightarrow f\,PLV^2 = wAL\sin S_0$$

$$\Rightarrow V^2 = \frac{wAL\sin S_0}{f\,PL}$$

$$\Rightarrow V^2 = \frac{w}{f} \times \frac{A}{P} \times \sin S_0$$

$$V = \sqrt{\frac{w}{f}} \times \sqrt{\frac{A}{P} \times \sin S_0}$$

$$V = \sqrt{\frac{w}{f}} \times \sqrt{\frac{A}{P} S_0} \quad [\because i \text{ is small, so } \sin i = i]$$

Here, $\frac{A}{P} = m$ = hydraulic mean depth or hydraulic radius.

and, $\sqrt{\frac{w}{f}} = C = $ Chezy's constant

$\therefore V = C\sqrt{mi}$

C = Chezy's constant is a variable which depends on the roughness of the channel surface.

$\therefore$ Discharge, Q = Area (A) x Velocity (V)

$$Q = AC\sqrt{mi}$$

The Chezy's equation can also be written as:-

$$Q = K\sqrt{i}$$

where, $K = AC\sqrt{m}$ and it is called the conveyance of the channel section, and is a measure of the carrying capacity of the channel.

For a channel of constant slope, conveyance is directly proportional to discharge Q.

Q. Find the velocity of flow and rate of flow of water through a rectangular channel of 6m wide and 3m deep, when it is running full. The channel is having bed slope as 1 in 2000. Take Chezy's constant, C = 55.

Solⁿ:- Given:

Width of rectangular channel, b = 6m

Depth of channel, d = 3m.

$\therefore$ Area, $A = 6 \times 3 = 18\ m^2$

Bed slope, $i = 1$ in $2000 = \frac{1}{2000}$

Chezy's constant, C = 55.

Perimeter, $P = b + 2d = 6 + 2 \times 3 = 12\ m$.

$\therefore$ Hydraulic mean depth, $m = \frac{A}{P} = \frac{18}{12} = 1.5$ m.

Velocity of flow is given by:

$$V = C\sqrt{mi} = 55\sqrt{1.5 \times \frac{1}{2000}} = 1.506 \text{ m/sec.}$$

$\therefore$ Rate of flow, Q = Area $\times$ Velocity $= 18 \times 1.506 = 27.108 \text{ m}^3/\text{sec}$

Q. Find the slope of the bed of a rectangular channel of width 5m when depth of water is 2m and rate of flow is given as 20m³/s. Take Chezy's constant, C = 50.

Solⁿ:- Given,

Width of channel, $b = 5m$

Depth of water, $d = 2m$

Rate of flow, $Q = 20 \text{ m}^3/\text{sec}$

Chezy's constant, $C = 50$

Let the bed slope be 'i'

We know,

$$Q = AC\sqrt{mi} \quad \text{—(i)}$$

Here, $A = b \times d = 5 \times 2 = 10 \text{ m}^2$

$$m = \frac{A}{P} = \frac{10}{b+2d} = \frac{10}{5+2\times 2} = \frac{10}{9} \text{ m}$$

Substituting these values in eqⁿ (i), the value of 'i' is obtained.

$$20 = 10 \times 50 \times \sqrt{\frac{10}{9} \times i}$$

$$\Rightarrow \frac{10}{9} i = \frac{4}{2500}$$

$$\Rightarrow i = \frac{4}{2500} \times \frac{9}{10}$$

$$\Rightarrow i = \frac{36}{25000}$$

$$\Rightarrow i = \frac{1}{\frac{25000}{36}} = \frac{1}{694.44}$$

$\therefore$ Bed slope is 1 in 694.44

Q. A flow of water of 100 litres per second flows down in a rectangular flume of width 600 mm and having adjustable bottom slope. If Chezy's constant C is 56, find the bottom slope necessary for uniform flow with a depth of flow of 300 mm. Also find the conveyance K of the flume.

Soln:- Given :

Discharge, $Q = 100$ lt/sec $= \frac{100}{1000} = 0.10 \ m^3/sec$

Width of channel, $b = 600$ mm $= 0.60$ m

Depth of flow, $d = 300$ mm $= 0.30$ m

$\therefore$ Area of flow, $A = b \times d = 0.6 \times 0.3 = 0.18 \ m^2$

Chezy's constant, $C = 56$

Let the slope of bed be 'i'.

Hydraulic mean depth, $m = \frac{A}{P} = \frac{0.18}{b+2d} = \frac{0.18}{0.6 + 2 \times 0.30} = \frac{0.18}{1.2}$

$\Rightarrow m = 0.15$ mt.

We know, $Q = AC\sqrt{mi}$

$\Rightarrow 0.10 = 0.18 \times 56 \times \sqrt{0.15 \times i}$

$\Rightarrow \sqrt{0.15i} = \frac{0.10}{0.18 \times 56}$

$\Rightarrow 0.15i = \left(\frac{0.10}{0.18 \times 56}\right)^2$

$\Rightarrow 0.15i = 0.000098418$

$\Rightarrow i = \frac{0.000098418}{0.15} = 0.0006512$

$\Rightarrow i = \frac{1 \times 0.0006512}{1} = \frac{1}{\frac{1}{0.0006512}}$

$\Rightarrow i = \frac{1}{1524}$

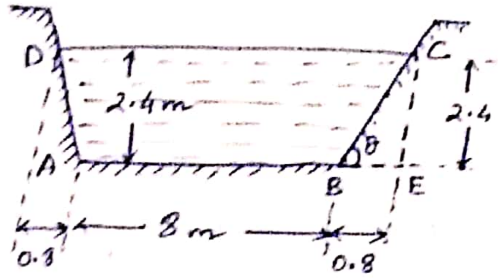
$\therefore$ Slope of the bed is 1 in 1524.

Also, $K = AC\sqrt{m}$

$= 0.18 \times 56 \times \sqrt{0.15} = 3.9039\ m^3/sec$

** Q. Find the discharge through a trapezoidal channel of width 8 m and side slope of 1 horizontal to 3 vertical. The depth of flow of water is 2.4m and value of Chezy's constant, C = 50. The slope of the bed of the channel is given 1 in 4000.

Soln:- Given,



Width, $b = 8m$

Side slope $= 1\ H\ to\ 3\ V$

Depth, $d = 2.4m$

Chezy's Constant, $C = 50$

Bed slope, $i = \frac{1}{4000}$

From the figure, $CE = 2.4$

From $\triangle CBE$, $BE = 2.4 \times \frac{1}{3} = 0.8m$

From the figure,

Top width of channel, $CD = AB + 2BE$

$= 8 + 2 \times 0.8 = 9.6m$

$\therefore$ Area of the trapezoidal channel ABCD is given as

Area, $A = \frac{AB + CD}{2} \times CE$

$= \frac{8 + 9.6}{2} \times 2.4 = 21.12\ m^2$

Wetted Perimeter, $P = DA + AB + BC$

$= AD + AB + BC$

$= AB + AD + BC$

$= AB + 2BC$

Here,

$BC = \sqrt{BE^2 + CE^2} = \sqrt{(0.8)^2 + (2.4)^2} = 2.529\ m.$

$\therefore P = AB + 2BC = 8.0 + 2 \times 2.529 = 13.058m$

Hydraulic mean depth, $m = \frac{A}{P} = \frac{12.12}{13.058} = 1.617\ m.$

$\therefore Q = AC\sqrt{mi} = 21.12 \times 50 \times \sqrt{1.617 \times \frac{1}{4000}} = 21.23\ m^3/sec$

Q. Find the bed slope of trapezoidal channel of bed width 6m, depth of water 3m and side slope of 3 horizontal to 4 vertical, when the discharge through the channel is $30\ m^3/sec$. Take Chezy's constant $C = 70$.

Solⁿ:- Given,

Bed width, $b = 6m$

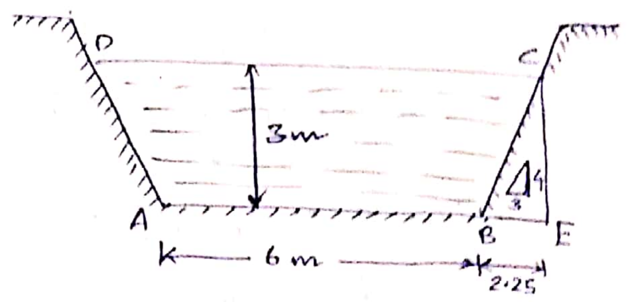
Depth of flow, $d = 3m$.

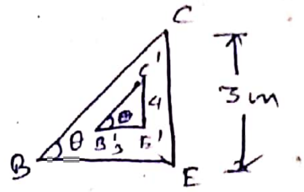
Side slope $= 3H$ to $4V$

Discharge, $Q = 30\ m^3/sec$

Chezy's constant, $C = 70$

Water depth, $CE = 3m$





From the figure, i.e, ΔCBE and the $\Delta C'B'E'$, both the triangles are similar in nature

In ΔCBE,

$$\tan\theta = \frac{3}{BE},$$

$$\Rightarrow BE = \frac{3}{\tan\theta}$$

Similarly, In $\Delta C'B'E'$, $\tan\theta = \frac{4}{3}$

$\because \Delta CBE \approx \Delta C'B'E'$

$$\therefore BE = \frac{3}{4/3} = 3 \times \frac{3}{4} = \frac{9}{4} = 2.25m$$

$\therefore$ Top width; $CD = AB + 2 \times BE = 6 + 2 \times 2.25 = 10.50m$

Wetted perimeter, $P = DA + AB + BC$

$= AD + AB + BC = AB + AD + BC = AB + 2BC$

But, $BC = \sqrt{BE^2 + CE^2} = \sqrt{2.25^2 + 3^2}$

$\therefore$ $P = 6 + 2 \times BC = 13.5\ m.$

Area of trapezoidal channel ABCD is :-

$$A = \frac{AB + CD}{2} \times CE$$

$$= \frac{6 + 10.5}{2} \times 3 = 24.75\ m^2$$

$\therefore$ Hydraulic mean depth, $m = \frac{A}{P} = \frac{24.75}{13.50} = 1.833$

<u>To find 'i'</u>

We know,

$$Q = AC\sqrt{mi}$$

$$\Rightarrow 30 = 24.75 \times 70 \times \sqrt{1.833 \times i}$$

$$\Rightarrow 30 = 2345.6\sqrt{i}$$

$$\Rightarrow \sqrt{i} = \frac{30}{2345.6}$$

$$\Rightarrow i = \left(\frac{30}{2345.6}\right)^2.$$

$$\Rightarrow i = 1 \times \frac{\left(\frac{30}{2345.6}\right)^2}{1.}$$

$$\Rightarrow i = \frac{1}{\left(\frac{2345.6}{30}\right)^2} = \frac{1}{6133}$$

$\therefore$ Bed slope is 1 in 6133.

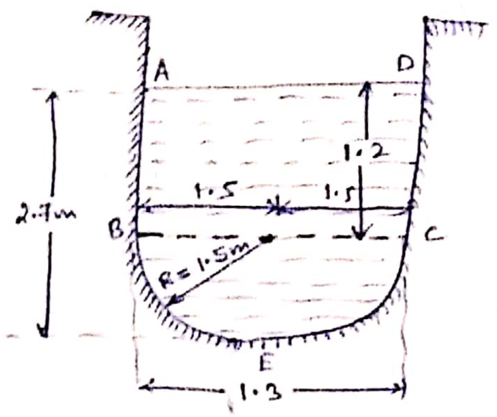
Q. Find the discharge of water through the channel shown in the figure. Take the value of Chezy's constant = 60 and slope of the bed as 1 in 2000

P.T.O

Sol^n:- Given

Chezy's constant, $C = 60$

Bed slope, $i = \frac{1}{2000}$



From the figure,

Area of ABECD = Area of ABCD + Area of BEC

$$= (1.2 \times 3.0) + \frac{\pi R^2}{2}$$

$$= 3.6 + \frac{\pi \times (1.5)^2}{2}$$

$$= 7.134 \ m^2$$

$\therefore$ Hydraulic mean depth, $m = \frac{A}{P}$ =

Here, P = Wetted perimeter = AB + BEC + CD

$= 1.2 + \pi R + 1.2$

$= 1.2 + 1.2 + \pi \times 1.5 = 7.1124 \ m.$

$$\therefore m = \frac{7.134}{7.1124} = 1.003.$$

We know,

$$Q = AC\sqrt{mi}$$

$$= 7.134 \times 60 \times \sqrt{1.003 \times \frac{1}{2000}}$$

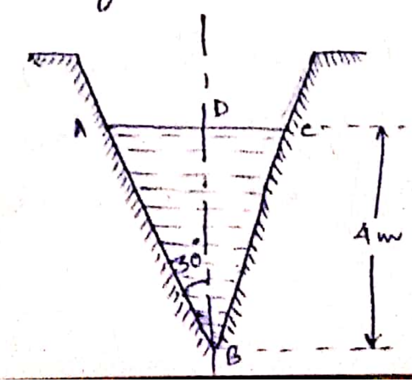
$$Q = 9.585 \ m^3/sec$$

Q. Find the rate of flow of water through a V-shaped channel as shown in the figure. Take the value of $C = 55$ and slope of the bed as 1 in 2000.

Sol^n:- Given:

Chezy's constant, $C = 55$

Bed slope, $i = \frac{1}{2000}$



Depth of flow, $d = 4\,m$

Angle made by each side with vertical,

$$\angle ABD = \angle CBD = 30^\circ$$

From the figure,

$$\text{Area of } ABC,\ A = 2 \times \text{Area of } ABD$$

$$= 2 \times \frac{1}{2} \times AD \times BD$$

$$= AD \times BD.$$

From the figure,

$$\tan 30^\circ = \frac{AD}{BD} \Rightarrow AD = BD \tan 30^\circ.$$

$$\therefore A = BD \tan 30^\circ \times BD = 4 \tan 30^\circ \times 4 = 9.2376\,m^2$$

Wetted Perimeter, $P = AB + BC = 2AB$

$$= 2\sqrt{BD^2 + AD^2}$$

$$= 2\sqrt{4^2 + (4\tan 30^\circ)^2}$$

$$P = 9.2375\,m.$$

$\therefore$ Hydraulic mean depth, $m = \frac{A}{P} = \frac{9.2376}{9.2375} = 1\,m$

$$\therefore Q = AC\sqrt{mi} = 9.2376 \times 55 \times \sqrt{1 \times \frac{1}{1000}} = 16.066\,m^3/sec.$$

EMPERICAL FORMULAE FOR THE VALUE OF CHEZY'S CONSTANT :-

In Chezy's Formula, the term C is known as Chezy's constant, which is not a dimensionless coefficient. The dimension of C is:-

$$V = C\sqrt{mi}$$

$$\Rightarrow C = \frac{V}{\sqrt{mi}} = \frac{L/T}{\sqrt{Li}}$$

$$= \frac{L/T}{\sqrt{\frac{A}{P}i}} = L^{1-1/2}\,T^{-1}$$ [since i is dimensionless, so it is neglected]

$$= \frac{L/T}{\sqrt{\frac{L^2}{L}i}}$$

Thus, the value of C depends upon the system of units.

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The followings are the empirical formulae, used to determine the value of C.

1. Bazin's formula:

$$C = \frac{157.6}{1.81 + \frac{K}{\sqrt{m}}}$$

Here,

K = Bazin's constant which depends upon the roughness of the channel surface

m = Hydraulic mean depth or Hydraulic radius.

2. Manning's formula: $V = \frac{1}{N} m^{1/2} i^{2/3}$

$$C = \frac{1}{N} m^{1/6}$$

Here,

m = Hydraulic mean depth.

N = Manning's constant, called coefficient of roughness

The values of K are given in the below table:-

S.L NO.	NATURE OF CHANNEL INSIDE SURFACE	VALUE OF K
1.	Smooth cemented or planned wood	0.11
2.	Brick or concrete or unplanned wood	0.21
3.	Rubble masonry or Ashlar or poor brick work	0.83
4.	Earthen channel of very good surface	1.54
5.	Earthen channel of ordinary surface	2.36
6.	Earthen channel of rough surface	3.17

S = m R = i

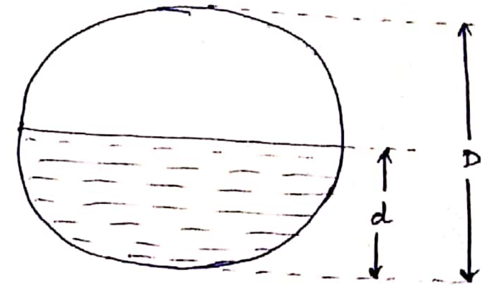
Contd....

The values of N are given in the below table:—

SL NO.	NATURE OF CHANNEL INSIDE SURFACE	VALUE OF N
1.	Very smooth surface of glass, plastic or brass	0.010
2.	Smooth surface of concrete	0.012
3.	Rubble masonry or poor brick work	0.017
4.	Earthen channels neatly excavated.	0.018
5.	Earthen channels of ordinary surface	0.027
6.	Earthen channels of rough surface	0.030
7.	Natural streams, clean and straight	0.030
8.	Natural streams with weeds, deep pools etc	0.075 to 0.15

Q. Find the diameter of a circular sewer pipe which is laid at a slope of 1 in 8000 and carries a discharge of 800 litres/sec when flowing half full. Take the value of Manning's N = 0.020.

Sol^n:- given:



Slope of pipe, $i = \frac{1}{8000}$

Discharge, $Q = 800$ Lt/sec $= \frac{800}{1000}$ m³/sec

$\Rightarrow Q = 0.8$ m³/sec

Manning's $N = 0.020$.

Let the dia of the sewer pipe be D

Depth of flow, $d = \frac{D}{2}$ [∵ the pipe is flowing half full, as given in the question]

∴ Area of flow, $A = \frac{\text{Area of pipe}}{2} = \frac{\pi/4\, D^2}{2} = \frac{\pi}{4} D^2 \times \frac{1}{2} = \frac{\pi D^2}{8}$

and Wetted Perimeter, $P = \frac{\text{Perimeter of pipe}}{2} = \frac{\pi D}{2}$

∴ Hydraulic mean depth, $m = \frac{A}{P} = \frac{\pi D^2/8}{\pi D/2} = \frac{D}{4}$

Using Manning's formula, we have

$C = \frac{1}{N} m^{1/6}$

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We know that,

$$Q = AC\sqrt{mi}$$

$$\Rightarrow 0.8 = \frac{\pi D^2}{8} \times \frac{1}{N} m^{1/6} \times \sqrt{D/4 \times \frac{1}{8000}}$$

$$\Rightarrow 0.8 = \frac{\pi D^2}{8} \times \frac{1}{0.020} \times \left(\frac{D}{4}\right)^{1/6} \times \left(D/4\right)^{1/2} \times \left(\frac{1}{8000}\right)^{1/2}$$

$\therefore D = 2.296$ m.

* GRADUALLY VARIED FLOW (GVF):→

✓ A steady (fluid properties, like depth does not vary with time), non-uniform flow (depth of flow varies with distance) in a prismatic channel (artificial channel) with gradual changes in its water surface elevation is named as gradually varied flow (GVF).

→ Analysis of GVF:

The following assumptions are made:

1. The pressure distribution at any section is assumed to be hydrostatic.
2. The velocity of flow at a given depth is assumed to be given by the corresponding uniform flow equation, such as the Manning equation, with the condition that the slope term to be used in the equation is the energy line slope (i_e or S_f) instead of bed slope (i_b or S_0)

i.e. For uniform flow,

$v = C\sqrt{m\, i_b}$ (Chezy's equation)

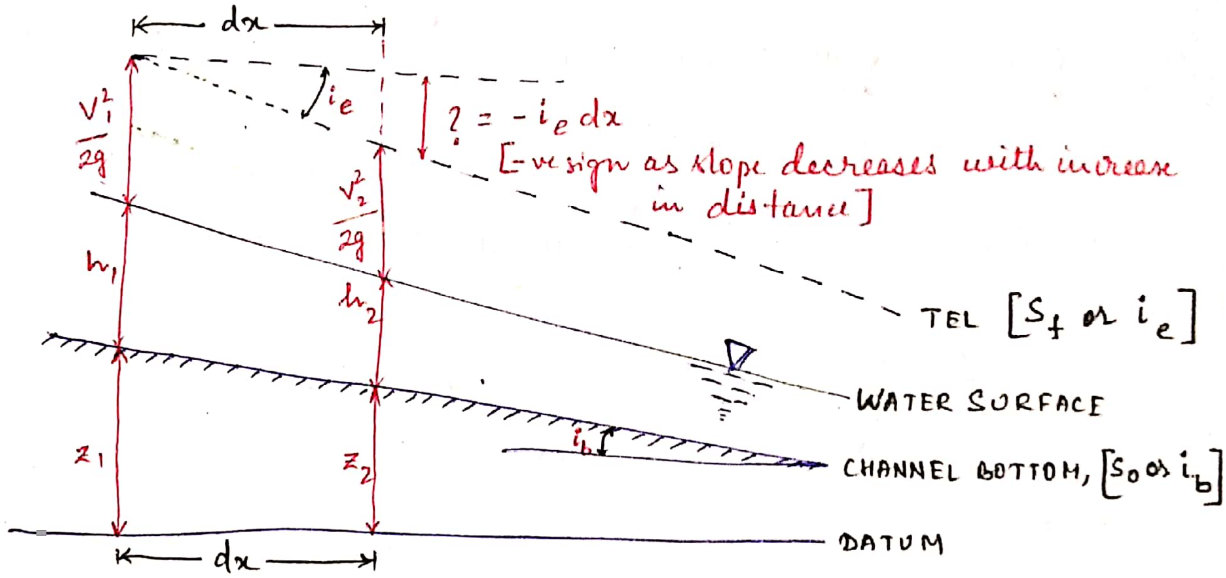
$v = \frac{1}{N} m^{2/3} i_b^{1/2}$ (Manning's equation)

Now, for a GVF.

$v = C\sqrt{m\, i_e}$ (Chezy's equation)

$v = \frac{1}{N} m^{2/3} i_e^{1/2}$ (Manning's equation)

3. The bed slope of the channel is small.
4. The flow is steady and hence discharge Q is constant
5. Accelerative effect is negligible and hence hydrostatic pressure distribution prevails over channel cross-section.
6. The roughness coefficient is constant for the length of the channel and it does not depend on the depth of flow.
7. The channel is prismatic.



Let, z_1 = height of bottom of channel above datum @ 1-1
h_1 = depth of flow @ 1-1
z_2 = height of bottom of channel above datum @ 2-2
h_2 = depth of flow @ 2-2.

V = Mean velocity of flow.
V_1 = Velocity of flow @ 1-1
V_2 = Velocity of flow @ 2-2.
b = width of the channel
Q = discharge through the channel
q = discharge per unit width
i_b = slope of the channel bed
i_e = slope of the energy line.

From the figure,

$[\because \tan\theta = \frac{P}{b}]$ $\tan i_b = \frac{\text{distance b/w horizontal line and TEL}}{dx}$

$\because$ The slope i_e is very small, so $\tan i_e = i_e$

$\therefore i_e = \frac{\text{distance b/w horizontal line and TEL}}{dx}$

$\Rightarrow$ distance b/w horizontal line and TEL $= i_e dx$.

From the figure, since slope decreases with increase in distance, so it is denoted with negative sign, i.e $-i_e dx$.

The energy equation at any section is given by Bernoulli's equation:

$$E = Z + h + \frac{V^2}{2g}$$

From the figure, the energy value at section 1-1 and 2-2 can be equated as:-

$$Z_1 + h_1 + \frac{V_1^2}{2g} = Z_2 + h_2 + \frac{V_2^2}{2g} + (-i_e dx)$$

$$\Rightarrow Z_1 + h_1 + \frac{V_1^2}{2g} = Z_2 + h_2 + \frac{V_2^2}{2g} - i_e dx$$

$$\Rightarrow (Z_1 - Z_2) + (h_1 - h_2) + \left(\frac{V_1^2}{2g} - \frac{V_2^2}{2g}\right) = -i_e dx$$

Here, $Z_1 - Z_2 = dz$ and $h_1 - h_2 = dh$ and $\frac{V_1^2}{2g} - \frac{V_2^2}{2g} = d\left(\frac{V^2}{2g}\right)$

$$\therefore dz + dh + d\left(\frac{V^2}{2g}\right) = -i_e dx$$

Dividing throughout by dx, we get:-

$$\frac{dz}{dx} + \frac{dh}{dx} + \frac{d}{dx}\left(\frac{V^2}{2g}\right) = -i_e \frac{\cancel{dx}}{\cancel{dx}}$$

$$\Rightarrow \frac{dz}{dx} + \frac{dh}{dx} + \frac{d}{dx}\left(\frac{V^2}{2g}\right) = -i_e$$

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Here,

$$\frac{dz}{dx} = \frac{z_1 - z_2}{dx} = \text{slope of the channel bed or bottom of channel}$$
$$= i_b$$

From the figure, since slope decreases with increase in distance, so it is denoted with negative sign, i.e $-i_b$

$$\therefore \frac{dz}{dx} = -i_b$$

Also,

$$Q = A \times V \Rightarrow V = \frac{Q}{A} = \frac{Q}{b \times h} = \frac{q}{h} \quad \left[\because q = \frac{Q}{b}\right]$$

So, the equation becomes:

$$-i_b + \frac{dh}{dx} + \frac{d}{dx}\left(\frac{q^2}{2gh^2}\right) = -i_e$$

$$\Rightarrow \frac{dh}{dx} + \frac{d}{dx}\left(\frac{q^2}{2gh^2}\right) = -i_e + i_b$$

$$\Rightarrow \frac{dh}{dx} + \frac{d}{dx}\left(\frac{q^2}{2gh^2}\right) = i_b - i_e$$

$$\Rightarrow \frac{dh}{dx} + \frac{q^2}{2g}\frac{d}{dx}\left(\frac{1}{h^2}\right) = i_b - i_e$$

$$\Rightarrow \frac{dh}{dx} + \frac{q^2}{2g}\frac{d}{dx}\left(h^{-2}\right) = i_b - i_e$$

$$\Rightarrow \frac{dh}{dx} + \frac{q^2}{2g}\left(-2h^{-3}\right)\frac{dh}{dx} = i_b - i_e$$

$$\Rightarrow \frac{dh}{dx} - \frac{2q^2}{2gh^3}\frac{dh}{dx} = i_b - i_e$$

$$\Rightarrow \frac{dh}{dx}\left(1 - \frac{q^2}{gh^3}\right) = i_b - i_e$$

$$\Rightarrow \frac{dh}{dx} = \frac{i_b - i_e}{\left(1 - \frac{q^2}{gh^3}\right)}$$

Here, $\frac{dh}{dx}$ is the rate of change of flow depth along with distance through the bottom of the channel.

Now,

$$\frac{q^2}{gh^3} = \frac{Q^2}{b^2 g h^3} = \frac{Q^2}{b^2 . gh^2 . h} = \frac{Q^2}{b^2 h^2 . gh} = \frac{v^2}{gh}$$

$$= \frac{v^2}{(\sqrt{gh})^2} = \left(\frac{V}{\sqrt{gh}}\right)^2 = F_r^2$$

$$\therefore \frac{dh}{dx} = \frac{(i_b - i_e)}{[1 - (F_r)^2]}$$

This equation is the final form of the gradually varied flow analysis, and is known as dynamic equation for a GVF.

where, $\frac{dh}{dx}$ is called the slope of the free water surface.

The three different forms of GVF equation are:-

1. $\frac{dh}{dx} = \frac{i_b - i_e}{(1 - F_r^2)}$

2. $\frac{dh}{dx} = \frac{i_b - i_e}{\left(1 - \frac{q^2}{gh^3}\right)}$

3. $\frac{dh}{dx} = \frac{i_b - i_e}{\left(1 - \frac{v^2}{gh}\right)}$

Points to remember:

1. When $\frac{dh}{dx} = 0$, h is constant, i.e depth of the water above the bottom of the channel is constant. It means that the free surface of water is parallel to the bed of the channel.

2. When $\frac{dh}{dx} > 0$ or $\frac{dh}{dx}$ is +ve, it means the depth of water increases in the direction of flow. The profile of the water so obtained is called back water curve.

3. When $\frac{dh}{dx} < 0$ or $\frac{dh}{dx}$ is -ve, it means the depth of water decreases in the direction of flow. The profile of the water so obtained is called drop down curve.

Q. Find the rate of change of depth of water in a rectangular channel of 10m width and 1.5m depth, when the water is flowing with a velocity of 1 m/sec. The flow of water through the channel of bed slope ~~is~~ 1 in 4000, is regulated in such a way that the energy line slope is 0.00004 ?

Solⁿ:- Given :

Width, $b = 10\,m$

Depth, $h = 1.5\,m$

Velocity, $V = 1\,m/sec$

Bed slope, $i_b = 1$ in $4000 = 0.00025$

Energy line slope, $i_e = 0.00004$

To find, rate of change of depth of water, i.e $\frac{dh}{dx}$

We know,

$$\frac{dh}{dx} = \frac{i_b - i_e}{\left(1 - \frac{V^2}{gh}\right)}$$

$$= \frac{0.00025 - 0.00004}{\left(1 - \frac{1^2}{9.81 \times 1.5}\right)}$$

$$\therefore \frac{dh}{dx} = 2.253 \times 10^{-4}\ m$$

Q. Find the slope of free water surface in a rectangular channel of width 20m, having depth of flow 5m. The discharge through the channel is 50 m³/sec. The bed slope of the channel is given by 1 in 4000, Take Chezy's $C = 60$.

Solⁿ:- Given,

$b = 20\,m.$

$h = 5\,m$

P.T.O

$Q = 50\,m^3/sec$

$\therefore\ q = \frac{Q}{b} = \frac{50}{20} = 2.5\,m^2/sec$

$i_b = 1 \text{ in } 4000 = 0.00025$

$C = 60$

For uniform flow (GVF),

$$V = C\sqrt{m\,i_e}$$

$$\therefore Q = AC\sqrt{m\,i_e}$$

$$\Rightarrow 50 = (20 \times 5) \times 60 \times \sqrt{\frac{A}{P} i_e}$$

$$\Rightarrow 50 = 100 \times 60 \times \sqrt{\left(\frac{20 \times 5}{20 + 2 \times 5}\right) i_e}$$

$$\Rightarrow i_e = 0.0000208$$

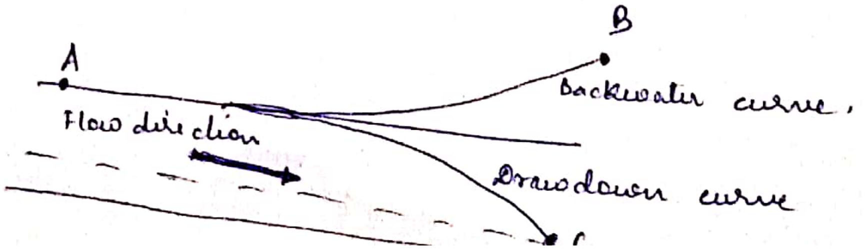
We know that,

$$\frac{dh}{dx} = \frac{i_b - i_e}{\left(1 - \frac{q^2}{gh^3}\right)}$$

$$= \frac{0.00025 - 0.0000208}{\left(1 - \frac{2.5^2}{9.81 \times 5^3}\right)}$$

$$\therefore \frac{dh}{dx} = 2.304 \times 10^{-4}\ m.$$

Backwater curve & Drawdown curve:-



Dynamic Equation for Gradually Varied Flow (GVF) in a very wide rectangular channel :

Dynamic equation for GVF is given by:

$$\frac{dh}{dx} = \frac{i_b - i_e}{\left[1 - \frac{q^2}{gh^3}\right]}$$

We know,

$$\text{Critical depth } (h_c) = \left(\frac{q^2}{g}\right)^{1/3}$$

$$\Rightarrow [h_c]^{3 \times \frac{1}{3}} = \left[\frac{q^2}{g}\right]^{1/3}$$

$$\Rightarrow h_c^3 = \frac{q^2}{g}$$

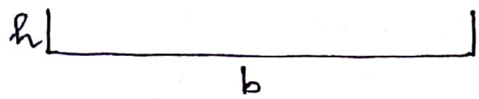
Substituting the value of h_c^3 in the dynamic equation for GVF, we get :-

$$\frac{dh}{dx} = \frac{i_b - i_e}{1 - \frac{h_c^3}{h^3}}$$

$$\frac{dh}{dx} = \frac{i_b - i_e}{1 - \left(\frac{h_c}{h}\right)^3}$$

In case of a very wide rectangular channel, the width of the channel is much greater than the depth of the channel.

i.e, $b >> h$



We know that,

$$m = \frac{A}{P}$$

$$m = \frac{b \times h}{b + 2h}$$

$\because$ $b >> h$, so the denominator $(b + 2h) \approx b$

$$\therefore m = \frac{\cancel{b} \times h}{\cancel{b}} \Rightarrow m = h.$$

From Manning's equation, the expression for discharge of a GVF is given by :

$$Q = A \times V = \frac{A}{N} m^{2/3} i_e^{1/2}$$

When the flow is uniform, i.e at normal depth –

$$h = h_n \text{ and } i_e = i_b$$

$$\therefore \quad Q = \frac{b \times h_n}{N} (h_n)^{2/3} i_b^{1/2} \quad \text{——— (ii)}$$

In case of GVF, i.e at a general depth –

$$Q = \frac{b \times h}{N} (h)^{2/3} i_e^{1/2} \quad \text{——— (iii)}$$

Dividing eqn (ii) by eqn (iii), we get :–

$$1 = \left(\frac{h_n}{h}\right)^{5/3} \times \left(\frac{i_b}{i_e}\right)^{1/2}$$

$$\Rightarrow \left(\frac{i_e}{i_b}\right)^{1/2} = \left(\frac{h_n}{h}\right)^{5/3}$$

$$\Rightarrow \left(\frac{i_e}{i_b}\right)^{\frac{1}{2} \times 2} = \left(\frac{h_n}{h}\right)^{\frac{5}{3} \times 2}$$

$$\Rightarrow \frac{i_e}{i_b} = \left(\frac{h_n}{h}\right)^{\frac{10}{3}}$$

$$\Rightarrow 1 - \frac{i_e}{i_b} = 1 - \left(\frac{h_n}{h}\right)^{\frac{10}{3}}$$

$$\Rightarrow \frac{i_b - i_e}{i_b} = 1 - \left(\frac{h_n}{h}\right)^{\frac{10}{3}}$$

$$\Rightarrow i_b - i_e = i_b \left[1 - \left(\frac{h_n}{h}\right)^{\frac{10}{3}}\right]$$

Substituting the value of $(i_b - i_e)$ in the dynamic equation of GVF, we get :–

$$\frac{dh}{dx} = \frac{i_b\left[1-\left(\frac{h_n}{h}\right)^{\frac{10}{3}}\right]}{\left[1-\left(\frac{h_c}{h}\right)^{3}\right]}$$

This is the dynamic equation for GVF in a very wide rectangular channel.

Instead of using Manning's equation to derive the dynamic eqn of GVF for a wide rectangular channel, if Chezy's equation is used, then the dynamic eqn is :

$$\frac{dh}{dx} = \frac{i_b\left[1-\left(\frac{h_n}{h}\right)^{3}\right]}{\left[1-\left(\frac{h_c}{h}\right)^{3}\right]}$$

The difference in expression for both the conditions are different because the dynamic eqn for GVF in a wide rectangular channel is basically an approximation.

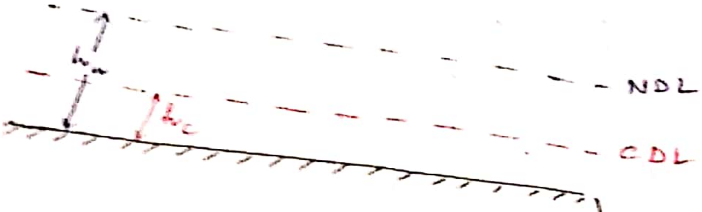
* CLASSIFICATION OF CHANEEL BOTTOM SLOPES :→

The channel bottom slopes are classified into 5 types. They are:-

1. Mild slope
2. Steep slope
3. Critical slope
4. Horizontal slope
5. Adverse slope.

The classification is based on the relative positions of the normal depth (h_n) and critical depth (h_c) of the flow through the channel. [If the flow is taking place @ normal depth, then depth is h_n and if the flow is taking place @ critical depth, then depth is h_c]

① Mild Slope : When the normal depth of flow is greater than the critical depth of flow, the channel bottom slope is known as a mild slope.



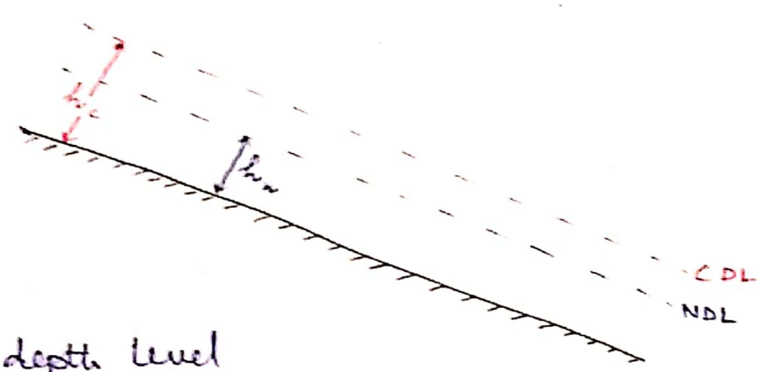
where,

NDL – normal depth level (uniform flow)

CDL – critical depth level.

↳ This a mild sloped channel. Here, $h_n > h_c$

② **Steep Slope:** When the normal depth of flow is lesser than the critical depth of flow, then the channel bottom slope is known as a steep slope.



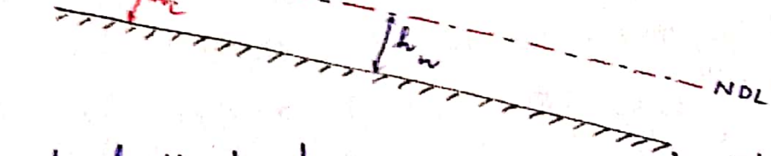
where,

NDL – normal depth level

CDL – critical depth level

↳ This is a steep sloped channel. Here, $h_c > h_n$

③ **Critical Slope:** When the normal depth of flow is equal to the critical depth of flow, the channel bottom slope is known as a critical slope.



where,

NDL – normal depth level

CDL – critical depth level

↳ This a critical slope channel.

Here, $h_c = h_n$

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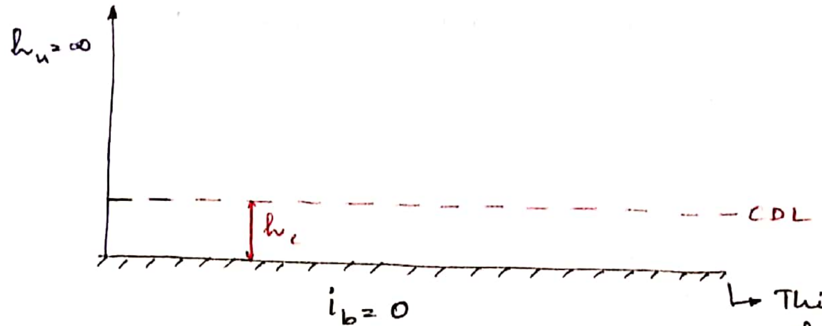
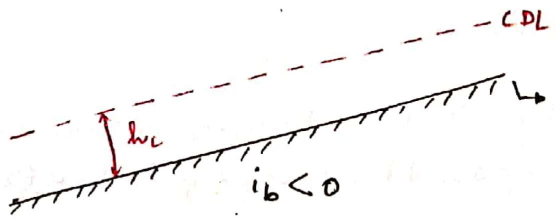
④ **Horizontal Slope:** When the channel bottom slope (i_b) is equal to zero, then the slope is known as a horizontal slope.
For a horizontal sloped channel, normal depth lies to infinity, i.e $h_n = \infty$. [normal depth cannot be attained].



↳ This is a horizontal sloped channel.
Here, $h_n = \infty$

where,
CDL – critical depth level.

⑤ **Adverse Slope:** When the channel bottom slope instead of falling, rises in the direction of flow ($i_b < 0$), it is called an adverse slope.
[the bottom slope instead of aiding the flow, is going in the opposite direction of the direction of flow, which means that as flow takes place upstream to down stream, so the bed slope goes downstream to upstream.]
For an adverse sloped channel, normal depth is imaginary or non-existant.



↳ This is a adverse sloped channel.
Here, h_n = imaginary

where, CDL – critical depth level.

* **CLASSIFICATION OF WATER SURFACE PROFILES:**
The various water surface profiles occuring in the channels are designated with reference to the bottom slopes of the channels.

✓ The surface profiles occuring in mild channels are called M curves

✓ The surface profiles occuring in steep channels are called S curves.

✓ The surface profiles occuring in critical channels are called C curves.

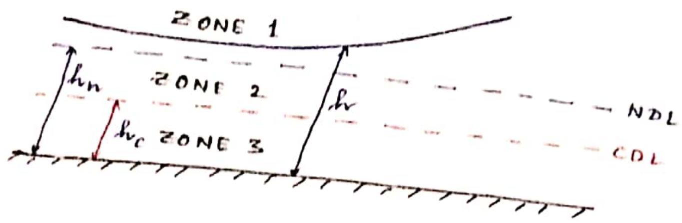
✓ The surface profiles occuring in horizontal channels are called H curves.

✓ The surface profiles occuring in adverse sloped channels are called A curves.

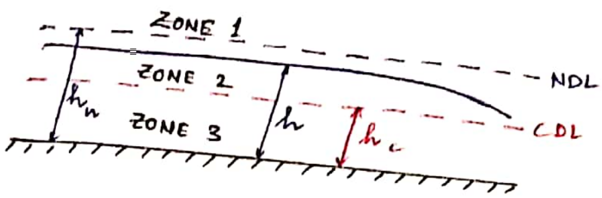
• M CURVES :→

There are 3 zones in a mild sloped channel.

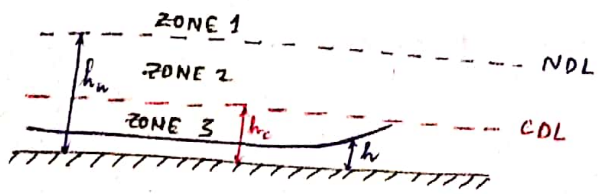
Case I: If water depth occurs in zone 1, then $h > h_n > h_c$. The water surface profile is called an M1 curve (M1 stands for mild channel zone 1)



Case II: If water depth occurs in zone 2, then $h_n > h > h_c$. The water surface profile is called an M2 curve (M2 stands for mild channel zone 2).



Case III: If water depth occurs in zone 3, then $h_n > h_c > h$. The water surface profile is called an M3 curve (M3 stands for mild channel zone 3)

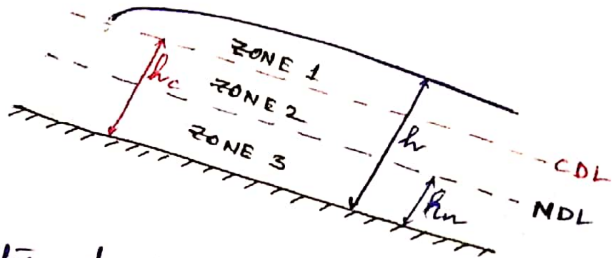


N:B: Water will be individually flowing either in zone 1 or in zone 2 or in zone 3.

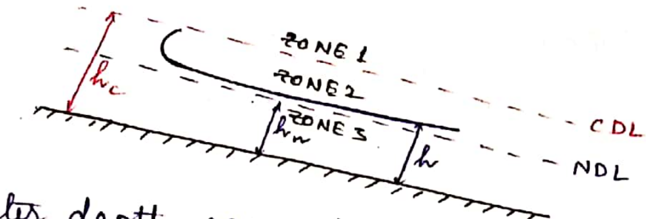
• S CURVES :→

There are 3 zones in a steep sloped channel.

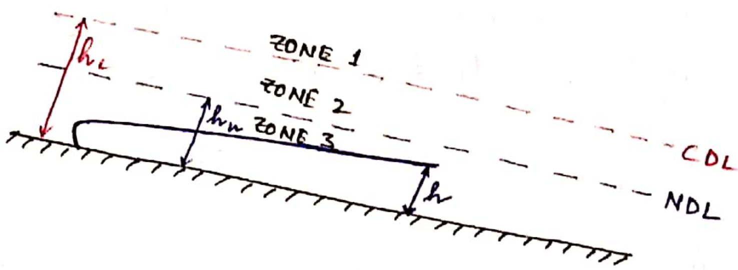
Case I: If the water depth occurs in zone 1, then $h > h_c > h_n$. The water surface profile is called an S1 curve.



Case II: If water depth occurs in zone 2, then $h_c > h > h_n$. The water surface profile is called an S2 curve.



Case III: If water depth occurs in zone 3, then $h_c > h_n > h$. The water surface profile is called an S3 curve.

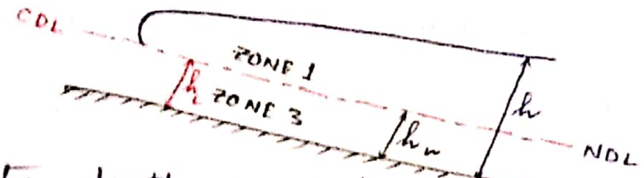


• C CURVES :→

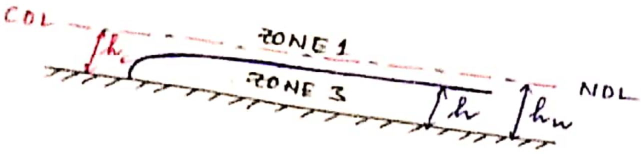
There are 2 zones in a critical sloped channel. Since NDL and CDL coincides, therefore zone 2 is sandwiched between them and thus not visible.

Case I: If water depth occurs in zone 1, then $h > h_c = h_n$. The water surface profile is called a C1 curve

P.T.O.



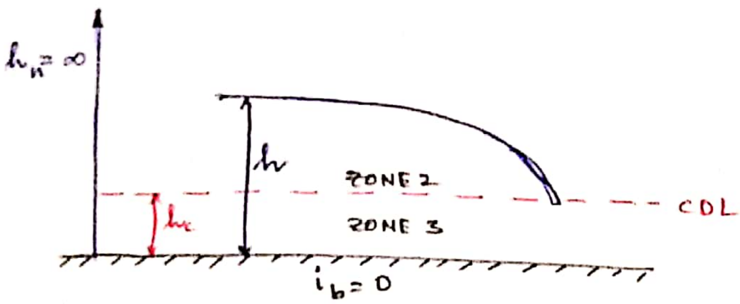
Case II: If water depth occurs in zone 3, then $h_c = h_n > h$. The water surface profile is called a C3 curve.



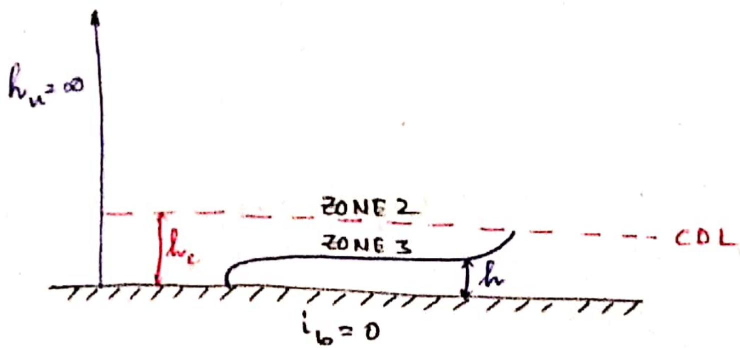
- **H CURVES:**

There are two zones in a horizontal sloped channel. Since NDL is at ∞, hence zone 1 can't be seen.

Case I: If water depth occurs in zone 2, then $h_n > h \geq h_c$. The water surface profile is called an H2 curve.



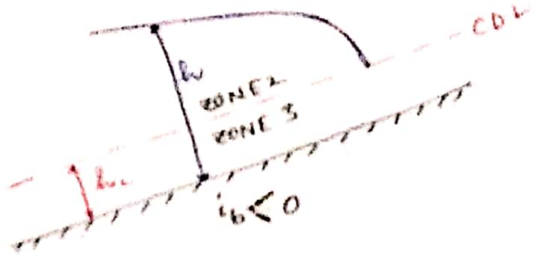
Case II: If water depth occurs in zone 3, then $h_n > h_c > h$. The water surface profile is called an H3 curve.



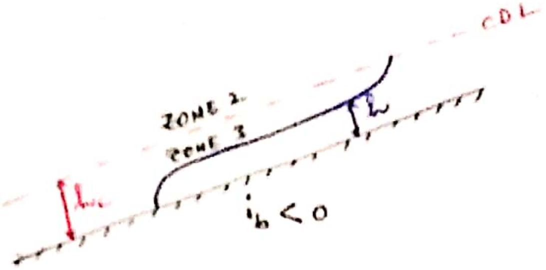
- **A CURVES:**

There are 2 zones in an adverse slope channel. Since normal depth doesnot exist, thus zone 1 doesnot exist.

Case I: If water depth occurs in zone 2, then $h > h_c$. The water surface profile is called an A2 curve.



Case II: If water depth occurs in zone 3, then $h_c > h$. The water surface profile is called an A3 curve.

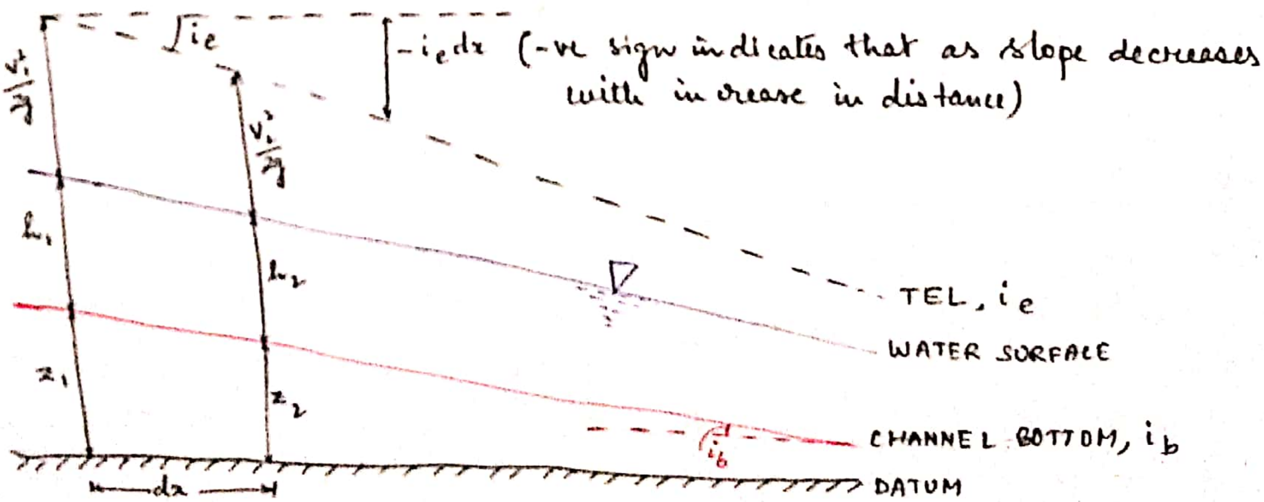


• DIRECT STEP METHOD :→

→ In practice, it is often required to determine the distance upto which the surface profile of the GVF extends.

→ In order to do this, it is necessary to integrate the dynamic equation of GVF with numerical methods to obtain the solution.

→ The direct step method is a numerical method used to determine the length of the GVF water surface profile.



P.T.O.

From the given figure, we get:

$$z_1 + h_1 + \frac{V_1^2}{2g} = z_2 + h_2 + \frac{V_2^2}{2g} - i_e dx$$

$$\Rightarrow (z_1 - z_2) + (h_1 - h_2) + \left(\frac{V_1^2}{2g} - \frac{V_2^2}{2g}\right) = -i_e dx$$

$$\Rightarrow (z_1 - z_2) + \left(h_1 + \frac{V_1^2}{2g}\right) - \left(h_2 + \frac{V_2^2}{2g}\right) = -i_e dx$$

Here,

$\left(h_1 + \frac{V_1^2}{2g}\right)$ is the specific energy at section 1-1 $= E_1$

and $\left(h_2 + \frac{V_2^2}{2g}\right)$ is the specific energy at section 2-2 $= E_2$

$$\therefore (z_1 - z_2) + E_1 - E_2 = -i_e dx$$

From the figure, $\frac{(z_1 - z_2)}{dx}$ = Slope of the channel bottom

$$\Rightarrow \frac{(z_1 - z_2)}{dx} = i_b$$

$$\Rightarrow (z_1 - z_2) = i_b dx$$

$\because$ the slope of the channel bottom decreases with the increase in distance, so it is given as negative.

$$\therefore (z_1 - z_2) = -i_b dx.$$

So,

$$-i_b dx + E_1 - E_2 = -i_e dx$$

$$\Rightarrow E_1 - E_2 = i_b dx - i_e dx$$

$$\Rightarrow E_1 - E_2 = (i_b - i_e) dx.$$

$$\Rightarrow dx = \frac{E_1 - E_2}{i_b - i_e}$$

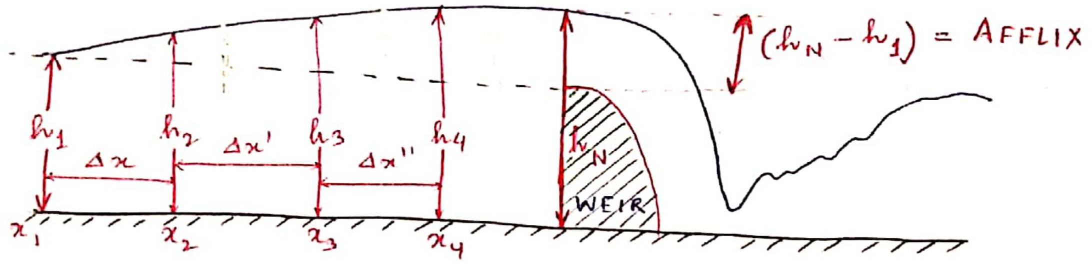
This is the dynamic equation in differential equation form, which is further converted into numerical equation form.

$$\Delta x = \frac{\Delta E}{i_b - \bar{i}_e}$$

This equation is implemented in solving problems in direct step method.

where,
$$\bar{i}_e = \frac{i_{e_1} + i_{e_2}}{2}$$

A horizontal dotted line is drawn which shows the gradual variation in the depth of the water, as shown in the figure. Let the initial depth of the flowing water be h_1



and, when the water tends to pass the weir, let the depth be h_N. This is the depth at the end of the GVF profile.

The objective is to get the horizontal distance between h_1 and h_N, which will give the length of the GVF profile.

From the figure, $(h_N - h_1)$ = Afflux, which is the amount by which the water will rise behind the obstruction along its flowpath. It is the difference between the final depth and the initial depth of a GVF.

In order to find out the length between h_1 and h_N, Let us consider an intermediate depth h_2, as shown in the figure. Let the x coordinate of h_1 be x_1 and the x coordinate of h_2 be x_2, as shown in the figure.

Let the horizontal distance between h_1 and h_2 be Δh.

To find Δh, the below formula is used

$$\Delta h = \frac{\Delta E}{i_b - \bar{i_e}}$$

where,

$$\Delta E = E_1 - E_2$$

Here, $E_1 = h_1 + \frac{V_1^2}{2g}$ and $E_2 = h_2 + \frac{V_2^2}{2g}$

E_1 is the specific energy corresponding to depth h_1,
E_2 is the specific energy corresponding to depth h_2

The value of bed slope i_b will be provided as it is a constant value.

$\bar{i_e}$ is the average energy line slope, whose value is:-

$$\bar{i_e} = \frac{i_{e_1} + i_{e_2}}{2}.$$

where, $i_{e_1} = \frac{N^2 V_1^2}{m_1^{4/3}}$ $\left[\because V_1 = \frac{1}{N} m_1^{2/3} i_{e_1}^{1/2}\right]$

Similarly, $i_{e_2} = \frac{N^2 V_2^2}{m_2^{4/3}}$ $\left[\because V_2 = \frac{1}{N} m_2^{2/3} i_{e_2}^{1/2}\right]$

Now, let us consider another intermediate flow depth h_3 between h_1 and h_N, as shown in the figure.
The horizontal distance between h_2 and h_3 be $\Delta x'$.
Let the x-coordinate of h_3 be x_3.

Similarly, on considering another intermediate flow depth h_4, as shown in the figure, whose x coordinate is x_4, the horizontal distance between h_3 and h_4 be $\Delta x''$.

This procedure is continued, till the final vertical depth h_N is reached.

The values of $\Delta x'$, $\Delta x''$, ... are calculated following the same calculation procedure as in case of Δx.

By adding all these Δx values, the total length x, i.e the horizontal distance between h_1 and h_N is obtained.

[N.B: Since the curve is not a straight line and is varying very slowly, so only if h_1 and h_N is considered, then the approximation for calculation will be very large and difficult to compute the entire length 'x' in one go].

Q. A rectangular channel 10m wide, carries a discharge of $30\ m^3/sec$. It is laid at a slope of 0.0001. If at a section in this channel the depth is 1.6m, how far from the section, will the depth be 2m. Take Manning's $N = 0.015$.

Sol^n:- Given,

$b = 10\ m$

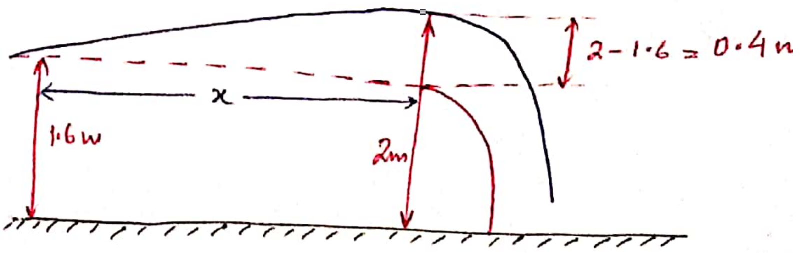
$Q = 30\ m^3/sec$.

$h_1 = 1.6m$

$h_N = 2m$ (final depth)

S_0 (or) $i_b = 0.0001$

$N = 0.015$



We will be following a tabular column approach to solve the problem:—

Referring the tabular calculation, we get:

The length of the length of the water profile, $x = 1047.4271\ m$

or $x = 1.047\ km$.

Note: We should try to keep the number of steps involved not more than 5 or 6. Increase in more steps will lead to more calculations.

However, steps less than 5 will provide an accurate answer. Thus 5 to 6 steps is the optimum number of steps to be assumed.

h (m)	A (m^2)	P (m)	m (m)	V (m/s)	E (m)	ΔE (m)	S_f	$\bar{S}_f$	$S_o - \bar{S}_f$	Δx (m)	x (m)
	$b \times h_i$	$b + 2h_i$	A/P	Q/A	$h_i + \frac{V_i^2}{2g}$	$E_1 - E_2$	$\frac{N^2 V_i^2}{m_i^{4/3}}$	$\frac{S_{f_1} + S_{f_2}}{2}$		$\frac{\Delta E}{S_o - \bar{S}_f}$	Sum of Δx
1.60	16	13.2	1.21	1.88	1.7792	—	0.00061	—	—	—	—
1.70	17	13.4	1.27	1.76	1.8587	-0.0795	0.00051	0.00056	-0.00046	172.49005	172.49005
1.80	18	13.6	1.32	1.67	1.9416	-0.0829	0.00043	0.00047	-0.00037	223.84055	396.33060
1.90	19	13.8	1.38	1.58	2.0271	-0.0855	0.00037	0.00040	-0.00030	286.71885	683.04945
2.00	20	14.0	1.43	1.50	2.1147	-0.0876	0.00031	0.00034	-0.00024	364.37786	1047.4271

Q.2. In the previous problem, determine the type of water surface profile.

Solⁿ:- In order to determine the type of water surface profile, we need to first determine the relative positions of normal depth level and critical depth level, i.e h_n and h_c.

Given. $Q = 30\ m^3/s$, $N = 0.015$, $b = 10\ m$, $q = \frac{Q}{b} = 3\ m^2/s$.

We know that,

$$h_c = \left(\frac{q^2}{g}\right)^{1/3} = \left(\frac{3^2}{9.81}\right)^{1/3} = 0.971\ m$$

To determine the position of y_n, Manning's equation is adopted.

$$Q = \frac{A}{N} m^{2/3} S_o^{1/2}$$

[∵ the depth is normal depth, i.e h_n, so the slope involved is S_o]

Here,

$$A = b \times h_n = 10 \times h_n$$

$$P = b + 2h_n = 10 + 2h_n$$

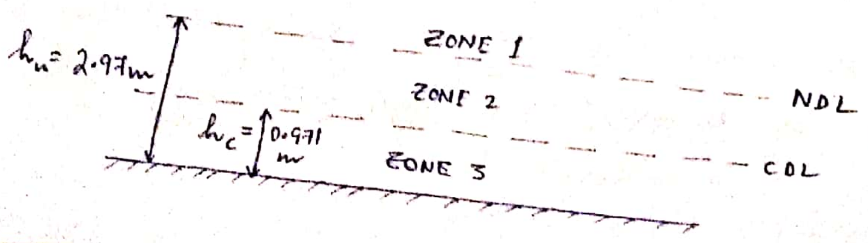
$$\therefore m = \frac{10h_n}{10 + 2h_n} = \frac{5h_n}{5 + h_n}$$

So, substituting the available values in Manning's equation, we get:-

$$30 = \frac{10h_n}{0.015} \times \left(\frac{5h_n}{5 + h_n}\right)^{2/3} \times (0.0001)^{1/2}$$

By trial and error method, $h_n = 2.97\ m$.

Since, $h_n > h_c$, hence it is a mild slope, i.e M curve.



From the figure, it is evident that the depth of zone 1 is 2.97 m and the depth of zone 3 is 0.972 m.
Thus the zone 2 is less than 2.97 m and greater than 0.972 m, Since it lies between zone 1 and zone 3.
Here, we have analysed the depth values from 1.6 m to 2 m. This falls in zone 2 of the channel. Hence the curve is a M2 curve.

* **MOST ECONOMICAL SECTION OF CHANNELS :→**

→ A section of a channel is said to be most economical when the cost of construction of the channel is minimum.

→ A minimum area of flow of a channel should be maintained, such that cost of excavation is minimum.

→ Concrete bed linings are generally provided along the surface of a channel to reduce the infiltration or percolation losses. The cost of providing this lining can also be reduced by maintaining a minimum wetted perimeter for a given discharge.

→ From continuity equation, we know that $Q = A \times V$. In order to maintain high discharge, the velocity should also be increased by maintaining a small cross sectional area.

→ Also we know that, from Chezy's formula, $V = C\sqrt{mS}$ and from Manning's equation, $V = \frac{1}{N} m^{2/3} \cdot S^{1/2}$.
From the above two formula it is clear that to maintain high velocity, the hydraulic mean depth 'm' should be increased. We know that $m = \frac{A}{P}$. Therefore hydraulic mean depth can be increased by maintaining a minimum wetted perimeter.

→ Most economical section is also called the best section or most efficient section

→ All the above conditions are utilized to determine the dimensions of economical sections of different forms of channels.

The conditions to be most economical for the following shapes of the channels will be considered:

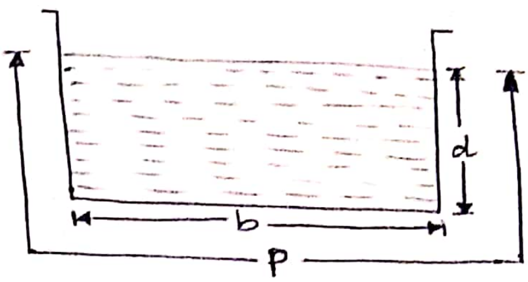
1. Rectangular Channel 2. Trapezoidal channel 3. Circular channel.

① Most Economical Rectangular Channel:

Consider a rectangular channel as shown in the figure:

Let b = width of channel

d = depth of flow in the channel



∴ Area of flow, $A = b \times d$ —(i)

Wetted Perimeter, $P = d + b + d = b + 2d$ —(ii)

From eqⁿ (i), $b = \frac{A}{d}$

Substituting the value of 'b' in eqⁿ (ii), we get:

$$P = \frac{A}{d} + 2d \quad \text{—(iii)}$$

Assuming the area A to be constant, eqⁿ (iii) can be differentiated with respect to 'd' and equated to zero for obtaining the condition for minimum P.

$$\frac{dP}{d(d)} = 0$$

$$\Rightarrow -\frac{A}{d^2} + 2 = 0.$$

$$\Rightarrow -\frac{A}{d^2} = -2 \quad \Rightarrow \frac{A}{d^2} = 2 \quad \Rightarrow A = 2d^2$$

From eqⁿ (i),

$$b \times d = 2d^2$$

$$\Rightarrow b = 2d$$

$$\Rightarrow d = \frac{b}{2}$$

P.T.O

Hydraulic mean depth $m = \frac{A}{P}$

$$m = \frac{b \times d}{b + 2d} = \frac{d}{2}$$

Therefore, a rectangular channel section will be most economical when either the depth of flow is equal to half of the bottom width, (or) the hydraulic mean depth is equal to half of the depth of flow.

Q. A rectangular channel 4m wide has depth of water 1.5m. The slope of the bed of the channel is 1 in 1000 and value of Chezy's constant C = 55. It is desired to increase the discharge to a maximum by changing the dimensions of the section for constant area of cross-section, slope of the bed and roughness of the channel. Find the new dimensions of the channel and increase in discharge?

Solⁿ:- Given:

Width of channel, $b = 4m$

Depth of flow, $d = 1.5m$

$\therefore$ Area of flow, $A = b \times d = 4 \times 1.5 \ m^2 = 6m^2$

Slope of bed, $i = \frac{1}{1000}$

Chezy's constant, $C = 55$.

Wetted Perimeter, $P = d + b + d = 1.5 + 4 + 1.5 = 7m$.

$\therefore$ Hydraulic mean depth, $m = \frac{A}{P} = \frac{6}{7} = 0.857 \ m$

We know,

$$Q = AC\sqrt{mi}$$

$$\therefore Q = 6 \times 55 \times \sqrt{0.857 \times \frac{1}{1000}} = 9.66 \ m^3/sec \quad \text{---(i)}$$

As per the question, the discharge is desired to get increased to maximum by changing the dimensions of the section for a constant area of cross-section (A), slope of the bed (i) and roughness of the channel.

Let b' = New width of channel

d' = New depth of flow.

We know, Area $= b' \times d'$

$\Rightarrow A = b' \times d'$

But, A = constant = 6 m

$\therefore \quad 6 = b' \times d'$ —— (ii)

Also for maximum discharge, the condition is : $b' = 2d'$ —— (iii)

Substituting the value of b' in eqn (ii),

$$6 = 2d' \times d' \Rightarrow 6 = 2d'^2 \Rightarrow d'^2 = \frac{6}{2} = 3$$

$$\therefore d' = \sqrt{3} = 1.732 \text{ m.}$$

Substituting the value of d' in eqn (iii),

$$b' = 2d' = 2 \times 1.732 \text{m} = 3.464 \text{ m}$$

$\therefore$ New dimensions of the channel are:-

$b' = 3.464$ m and $d' = 1.732$ m.

So, New wetted perimeter, $P' = d' + b' + d' = 1.732 + 3.464 + 1.732$

$$P' = 6.928 \text{ m}$$

$\therefore$ New Hydraulic mean depth, $m' = \frac{A}{P'}$ [$\because$ A is constant]

$$\Rightarrow m' = \frac{6}{6.928} = 0.866 \text{ m.}$$

[N:B:

This new hydraulic mean depth (m') corresponds to maximum discharge, so also equal to $\frac{d'}{2}$, i.e., $m' = \frac{d'}{2} = \frac{1.732}{2} = 0.866$ m

$\therefore$ Maximum discharge, $Q' = AC\sqrt{m' i}$

$$Q' = 6 \times 55 \times \sqrt{0.866 \times \frac{1}{1000}} = 9.71 \text{ m}^3/\text{sec}$$ —— ii

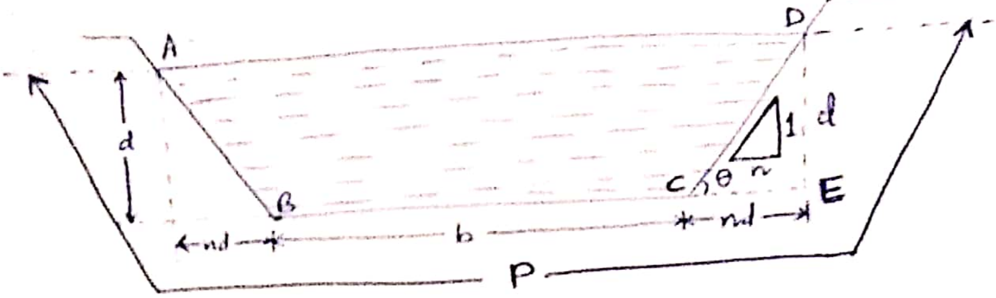
$\therefore$ Increase in discharge $= Q' - Q = 9.71 - 9.66 = 0.05 \text{ m}^3/\text{sec}$.

(2) Most Economical Trapezoidal Channel :

Consider a trapezoidal section of channel of bottom width 'b', depth of flow 'd' and side slope n horizontal to 1 vertical. Let θ be the angle made by the sides with horizontal.

For 1 vertical, we are taking $1 \times n = n$ horizontal. Similarly, for d vertical, it will be $n \times d = nd$ horizontal.

P.T.O



Let b = width of channel at bottom
d = depth of flow
θ = angle made by the sides with horizontal

From the figure, $BC = b$ and $AD = b + nd + nd = b + 2nd$

Let the area of the flow be A and
the wetted perimeter be P

Case I

$\therefore$ Area of flow, $A = \frac{(BC + AD)}{2} \times d = \frac{b + b + 2nd}{2} \times d = \frac{2b + 2nd}{2} \times d$

$$A = \frac{\cancel{2}(b + nd)}{\cancel{2}} \times d = (b + nd) \times d \quad \text{—— (i)}$$

$$\Rightarrow \frac{A}{d} = (b + nd)$$

$$\Rightarrow \frac{A}{d} - nd = b \qquad \Rightarrow b = \frac{A}{d} - nd \quad \text{—— (ii)}$$

and, Wetted Perimeter, $P = AB + BC + CD = BC + AB + CD = BC + 2CD$

In $\triangle DCE$, by using phythagorous theorem,

$$(\text{hypotenuse})^2 = (\text{base})^2 + (\text{altitude})^2$$

$$\Rightarrow CD^2 = CE^2 + DE^2$$

$$\Rightarrow CD = \sqrt{CE^2 + DE^2} = \sqrt{(nd)^2 + d^2} = \sqrt{n^2 d^2 + d^2}$$

$$= \sqrt{d^2 (n^2 + 1)}$$

$$= d\sqrt{(n^2 + 1)}$$

$$\therefore P = b + 2d\sqrt{n^2 + 1}$$

Substituting the value of b from eqⁿ (ii), we get :-

$$P = \frac{A}{d} - nd + 2d\sqrt{n^2+1} \quad \text{——— (iii)}$$

Assuming the area A to be constant, eqⁿ (iii) can be differentiated with respect to 'd' and equated to zero for obtaining the condition for minimum P.

$$\frac{dP}{d(d)} = 0$$

$$\Rightarrow \frac{d}{d(d)}\left[\frac{A}{d} - nd + 2d\sqrt{n^2+1}\right] = 0$$

$$\Rightarrow -\frac{A}{d^2} - n + 2\sqrt{n^2+1} = 0$$

$$\Rightarrow \cancel{-}\left(\frac{A}{d^2} + n\right) = \cancel{-}\,2\sqrt{n^2+1}$$

$$\Rightarrow \frac{A}{d^2} + n = 2\sqrt{n^2+1}$$

Substituting the value of A from eqⁿ (i), we get :-

$$\frac{(b+nd)\cancel{d}}{d^{\cancel{2}}} + n = 2\sqrt{n^2+1}$$

$$\Rightarrow \frac{(b+nd)}{d} + n = 2\sqrt{n^2+1}$$

$$\Rightarrow \frac{b+nd+nd}{d} = 2\sqrt{n^2+1}$$

$$\Rightarrow \frac{b+2nd}{d} = 2\sqrt{n^2+1}$$

$$\Rightarrow \frac{b+2nd}{2} = d\sqrt{n^2+1} \quad \text{——— (iv)}$$

In the above expression,

$\frac{b+2nd}{2}$ = half of top width = half of AD [from the figure]

and $d\sqrt{n^2+1}$ = CD = one of the sloping side

Equation (iv) is the required condition for a trapezoidal section to be most economical, which can be stated as, " half of top width must be equal to one of the sloping sides of the channel."

Case II

Hydraulic mean depth, $m = \frac{A}{P}$

Here, Value of $A = (b+nd) \times d$

and Value of $P = b + 2d\sqrt{n^2+1}$

From eq^n (iv), $2d\sqrt{n^2+1} = b + 2nd$

$\therefore P = b + b + 2nd = 2b + 2nd = 2(b+nd)$

So, $m = \frac{(b+nd)d}{2(b+nd)} = \frac{d}{2}$

Hence for a trapezoidal section to be most economical, the hydraulic mean depth must be equal to half the depth of flow.

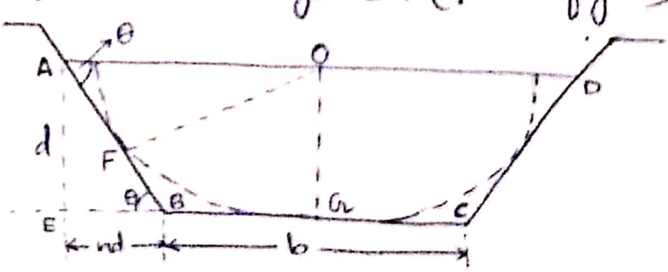
Case III

The three sides of the trapezoidal section of most economical section are tangential to the semi-circle, described by the dotted lines, as shown in the figure.

Let θ = angle made by the sloping side with the horizontal (from figure)

O = the centre of the top width AD

Draw a perpendicular line OF to the sloping line AB. This makes a right angled triangle OAF, where $\angle OAF = \theta$



$\therefore \sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{OF}{OA} \Rightarrow OF = OA \sin\theta$ —— (v)

In $\triangle ABE$, $\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{AE}{AB} = \frac{d}{d\sqrt{n^2+1}} = \frac{1}{\sqrt{n^2+1}}$

Substituting the value of $\sin\theta$ in eq^n (v), we get:-

$OF = OA \times \frac{1}{\sqrt{n^2+1}}$

Here,

OA = half of Top width

$\therefore OA = \dfrac{b+2nd}{2} = d\sqrt{n^2+1}$ (from eq^n iv)

So,

$$OF = OA \times \frac{1}{\sqrt{n^2+1}}$$

$$= \frac{d\cancel{\sqrt{n^2+1}}}{\cancel{\sqrt{n^2+1}}} = d$$

$\therefore OF = d$ = depth of the flow.

Thus, if a semi-circle is drawn with O as centre and radius equal to the depth of flow d, then the 3 sides of the most economical trapezoidal section will be tangential to the semi-circle.

Hence the three cases or conditions for the most economical trapezoidal section are:

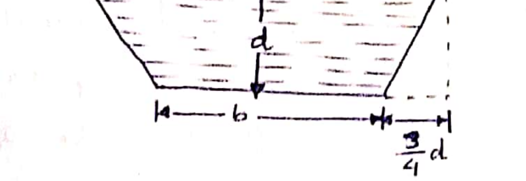
I. $\dfrac{b+2nd}{2} = d\sqrt{n^2+1}$

II. $m = \dfrac{d}{2}$

III. A semi-circle drawn from O with radius equal to the depth of flow will touch the three sides of the channel.

Q. A trapezoidal channel has side slopes of 3 horizontal to 4 vertical and slope of its bed is 1 in 2000. Determine the optimum dimensions of the channel, if it is to carry water at 0.5 m^3/sec. Take Chezy's constant as 80.

Sol^n:- Given,



Side slopes, $n = \dfrac{\text{Horizontal}}{\text{Vertical}} = \dfrac{3}{4}$

Slope of bed, $i = \dfrac{1}{2000}$

Discharge, $Q = 0.5\ m^3/sec$

Chezy's constant, $C = 80$

For the most economical trapezoidal section, the condition is:

$$\frac{b+2nd}{2} = d\sqrt{n^2+1}$$

$$\Rightarrow \frac{b+2\times\frac{3}{4}\times d}{2} = d\sqrt{\left(\frac{3}{4}\right)^2+1}$$

$$\Rightarrow \frac{b+1.5d}{2} = \frac{5}{4}d$$

$$\Rightarrow \frac{b+1.5d}{2} = 1.25d$$

$$\Rightarrow b = d \quad \text{——(i)}$$

We know that,

$$Q = AC\sqrt{mi} \quad \text{——(ii)}$$

But for a most economical trapezoidal section, the condition is:

$$m = \frac{d}{2}$$

$$\therefore 0.50 = A\times 80\times\sqrt{\frac{d}{2}\times\frac{1}{2000}} \quad \text{——(iii)}$$

Here, $A = (b+nd)d = \left(d+\frac{3}{4}d\right)d = 1.75d^2$ $\left[\because b = d \text{ and } n = \frac{3}{4}\right]$

Substituting the value of A in eqn (iii),

$$\Rightarrow 0.50 = 1.75d^2\times 80\times\sqrt{\frac{d}{2}\times\frac{1}{2000}}$$

$$\Rightarrow d = 0.55m$$

$$\therefore b = d = 0.55m$$

So, optimum dimensions of the channel are width = depth = 0.55m.

Q. A trapezoidal channel has side slopes 1 to 1. It is required to discharge 13.75 m³/sec of water with a bed gradient of 1 in 1000. If unlined, then the value of Chezy's C is 44. If lined with concrete, its value is 60. The cost per m³ of excavation is four times the cost per m² of lining. The channel is to be the most efficient one.

Find whether the lined canal or the unlined canal will be cheaper. What will be the dimensions of that economical canal?

Sol^n:- Given,

Side slope, $n = \frac{1}{1} = 1$

Discharge, $Q = 13.75 \ m^3/s$

Slope of bed, $i = \frac{1}{1000}$

For unlined, $C = 44$

For lined, $C = 60$

Cost per m^3 of excavation $= 4 \times$ cost per m^2 of lining

Let the cost per m^2 of lining be 'x'.

Then the cost per m^3 of excavation will be equal to $4x$

As the channel is considered to be most efficient or economical,

$\therefore$ Hydraulic mean depth, $m = \frac{d}{2}$

Here, d = depth of the channel and b = width of channel (say)

Also for the most efficient trapezoidal channel,

$$\frac{b + 2nd}{2} = d\sqrt{n^2+1}$$

$$\Rightarrow \frac{b + 2 \times 1 \times d}{2} = d\sqrt{1^2+1}$$

$$\Rightarrow b = 0.828d$$

Also, Area of the trapezoidal channel, $A = (b+nd)d$

$$A = (0.828d + 1 \times d)d$$

$$\therefore A = 1.828d^2$$

1. For unlined channel

$C = 44$

We know, $Q = AC\sqrt{mi}$

$$\Rightarrow 13.75 = 1.828d^2 \times 44 \times \sqrt{\frac{d}{2} \times \frac{1}{1000}}$$

$$\therefore d = 2.256 m.$$

So, $b = 0.828d = 0.828 \times 2.256 = 1.868 \ m.$

Now,

Cost of excavation per running metre length of unlined channel is:

$= $ Volume of Channel $\times$ cost per m^3 of excavation

$= $ (Area $\times$ Thickness) $\times$ cost per m^3 of excavation

$= \left[(b+nd)d \times 1\right] \times 4x$ [∵ In the problem, no value of thickness is given, so it is considered as unity]

$= \left[(1.868 + 1 \times 2.256)2.256 \times 1\right] \times 4x$

$= 37.215x$

2. For Lined channel

Value of $C = 60$

We know,

$$Q = AC\sqrt{mi}$$

$$\Rightarrow 13.75 = 1.828d^2 \times 60 \times \sqrt{\frac{d}{2} \times \frac{1}{1000}}$$

$\therefore d = 1.992\ m$

So, $b = 0.828d = 0.828 \times 1.992 = 1.649\ m$

In case of lined channel, the cost of lining as well as the cost of excavation, both should be considered.

Now,

Cost of excavation per running metre length of lined channel is:

$= $ Volume of channel $\times$ cost per m^3 of excavation

$= \left[(b+nd)d \times 1\right] \times 4x$

$= \left[(1.649 + 1 \times 1.992)1.992 \times 1\right] \times 4x = 7.383\ m.$

Cost of Lining $=$ Area of lining $\times$ Cost per m^2 of lining

$= $ (Perimeter of lining $\times$ 1) $\times x$

$= \left[\left(b + 2d\sqrt{n^2+1}\right) \times 1\right] \times x$

$= \left(1.649 + 2 \times 1.992 \sqrt{1+1^2}\right) \times 1 \times x$

$= 7.283x.$

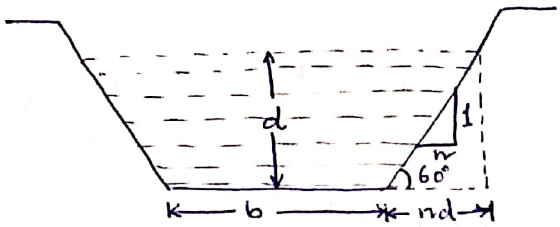
$\therefore$ Total cost $= 29.01x + 7.283x = 36.293x$

So, the lined canal is cheaper.

The dimensions are: $b = 1.649$ m and $d = 1.992$ m.

Q. An open channel of most economical section, having the form of a half hexagon with horizontal bottom is required to give a maximum discharge of 20.2 m³/sec of water. The slope of the channel bottom is 1 in 2500. Taking $C = 60$ in Chezy's equation determine the dimensions of the cross-section.

Sol^n:- Given,



Maximum discharge, $Q = 20.2\ m^3/s$

Bed slope, $i = \frac{1}{2500}$

Chezy's constant, $C = 60$

As per the question,

Channel is the form of a half hexagon, which means that the angle made by the sloping side with horizontal will be 60°.

From the figure, $\tan\theta = \frac{1}{n}$

$\because \theta = 60°$, so $\tan 60° = \sqrt{3}$

i.e, $\sqrt{3} = \frac{1}{n}$

$\Rightarrow n = \frac{1}{\sqrt{3}}$

Let the width of the channel be 'b' and the depth of the channel be 'd'.

For most economical condition of a trapezoidal channel section,

$$\frac{b+2nd}{2} = d\sqrt{n^2+1}$$

P.T.O.

$$\Rightarrow \frac{b + 2 \times \frac{1}{\sqrt{3}} \times d}{2} = d\sqrt{\left(\frac{1}{\sqrt{3}}\right)^2 + 1}$$

$$\Rightarrow b = \frac{2d}{\sqrt{3}}$$

Area of flow, $A = (b + nd)\,d = \left(\frac{2d}{\sqrt{3}} + \frac{1}{\sqrt{3}}d\right)d = \sqrt{3}\,d^2$

Also, as per condition of an economical trapezoidal section,

$$m = \frac{d}{2}$$

We know,

$$Q = AC\sqrt{mi}$$

$$\Rightarrow 20.2 = \sqrt{3}\,d^2 \times 60 \times \sqrt{\frac{d}{2} \times \frac{1}{2500}}$$

$$\therefore d = 2.852 \text{ m}$$

So, $b = \frac{2d}{\sqrt{3}} = \frac{2}{\sqrt{3}} \times 2.852 = 3.293$ m.

Assignment no. 1 :

A trapezoidal channel to carry 142 m^3/minute of water is designed to have a minimum cross-section. Find the bottom width and depth if the bed slope is 1 in 1200, the side slopes at 45° and $C = 55$.

Hint :- $Q = 142 \text{ m}^3/\text{min} = \frac{142}{60} = 2.367 \text{ m}^3/\text{sec}$

Assignment no. 2 :

A trapezoidal channel with side slopes of 3 horizontal to 2 vertical has to be designed to convey 10 m^3/sec at a velocity of 1.5 m/sec, so that the amount of concrete lining for the bed and sides is minimum. Find :

(i) The wetted perimeter and

(ii) Slope of the bed if Manning's $N = 0.014$ in the formula $C = \frac{1}{N} m^{1/6}$

Hint :- $n = \frac{\text{Horizontal}}{\text{Vertical}}$ and $Q = AV \Rightarrow A = \frac{Q}{V}$

* BEST SIDE SLOPE FOR MOST ECONOMICAL TRAPEZOIDAL SECTION :

Area of trapezoidal section, $A = (b+nd)d$ ——— (i)

Here,

b = width of the trapezoidal channel

d = depth of flow.

n = slope of the side of the channel.

From eqn (i)

$$A = bd + nd^2$$

$$\Rightarrow bd + nd^2 = A.$$

$$\Rightarrow bd = A - nd^2$$

$$\Rightarrow b\cancel{d} = \cancel{d}\left(\frac{A}{d} - nd\right)$$

$$\Rightarrow b = \frac{A}{d} - nd \quad \text{——— (ii)}$$

Wetted Perimeter of channel, $P = b + 2d\sqrt{n^2+1}$

Substituting the value of 'b' from eqn (ii), in the above eqn, we get:–

$$P = \left(\frac{A}{d} - nd\right) + 2d\sqrt{n^2+1} \quad \text{——— (iii)}$$

We know that for a most economical trapezoidal section, the depth of flow (d) and the area (A) are constant.

In this case, in eqn (iii), the terms d and A are constant. So only the term n is a variable.

As the topic speaks of "Best Side Slope", so a section is said to have best side slope only when it is most economical, which mean that best side slope means that the wetted perimeter P will be minimum.

For wetted perimeter to be minimum, we have to differentiate eqn (iii) with respect to 'n' and equate to 0.

i.e, $\frac{dP}{dn} = 0$

$$\Rightarrow \frac{d}{dn}\left[\frac{A}{d} - nd + 2d\sqrt{n^2+1}\right] = 0$$

$$\Rightarrow 0 - d + 2d \times \frac{1}{2} \times (n^2+1)^{1/2 - 1} \times 2n = 0.$$

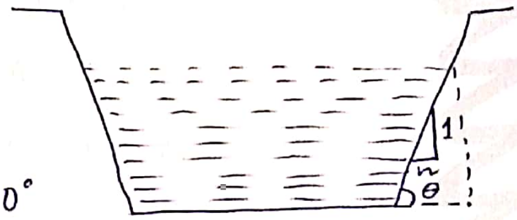
$$\Rightarrow 2n = \sqrt{n^2+1}$$

Squaring on both sides,

$$4n^2 = n^2 + 1$$

$$\Rightarrow 3n^2 = 1 \quad \Rightarrow n^2 = \frac{1}{3} \quad \Rightarrow n = \frac{1}{\sqrt{3}}$$

If the sloping sides makes an angle θ, with the horizontal, as shown in the figure, then we have:



$$\tan\theta = \frac{1}{n} = \frac{1}{1/\sqrt{3}} = \sqrt{3}$$

$$\Rightarrow \tan\theta = \sqrt{3} \quad \Rightarrow \theta = \tan^{-1}(\sqrt{3}) = 60^\circ$$

Hence the best side slope is at 60° to the horizontal

(OR)

The value of n for the best side slope is given as equal to $\frac{1}{\sqrt{3}}$

We know that for the most economical trapezoidal section,

$$\frac{b + 2nd}{2} = d\sqrt{n^2+1}$$

$$\Rightarrow \frac{b + 2 \times \frac{1}{\sqrt{3}} \times d}{2} = d\sqrt{\left(\frac{1}{\sqrt{3}}\right)^2 + 1}$$

$$\Rightarrow b = \frac{2d}{\sqrt{3}} \quad \text{——— (iv)}$$

Now, Wetted Perimeter, $P = b + 2d\sqrt{n^2+1}$

$$\Rightarrow P = \frac{2d}{\sqrt{3}} + 2d\sqrt{\left(\frac{1}{\sqrt{3}}\right)^2 + 1}$$

$$\Rightarrow P = \frac{6d}{\sqrt{3}} = \frac{3 \times 2d}{\sqrt{3}}$$

$$\Rightarrow P = 3b \quad \left[\because b = \frac{2d}{\sqrt{3}}\right]$$

For a slope of 60°, the length of the sloping side is equal to the width of the trapezoidal section.

Q. A power canal of trapezoidal section has to be excavated through hard clay at the least cost. Determine the dimensions of the channel given, discharge equal to 14 m^3/sec, bed slope as 1 : 2500 and Manning's N = 0.020.

Solⁿ:- given,

Discharge, $Q = 14\ m^3/sec$

Bed Slope, $i = \frac{1}{2500}$

Manning's $N = 0.020$

In the question, the value of 'n' is not given. However it is asked to excavate the canal at a least cost, which means the trapezoidal section should be most economical. For a most economical trapezoidal section, $n = \frac{1}{\sqrt{3}}$

Let b = width of channel and d = depth of flow.

For most economical section,

$$\frac{b + 2nd}{2} = d\sqrt{n^2+1}$$

$$\Rightarrow \frac{b + 2 \times \frac{1}{\sqrt{3}} \times d}{2} = d\sqrt{\left(\frac{1}{\sqrt{3}}\right)^2 + 1}.$$

$$\Rightarrow b = \frac{2d}{\sqrt{3}}$$

Area of trapezoidal section, $A = (b+nd)d = \left(\frac{2d}{\sqrt{3}} + \frac{1}{\sqrt{3}}d\right)d = \sqrt{3}d^2$

Also, for most economical condition, $m = \frac{d}{2}$

We know that,

$$Q = AC\sqrt{mi}$$

Also in the given question, the value of C is not given, but we know from Chezy's equation that,

$$C = \frac{1}{N} m^{1/6}$$

$$\therefore Q = \sqrt{3}d^2 \times \frac{1}{N} m^{1/6} \times \sqrt{\frac{d}{2} \times \frac{1}{2500}}$$, where $N = 0.020$

$$\Rightarrow 14 = \sqrt{3}d^2 \times \frac{1}{0.020} \times \left(\frac{d}{2}\right)^{1/6} \times \sqrt{\frac{d}{2 \times 2500}}$$

$\Rightarrow d = 2.605\,m$

$\therefore b = \frac{2d}{\sqrt{3}} = \frac{2 \times 2.605}{\sqrt{3}} = 3.008\,m$

* FLOW THROUGH CIRCULAR CHANNEL :-

→ The flow of a liquid through a circular pipe when the level of the liquid in the pipe is below the top of the pipe is classified as an open channel flow.

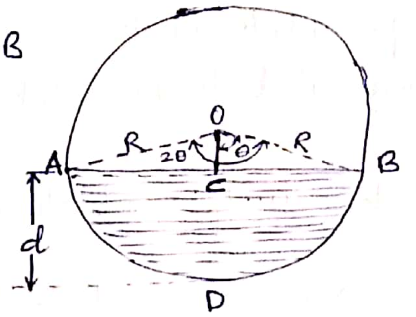
→ The rate of flow or discharge through a circular channel is determined from the depth of flow and angle subtended by the liquid surface at the centre of the circular channel.

→ The figure shows a circular channel through which water is flowing.

Let d = depth of water

2θ = angle subtended by water surface AB at the centre in radians.

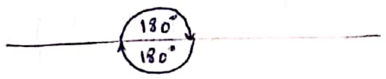
R = Radius of the channel.



Wetted Perimeter (P) :

Total angle of a circle is 360° i.e, 2π.

[N.B: Opening a circle becomes a straight line whose angle is 180° or π.



$= \pi + \pi = 2\pi = 360^\circ$

So, $2\pi \longrightarrow 2\pi R$

$\therefore \quad 2\theta \longrightarrow \dfrac{2\pi R}{2\pi} \times 2\theta = 2R\theta$]

$\therefore$ Wetted Perimeter of a circle, $P = 2R\theta$

Wetted Area (A):

Area of a circle $= \pi R^2$

Total angle of a circle $= 2\pi \cdot (360^\circ)$

$2\pi \longrightarrow \pi R^2$

$2\theta \longrightarrow \dfrac{\pi R^2}{2\pi} \times 2\theta = R^2\theta$

$\therefore$ Wetted area A = Area of ADBA

= Area of sector OADBO − Area of Δ ABO

$= R^2\theta - \frac{1}{2} \times$ base $\times$ height.

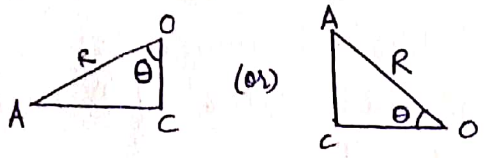
Scanned with CamScanner

$$= R^2\theta - \frac{1}{2} \times AB \times CO$$

$$= R^2\theta - \frac{1}{2} \times 2BC \times CO$$ [∵ AB is diameter, which is equal to 2-times of radius, either AC or BC]

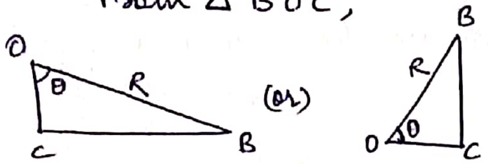
$$= R^2\theta - \frac{2BC \times CO}{2}$$

From ΔAOC, $\cos\theta = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{CO}{AO} = \frac{CO}{R}$



$\Rightarrow CO = R\cos\theta$

From ΔBOC,



$\sin\theta = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{BC}{BO} = \frac{BC}{R}$

$\Rightarrow BC = R\sin\theta$

$$\therefore A = R^2\theta - \frac{2 \times R\sin\theta \times R\cos\theta}{2}$$

$$= R^2\theta - R^2 \times \frac{2\sin\theta\cos\theta}{2}$$

$$= R^2\theta - R^2 \times \frac{\sin 2\theta}{2}$$ [∵ $\sin 2\theta = 2\sin\theta\cos\theta$]

$$= R^2\left(\theta - \frac{\sin 2\theta}{2}\right)$$

Then, hydraulic mean depth, $m = \frac{A}{P} = \frac{R^2\left(\theta - \frac{\sin 2\theta}{2}\right)}{2R\theta}$

$$\Rightarrow m = \frac{R}{2\theta}\left(\theta - \frac{\sin 2\theta}{2}\right)$$

And discharge, $Q = AC\sqrt{mi}$

* **Most Economical Circular Section:**

We know that for a most economical section, the discharge is maximum for a constant cross-sectional area, slope of bed and resistant coefficient.

However, in case of cirular channels, the area of flow cannot be maintained constant. With the change of depth of flow in a circular channel of any radius, the wetted area and the wetted perimeter changes.

Thus in case of circular channels, for most economical section, two seperate conditions are obtained. They are:

1. Condition for Maximum Velocity for Circular Section →

Let i be the slope of the channel bed.

The velocity of flow through a circular channel will be maximum, when the hydraulic mean depth 'm' (A/P) is maximum for a given value of C and i. In this case, the value of θ will also vary with the variation in the value of m.

Hence for maximum value of A/P, we have the condition:

$$\frac{d(A/P)}{d\theta} = 0 \quad \text{—— (i)}$$

where, both A and P are functions of θ.

Differentiating eq^n (i) w.r.t θ, we have:

$$\frac{P\frac{dA}{d\theta} - A\frac{dP}{d\theta}}{P^2} = 0$$

$$\Rightarrow P\frac{dA}{d\theta} - A\frac{dP}{d\theta} = 0 \quad \text{—— (ii)}$$

We know, $A = R^2\left(\theta - \frac{\sin 2\theta}{2}\right)$

$$\therefore \frac{dA}{d\theta} = R^2\left(1 - \frac{\cos 2\theta}{2} \times 2\right) = R^2(1-\cos 2\theta)$$

and $P = 2R\theta$

$$\therefore \frac{dP}{d\theta} = 2R$$

So, substituting all the values in eq^n (ii), we get:-

$$2R\theta\left[R^2(1-\cos 2\theta)\right] - R^2\left(\theta - \frac{\sin 2\theta}{2}\right)2R = 0$$

$$\Rightarrow 2R^3\theta(1-\cos 2\theta) - 2R^3\left(\theta - \frac{\sin 2\theta}{2}\right) = 0.$$

$$\Rightarrow \theta(1-\cos 2\theta) - \left(\theta - \frac{\sin 2\theta}{2}\right) = 0.$$

$$\Rightarrow \theta - \theta\cos 2\theta - \theta + \frac{\sin 2\theta}{2} = 0$$

$$\Rightarrow \theta\cos 2\theta = \frac{\sin 2\theta}{2}$$

$$\Rightarrow 2\theta \cos 2\theta = \sin 2\theta$$

$$\Rightarrow \sin 2\theta = 2\theta\cos\theta$$

$$\Rightarrow \frac{\sin 2\theta}{\cos 2\theta} = 2\theta$$

$$\Rightarrow \tan 2\theta = 2\theta \qquad \Rightarrow 2\theta = \tan^{-1}(2\theta)$$

By hit and trial method,

$$2\theta = 257^\circ 30'$$

$$\theta = 128^\circ 45'$$

From the figure,

The depth of flow for maximum velocity is :-

$$d = OD - OC = R - R\cos\theta \quad \left[\because OD = \text{Radius of the circular channel} = R\right]$$

$$\Rightarrow d = R[1-\cos\theta]$$

$$= R[1-\cos 128^\circ 45']$$

$$= R\left[1 - \cos(180^\circ - 51^\circ 15')\right]$$

$$= R\left[1 - (-\cos 51^\circ 15')\right] \quad \left[\because \cos(180^\circ - \theta) = -\cos\theta\right].$$

$$= R[1 + \cos 51^\circ 15'].$$

$$\therefore d = R[1 + 0.62] = 1.62R = 1.62 \times \frac{D}{2} = 0.81D$$

where, D = diameter of the circular channel.

P.T.O.

Thus, for maximum velocity of flow, the depth of water in the circular channel should be equal to 0·81 times the diameter of the channel.

Hydraulic mean depth for maximum velocity is:

$$m = \frac{A}{P} = \frac{R^2\left(\theta - \frac{\sin 2\theta}{2}\right)}{2R\theta} = \frac{R}{2\theta}\left[\theta - \frac{\sin 2\theta}{2}\right]$$

where, $\theta = 128^\circ 45' = 128·75^\circ$

$$= 128·75 \times \frac{\pi}{180} = 2·247 \text{ radians.}$$

$$\therefore m = \frac{R}{2 \times 2·247}\left[2·247 - \frac{\sin 257^\circ 30'}{2}\right]$$

$$= \frac{R}{4·494}\left[2·247 - \frac{\sin(180^\circ + 87·5^\circ)}{2}\right]$$

$$= \frac{R}{4·494}\left[2·247 - \left(\frac{-\sin 87·5^\circ}{2}\right)\right] \quad \left[\because \sin(180^\circ + \theta) = -\sin\theta\right]$$

$$= \frac{R}{4·494}\left[2·247 + \frac{\sin 87·5^\circ}{2}\right]$$

$$= 0·611 R = 0·611 \times \frac{D}{2} = 0·3D$$

Thus, for maximum velocity, the hydraulic mean depth is equal to 0·3 times the diameter of circular channel.

2. <u>Condition for Maximum Discharge for Circular Section</u> :→

The discharge through a channel is given by:

$$Q = AC\sqrt{mi}$$

$$= AC\sqrt{\frac{A}{P}i} \quad \left[\because m = \frac{A}{P}\right]$$

$$= \sqrt{A^2} \cdot C \frac{\sqrt{A}}{\sqrt{P}} \times \sqrt{i}$$

$$Q = C\sqrt{\frac{A^3}{P}i}$$

The discharge will be maximum for constant values of C and i, when $\frac{A^3}{P}$ is maximum.

$\frac{A^3}{P}$ will be maximum, when $\frac{d}{d\theta}\left(\frac{A^3}{P}\right) = 0$

$$\Rightarrow \frac{P \times 3A^2 \frac{dA}{d\theta} - A^3 \frac{dP}{d\theta}}{P^2} = 0$$

$$\Rightarrow 3PA^2 \frac{dA}{d\theta} - A^3 \frac{dP}{d\theta} = 0$$

Dividing by A^2,

$$\Rightarrow 3P \frac{dA}{d\theta} - A \frac{dP}{d\theta} = 0 \quad \text{---(i)}$$

We know,

$$P = 2R\theta$$

$$\Rightarrow \frac{dP}{d\theta} = 2R$$

Also,

$$A = R^2\left(\theta - \frac{\sin 2\theta}{2}\right)$$

$$\Rightarrow \frac{dA}{d\theta} = R^2(1 - \cos 2\theta)$$

Substituting the value of A, P, $\frac{dA}{d\theta}$ and $\frac{dP}{d\theta}$ in eqⁿ (i), we get:-

$$3 \times 2R\theta \times R^2(1 - \cos 2\theta) - R^2\left(\theta - \frac{\sin 2\theta}{2}\right) \times 2R = 0$$

$$\Rightarrow 6R^3\theta(1 - \cos 2\theta) - 2R^3\left(\theta - \frac{\sin 2\theta}{2}\right) = 0$$

Dividing by $2R^3$,

$$\Rightarrow 3\theta(1 - \cos 2\theta) - \left(\theta - \frac{\sin 2\theta}{2}\right) = 0$$

$$\Rightarrow 3\theta - 3\theta\cos 2\theta - \theta + \frac{\sin 2\theta}{2} = 0$$

$$\Rightarrow 2\theta - 3\theta\cos 2\theta + \frac{\sin 2\theta}{2} = 0.$$

$\Rightarrow 4\theta - 6\theta\cos 2\theta + \sin 2\theta = 0$

By hit and trial method,

$$2\theta = 308^\circ$$

$$\Rightarrow \theta = \frac{308^\circ}{2} = 154^\circ$$

From the figure,

Depth of flow for maximum discharge, $d = OD - OC$

$$= R - R\cos\theta$$

$$= R(1-\cos\theta)$$

$$= R(1-\cos 154^\circ)$$

$$= R[1-\cos(180^\circ - 26^\circ)]$$

$$= R[1-(-\cos 26^\circ)]$$

$$[\because \cos(180^\circ - \theta) = -\cos\theta]$$

$$= R[1+\cos 26^\circ]$$

$$= 1.898\,R$$

$$\therefore d = 1.898 \times \frac{D}{2} = 0.948\,D \approx 0.95D$$

where, D = diameter of the circular channel.

Thus, for maximum discharge through a circular channel, the depth of flow is equal to 0.95 times its diameter.

Q. Find the discharge through a circular pipe of diameter 3m, if the depth of water in the pipe is 1m. and the pipe is laid at a slope of 1 in 1000. Take the value of Chezy's constant as 70.

If the depth of water in the pipe increases to 2.5m, then find the rate of flow through the pipe?

Soln:- Given,

Diameter of pipe, $D = 3m$.

$\therefore$ Radius of pipe, $R = \frac{D}{2} = \frac{3}{2} = 1.5m$.

Depth of water in pipe, $d = 1m$

Bed slope, $i = \frac{1}{1000}$

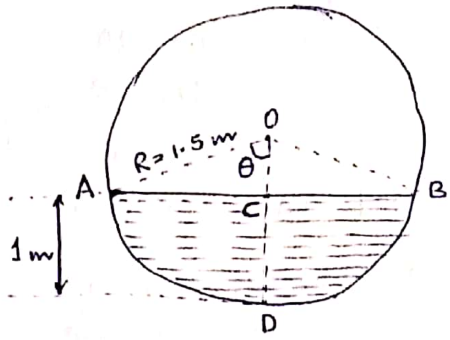
Chezy's constant, $C = 70$.

From the figure, $OC = OD - CD$

$= R - 1$

$OC = 1.5 - 1 = 0.5\,m$

and $OA = 1.5\,m = R$



In ΔAOC, $\cos\theta = \dfrac{OC}{AO} = \dfrac{0.5}{1.5} = \dfrac{1}{3}$

$\Rightarrow \theta = \cos^{-1}\left(\frac{1}{3}\right) = 70.53^\circ = 70.53 \times \frac{\pi}{180^\circ} = 1.23$ radians.

We know,

Wetted area, $A = R^2\left(\theta - \dfrac{\sin 2\theta}{2}\right)$

$$= 1.5^2\left[1.23 - \frac{\sin(2 \times 1.23)}{2}\right]$$

$$= 2.25\left(1.23 - \frac{\sin 141.08^\circ}{2}\right)$$

$$= 2.25\left[1.23 - \frac{\sin(180^\circ - 38.94^\circ)}{2}\right]$$

$$= 2.25\left[1.23 - \frac{\sin 38.94^\circ}{2}\right] \quad \left(\because \sin(180^\circ - \theta) = \sin\theta\right)$$

$$= 2.06\,m^2$$

and, Wetted Perimeter, $P = 2R\theta$

$$P = 2 \times 1.5 \times 1.23 = 3.69\,m$$

$\therefore$ Hydraulic mean depth, $m = \dfrac{A}{P}$

$$= \frac{2.06}{3.69} = 0.5582\,m.$$

So, discharge, $Q = AC\sqrt{mi}$

$$= 2.06 \times 70 \times \sqrt{0.5582 \times \frac{1}{1000}}$$

$$= 3.407\,m^3/sec.$$

If the depth of water increases to $2.5\,m$, i.e $d = 2.5\,m$.

P.T.O.

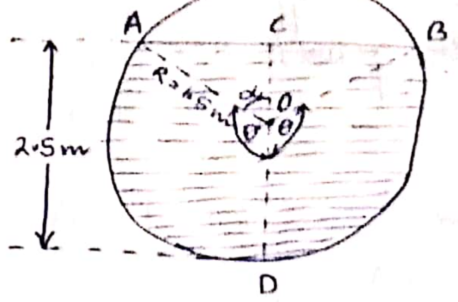
From the figure,

$OC = CD - OD$

$= 2.5 - R$

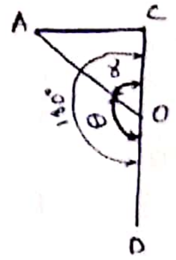
$= 2.5 - 1.5 = 1\ m$

and $OA = R = 1.5\ m.$



From Δ AOC, $\cos\alpha = \dfrac{OC}{OA} = \dfrac{1.0}{1.5} = 0.667$

$\Rightarrow \alpha = \cos^{-1}(0.667) = 48.16^{\circ}$



So, $\theta = 180^{\circ} - \alpha = 180^{\circ} - 48.16^{\circ} = 131.84^{\circ}$

$= 131.84 \times \dfrac{\pi}{180}$

$= 2.30$ radians.

We know,

Wetted area, $A = R^2\left(\theta - \dfrac{\sin 2\theta}{2}\right)$

$= 1.5^2\left[2.30 - \dfrac{\sin(2 \times 131.84^{\circ})}{2}\right]$

$= 2.25\left[2.30 - \dfrac{\sin 263.68^{\circ}}{2}\right]$

$= 2.25\left[2.30 - \dfrac{\sin(180^{\circ} + 83.68^{\circ})}{2}\right]$

$= 2.25\left(2.30 + \dfrac{\sin 83.68^{\circ}}{2}\right)$ $\left[\because \sin(180^{\circ} + \theta) = -\sin\theta\right]$

$A = 6.293\ m^2$

Also, Wetted Perimeter, $P = 2R\theta = 2 \times 1.5 \times 2.30 = 6.90\ m.$

$\therefore$ Hydraulic mean depth, $m = \dfrac{A}{P} = \dfrac{6.293}{6.90} = 0.912\ m.$

So, Discharge or rate of flow is :-

$Q = AC\sqrt{mi}$

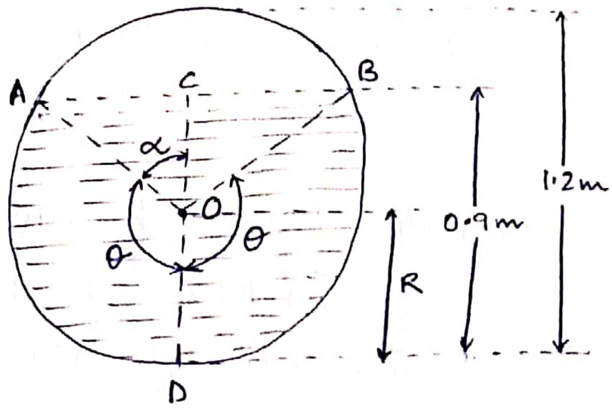
$= 6.293 \times 70 \times \sqrt{0.912 \times \dfrac{1}{1000}}$

$= 13.303\ m^3/sec.$

Q. Calculate the quantity of water that will be discharged at a uniform depth of 0.9 m in a 1.2 m diameter pipe which is laid at a slope 1 in 1000. Assume Chezy's $C = 58$.

Sol^n:- Given,

Dia of pipe $= 1.2\,m$



$\therefore$ Radius of pipe, $R = \dfrac{\text{Dia of pipe}}{2}$

$= \dfrac{1.2}{2} = 0.6\,m$

Depth of water, $d = 0.9\,m$

Slope, $i = \dfrac{1}{1000}$

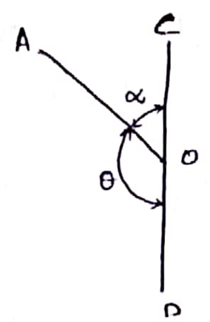
Chezy's $C = 58$

From the figure, $OC = CD - OD$

$= 0.9 - R = 0.9 - 0.6 = 0.3\,m.$

Similarly, $OA = R = 0.6\,m.$

In ΔAOC, $\cos\alpha = \dfrac{OC}{OA} = \dfrac{0.3}{0.6} = \dfrac{1}{2}$



$\Rightarrow \alpha = \cos^{-1}\left(\frac{1}{2}\right) = 60^\circ$

Also, from the figure, $\theta + \alpha = 180^\circ$

$\Rightarrow \theta = 180^\circ - \alpha$

$= 180^\circ - 60^\circ$

$\theta = 120^\circ = 120 \times \dfrac{\pi}{180}$ radians.

$\theta = 0.667\,\pi$ radians.

Area of flow, $A = R^2\left(\theta - \dfrac{\sin 2\theta}{2}\right)$

$= 0.6^2\left[0.667\pi - \dfrac{\sin(2\times 120^\circ)}{2}\right]$

$= 0.36\left(0.667\pi - \dfrac{\sin 240^\circ}{2}\right)$

$= 0.36\left[0.667\pi - \dfrac{(-0.866)}{2}\right]$

$= 0.36\,(0.667\pi + 0.433) = 0.913\ m^2$

Now, $Q = A \times V = AC\sqrt{mi} = 0.913 \times 58\sqrt{\frac{A}{P} \times \frac{1}{1000}}$

$\Rightarrow Q = 0.913 \times 58\sqrt{\frac{0.913}{2.256} \times \frac{1}{1000}} = 1.007\ m^3/sec.$

Q. The rate of flow of water through a circular channel of diameter 0.6 m is 150 lts/sec. Find the slope of the bed of the channel for maximum velocity. Take C = 60.

Solⁿ:- Given:

Discharge, $Q = 150\ lt/sec = \frac{150}{1000}\ m^3/sec = 0.15\ m^3/sec.$

Diameter of channel, $D = 0.6\ m$

Value of $C = 60.$

Let the slope of the channel bed for maximum velocity be 'i'

We know that for maximum velocity through a circular channel,

→ depth of flow is given by:

$$d = 0.81 \times D = 0.81 \times 0.6 = 0.486\ m.$$

and

→ $\theta = 128°45' = 128.45 \times \frac{\pi}{180} = 2.247$ radians.

→ Also for maximum velocity, the hydraulic mean depth is:

$$m = 0.3D = 0.3 \times 0.6\ m = 0.18\ m.$$

We know that, wetted perimeter for a circular pipe is:

$$P = 2R\theta$$
$$= D \times \theta$$
$$= 0.6 \times 2.247 = 1.3482\ m$$

But, $m = \frac{A}{P}$

$\Rightarrow A = m \times P = 0.18 \times 1.3482 = 0.2426\ m^2.$

We know, $Q = AC\sqrt{mi}$

$\Rightarrow 0.15 = 0.2426 \times 60 \times \sqrt{0.18 \times i}$

$\therefore i = \left(\frac{0.15}{6.175}\right)^2 = \frac{1}{1694.7}.$

Q. Determine the maximum discharge of water through a circular channel of diameter 1.5m when the bed slope of the channel is 1 in 1000. Take C = 60.

Soln:- Given,

Dia of channel, D = 1.5 m.

∴ Radius of channel, $R = \frac{D}{2} = \frac{1.5}{2} = 0.75\,m$

Bed slope, $i = \frac{1}{1000}$

Chezy's constant, C = 60

For maximum discharge condition,

$\theta = 154^\circ = 154 \times \frac{\pi}{180} = 2.6878$ radians.

We know,

Wetted Area (A) $= R^2\left(\theta - \frac{\sin 2\theta}{2}\right)$

$= 0.75^2\left[2.6878 - \frac{\sin(2 \times 154^\circ)}{2}\right]$

$= 0.75^2\left(2.6878 - \frac{\sin 308^\circ}{2}\right)$

$= 0.75^2\left[2.6878 - \frac{\sin(360 - 52)}{2}\right]$

$= 0.75^2\left(2.6878 + \frac{\sin 52^\circ}{2}\right)$

$= 1.7335\ m^2$

Also, Wetted Perimeter (P) $= 2R\theta$

$= 2 \times 0.75 \times 2.6878$.

$= 4.0317$ m.

∴ Hydraulic mean depth, $m = \frac{A}{P} = \frac{1.7335}{4.0317} = 0.4299$ m.

∴ Max. discharge, $Q = AC\sqrt{mi} = 1.7335 \times 60 \times \sqrt{0.4299 \times \frac{1}{1000}}$

∴ $Q = 2.1565\ m^3/sec$

Q. A concrete lined circular channel of diameter 3m has a bed slope of 1 in 500. Work out the velocity and flow rate for the conditions of i) Maximum velocity and ii) Maximum discharge. Assume, $C = 50$?

[ASSIGNMENT]

MODULE II

UNIT - II.

DIMENSIONAL & MODEL ANALYSIS

* DIMENSIONAL ANALYSIS:

→ This method or approach is mainly used in research and developement field to conduct model tests.

→ It deals with dimensions of the physical quantities involved in the phenomenon.

→ In Fluid Mechanics, length (L), mass (M) and time (T) are the three fixed dimensions which are of basic importance.

→ In case, heat is involved in any problem, then temperature is also taken as fixed dimension

→ These fixed dimensions are called fundamental dimensions or fundamental quantity.

P.T.O

* LIST OF FUNDAMENTAL DIMENSIONS:-

PHYSICAL QUANTITY	SYMBOL	SI UNIT
Length	L	meter
Mass	M	Kilogram
Time	T	Seconds
Temperature	θ	Kelvin
Amount of Substance	N	Moles
Light intensity	J	Candela
Electric Current	I	Ampere.

* SECONDARY OR DERIVED DIMENSIONS (QUANTITIES):

Secondary or derived dimensions are those dimensions which possess more than one fundamental dimension.

For Eg:- Velocity is denoted by distance per unit time, (L/T)

Density as mass per unit volume (M/L^3)

Accel^n as distance per second square (L/T^2)

Here, the expressions (L/T), (M/L^3) and (L/T^2) are called the dimensions of velocity, density and acceleration respectively.

(Or) Force = Mass × Accel^n

Here, Mass is fundamental dimension (M).

Accl^n is a derived dimension which is equal to (L/T^2).

∴ In Force, since there is more than one fundamental dimension, so it is a derived dimension or quantity.

$F = m.a.$

$F = M.\frac{L}{T^2} = \underbrace{MLT^{-2}}$ ⟶ Dimensional formula.

* DIMENSIONAL HOMOGENEITY :

" The power of fundamental dimension (M, L, T etc) on both sides of the equation will be identical for dimensionally homogeneous equation."

Such equations are independent of the system of units.

Let us consider the equation $V = \sqrt{2gH}$.

$L.H.S = V = \frac{L}{T} = LT^{-1}$

$R.H.S = \sqrt{2gH} = \sqrt{\frac{L}{T^2} \times L} = \sqrt{\frac{L^2}{T^2}} = \frac{L}{T} = LT^{-1}$

∴ L.H.S = R.H.S.

So, the eq^n $V = \sqrt{2gH}$ is dimensionally homogeneous.

* METHODS OF DIMENSIONAL ANALYSIS :-

If the no. of variables involved in a physical phenomen are known, then the relation among the variables can be determined by the following two methods :→

1. Rayleigh's method

&

2. Buckingham's π- theorem.

*** RAYLEIGH'S METHOD :→**

→ This method is used for determining the expression for a variable which depends upon maximum 3 or 4 variables only.

VARIABLE

- DEPENDENT VARIABLE (A physical quantity which depends on other physical quantities)
- INDEPENDENT VARIABLE. (A physical quantity which doesnot depend on other physical quantities)

→ If the number of variables on which another variable is depending becomes more than four, then it is bit tideous to find the expression for the dependent variable through Rayleigh's method.

→ Let X is a variable, which depends on other 3 variables X_1, X_2 and X_3

→ So, here X is a dependent variable.

→ According to Rayleigh's method, X is a function of X_1, X_2 and X_3, mathematically written as:-

$$X = f[X_1, X_2, X_3]$$

→ This can also be written as:-

$X = K X_1^a \cdot X_2^b \cdot X_3^c$

where K is a dimensionless constant

a, b, c are arbitrary powers.

The values of a, b and c are obtained by comparing the powers of the fundamental dimension on both sides.

Thus this expression is obtained for fundamental Variable.

Problem: Find the expression for the power (P) developed by a pump when P depends upon the head (H), the discharge (Q) and the specific wt. (ω) of the fluid.

Sol^n:- From the question,

Power (P) is a function of :—

(i) Head (H) (ii) Discharge (Q) (iii) Specific wt. (ω)

$\therefore P = f(H, Q, \omega)$

WKT, $X = K \cdot X_1^a \cdot X_2^b \cdot X_3^c$

Here, $X_1 = H$ $X_2 = Q$ $X_3 = \omega$.

$X_1 = H$, here head is measured in terms of length whose symbol is L

$X_2 = Q$, here discharge is measured in terms of $m^3/sec = L^3/T = L^3 T^{-1}$

$X_3 = w$, which is specific weight.

$$= \frac{\text{Weight}}{\text{Volume}} = \frac{\text{Mass} \times \text{Accl}^n}{\text{Vol}^m} = \frac{\text{Force}}{\text{Volume}}$$

[$\because$ Force = mass x Accl^n]

$$= \frac{MLT^{-2}}{L^3}$$

$$= ML^{-2}T^{-2}$$

$X = P$, power is Rate of doing work. whose S.I unit is watt

$$\text{Watt} = \frac{\text{Joules}}{\text{sec}} = \frac{\text{Newton} \times \text{meter}}{\text{sec}}$$

Here, meter (L) is a fundamental dimension
second (T) is a fundamental dimension

But, Newton which represents force is a derived dimension. Its fundamental dimension is MLT^{-2}

$$\therefore \text{Power}(P) = \frac{MLT^{-2} \times L}{T} = ML^2T^{-3}$$

$$\therefore P = K.H^a.Q^b.w^c \longrightarrow (i)$$

Substituting the dimensions on both sides, we get :-

$$ML^2T^{-3} = K.L^a.(L^3T^{-1})^b.(ML^{-2}T^{-2})^c$$

$$ML^2T^{-3} = K.L^a.(L^{3b}.T^{-b}).(M^c.L^{-2c}.T^{-2c})$$

$$M^1L^2T^{-3} = K.M^c.L^{(a+3b-2c)}.T^{(-b-2c)}$$

Equating the powers of M, L and T on both sides,

Power of M, $c = 1$ —— (ii)

Power of L, $a + 3b - 2c = 2$ —— (iii)

Power of T, $-b - 2c = -3$ —— (iv)

Substitute (ii) in (iv),

$$-b - 2(1) = -3$$

$$\Rightarrow -b - 2 = -3$$

$$\Rightarrow b + 2 = 3$$

$$\Rightarrow b = 3 - 2 = 1. \quad \text{—— (v)}$$

Substitute (ii) and (v) in (iii),

$$a + 3(1) - 2(1) = 2$$

$$\Rightarrow a + 3 - 2 = 2$$

$$\Rightarrow a + 1 = 2$$

$$\Rightarrow a = 2 - 1$$

$$\Rightarrow a = 1.$$

Substituting the value of a, b and c in eqn (i),

$$P = K.H^1.Q^1.\omega^1$$

$$\Rightarrow \boxed{P = KHQ\omega}$$

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Problem:- The efficiency η of a fan depends on density ρ, the dynamic viscosity μ of the fluid, the angular velocity ω, diameter D of the rotor and the discharge Q. Express η in terms of dimensionless parameters?

Soln:- The efficiency η depends on:—

(i) Density ρ (ii) Viscosity μ

(iii) Angular velocity ω (iv) Diameter D

(v) Discharge Q.

$$\therefore \eta = K.\rho^a.\mu^b.\omega^c.D^d.Q^e \quad \text{—— (i)}$$

where, K = Non-dimensional constant

$$\text{Efficiency } \eta = \frac{O/P}{I/P} = \frac{J}{J} = 1$$

So, it is unitless or unity (dimensionless)

So, it is $M^0 L^0 T^0$.

$$\therefore \eta = M^0.L^0.T^0 \qquad \omega = T^{-1}$$

$$\rho = ML^{-3} \qquad D = L$$

$$\mu = ML^{-1}T^{-1} \qquad Q = L^3 T^{-1}$$

Substituting the dimensions on both sides of equation —— (i),

$$M^0L^0T^0 = K.(ML^{-3})^a.(ML^{-1}T^{-1})^b.(T^{-1})^c.(L)^d.(L^3T^{-1})^e$$

Equating powers of M, L and T on both sides,

$$M^0 . L^0 . T^0 = K . (M)^a . (L^{-3})^a . (M)^b . (L^{-1})^b . (T^{-1})^b . (T^{-1})^c . (L)^d . (L^3)^e . (T^{-1})^e$$

Power of M, $0 = a + b$

Power of L, $0 = -3a - b + d + 3e$

Power of T, $0 = -b - c - e$

As we can observe here, there are 3 equations but 5 unknowns.

WKT, Efficiency (η) mainly depends on Viscosity (μ) and Discharge (Q).

Hence, here we need to express the other three unknowns in terms of the other 2 unknowns of μ and Q.

$\therefore$ Expressing a, c and d in terms of b and e, we get :→

In the above 3 equations,

$$a = -b$$

$$-d = -3a - b + 3e \Rightarrow d = 3a + b - 3e$$

$$= 3(-b) + b - 3e$$

$$\Rightarrow d = -3b + b - 3e$$

$$\Rightarrow d = -2b - 3e$$

$$c = -b - e \Rightarrow c = -(b + e)$$

Substituting these values in eqn (i), we get :-

$$\eta = K.\rho^{-b}.\mu^{b}.\omega^{-(b+e)}.D^{-2b-3e}.Q^{e}$$

$$= K.\rho^{-b}.\mu^{b}.\omega^{-b}.\omega^{-e}.D^{-2b}.D^{-3e}.Q^{e}$$

$$= K.\frac{1}{\rho^{b}}.\mu^{b}.\frac{1}{\omega^{b}}.\frac{1}{(D^{2})^{b}}.\frac{1}{\omega^{e}}.\frac{1}{(D^{3})^{e}}.Q^{e}$$

$$\eta = K.\left(\frac{\mu}{\rho\omega D^{2}}\right)^{b}.\left(\frac{Q}{\omega D^{3}}\right)^{e}$$

* **BUCKINGHAM'S π THEOREM** :→

" If there are 'n' variables (including both dependent and independent variable) in a physical phenomenon if the variable contains 'm' fundamental dimensions then the variable are arranged into $(n-m)$ dimensionless terms. Each term is called π-term.

n = No. of variable (Dependent & Independent variable)
m = No. of fundamental dimension.

Then,

$(n-m)$ = No. of dimensionless π- term

Let $X_1, X_2, X_3, X_4, X_5, X_6$ are the variables involved in a physical problem.

Let X_1 be the dependent variable and X_2, X_3, X_4, X_5 and X_6 are the variables on which X_1 depends i.e,

$$X_1 = f(X_2, X_3, X_4, X_5, X_6)$$

The above equation can also be written as:—

$$f(X_1, X_2, X_3, X_4, X_5, X_6) = 0.$$

[Transferring X_1 to the other variables, it forms a function as mentioned above].

[This is the general form of Buckingam's π-theorem)

Here,

$n = 6$

In fluid mechanics, most of the cases, it depends on three fundamental dimensions; M, L and T.

$m = 3$

Then,

$n - m = 6 - 3 = 3$ numbers of π-term

Let the 3 π-term are π_1, π_2 and π_3

As per Buckingham's π-theorem,

$$f(\pi_1, \pi_2, \pi_3) = 0.$$

π_1, π_2, π_3 are dimensionless terms, which can be written in the form of $M^0 L^0 T^0$.

GENERAL PROCEDURE :-

$$X_1 = f(X_2, X_3, X_4, \ldots, X_n)$$

$$f(X_1, X_2, X_3, X_4, \ldots, X_n) = 0.$$

n = No. of variables.

m = No. of fundamental dimension.

$n - m$ = π-terms.

If there are $(n-m)$ π terms, then each π term will have $(m+1)$ variable.

Here,

m = no. of fundamental dimension which is also called as no. of repeating variables

For example, let us consider a total of 7 variables.

$\therefore n = 7.$

And let us assume a total of 4 no. of fundamental dimension.

$\therefore m = 4$ no. of fundamental dimension
$= 4$ no. of repeating variables.

Let $x_1, x_2, x_3, x_4, x_5, x_6, x_7$ be all the 7 variables, out of which x_1, x_2, x_3, x_4 are the no. of repeating variables.

$\therefore n - m = 7 - 4 = 3$ π terms; where each π-term should have $(m+1)$ variables.

<u>Proof</u>:

$$\pi_1 = f(x_1, x_2, x_3, x_4, x_5)$$

$$\pi_2 = f(x_1, x_2, x_3, x_4, x_6)$$

$$\pi_3 = f(x_1, x_2, x_3, x_4, x_7)$$

Here, All the π terms are in the form of $m+1$, where m are the repeating variables x_1, x_2, x_3, and x_4 and other 3 variables are arranged each in every π term.

Finally, the expression becomes:

$$\pi_1 = \phi[\pi_2, \pi_3, \ldots, \pi_{n-m}]$$

$$\pi_2 = \phi[\pi_1, \pi_3, \ldots, \pi_{n-m}]$$

* METHOD OF SELECTING REPEATING VARIABLE :→

RULE I :- As far as possible, no dependent variable should be selected as a repeating variable.

RULE II :- While selecting repeating variables, preference should be given based on following sequence :-

I. Variables with Geometric Property like Diameter, Radius, length, Area.

In case in a problem, if three geometric variables are given, in that case, the variable with the lowest power of dimension should be considered as the geometric variable.

For eg :- Area (L^2), Radius (L), Volume (L^3)

in this case, Radius should be considered as the geometric variable, since it has least no. of dimension

II. Variables with flow Property like Discharge, Velocity, Acceleration.

III. Variables with Fluid Property like Density (ρ), Surface tension (σ), Viscosity (μ), kinematic Viscosity (ν).

RULE III :- Repeating variables selected should not form a dimensionless group within themselves.

RULE IV :- All the repeating variables together must have the same number of fundamental dimensions.

RULE V :- No two repeating variables should not have same dimensional formula.

For eg :- Potential energy $= ML^2T^{-2}$

Kinetic energy $= ML^2T^{-2}$

Here, both P.E & K.E cannot be considered as 2 repeating variables.

Problem :- The resisting force R of a supersonic plane during flight can be considered as dependent upon the length of the aircraft l, velocity V, air viscosity μ, air density ρ and bulk modulus of air K. Express the functional relationship between these variables and the resisting force.

Solⁿ :- STEP 1 :

Let,

$$X_1 = f(X_2, X_3, X_4, X_5, X_6, ----, X_n).$$

$$R = f(l, V, \mu, \rho, K)$$

As per Buckingham's π-theorem,

$$f(R, l, V, \mu, \rho, K) = 0$$

Here, $n = 6$

All the 6 variables here are having dimensional formula in the form of 3 fundamental dimension M, L and T.

So, $m = 3$.

$[R = MLT^{-2}, \ l = L, \ V = LT^{-1}, \ \mu = ML^{-1}T^{-1}, \ \rho = ML^{-3},$

$K = ML^{-1}T^{-2}]$

No. of dimensionless π terms $= n - m = 6 - 3 = 3$.

Thus 3 π-terms say, π_1, π_2 and π_3 are formed.

Therefore,

$$f(\pi_1, \pi_2, \pi_3) = 0.$$

STEP 2:

$$\pi_1 = M^0 L^0 T^0$$

$$\pi_2 = M^0 L^0 T^0$$

$$\pi_3 = M^0 L^0 T^0$$

Here,

$m = 3$ = no. of repeating variables.

Now, Repeating variables should be selected out of the 6 variables R, l, V, μ, ρ and K.

As per the rules of selecting repeating variables:-

* R is a dependent variable and should not be selected as a repeating variable.
* Out of the remaining 5 variables, length (l) is a geometric property, hence should be given first preference as repeating variable.
* Velocity (V) is a flow property, and hence should be given 2nd preference as a repeating variable.
* The remaining 3 variables viscosity (μ), air density (ρ) and bulk modulus of air (K) are all fluid properties.

As per the rules of selection, since air density (ρ) has the least no. of dimension out of the 3 variables, hence it is considered as the third repeating variable.

$$\therefore \pi_1 = f(l, V, \rho, \underline{R})$$

$$\pi_2 = f(l, V, \rho, \underline{\mu})$$

$$\pi_3 = f(l, V, \rho, \underline{K})$$

Thus, if we substitute remaining 3 variables and distribute them among all the π-terms, then we can observe that each π-term is having $m+1$ variable, i.e. $3+1=4$ variables. [Since, $m=3$].

* Check for satisfying Rule 3 of selection of repeating variable:

$l = L^1$

$V = LT^{-1}$

$\rho = ML^{-3}$

Multiplying their dimensions,

$$L^1 \times LT^{-1} \times ML^{-3}$$

$$= L^1 \times \frac{L}{T} \times \frac{M}{L^3} = \frac{M}{LT} = ML^{-1}T^{-1}$$

Thus, these 3 repeating variables are not formin a dimensionless group within themselves., thus Satisfying Rule no. 3.

* Check for Satisfying Rule 4 of selection of repeating variable.

$$l = L^1 = M^0 L^1 T^0$$

$$V = LT^{-1} = M^0 L^1 T^{-1}$$

$$\rho = ML^{-3} = M^1 L^{-3} T^0$$

Thus, all these 3 variables are having a same of 3 dimensions M, L and T, thus satisfying Rule no. 4.

* All the 3 repeating variables are having 3 different dimensional formula, thus Satisfying Rule no. 5

Thus, all the 5 Rules for the selection of the repeating variables are Satisfied.

STEP 3:

To Solve the 3 π terms, the functional form of all the 3 π- terms are converted in the form of equations by considering arbitrary constants for all the 3 repeating variables.

$$\pi_1 = l^{a_1} V^{b_1} \rho^{c_1} . R$$

$$\pi_2 = l^{a_2} V^{b_2} \rho^{c_2} . K.$$

$$\pi_3 = l^{a_3} V^{b_3} \rho^{c_3} . \mu$$

We know, that π_1 π_2 and π_3 are dimensionless terms and can be written as $M^0 L^0 T^0$.

STEP 4:

Each π - term is now solved by the principle of dimensional homogeneity in the same process of Rayleigh's method.

IST π- TERM

$$M^0 L^0 T^0 = (M^0 L T^0)^{a_1} \cdot (M^0 L T^{-1})^{b_1} \cdot (M L^{-3} T^0)^{c_1} \cdot (M L T^{-2}).$$

$$\Rightarrow M^0 L^0 T^0 = L^{a_1} \cdot L^{b_1} \cdot T^{-b_1} \cdot M^{c_1} \cdot L^{-3c_1} \cdot M^1 \cdot L^1 \cdot T^{-2}$$

$$\Rightarrow M^0 . L^0 T^0 = M^{c_1+1} \cdot L^{a_1+b_1-3c_1+1} \cdot T^{-b_1-2}$$

Power of M, $\quad 0 = c_1 + 1 \quad \Rightarrow c_1 = -1$

Power of L, $\quad 0 = a_1 + b_1 - 3c_1 + 1 = a_1 - 2 + 3 + 1 = 0$
$\Rightarrow a_1 + 2 = 0 \Rightarrow a_1 = -2$

Power of T, $\quad 0 = -b_1 - 2 \quad \Rightarrow b_1 = -2$

Substituting the values of a_1, b_1 and c_1 in the equation of π_1

$$\pi_1 = l^{-2} \cdot V^{-2} \cdot \rho^{-1} \cdot R.$$

$$\Rightarrow \pi_1 = \frac{R}{l^2 V^2 \rho} = \frac{R}{\rho l^2 V^2}$$

IInd π- TERM

$$M^0L^0T^0 = (M^0LT^0)^{a_2}.(M^0LT^{-1})^{b_2}.(ML^{-3}T^0)^{c_2}.(ML^{-1}T^{-1})$$

$$\Rightarrow M^0L^0T^0 = L^{a_2}.L^{b_2}.T^{-b_2}.M^{c_2}.L^{-3c_2}.M^1L^{-1}.T^{-1}$$

$$\Rightarrow M^0.L^0.T^0 = M^{c_2+1}.L^{a_2+b_2-3c_2-1}.T^{-b_2-1}$$

Power of M, $\quad 0 = c_2+1 \Rightarrow c_2 = -1$

Power of L, $\quad 0 = a_2+b_2-3c_2-1 \Rightarrow a_2 = -b_2+3c_2+1$

$\Rightarrow a_2 = 1-3+1 = -1$

Power of T, $\quad 0 = -b_2-1 \Rightarrow b_2 = -1$

Substituting the values of a_2, b_2 and c_2 in the equation of π_2,

$$\pi_2 = l^{-1}.V^{-1}.\rho^{-1}.\mu$$

$$\Rightarrow \pi_2 = \frac{\mu}{lV\rho}$$

IIIrd π- TERM

$$M^0L^0T^0 = (M^0LT^0)^{a_3}.(M^0LT^{-1})^{b_3}.(ML^{-3}T^0)^{c_3}.(ML^{-1}T^{-2})$$

$$\Rightarrow M^0L^0T^0 = L^{a_3}.L^{b_3}.T^{-b_3}.M^{c_3}.L^{-3c_3}.M^1.L^{-1}.T^{-2}$$

$$\Rightarrow M^0L^0T^0 = M^{c_3+1}.L^{a_3+b_3-3c_3-1}.T^{-b_3-2}$$

Power of M, $\quad 0 = c_3+1 \Rightarrow c_3 = -1$

Power of L, $\quad 0 = a_3+b_3-3c_3-1 \Rightarrow a_3 = -b_3+3c_3+1$

$\Rightarrow a_3 = 2-3+1 = 0$

Power of T, $\quad 0 = -b_3-2 \Rightarrow b_3 = -2$

predicted with the help of model testing. The most economical and safe design may be finally adopted.

*** LIMITATIONS OF DIMENSIONAL AND MODEL ANALYSIS:-**

1. The tests performed on the models can only be used to obtain information about prototype performance, if a complete similarity exist between a model and a prototype.

2. Dimensional analysis does not give any information about the constant appearing in the formula.

3. This method is useful only when a physical quantity depends on other quantities by multiplication and power relations. It fails if a physical quantity depends on sum or difference of other physical quantities.

4. This method fails if a physical quantity depends on another quantity as sine, cosine or logarithmic or exponential relations.

* SIMILITUDE - TYPES OF SIMILARITIES:

Similitude is defined as the similarity between the model and its prototype in every respect, which means that the model and prototype have similar properties or model and prototype are completely similar.

The 3 basic types of conditions or similarities for a model to be similar to a prototype are as follows:

1. Geometric Similarity
2. Kinematic Similarity
3. Dynamic Similarity.

1. Geometric Similarity:→

The geometric similarity is said to exist between the model and the prototype when the ratio of all corresponding linear dimension in model and prototype are equal.

Let L_m = Length of model

b_m = Breadth of model

D_m = Diameter of model

A_m = Area of model

$\forall_m$ = Volume of model

$L_p, b_p, D_p, A_p, \forall_p$ are corresponding values of the prototype.

for geometric Similarity between model and prototy
the relation is :-

$$\frac{L_p}{L_m} = \frac{b_p}{b_m} = \frac{D_p}{D_m} = L_r$$

where, L_r is called the Scale ratio.

[Scale Ratio :→ The ratio of measurement of a ~~model~~ prototype to the measurement of a model.]

For area's ratio and volume's ratio, the relation is given as :-

$$\frac{A_p}{A_m} = \frac{L_p \times b_p}{L_m \times b_m} = L_r \times L_r = L_r^2$$

$$\frac{V_p}{V_m} = \left(\frac{L_p}{L_m}\right)^3 = \left(\frac{b_p}{b_m}\right)^3 = \left(\frac{D_p}{D_m}\right)^3$$

[Similitude is the theory and art of predicting prototype performance from model observations]

- In Geometric Similarity, model and prototype are of same shape.
- All the linear dimensions of the model are related to the corresponding dimensions of the prototype by a constant scale factor.

2. Kinematic Similarity :→

Kinematic Similarity means the similarity of motion between model and prototype.

, Thus Kinematic similarity is said to exist between the model and the prototype if the ratios of the velocity and acceleration at the corresponding points in the model and at the corresponding points in the prototype are the same.

Since velocity and acceleration are vector quantities, hence not only the ratio of magnitude of velocity and acceleration at the corresponding points in model and prototype should be same; but the directions of velocity and accelerations at the corresponding points in the model and prototype should also be same and parallel.

Let, V_{P_1} = Velocity of fluid @ pt. 1 in prototype.

V_{P_2} = Velocity of fluid @ pt. 2 in prototype.

a_{P_1} = Accl^n of fluid @ pt. 1 in prototype.

a_{P_2} = Accl^n of fluid @ pt. 2 in prototype.

$V_{m_1}, V_{m_2}, a_{m_1}, a_{m_2}$ = Corresponding values @ the corresponding points of fluid velocity and accl^n in the model.

For Kinematic similarity,

$$\frac{V_{P_1}}{V_{m_1}} = \frac{V_{P_2}}{V_{m_2}} = V_r$$

where,

V_r is the velocity ratio.

For accl^n, $\frac{a_{p_1}}{a_{m_1}} = \frac{a_{p_2}}{a_{m_2}} = a_r$

where, a_r is the acceleration ratio.

Also, the direction of the velocities and accl^n in the model and prototype should be same.

3. Dynamic Similarity :→

Dynamic Similarity means the similarity of forces b/w the model and prototype. Thus dynamic similarity is said to exist between the model and prototype if the ratios of the corresponding forces acting at the corresponding points are equal.

Also the directions of the corresponding forces at the corresponding points should be same.

Let, $(F_i)_p$ = Inertia force at a point in prototype.

$(F_v)_p$ = Viscous force at a point in prototype.

$(F_g)_p$ = Gravity force at a point in prototype.

and $(F_i)_m$, $(F_v)_m$ & $(F_g)_m$ = Corresponding values of forces at the corresponding point in model.

Then for dynamic similarity; we have :—

$$\frac{(F_i)_p}{(F_i)_m} = \frac{(F_v)_p}{(F_v)_m} = \frac{(F_g)_p}{(F_g)_m} = F_r$$

where, F_r is the force ratio.

Also, the directions of the corresponding forces at the corresponding points in the model and prototype should be same.

* DIMENSIONLESS NUMBERS :—

→ Dimensionless numbers are those numbers which are obtained by dividing the inertia force by viscous force or gravity force or pressure force or surface tension force or elastic force.

→ As it is a ratio of one force to the other force, hence it will be a dimensionless number.

→ These dimensionless numbers are called also as non-dimensional number.

→ The following are the important dimensionless numbers:

1. Reynold's number.
2. Froude's number.
3. Euler's number.
4. Weber's number.
5. Mach's number.

1. Reynold's number :→ (Re)

It is defined as the ratio of inertia force of a flowing fluid and the viscous force of the fluid. The expression for Reynold's number is obtained as:-

WKT, Inertia force (Fi) = Mass x Accl^n of flowing fluid

$$= (\rho \times \text{Volume}) \times \left(\frac{\text{Velocity}}{\text{Time}}\right)$$

$$= \rho \times \frac{\text{Volume}}{\text{Time}} \times \text{Velocity}$$

$= \rho \times (\text{Area} \times \text{Velocity}) \times \text{Velocity}$

$\left[\because \frac{\text{Volume}}{\text{Time}} = \text{Volume per sec} = \text{Area} \times \text{Velocity}\right]$

$= \rho \times (A \times V) \times V.$

$F_i = \rho A V^2$

Viscous force (F_v) = Shear Stress x Area

WKT,

Shear Stress, $\tau = \mu \frac{du}{dy}$

$\therefore F_v = \tau \times A = \mu \frac{du}{dy} \times A$

$= \mu \cdot \frac{V}{L} \times A \left[\because \frac{du}{dy} = \frac{V}{L}\right]$

By defination,

Reynold's no. $R_e = \frac{F_i}{F_v} = \frac{\rho A V^2}{\mu \cdot \frac{V}{L} \cdot A} = \frac{\rho V L}{\mu}$

$= \frac{V \times L}{\mu / \rho} = \frac{V \times L}{\nu} \left[\because \frac{\mu}{\rho} = \nu\right]$

ν = Kinematic viscosity.

$\therefore R_e = \frac{V \times L}{\nu}$

Here, L = Linear dimension, which is taken as diameter d in case of pipe flow.

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Therefore,
Reynold's number for pipe flow is :—

$$\boxed{R_e = \frac{Vd}{\nu} \quad \text{or} \quad \frac{\rho V d}{\mu}}$$

2. Froude's Number (Fe) :→

The Froude's number is defined as the square root of the ratio of inertia force of a flowing fluid to the gravity force.

Mathematically, it is expressed as:-

$$F_e = \sqrt{\frac{F_i}{F_g}}$$

WKT,

$$F_i = \rho A V^2$$

and F_g = Force due to gravity

= Mass × Accl^n due to gravity

= (ρ × Volume) × g.

$= \rho \times L^3 \times g$ [∵ Volume $= L^3$]

$= \rho \times L^2 \times L \times g$

$= \rho \times A \times L \times g$ [∵ Area(A) $= L^2$]

$$\therefore \boxed{F_e = \sqrt{\frac{F_i}{F_g}} = \sqrt{\frac{\rho A V^2}{\rho A L g}} = \sqrt{\frac{V^2}{Lg}} = \frac{V}{\sqrt{Lg}}}$$

P.T.O

3. Euler's Number (E_u):→

It is defined as the square root of the ratio of the inertia force of a flowing fluid to the pressure force.

Mathematically, it is expressed as:—

$$E_u = \sqrt{\frac{F_i}{F_p}}$$

where,

$F_i = \rho A V^2$

F_p = ~~Intensity~~ Force due to pressure intensity

= Intensity of Pressure × Area

$= p \times A$

$$\therefore \boxed{E_u = \sqrt{\frac{\rho A V^2}{p \times A}} = \sqrt{\frac{V^2}{p/\rho}} = \frac{V}{\sqrt{p/\rho}}}$$

4. Weber's Number (W_e):→

It is defined as the square root of the ratio of the inertia force of a ~~flowing~~ fluid to the surface tension force.

Mathematically, it is expressed as:—

$$W_e = \sqrt{\frac{F_i}{F_s}}$$

where, $F_i = \rho A V^2$

F_s = Surface tension force

$= \text{Surface tension per unit length} \times \text{Length}$

$= \sigma \times L$

$$\therefore \; W_e = \sqrt{\frac{\rho A V^2}{\sigma \times L}} = \sqrt{\frac{\rho L^2 V^2}{\sigma \times L}} \qquad [\because A = L^2]$$

$$= \sqrt{\frac{\rho \times L \times V^2}{\sigma}}$$

$$= \sqrt{\frac{V^2}{\sigma / \rho L}}$$

$$\therefore \; \boxed{W_e = \frac{V}{\sqrt{\sigma / \rho L}}}$$

5. Mach's Number (M):→

Mach's Number is defined as the square root of the ratio of the inertia force of a flowing fluid to the elastic force.

Mathematically, it is expressed as:–

$$M = \sqrt{\frac{F_i}{F_e}}$$

where, $F_i = \rho A V^2$

F_e = Elastic force

= Elastic Stress × Area

$= K \times A$

$F_e = K \times L^2 \quad [\because K = \text{Elastic Stress}; \; A = \text{Area} = L^2]$

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$$M = \sqrt{\frac{\rho A V^2}{K \times L^2}}$$

$$= \sqrt{\frac{\rho \times L^2 \times V^2}{K \times L^2}} \quad [\because A = L^2]$$

$$= \sqrt{\frac{V^2}{K/\rho}}$$

$$= \frac{V}{\sqrt{\frac{K}{\rho}}}$$

But, $\sqrt{\frac{K}{\rho}} = C =$ Velocity of Sound in the fluid.

$$\therefore \boxed{M = \frac{V}{C}}$$

* **CLASSIFICATION OF MODELS:** →

1) **Undistorted Models:**

Undistorted models are those models which are geometrically similar to their prototypes. In other words, the scale ratio for the linear dimensions of the model and its prototype are the same.

2) **Distorted Models:**

Distorted models are those models which are geometrically not similar to its prototype. In other words, the scale ratio for the linear dimensions of the model and its prototype are not same.

→ Advantages of distorted models:

1. Horizontal and vertical dimensions of the model can be measured accurately.
2. The cost of the model can be reduced.
3. Turbulent flow in the model can be maintained.

→ Scale ratios for Distorted Models:

In case of Distorted Models, 2 different scale ratios, one for horizontal dimensions and other for vertical dimensions are taken.

Let $(L_r)_H$ = Scale ratio for horizontal dimension

$$= \frac{L_P}{L_m} = \frac{b_P}{b_m}$$

$$= \frac{\text{Linear horizontal dimension of prototype}}{\text{Linear horizontal dimension of model}}$$

$(L_r)_V$ = Scale ratio for vertical dimension

$$= \frac{h_P}{h_m}$$

$$= \frac{\text{Linear vertical dimension of prototype}}{\text{Linear vertical dimension of model}}$$

Then the scale ratios of velocity, area of flow, discharge etc. in terms of $(L_r)_H$ and $(L_r)_V$ can be obtained for distorted modelsas given below:

1. Scale ratio for velocity:

Let V_p = Velocity in prototype

V_m = Velocity in model.

Then, $$\frac{V_p}{V_m} = \frac{\sqrt{2gh_p}}{\sqrt{2gh_m}} = \frac{\sqrt{h_p}}{\sqrt{h_m}} = \sqrt{\frac{h_p}{h_m}} = \sqrt{(L_r)_V}$$

$$\left[\because V = \sqrt{2gh} \text{ and } \frac{h_p}{h_m} = (L_r)_V\right]$$

2. Scale ratio for area of flow:

Let, A_p = Area of flow in prototype = $b_p \times h_p$

A_m = Area of flow in model = $b_m \times h_m$

$$\therefore \frac{A_p}{A_m} = \frac{b_p \times h_p}{b_m \times h_m} = \frac{b_p}{b_m} \times \frac{h_p}{H_m} = (L_r)_H \times (L_r)_V$$

3. Scale ratio for discharge:

Let Q_p = Discharge through prototype = $A_p \times V_p$

Q_m = Discharge through model = $A_m \times V_m$

$$\therefore \frac{Q_p}{Q_m} = \frac{A_p \times V_p}{A_m \times V_m} = \frac{A_p}{A_m} \times \frac{V_p}{V_m} = (L_r)_H \times (L_r)_V \times \sqrt{(L_r)_V}$$

$$= (L_r)_H \times \left[(L_r)_V\right]^{3/2}$$

<u>Problem</u>: The discharge through a weir is $1.5\,m^3/sec$. Find the discharge through the model of the weir if the horizontal dimension of the model is equal to $1/50$ of the horizontal dimension of the prototype and vertical dimension of the model is equal to

1/10 of the vertical dimension of the prototype.

Solⁿ:- Given,

Discharge through a weir (prototype) $Q_p = 1.5\,m^3/s$

Horizontal dimension of model $= \frac{1}{50} \times$ Horizontal dimension of prototype

$$\Rightarrow \frac{\text{Horizontal dimension of prototype}}{\text{Horizontal dimension of model}} = 50$$

$$\Rightarrow (L_r)_H = 50$$

Vertical dimension of model $= \frac{1}{10} \times$ Vertical dimension of prototype.

$$\Rightarrow \frac{\text{Vertical dimension of prototype}}{\text{Vertical dimension of model}} = 10$$

$$\Rightarrow (L_r)_V = 10.$$

WKT,

$$\frac{Q_P}{Q_m} = (L_r)_H \times \left[(L_r)_V\right]^{3/2}$$

$$\Rightarrow \frac{Q_P}{Q_m} = 50 \times 10^{3/2} = 1581.14$$

$$\Rightarrow Q_m = \frac{Q_P}{1581.14} = \frac{1.5\ m^3/s}{1581.14}$$

$$\therefore Q_m = 0.000948\,m^3/sec$$

(or) $Q_m = 0.948\ lt/sec$ $[\because 1\,m^3 = 1000\ lt]$

Substituting the values of a_3, b_3 and c_3 in the equation of π_3,

$$\pi_3 = l^0 . V^{-2} . \rho^{-1} . K.$$

$$\Rightarrow \pi_3 = \frac{K}{V^2 \rho}$$

STEP 5:-

According to Buckingham's π-theorem,

$$f(\pi_1, \pi_2, \pi_3) = 0.$$

$$f\left(\frac{R}{\rho l^2 V^2}, \frac{\mu}{l V \rho}, \frac{K}{V^2 \rho}\right) = 0$$

$$\Rightarrow \frac{R}{\rho l^2 V^2} = f\left(\frac{\mu}{l V \rho}, \frac{K}{V^2 \rho}\right).$$

$$\Rightarrow \frac{R}{\rho l^2 V^2} = \phi\left(\frac{\mu}{l V \rho}, \frac{K}{V^2 \rho}\right).$$

$$\Rightarrow R = \rho l^2 V^2 \phi\left(\frac{\mu}{l V \rho}, \frac{K}{V^2 \rho}\right).$$

* MODEL ANALYSIS:-

For predicting the performance of the hydraulic structures (such as dams, spillways etc.) or hydraulic machines (such as turbines, pumps etc.), before actually constructing or manufacturing, are made replica of the structures or machines and tests are performed on them to obtain the desired information.

The model is the small replica of the actual structure or machine. The actual structure or machine is called as Prototype.

It is not necessary that the models should be smaller than the prototypes (though in most of the cases it is), they may be larger than the prototype.

This study of models of actual machines is called as Model Analysis. Model Analysis is actually an experimental method of finding solutions of complex flow problems.

However, exact analytical solutions are possible only for a limited number of flow problems.

* **ADVANTAGES OF DIMENSIONAL AND MODEL ANALYSIS:-**

1. The performance of the hydraulic structures or hydraulic machine can be easily predicted in advance, from its model.

2. With the help of dimensional analysis, a relationship between the variables including a flow problem in terms of dimensionless parameters is obtained. This relationship helps in conducting tests on the model.

3. The merits of alternative designs can be

MODULE III

* UNIT III *

* FLUID MACHINES :→

→ It is such a energy conversion device which converts the stored energy of a fluid into mechanical energy and vice versa.

→ Stored energy in fluid may be:-
(i) Potential energy (height of rise of fluid)
(ii) Kinetic energy (velocity of fluid)
(iii) Inter molecular energy (internal energy of fluid which depends on motion of fluid particles, temperature of fluid particles)

→ Hydraulic machines are those fluid machines which uses water as a source of medium for energy conversion.

* IMPACT OF JETS :→

→ Jet is a stream of fluid issued from a nozzle
→ Since the nozzle has less area at outlet, hence the jet velocity will be more, which means that the jet possesses more kinetic energy.

→ When the jet strikes any plate, then it exerts a force on the jet due to the change in momentum of the jet.

→ This force can be evaluated hydrodynamically by using Impulse – Momentum theorem of equation

→ **Impulse Momentum Equation:–**

This equation is based on the principle of conservation of momentum which states that, " The net force acting on a fluid mass is equal to rate of change of momentum of flow per unit time in that direction"

From Newton's IInd law,

Net force = Mass × Accl^n

$$\Sigma F = ma$$

⇒ Net force acting on a fluid mass = Mass of fluid × Accl^n

Also,

a = Change in Velocity w.r.t time

$$a = \frac{dV}{dt}.$$

$$\therefore F = m\frac{dV}{dt}$$

$$F = \frac{d(Vm)}{dt} \quad [\because m \text{ is a constant}]$$

This eq^n is termed as Momentum Principle.

$$\Rightarrow F.dt = d(Vm)$$

This eq^n is termed as Impulse Momentum Equation which states that the impulse force F acting on a

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short interval of time dt, on a fluid mass 'm' is equal to rate of change of momentum or velocity

⇒ for Angular consideration →

Angular Momentum Principle:

It states that, "The torque exerted by any body is equal to the rate of change of angular momentum"

$$T = I_m \alpha$$

Here,

T = Torque

I_m = Mass moment of inertia

α = Angular Acceleration

= Change in Angular velocity w.r.t time

$$\alpha = \frac{d\omega}{dt}$$

$$\therefore T = I_m \frac{d\omega}{dt}$$

$$\Rightarrow T = \frac{I_m}{dt} d\omega$$

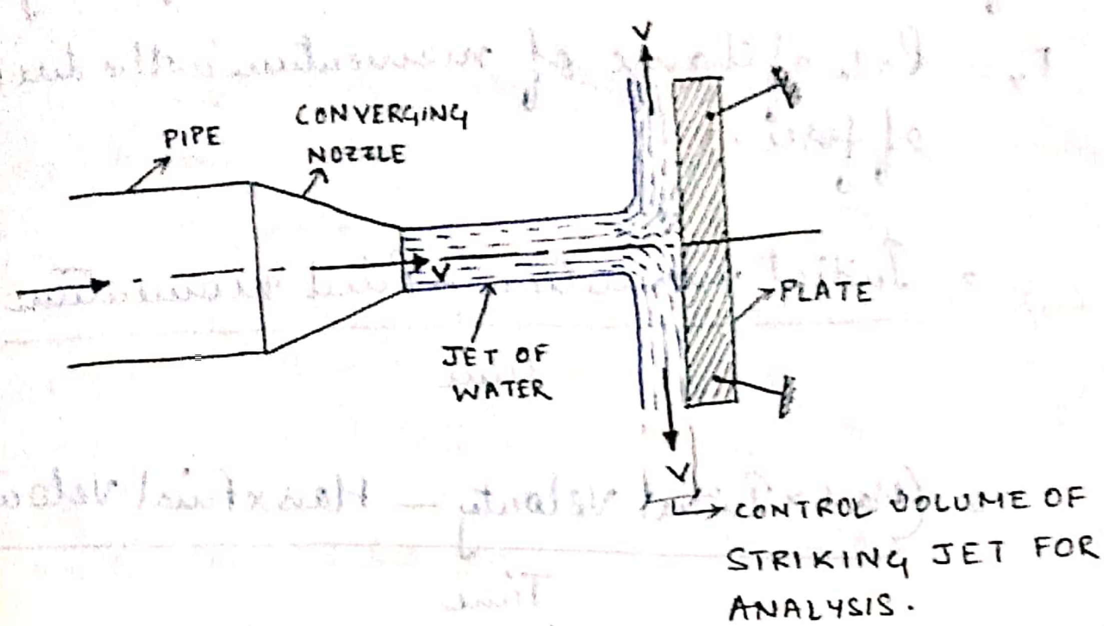
Here, $\frac{I_m}{dt}$ = Mass moment of inertia w.r.t time

$= \dot{I}_m$

$$\therefore T = \dot{I}_m d\omega$$

$$\Rightarrow T = \dot{I}_m (\omega_2 - \omega_1)$$

* FORCE EXERTED BY THE JET ON A STATIONARY VERTICAL PLATE :→



Consider a jet of water coming out from a nozzle, which strikes a flat vertical plate as shown in the figure.

Let V = Velocity of jet

d = diameter of jet

a = C/S area of the jet $= \frac{\pi}{4} d^2$

The jet after striking the plate, will move along the plate. But the plate is vertically placed w.r.t the jet flow, hence it is at right angles to the jet. Hence, the jet after striking, will get deflected through 90°.

Initially, as the jet moves out of the nozzle, it possesses a certain velocity, but since the plate is fixed and is not moving, hence after striking the plate, the final velocity of the jet becomes zero.

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Let F_x be the force exerted by the jet on the plate in the direction of jet.

Therefore, as per Impulse momentum principle,

F_x = Rate of change of momentum in the direction of force.

$$= \frac{\text{Initial momentum} - \text{Final momentum}}{\text{Time}}$$

$$= \frac{(\text{Mass} \times \text{Initial Velocity} - \text{Mass} \times \text{Final Velocity})}{\text{Time}}$$

$$= \frac{\text{Mass}}{\text{Time}} \times (\text{Initial Velocity} - \text{Final Velocity})$$

$$= (\text{Mass per sec}) \times (V - 0)$$

$$= (\text{Density} \times \text{Volume per sec}) \times V$$

$$= (\rho \times \text{area} \times \text{Velocity}) \times V$$

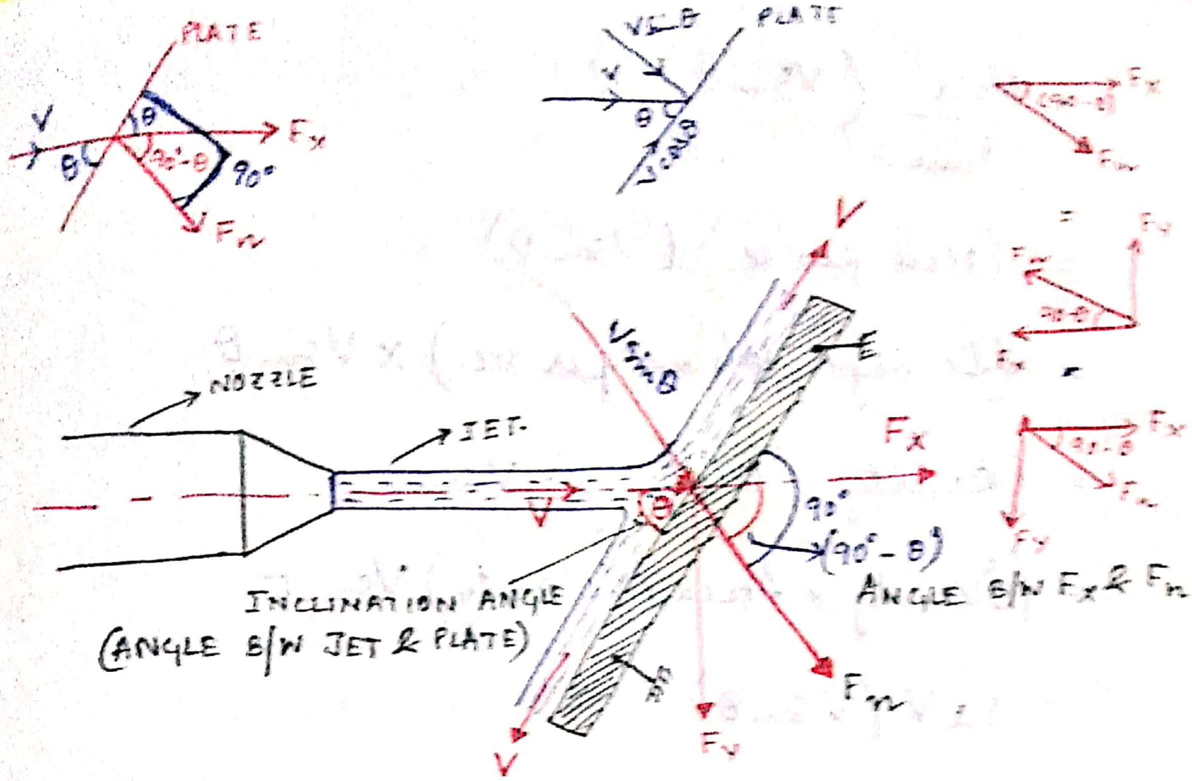
$$F_x \; \cancel{=\rho t} = \rho a V \times V = \rho a V^2$$

* **FORCE EXERTED BY A JET ON STATIONARY INCLINED FLAT PLATE :→**

Let a jet of water coming out from the nozzle, strike an inclined stationary flat plate as shown in the figure.

Let V = Velocity of jet in the direction of x

θ = Inclination angle b/w jet and plate.



a = c/s area of the jet.

If the plate is smooth and it is assumed that there is no loss of energy due to impact of the jet (since there is no frictional loss), there is no frictional resistance force.

Let us find the force exerted by the jet on the plate in the direction normal to the plate. Let the force be represented by F_n.

F_n = Rate of change of momentum in the direction of force

$$= \frac{\text{Initial momentum} - \text{Final momentum}}{\text{Time}}$$

$$= \frac{\text{Mass} \times \text{Initial velocity in the direction of } n - \text{Mass} \times \text{Final velocity in the direction of } n}{\text{Time}}$$

$$= \frac{Mass}{Time}(V\sin\theta - 0)$$

$$= (\text{Mass per sec})(V\sin\theta)$$

$$= (\text{Density} \times \text{Volume per sec.}) \times V\sin\theta$$

$$= (\text{Density} \times \text{Discharge})\, V\sin\theta$$

$$= (\text{Density} \times \text{Area} \times \text{Velocity})\, V\sin\theta$$

$$= \rho a V (V\sin\theta)$$

$$F_n = \rho a V^2 \sin\theta$$

This force F_n can be resolved into 2 components, one in the direction of the jet and the other perpendicular to the direction of jet.

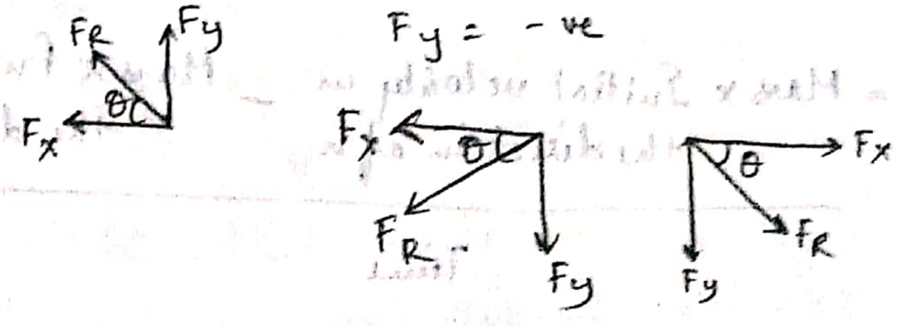
F_x = Component of F_n in the direction of flow

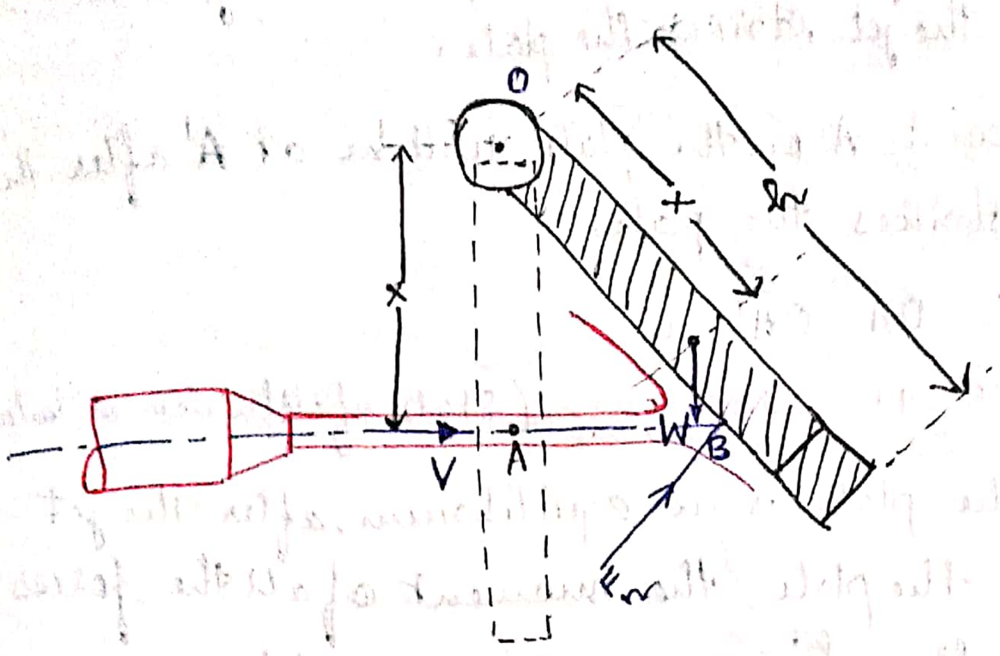
$$= F_n \cos(90^\circ - \theta) = F_n \sin\theta = \rho a V^2 \sin^2\theta$$

F_y = Component of F_n perpendicular to the direction of flow.

$$= F_n \sin(90^\circ - \theta) = F_n \cos\theta = \rho a V^2 \sin\theta \cos\theta$$

* **FORCE EXERTED BY A JET ON STATIONARY CURVED PLATE:**





→ Consider a jet of water striking a vertical plate @ centre which is hinged at O.

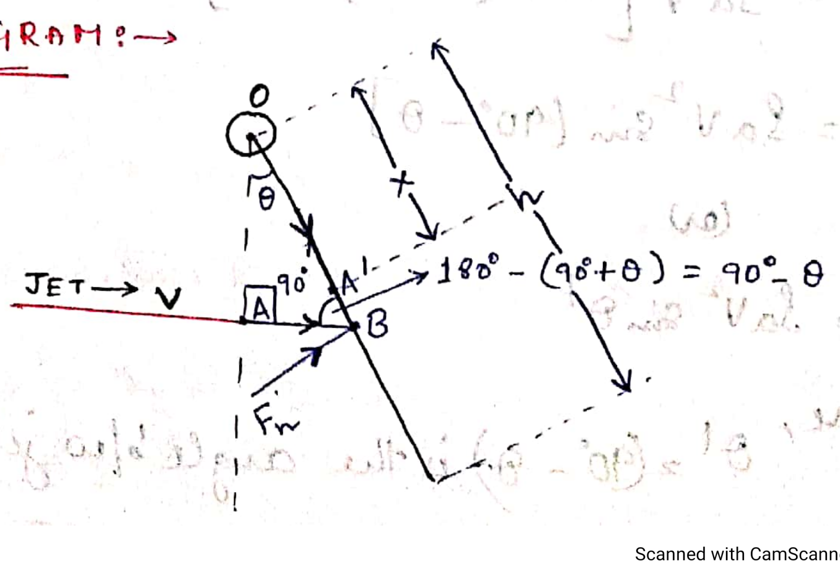
→ Due to the force exerted by the jet on the plate, the plate will swing through some angle about the hinge

→ Let x = distance of the centre of jet from hinge point O.

θ = Angle of swing about hinge

W = Weight of plate acting at C.G. of the plate

LINE DIAGRAM :→



→ The dotted lines shows the position of the plate, before the jet strikes the plate.

→ The point A on the plate will be at A' after the jet strikes the plate.

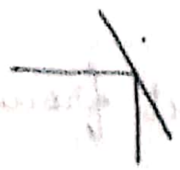
$\therefore OA = OA' = x$.

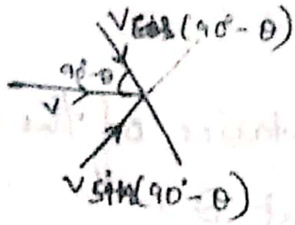
EQUILIBRIUM CONDITION:- (State of stableness or balance)

When the plate is in equilibrium, after the jet strikes the plate, the moment of all the forces about the hinge must be equal to zero.

→ Two forces are acting on the plate. They are:-

1. Force due to jet of water, normal to the plate





$$F_n = \frac{\text{mass}}{\text{sec}} [I.V - F.V]$$

$$= \rho a V [V \sin(90° - \theta) - 0]$$

$$F_n = \rho a V^2 \sin(90° - \theta)$$

(or)

$$F_n = \rho a V^2 \sin \theta'$$

where, $\theta' = (90° - \theta)$ is the angle b/w jet & plate.

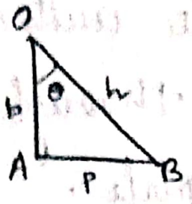
2. Weight of the plate, W

MOMENT CALCULATION:-

→ Moment of force F_n about hinge $= F_n \times OB$

$$= \rho a V^2 \sin(90^\circ - \theta) \times OB$$

$$= \rho a V^2 \cos\theta \times OB.$$



$$\cos\theta = \frac{b}{h} = \frac{OA}{OB}.$$

$$\Rightarrow \cos\theta \, OB = OA$$

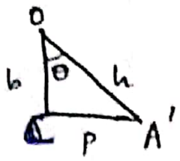
$$\Rightarrow OB = \frac{OA}{\cos\theta}$$

∴ Moment of force F_n about hinge

$$= \rho a V^2 \cos\theta \times \frac{OA}{\cos\theta}$$

$$= \rho a V^2 \, OA = \rho a V^2 x$$

Moment of weight W about hinge $= W \times AA'$



$$\sin\theta = \frac{p}{h} = \frac{AA'}{OA'}$$

$$\Rightarrow AA' = OA' \sin\theta$$

∴ Moment of weight W about hinge $= W \times OA' \sin\theta$

$$= W \times x \times \sin\theta$$

For equilibrium of the plate,

Moment of force F_n = Moment of weight W.

$$\Rightarrow \rho a V^2 x = W x \sin\theta$$

$$\Rightarrow \sin\theta = \frac{\rho a V^2}{W}$$

From the above eqⁿ, the angle of swing of the plate about the hinge can be calculated.

Problem:- A jet of water of 30 mm diameter strikes a hinged square plate at its centre with a velocity of 20 m/s. The plate is deflected through an angle of 20°. Find the weight of the plate.

If the plate is not allowed to swing, then what will be the force required at the lower edge of the plate to keep the plate in vertical position.

Solution:- Dia of jet, $d = 30\,mm = 0.03\,m$

$\therefore$ Area, $a = \frac{\pi}{4} d^2 = 0.0007068\,m^2$

Velocity of jet, $V = 20\,m/s$.

Angle of swing, $\theta = 20°$.

WKT,

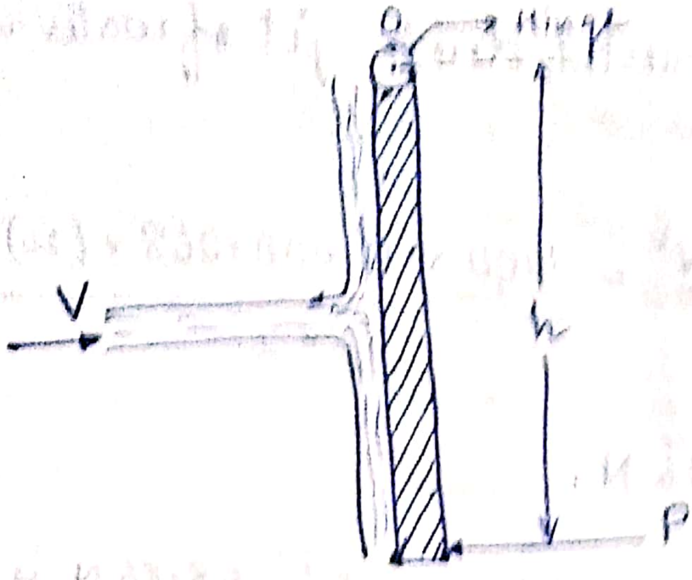
$$\sin\theta = \frac{\rho a V^2}{W}$$

$$\Rightarrow W = 826.6\,N.$$

In order to restrict the plate to swing, a force P will be applied at the lower edge of the plate as shown in the figure. The weight of the plate will be acting vertically downward through the C.G. of the plate.

Let F = force exerted by the jet of water.

h = Height of the plate.
= Distance of P from the hinge.



The jet strikes the plate @ centre and hence distance of the centre of the jet from hinge = $\frac{h}{2}$

After restricting the swing of the plate, it is observed that the weight of the plate W is passing through the hinge O. Therefore moment of force due to weight W about the hinge is zero.

Moment of force F due to jet of water is :-

$$F \times \frac{h}{2}$$

Moment of force P for restricting the swing is:

$$P \times h.$$

$$\therefore P \times h = F \times \frac{h}{2}$$

$$\Rightarrow P = \frac{F}{2}$$

After restricting the swing of the plate, it is observed that the plate behaves almost like a stationary vertical plate.

$\therefore$ Force F exerted due to jet of water $= \rho a V$

So,

$$P = \frac{\rho a V^2}{2} = \frac{1000 \times 0.0007068 \times (20)^2}{2}$$

$$P = 141.36 \text{ N}.$$

<u>Problem</u>:- A rectangular plate, weighing 58.86 N is suspended vertically by a hinge on the top of a horizontal edge. The centre of gravity of the plate is 10 cm from the hinge. A horizontal jet of water 2 cm dia, whose axis is 15 cm below the hinge impinges normally on the plate with a velocity of 5 m/sec. Find the horizontal force applied at the centre of the gravity to maintain the plate in its vertical position. Find the corresponding velocity of jet, if the plate is deflected through 30° and the same force continues to act at the centre of gravity of the plate?

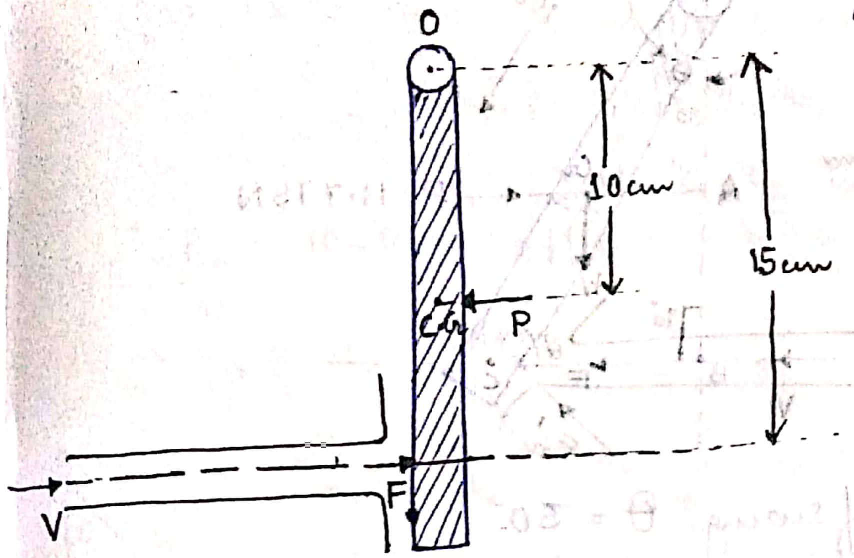
<u>Solution</u>:- Given:-

Weight of the plate, $W = 58.86$ N.

Distance of W from the hinge; $x = 10$ cm $= 0.1$ m.

Dia of the jet, $d = 2$ cm $= 0.02$ m

(ii) Let the force applied at the C.G. of the plate to keep the plate in vertical position be P as shown in the figure:



* $a = \frac{\pi}{4} d^2$
$= 0.000314 \, m^2$

In this case, since an attempt is made to keep the plate in vertical position by applying a force P at the C.G of the plate, the plate eventually becomes stationary.

∴ Force F exerted by the jet of water on the vertical plate $= \rho a V^2$

$$F = (1000) \times (0.000314) \times (5)^2$$

$$F = 7.85 \, N.$$

This force F is acting at a distance of 15 cm (0.15 m) from the hinge.

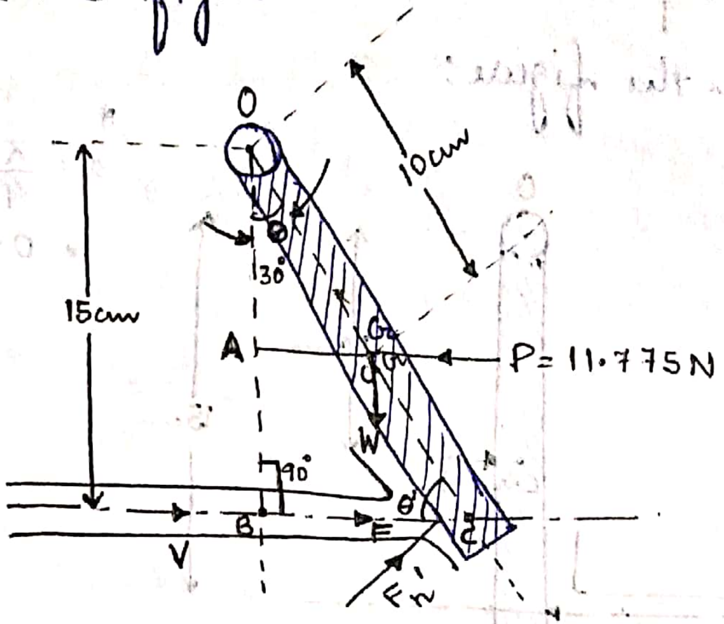
So, taking moments about the hinge, we get :-

$$F \times 0.15 = P \times 0.10$$

$$\Rightarrow P = \frac{F \times 0.15}{0.10} = \frac{7.85 \times 0.15}{0.10}$$

$$\therefore P = 11.775 \, N.$$

ii) The plate is deflected through an angle of 30° as shown in the figure:



Here, Angle of swing, $\theta = 30°$

The same force continuing to act on the C.G. of the plate, $P = 11.775\,N$.

Let the velocity of jet in this position be V m/sec.

In case of deflection condition of the plate, the various forces acting on the plate are:-

* Weight of the plate W acting at C.G. @ a distance of 10 cm from O.
* Horizontal force P acting at C.G.
* Normal force F_n due to jet of water acting on the plate at pt. C.

Therefore, for the plate to be in equilibrium taking moment of all forces about hinge O, we get:-

$$F_n \times OC = (P \times OA) + (W \times AG).$$

Here, $F_n = \rho a V^2 \sin\theta'$

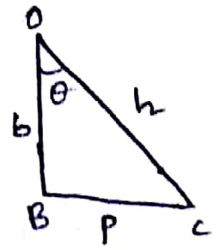
where, θ' = Angle b/w jet and plate

$= 90° - \theta$

$= 90° - 30° = 60°$

$\therefore F_n = 1000 \times (0.000314) \times V^2 \times \sin 60°$

$\Rightarrow F_n = 0.2717\, V^2$



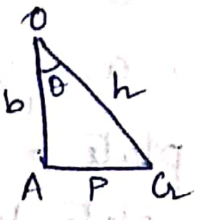
$\cos\theta = \frac{b}{h}$

$\Rightarrow \cos 30° = \frac{OB}{OC}$

$\Rightarrow OC = \frac{OB}{\cos 30°}$

$\Rightarrow OC = \frac{0.15}{\cos 30°}$

$\Rightarrow OC = 0.1732\, m$



$\cos\theta = \frac{b}{h}$

$\Rightarrow \cos 30° = \frac{OA}{OG}$

$\Rightarrow \cos 30° = \frac{OA}{0.1}$

$\Rightarrow OA = 0.0866\, m$

Also,

$\sin\theta = \frac{P}{h}$

$\Rightarrow \sin 30° = \frac{AG}{OG}$

$\Rightarrow AG = \sin 30° \times 0.1$

$\Rightarrow AG = 0.05\, m$

$$\therefore (0.2717 V^2) \times (0.1732) = (11.775 \times 0.0866) + (58.86 \times 0.05).$$

$\Rightarrow V = 9.175$ m/sec

Problem:- A jet of water of diameter 25 mm strikes a 20 cm × 20 cm square plate of uniform thickness with a velocity of 10 m/sec at the centre of the plate which is suspended vertically by a hinge on its top horizontal edge. The weight of the plate is 98.1 N.

The jet strikes normal to the plate. What force must be applied at the lower edge of the plate so that the plate is kept vertical? If the plate is allowed to deflect freely, what will be the inclination of the plate with vertical due to the force exerted by jet of water?

Soln:- Given,

Dia of jet, $d = 25\,mm = \frac{25}{1000}\,m = 0.025\,m$.

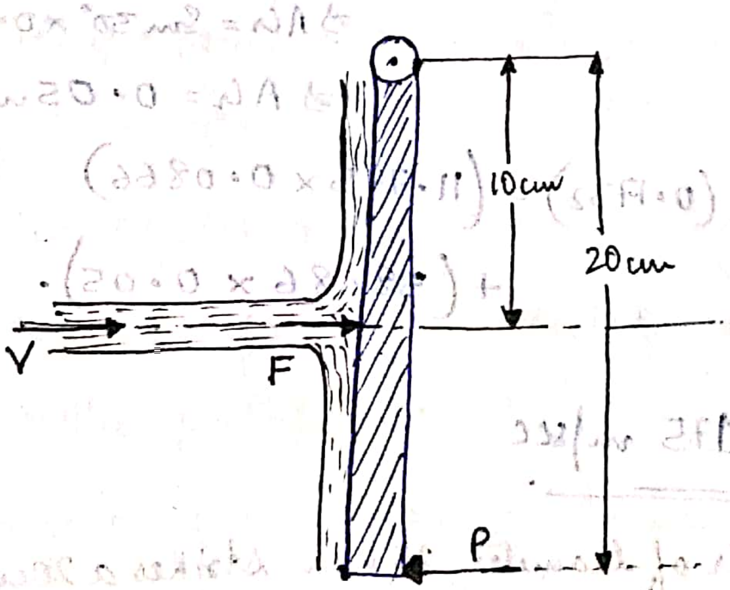
$\therefore$ Area of jet, $a = \frac{\pi}{4} d^2 = 0.00049\,m^2$

Size of the plate $= 20\,cm \times 20\,cm$

Weight of the plate, $W = 98.1\,N$.

Velocity of the jet, $V = 10\,m/sec$.

(i) Let the force applied at the lower edge of the plate to keep the plate in vertical position be P, as shown in the figure.



Since the plate is vertical, so, force exerted by the jet of water at the centre of the vertical plate, $F = \rho a V^2$

$= 1000 \times 0.0049 \times (10)^2$

$F = 49\,N.$

This force is acting at a distance of 10 cm from the hinge.

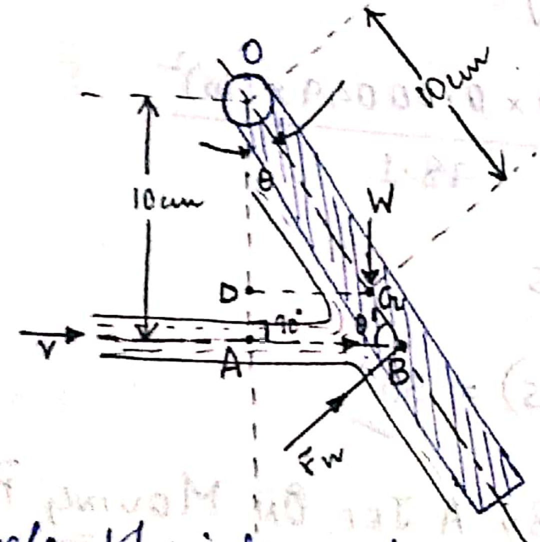
The force P is acting at a distance of 20 cm from the hinge.

∴ Taking moments about the hinge,

$$F \times 10 = P \times 20$$

$$\Rightarrow P = 24.5\,N.$$

(iii) When the plate is allowed to deflect freely about the hinge, then the inclination of the plate with vertical is θ.

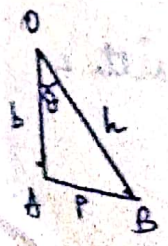


The angle b/w jet and plate is $\theta' = (90° - \theta)$

∴ Force exerted by water normal to the plate is :-

$$F_n = \rho a V^2 \sin\theta' = \rho a V^2 \sin(90° - \theta)$$

$$F_n = \rho a V^2 \cos\theta$$

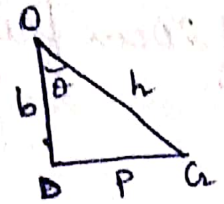


$$\cos\theta = \frac{b}{h} = \frac{OA}{OB} \Rightarrow OB = \frac{OA}{\cos\theta} = \frac{10}{\cos\theta}$$

$\therefore$ Moment of force $F_n = F_n \times OB = \rho a V^2 \cos\theta \times \frac{10}{\cos\theta}$

$= 10 \rho a V^2$

Also, Moment of force due to $W = W \times DG$



$\sin\theta = \frac{P}{h} = \frac{DG}{OG} \Rightarrow \sin\theta = \frac{DG}{10}$

$\Rightarrow DG = 10 \sin\theta$.

$\therefore$ Taking moments about hinge, we get:-

$F_n \times OB = W \times DG$

$\Rightarrow \rho a V^2 = W \sin\theta$

$\Rightarrow \sin\theta = \frac{\rho a V^2}{W}$

$\sin\theta = \frac{1000 \times 0.00049 \times (10)^2}{98.1}$

$\Rightarrow \sin\theta = 0.5$

$\Rightarrow \theta = \sin^{-1}(0.5) = 30^\circ$

* FORCE EXERTED BY A JET ON MOVING PLATES:-

(1) FORCE ON FLAT VERTICAL PLATE MOVING IN THE DIRECTION OF JET:

The figure given below shows a jet of water striking a flat vertical plate moving with a uniform velocity away from the jet.

[Uniform Velocity: Travelling at a constant speed along a particular direction].

Let V = Velocity of the jet (absolute)

[* ABSOLUTE VELOCITY:-

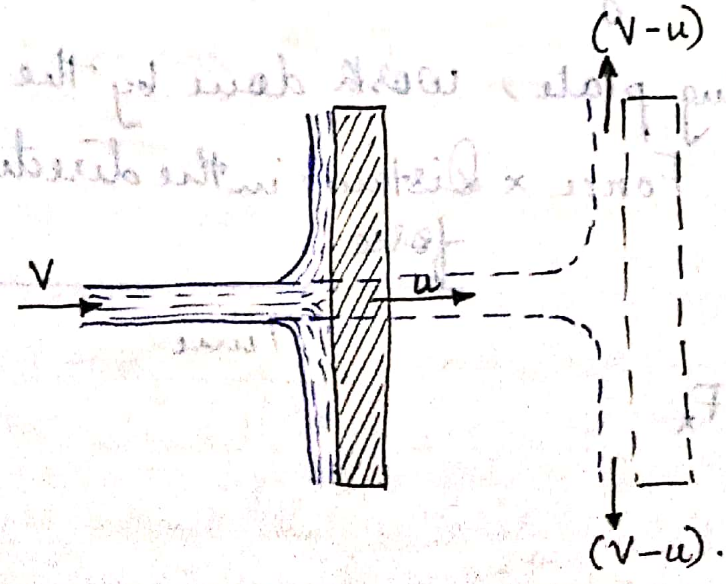
Absolute velocity is the velocity of an object w.r.t a fixed point (stationary)

Eg:- A passenger standing still at a point on a platform, and a train is moving. Here, the velocity of train w.r.t the still passenger is the absolute velocity of the train

* RELATIVE VELOCITY:-

It is the velocity of an object w.r.t a moving (moving) point.

Eg:- Two trains 1 and 2 are moving. The velocity of 1st train w.r.t 2nd train is the relative velocity of train 1 w.r.t train 2]



a = Area of c/s of the jet.
u = Velocity of the flat plate.

In this case, the jet does not strike the plate with a velocity V, but it strikes with a relative velocity which is equal to the absolute velocity of jet of water minus the velocity of the plate.

Hence,

Relative velocity of the jet w.r.t the plate $= (V-u)$

∴ Force exerted by the jet on the moving plate in the direction of the jet,

$$F_x = \frac{\text{Mass}}{\text{sec}} \times [\text{I.V with which water strikes} - \text{Final Velocity}]$$

$$= \rho a (V-u) [(V-u) - 0]$$

$$F_x = \rho a (V-u)^2$$

* For a stationary plate, work done by the jet on the plate is zero.

For a moving plate, work done by the jet on the plate $= \dfrac{\text{Force} \times \text{Distance in the direction of force}}{\text{Time}}$

$= F_x$

[* WORK DONE :-

Work results when a force acts upon an object to cause a (motion or displacement) or in some instances, hinders the motion or displacement.

These 3 variables are of equal importance in the defination of work done.

$\therefore$ Work = Force x Displacement x $\cos\theta$]

Here,

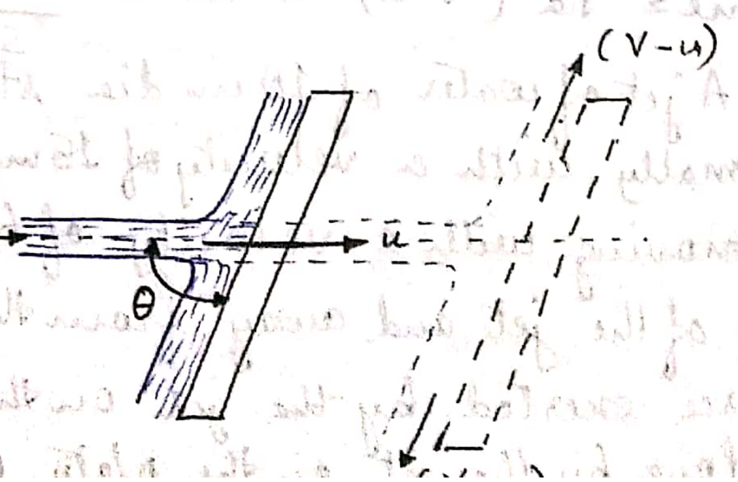
Work done = Force x Distance travelled by the plate in the direction of force

$= F_x \times u$

$$W = \rho a (V-u)^2 \times u$$

S.I unit of work done is $\frac{N.m}{s}$ equal to watt (W)

(2) FORCE ON THE INCLINED PLATE MOVING IN THE DIRECTION OF THE JET ::



Let a jet of water strikes an inclined plate, which is moving with a uniform velocity in the direction of the jet as shown in the figure.

Let V = Absolute velocity of jet of water

u = Velocity of the plate in the direction of jet.

a = c/s area of jet

θ = Angle b/w jet & plate.

Here,

$$F_n = \rho a(V-u)\left[(V-u)\sin\theta - 0\right]$$

$$F_n = \rho a (V-u)^2 \sin\theta$$

$$F_x = F_n \sin\theta = \rho a (V-u)^2 \sin^2\theta$$

$$F_y = F_n \cos\theta = \rho a (V-u)^2 \sin\theta \cos\theta$$

Work done = $F_x \times$ Distance travelled by the plate in the direction of x.

$$= \rho a (V-u)^2 \sin^2\theta \times u$$

Work done = $\rho a (V-u)^2 u \sin^2\theta$ $\frac{Nm}{s}$ or Watt.

Problem :- A jet of water of 10 cm dia strikes a flat plate normally with a velocity of 15 m/sec. The plate is moving with a velocity of 6 m/sec in the direction of the jet and away from the jet. Find:

(i) The force exerted by the jet on the plate.

(ii) Work done by the jet on the plate per second.

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(iii) ~~Power of the jet~~
(iv) Efficiency of the jet.

Sol^n:- Given :-

Dia of jet, $d = 10$ cm $= 0.1$ m.

Area, $a = \frac{\pi}{4} d^2 = \frac{\pi}{4}(0.1)^2 = 0.007854 \, m^2$

Velocity of jet, $V = 15$ m/sec.

Velocity of plate, $u = 6$ m/sec.

(i) The force exerted by the jet on a moving flat vertical plate is:-

$$F_x = \rho a (V-u)^2$$

$$F_x = 636.17 \text{ N}.$$

(ii) Work done by the jet on the plate per second

$$= F_x \times u$$

$$= 636.17 \times 6 \ \frac{Nm}{s} \text{ or Watt}$$

$$= 3817.02 \text{ Nm/s}$$

(iii) Efficiency of the jet:-

$$\eta = \frac{\text{Output of the jet per second}}{\text{Input of the jet per second}}$$

Here, Output of the jet /sec

= Work done by the jet per second

= 3817.02 Nm/sec.

Input of the jet /sec = Kinetic energy of the jet /sec.

$$= \frac{1}{2}(mV^2) \text{ /sec}$$

$$= \frac{1}{2} \times \left(\frac{mass}{sec}\right) \times V^2$$

$$= \frac{1}{2} (\rho a V) \times V^2 = \frac{1}{2} \rho a V^3$$

$$= \frac{1}{2} \times 1000 \times 0.007854 \times (15)^3$$

$$= 13253.6 \ Nm/sec.$$

$$\therefore \eta \text{ of the jet} = \frac{3817.02}{13253.6}$$

$$= 0.288 = 28.8\%.$$

Problem:- A 7.5 cm diameter jet having a velocity of 30 m/sec strikes a flat plate, the normal of which is inclined at 45° to the axis of the jet. Find the normal pressure on the plate:

(i) When the plate is stationary.

(ii) When the plate is moving with a velocity of 15 m/sec and away from the jet.

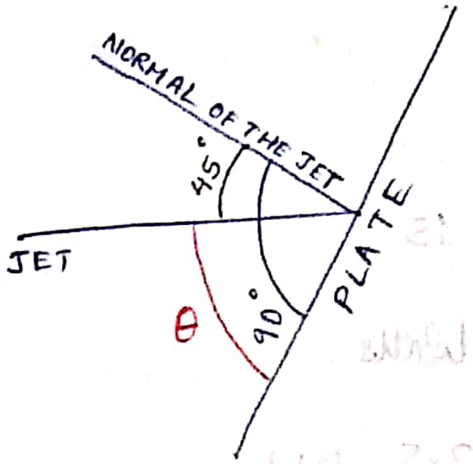
Also determine the power and efficiency of the jet when the plate is moving?

Solution:- Given,

Diameter of the jet, $d = 7.5 \ cm = 0.075 \ m$.

$\therefore$ Area, $a = \frac{\pi}{4} d^2 = 0.004417 \ m^2$

The pictorial presentation of the problem in the form of a line diagram is given below:-



Here, θ = Angle b/w jet and plate $= 90° - 45° = 45°$

Velocity of the jet, $V = 30$ m/sec.

(i) When the plate is stationary :-

The normal force on the plate is :-

$$F_n = \rho a V^2 \sin\theta$$

$$= 1000 \times 0.004417 \times (30)^2 \times \sin 45°$$

$$\therefore F_n = 2810.96 \text{ N.}$$

(ii) When the plate is moving with a velocity of 15 m/sec and away from the jet, then the normal force on the plate is :-

$$F_n = \rho a (V-u)^2 \sin\theta$$

Here, $u = 15$ m/sec

$$\therefore F_n = 1000 \times 0.004417 \times (30-15)^2 \sin 45°$$

$$= 702.74 \text{ N}$$

* Power in kW $= \dfrac{\text{Work done per second}}{1000}$

$\therefore$ Work done per sec = Force in the direction of jet × Distance travelled by the plate in the direction of jet

$= F_x \times u$

$= F_n \sin\theta \times u$

$= 702.74 \times \sin 45^\circ \times 15$

$= 7453.5$ Nm/s or Watts.

$\therefore$ Power in kW $= \frac{7453.5}{1000}$ kW

$= 7.453$ kW

* Efficiency of the jet $= \frac{O/P}{I/P}$

$= \frac{\text{Work done per sec}}{\text{K.E of the jet per sec}}$

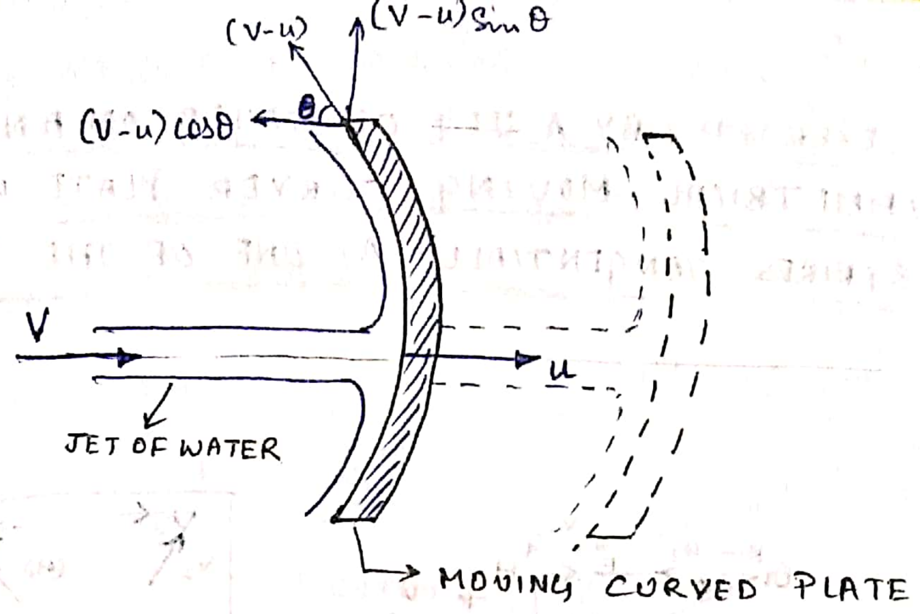
$= \frac{7453.5}{\frac{1}{2}(\rho a V) \times V^2}$

$= \frac{7453.5}{\frac{1}{2} \times 1000 \times 0.00447 \times (30)^3}$

$\eta = 0.1249 \approx 0.125 = 12.5\%$

(3) FORCE ON THE CURVED PLATE WHEN THE PLATE IS MOVING IN THE DIRECTION OF JET :

Let a jet of water strike a curved plate at the centre of the plate which is moving with a uniform velocity in the direction of the jet.



Let V = Absolute velocity of the jet
a = Area of the jet
u = Uniform velocity of the plate in the direction of jet.

∴ Relative Velocity = (V-u)

Hence, force exerted by the jet of water on the curved plate in the direction of the jet,

$$F_x = \frac{\text{Mass}}{\text{sec}} \left[I.V - F.V \right]$$

$$F_x = \rho a (V-u) \left[(V-u) + (V-u)\cos\theta \right]$$

$$F_x = \rho a (V-u)^2 (1+\cos\theta)$$

and, work done by the jet on the plate per second

$$= F_x \times u$$

$$= \rho a (V-u)^2 (1+\cos\theta) u$$

$$= \rho a (V-u)^2 u (1+\cos\theta)$$

* TANGENTIAL STRIKING WHEN PLATE IS SYMMETRICAL

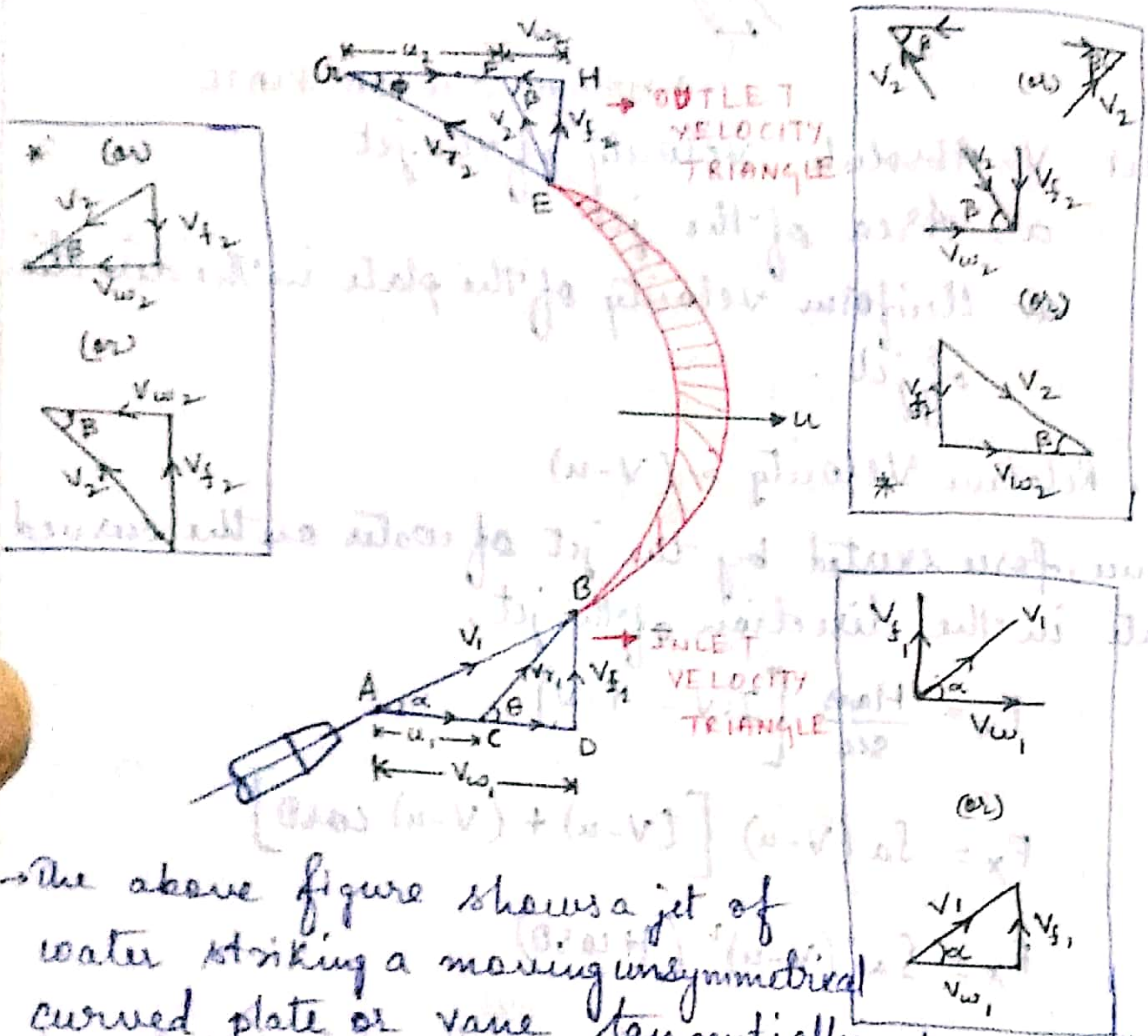
$F_x = 2\rho a (V-u)^2 \cos\theta$

$F_y = 0.$

Work done $= F_x \times u$

$W.D = 2\rho a (V-u)^2 u \cos\theta$ watts

* FORCE EXERTED BY A JET OF WATER ON AN UNSYMMETRICAL MOVING CURVED PLATE WHEN JET STRIKES TANGENTIALLY AT ONE OF THE TIPS:



→ The above figure shows a jet of water striking a moving unsymmetrical curved plate or vane tangentially, at one of its tips.

→ As the jet of water strikes the plate tangentially which is considered to be smooth, so the loss of energy due to the impact of the jet will be zero.

→ Since, the plate is moving, therefore the velocity with which the jet of water strikes the plate is equal to the relative velocity of the jet w.r.t the plate.

* Since velocity is a vector quantity, therefore the relative velocity at inlet is equal to the vector difference of the velocity of jet and velocity of the plate at inlet.

Let, V_1 = Velocity of the jet at inlet of the plate.
V_2 = Velocity of the jet at outlet of the plate or leaving the plate (vane)

u_1 = Velocity of the plate at inlet
u_2 = Velocity of the plate at outlet

V_{r_1} = Relative velocity of the jet with respect to the plate at inlet
V_{r_2} = Relative velocity of the jet with respect to the plate at outlet

α = Angle b/w the inlet velocity of the jet and direction of motion ~~inlet velocity~~ of the plate at inlet, called as guide blade angle.

β = Angle b/w the outlet velocity of the jet and direction of motion of the plate at outlet

θ = Angle made by V_{r_1} with the direction of motion of plate at inlet, called vane angle at inlet.

ϕ = Angle made by V_{r_2} with the direction of motion of plate at outlet, called vane angle at outlet

V_{w_1} = The component of inlet velocity of the jet V_1 in the direction of motion of the plate, Known as whirl velocity at inlet.

V_{w_2} = The component of outlet velocity of the jet V_2 in the direction of motion of the plate, Known as whirl velocity at outlet

V_{f_1} = The component of inlet velocity of the jet V_1, normal to the direction of the motion of the plate, Known as flow velocity at inlet

V_{f_2} = The component of outlet velocity of the jet V_2, normal to the direction of the motion of the plate, Known as flow velocity at outlet

If the plate is smooth and there is no loss of energy,

So, $V_{r_1} = V_{r_2}$

Also, since the plate is moving with a uniform velocity,

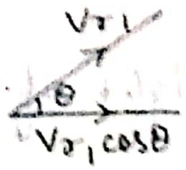
So, $u_1 = u_2 = u$, which is the velocity of the plate in the direction of motion.

∴ Force exerted by the jet of water in the direction of motion is:—

$F_x = \frac{\text{Mass}}{\text{sec}}$ [I.V with which jet strikes the plate in the direction of motion − F.V with which jet leaves the plate in the dire ction of motion]

$\Rightarrow F_x = \rho a V_{r_1} [I.V_x - F.V_x]$

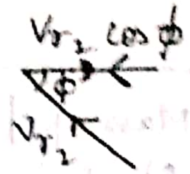
$\underline{I.V_x}$



From figure,

$V_{r_1}\cos\theta = V_{w_1} - u_1$

$\underline{F.V_x}$



$= -V_{r_2}\cos\phi = -(u_2 + V_{w_2})$

$\therefore F_x = \rho a V_{r_1} [(V_{w_1} - u_1) - \{-(u_2 + V_{w_2})\}]$

$\Rightarrow F_x = \rho a V_{r_1} [V_{w_1} - u_1 + u_2 + V_{w_2}]$

$\because u_1 = u_2$

$\therefore F_x = \rho a V_{r_1} [V_{w_1} + V_{w_2}]$

This eqn holds good, if $\beta < 90°$ (acute angle)

If $\beta = 90°$, $V_{w_2} = 0$

$\therefore F_x = \rho a V_{r_1} (V_{w_1})$

If $\beta > 90°$ (obtuse angle), then :-

$F_x = \rho a V_{r_1} [V_{w_1} - V_{w_2}]$

Thus in general,

$F_x = \rho a V_{r_1} [V_{w_1} \pm V_{w_2}]$

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$\therefore$ Work done per sec on the plate by jet

$$= F_x \times u$$

$$= \rho a V_{r_1} [V_{w_1} \pm V_{w_2}] u \quad Nm/s$$

$\therefore$ Work done per sec per unit weight of the fluid striking per sec

$$= \frac{F_x \times u}{\text{Accl}^n \text{ due to gravity} \times \text{Mass of fluid striking per sec}}$$

$$= \frac{\rho a V_{r_1} [V_{w_1} \pm V_{w_2}] u}{\rho a V_{r_1} \times g} \quad \frac{Nm/s}{N/s}$$

$$= \frac{1}{g} [V_{w_1} \pm V_{w_2}] \times u \quad Nm/N$$

$\therefore$ Work done per sec per unit mass of the fluid striking per sec.

$$= \frac{F_x \times u}{\text{Mass per sec}}$$

$$= \frac{\rho a V_{r_1} [V_{w_1} \pm V_{w_2}] u}{\rho a V_{r_1}} \quad \frac{Nm/s}{Kg/s}$$

$$= (V_{w_1} \pm V_{w_2}) u \quad Nm/Kg.$$

$\therefore$ Efficiency of jet, $\eta = \frac{O/P}{I/P}$

$$= \frac{\text{Work done per second on the plate}}{\text{Initial K.E of the jet per second.}}$$

$$\left[K.E = \frac{1}{2} m V^2 = Kg \times \frac{m^2}{s^2} \right.$$

$$\therefore K.E \text{ per sec} = \frac{Kgm^2}{s^2} / s = \frac{Kgm^2}{s^2} \times \frac{1}{s}$$

$$\left. = \frac{Kgm}{s^2} \times \frac{m}{s} = \frac{Nm}{s} \quad (\because 1\ Kgm/s^2 = 1N) \right]$$

$$\therefore \eta = \frac{Nm/s}{Nm/s} = \text{unitless}$$

$$= \frac{\rho a V_{r_1} (V_{w_1} \pm V_{w_2}) u}{\left(\frac{1}{2} \times m \times V^2\right) \text{per sec}}$$

$$= \frac{\rho a V_{r_1} (V_{w_1} \pm V_{w_2}) u}{\frac{1}{2} \times V^2 \times (\text{mass per sec})}$$

$$\eta = \frac{\rho a V_{r_1} (V_{w_1} \pm V_{w_2}) u}{\frac{1}{2} (\rho a V_1) \times V_1^2}$$

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Problem:- (a) A stationary vane having an inlet angle of zero degree and an outlet angle of 25°, receives water at a velocity of 50 m/s. Determine the components of force acting on it in the direction of the jet velocity and normal to it. Also find the resultant force in magnitude and direction per unit weight of the flow per second

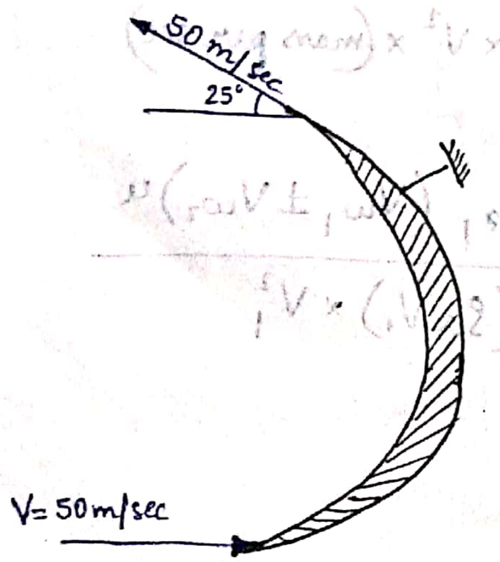
(b) If the vane stated above is moving with a velocity of 20 m/sec in the direction of the jet, calculate the force components in the direction of the vane velocity and across it, also the resultant force in magnitude and direction. Calculate the work done and power developed per unit weight of the flow per second.

Soln:- (a) STATIONARY CONDITION:→

Velocity of jet, V = 50 m/sec.

Angle of inlet = 0 (zero)

Angle at outlet = 25°.



For the stationary vane, the force in the direction of the jet is :-

$$F_x = \frac{\text{mass}}{\text{sec}}\left[I.V_x - F.V_x\right]$$

$I.V_x = V = 50 \, m/sec$

$F.V_x = -V\cos\theta = -50\cos 25^\circ = -45.315 \, m/sec.$

$\cancel{\frac{\text{mass}}{\text{sec}} = \rho a V = 1000 \times}$

$\therefore$ Force in the direction of jet per unit weight of water per sec

$$= \frac{F_x}{\text{Unit wt. of water per sec}}$$

$$= \frac{F_x}{\frac{\text{Mass} \times \text{accl}^n \text{ due to gravity}}{\text{sec}}}$$

$$= \frac{F_x}{\frac{\text{Mass}}{\text{sec}} \times g}$$

$$= \frac{\frac{\text{mass}}{\text{sec}}\left(I.V_x - F.V_x\right)}{\frac{\text{mass}}{\text{sec}} \times g}$$

$$= \frac{50 - (-50\cos 25^\circ)}{9.81}$$

* PELTON WHEEL (OR) PELTON TURBINE :→

→ The pelton wheel or Pelton turbine is a tangential flow impulse turbine.

→ The water strikes the bucket (vane) along the tangent of the runner.

→ The energy available at the inlet of the turbine is only kinetic energy.

→ This turbine is used for high heads and is named after L.A. Pelton, an American Engineer.

*

→ The layout of a hydroelectric power plant in which the turbine shown is a Pelton wheel turbine.

→ The water from the dam or reservoir flows through the penstock, at the outlet of which a nozzle is fitted.

→ This nozzle increases the kinetic energy of the water coming out from the penstock.

→ At the outlet of the nozzle, the water comes out in the form of a jet and strikes the bucket (vane) of the runner.

→ The main parts of the Pelton turbine are:—

1. Nozzle and flow regulating arrangement (spear)
2. Runner and buckets

3. Casing 4. Breaking jet.

→ Pelton wheel is a high head turbine

$$N_s \propto \frac{1}{H_{net}^{5/4}}.$$

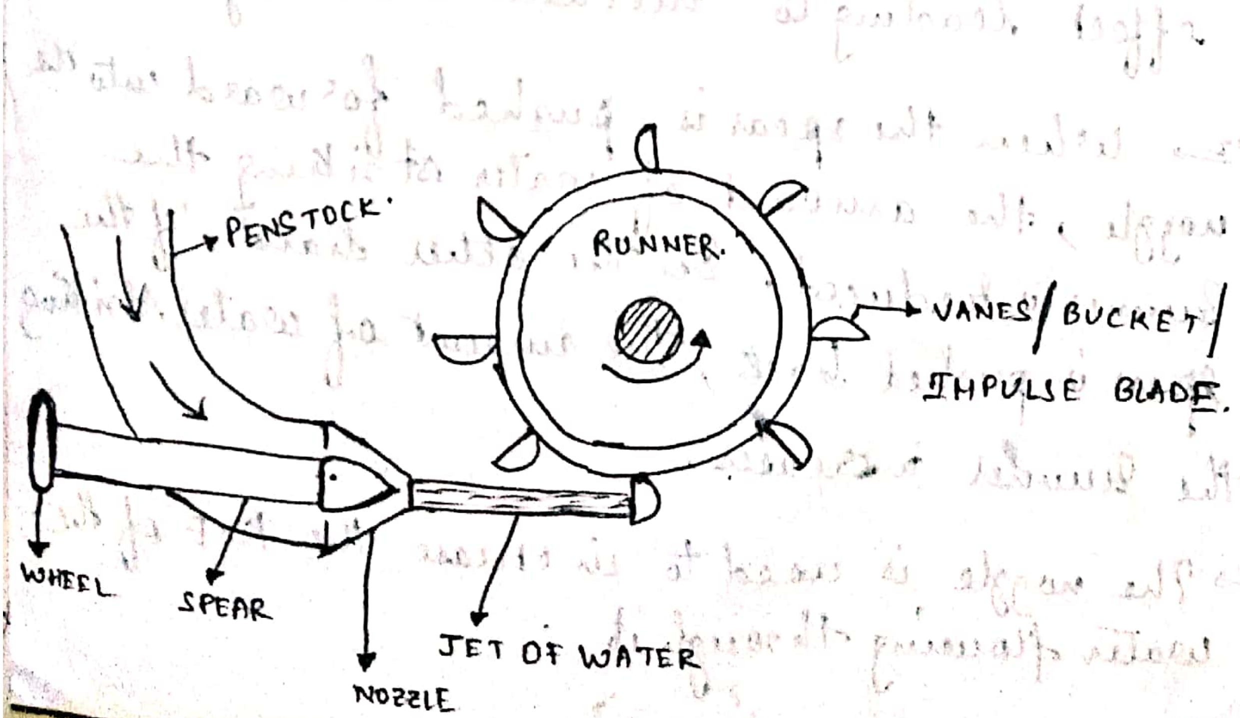
→ Therefore, pelton wheel is called also as low specific speed (N_s) turbine.

→ Pelton wheel is called also as low discharge turbine i.e, it is used when the discharge requirement is low.

→ Since the energy in pelton turbine is only kinetic energy, so there is no pressure difference at inlet and outlet. Throughout the pressure is atmospheric

$$\therefore P_{inlet} = P_{outlet} = P_{atm}.$$

Therefore, there is no cavitation in pelton wheel turbine.



1. Nozzle and Flow Regulating Arrangement :→

*1 → Due to the variation in load of the generator to which a turbine is coupled, we need to vary the rotating speed of the turbine. This can be done by varying the discharge or flow rate of the jet, which is controlled or regulated by providing a spear in the nozzle.

→ The spear is a conical needle which is operated either by a hand wheel or automatically in an axial direction depending upon the size of the unit.

*2 → The spear is not completely dragged inside to close the nozzle. If done so, then it may lead to hammering effect, since the water coming out high pressure through the penstock will strike the closed nozzle and create an hammering effect leading to mechanical damage.

*3 → When the spear is pushed forward into the nozzle, the amount of water striking the runner is reduced. On the other hand, if the spear is pushed back, the amount of water striking the runner increases.

*4 → The nozzle is used to increase the K.E of the water flowing through it.

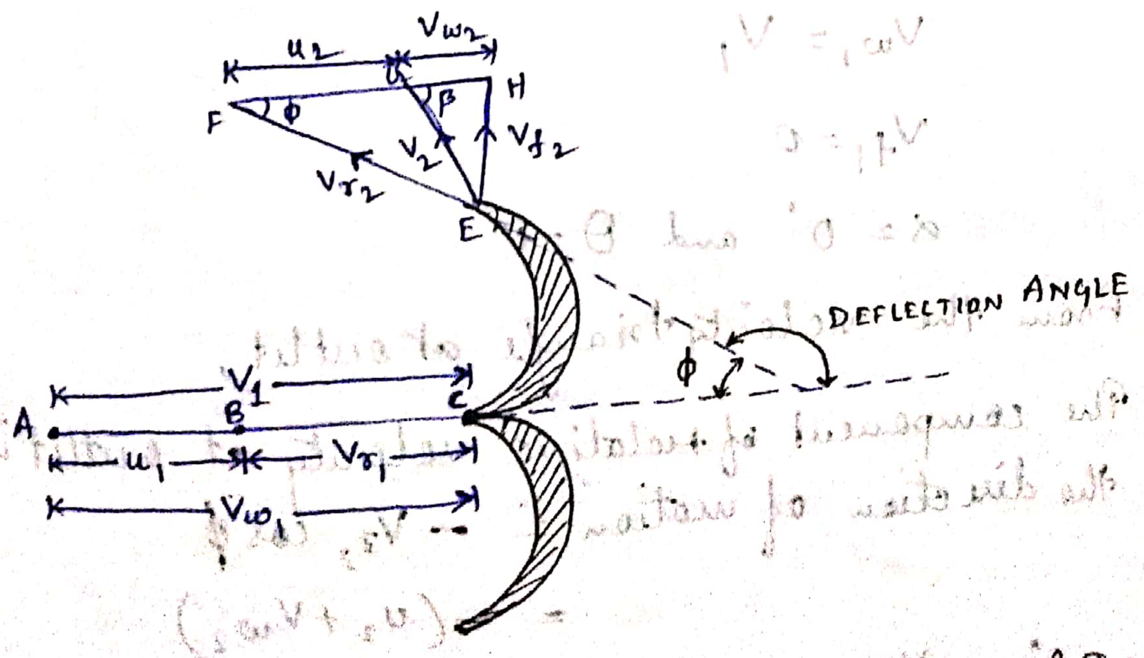
2. Runner with Buckets :→

* VELOCITY TRIANGLE & WORK DONE FOR PELTON WHEEL:→

→ The jet of water from the nozzle strikes the bucket at the splitter, which splits up the jet into two parts. These parts of the jet glides over the inner surfaces of the two symmetrical hemispherical cups of the bucket and comes out at the outer edges.

→ Since the bucket is symmetrically designed, hence the analysis of the velocity triangle holds good for any one of the cups.

→ The inlet velocity triangle is drawn at the splitter and the outlet velocity triangle is drawn at any of the outer edges of the bucket.



P.T.O

Let H_{net} = Net head acting on the Pelton wheel

$= H_g - h_f$.

where,

H_g = Gross head

$$h_f = \frac{4fLV^2}{2gD^*}$$

Here,

L = Sectional length of the penstock.

V = Velocity of the jet

D^* = Diameter of the penstock.

V_1 = Velocity of jet at inlet

= Theoritical velocity $= \sqrt{2g H_{net}}$

$$u_1 = u_2 = u = \frac{\pi DN}{60} \text{ (rpm)}$$

The velocity triangle at inlet will be a straight line, where

$$V_{r_1} = V_1 - u_1 = V_1 - u.$$

$$V_{w_1} = V_1$$

$$V_{f_1} = 0$$

$\alpha = 0°$ and $\theta = 0°$.

From the velocity triangle at outlet,

The component of relative velocity at outlet in the direction of motion $= -V_{r_2} \cos\phi$

$$= -(u_2 + V_{w_2})$$

$$\therefore -V_{r_2} \cos\phi = -(u_2 + V_{w_2})$$

$\Rightarrow V_{r_2} \cos\phi - u_2 = V_{w_2}$

$\therefore V_{w_2} = V_{r_2} \cos\phi - u_2$

Since there is no energy loss,

$V_{r_1} = V_{r_2}$ $\qquad \therefore V_{w_2} = V_{r_1} \cos\phi - u$

The force exerted by the jet of water in the direction of motion is given as:-

$$\cancel{F_x = \rho a V_1 [V_{w_1} + V_{w_2}]}$$

$$\cancel{F_x = \rho a V_1 [I.V_x - F.V_x]}$$

F_x = Mass of water coming out of the nozzle per second $\cdot [I.V_x - F.V_x]$.

$$= \rho a V_1 . [V_{r_1} - (-V_{r_2} \cos\phi)]$$

$$= \rho a V_1 [(V_1 - u_1) + (V_{r_2} \cos\phi)]$$

$$= \rho a V_1 [V_1 - u_1 + u_2 + V_{w_2}]$$

$$= \rho a V_1 [V_1 - \cancel{u_1} + \cancel{u_2} + V_{w_2}] \quad (\because u_1 = u_2 = u)$$

$$= \rho a V_1 [V_1 + V_{w_2}].$$

$$= \rho a V_1 [V_{w_1} + V_{w_2}] \quad (\because V_1 = V_{w_1}).$$

Now,

Work done by the jet on the runner per second

$$= F_x \times u.$$

$$= \rho a V_1 \left[V_{w_1} + V_{w_2} \right] u \quad Nm/sec.$$

$\therefore$ Power given to the runner by the jet is $= \dfrac{\rho a V_1 [V_{w_1} + V_{w_2}] u}{1000}$

* Work done by the jet on the runner per second per unit weight of water striking per second

$$= \frac{\rho a V_1 \left[V_{w_1} + V_{w_2} \right] u}{\text{Accl}^n \text{ due to gravity} \times \text{Mass of water coming out of the nozzle per second}} \quad \frac{Nm/s}{\frac{m}{s^2} \times \frac{kg}{\cancel{m^3}} \times \cancel{m^2} \times \frac{\cancel{m}}{sec}}$$

$$= \frac{\cancel{\rho a V_1} \left[V_{w_1} + V_{w_2} \right] u}{g \times \cancel{\rho a V_1}} \quad \frac{Nm/s}{\frac{Kgm}{s^2} \times \frac{1}{s}}$$

$$= \frac{\left[V_{w_1} + V_{w_2} \right] u}{g} \quad \frac{Nm/s}{N/s}$$

$$= \frac{1}{g} \left[V_{w_1} + V_{w_2} \right] u \quad \frac{Nm}{N}.$$

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Since pelton wheel turbine is an impulse turbine, therefore the jet possesses only kinetic energy, which is equal to $\frac{1}{2} m V^2$.

$\therefore$ K.E. of the jet per second $= \frac{1}{2} \times (\text{mass per sec}) \times V_1^2$.

$$= \frac{1}{2} \times (\rho a V_1) \times V_1^2$$

W

$\therefore$ Hydraulic efficiency, $\eta_h = \dfrac{\text{Work done per second}}{\text{K.E. per second}}$

$$\eta_h = \frac{\rho a V_1 [V_{w_1} \pm V_{w_2}] u}{\frac{1}{2} (\rho a V_1) \times V_1^2}$$

$$\eta_h = \frac{2 [V_{w_1} + V_{w_2}] u}{V_1^2}$$

$$= \frac{2 [V_1 + V_{r_2} \cos\phi - u_2] u}{V_1^2}$$

$$= \frac{2 [V_1 + V_{r_1} \cos\phi - u] u}{V_1^2}$$

$$= \frac{2 [V_1 + (V_1 - u) \cos\phi - u] u}{V_1^2}$$

$$= \frac{2 [(V_1 - u) + (V_1 - u) \cos\phi] u}{V_1^2}$$

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$$= \frac{2(V_1 - u)\left[1 + \cos\phi\right]u}{V_1^2}$$

For a given inlet jet velocity of V_1, the efficiency will be maximum, when:→

$$\frac{d}{du}(\eta_h) = 0.$$

$$\Rightarrow \frac{d}{du}\left[\frac{2u(V_1 - u)(1+\cos\phi)}{V_1^2}\right] = 0$$

$$\Rightarrow \left(\frac{1+\cos\phi}{V_1^2}\right)\frac{d}{du}(2uV_1 - 2u^2) = 0.$$

$$\Rightarrow \frac{d}{du}(2uV_1 - 2u^2) = 0.$$

$$\Rightarrow 2V_1 - 4u = 0.$$

$$\Rightarrow u = \frac{V_1}{2}.$$

→ It means that hydraulic efficiency of a Pelton wheel will be maximum when the velocity of the wheel is half the velocity of jet of water at inlet.

→ The expression for maximum efficiency will be obtained by substituting the value of $u = \frac{V_1}{2}$

in the equation of η_h.

$$\therefore \eta_{h_{max}} = \frac{2\left(V_1 - \frac{V_1}{2}\right)(1+\cos\phi) \times \frac{V_1}{2}}{V_1^2}$$

$$= \frac{2 \times \frac{V_1}{2}(1+\cos\phi)\frac{V_1}{2}}{V_1^2}$$

$$= \frac{(1+\cos\phi)}{2}$$

In some cases, if $V_{r_1} \neq V_{r_2}$, then

① $$\eta_{h_{max}} = \frac{(1+K\cos\phi)}{2}$$

Here, $$K = \frac{V_{r_2}}{V_{r_1}}$$

② If $K = 1$ and $\phi = 15^\circ$, then

$$\boxed{\eta_{h_{max}} = 98.6\%}$$

* Points to be remembered :-

① Speed ratio (ϕ or K_u) $= \frac{u}{V_1} = \frac{u}{\sqrt{2gH_{net}}}$

$\Rightarrow$ Velocity of wheel $(u) = K_u \sqrt{2gH_{net}}$

② If no angle of deflection is given, then the angle of deflection of the jet through buckets is assumed as 165°.

③ The mean diameter or the pitch diameter D of the Pelton wheel is given by :-

$$u = \frac{\pi D N}{60}$$

$$\Rightarrow D = \frac{60 u}{\pi N}$$

④ Jet Ratio :- It is defined as the ratio of the pitch diameter (D) of the Pelton wheel to the diameter of the jet (d). It is denoted by 'm'.

$$m = \frac{D}{d} = 12 \text{ in most of the cases.}$$

⑤ No. of buckets on a runner is given by :-

$$Z = 15 + \frac{D}{2d} = 15 + \frac{m}{2} = 15 + 0.5m$$

(6) Number of jets = $\dfrac{\text{Total discharge from Penstock}}{\text{Discharge through a single jet}}$

→ generally, if a pelton wheel is mounted on a horizontal shaft, then we can use a max. of 2 jets.

→ If a pelton wheel is mounted on a vertical shaft, then we can use a max. of 6 jets.

(7). The theoritical velocity of the jet is :-

$$V_1 = C_V \sqrt{2gh}$$

Here, C_V = Coefficient of velocity = 0.98 to 0.99

MODULE IV

Classification of Hydraulic turbines:

1.) According to the type of energy at Inlet:

a) Impulse turbine: If at the inlet of the turbine, the energy available is only kinetic energy, the turbine is known as impulse turbine.

b) Reaction turbine: If at the inlet of the turbine, the water possesses kinetic energy as well as pressure energy, the turbine is known as reaction turbine.

2.) According to the direction of flow through runner:

a) Tangential flow turbine: Water flows along the tangent of the runner.

b) Radial flow turbine: Water flows in the radial direction through the runner.

c) Axial flow turbine: Water flows through the runner along the direction parallel to the axis of rotation of the runner.

d) Mixed flow turbine: Water flows through the runner in the radial direction but leaves in the direction parallel to axis of rotation of the runner.

3.) According to the head at the Inlet of turbine:

a) High head turbine: Above 250m Ex: Pelton Turbine

b) Medium head turbine: 60m to 250m Ex: Francis Turbine

c) Low head turbine: Below 60m Ex: Kaplan & propeller Turbine

4) According to the Specific Speed of the turbine: (M.K.S Units)

a) Low Specific Speed turbine: 10 to 35 Ex: Pelton Turbine

b) Medium Specific Speed turbine: 35 to 300 Ex: Francis Turbine

c) High Specific Speed turbine: 300 to 1000 Ex: Kaplan & Propeller Turbine.

Pelton Wheel (or Turbine)

The Pelton wheel (or) Pelton turbine is a tangential flow impulse turbine. The water strikes the bucket along the tangent of the runner. The energy available at the inlet of the turbine is only kinetic energy. The pressure at the inlet & outlet of the turbine is atmospheric. This turbine is used for high heads & is named after L.A. Pelton, an American Engineer.

The main parts of the Pelton turbine are:

1.) Nozzle & flow Regulating arrangement (Spear)
2.) Runner and buckets
3.) Casing and
4.) Breaking jet.

1.) **Nozzle & Flow Regulating Arrangement:**

The amount of water striking the buckets (vanes) of the runner is controlled by providing a spear in the nozzle as shown in Figure (1). The spear is a conical needle which is operated either by a hand wheel (or) automatically in an axial direction depending upon the size of the unit. When the spear is pushed forward into the nozzle the amount of water striking the runner is reduced. On the other hand, if the spear is pushed back, the amount of water striking the runner increases.

2) <u>Runner with Buckets:</u>

Fig (2) shows the runner of the Pelton wheel. It consists of a circular disc on the periphery of which a number of buckets evenly spaced are fixed. The shape of the buckets is of a double hemispherical cup (or) bowl. Each bucket is divided into 2 symmetrical parts by a dividing wall which is known as splitter.

The jet of water strikes on the splitter. The splitter divides the jet into 2 equal parts & the jet comes out at the outer edge of the bucket. The buckets are shaped in such a way that the jet gets deflected through 160° to 170°. The buckets are made of cast iron, cast steel bronze (or) stainless steel depending upon the head at the inlet of the turbine.

3) <u>Casing:</u> Fig (3) shows a Pelton turbine with a casing. The function of the casing is to prevent the ~~Spash~~ splashing of the water & to discharge water to tail race. It is made of cast iron (or) fabricated steel plates. The casing of the Pelton wheel does not perform any hydraulic function.

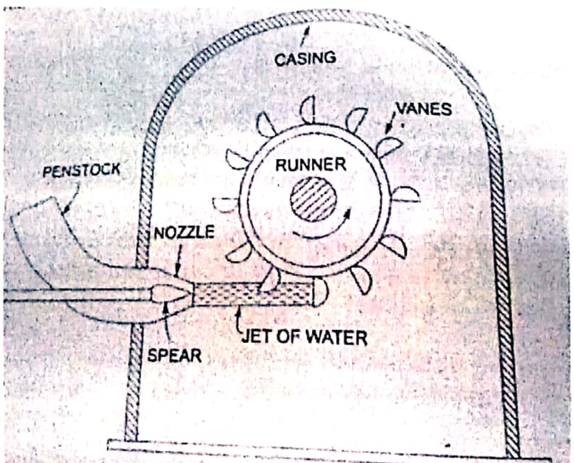

Fig. 18.4 Pelton turbine.

4) **Breaking Jet:**

When the nozzle is completely closed by moving the spear in the forward direction, the amount of water striking the runner reduces to zero. But the runner due to inertia goes on revolving for a long time. To stop the runner in a short time, a small nozzle is provided which directs the jet of water on the back of vanes. This jet of water is called breaking jet.

Velocity Triangles & Work done for Pelton Wheel:

Fig (4) shows the shape of the vanes (or) buckets of the Pelton wheel. The jet of water from the nozzle strikes the bucket at the splitter, which splits up the jet into 2 parts. These parts of the jet, glides over the inner surfaces & comes out at the outer edge. Fig (5) shows the section of the bucket at z-z. The splitter is the inlet tip & outer edge of the bucket is the outlet tip of the bucket. The inlet velocity triangle is drawn at the splitter & outlet velocity triangle is drawn at the outer edge of the bucket.

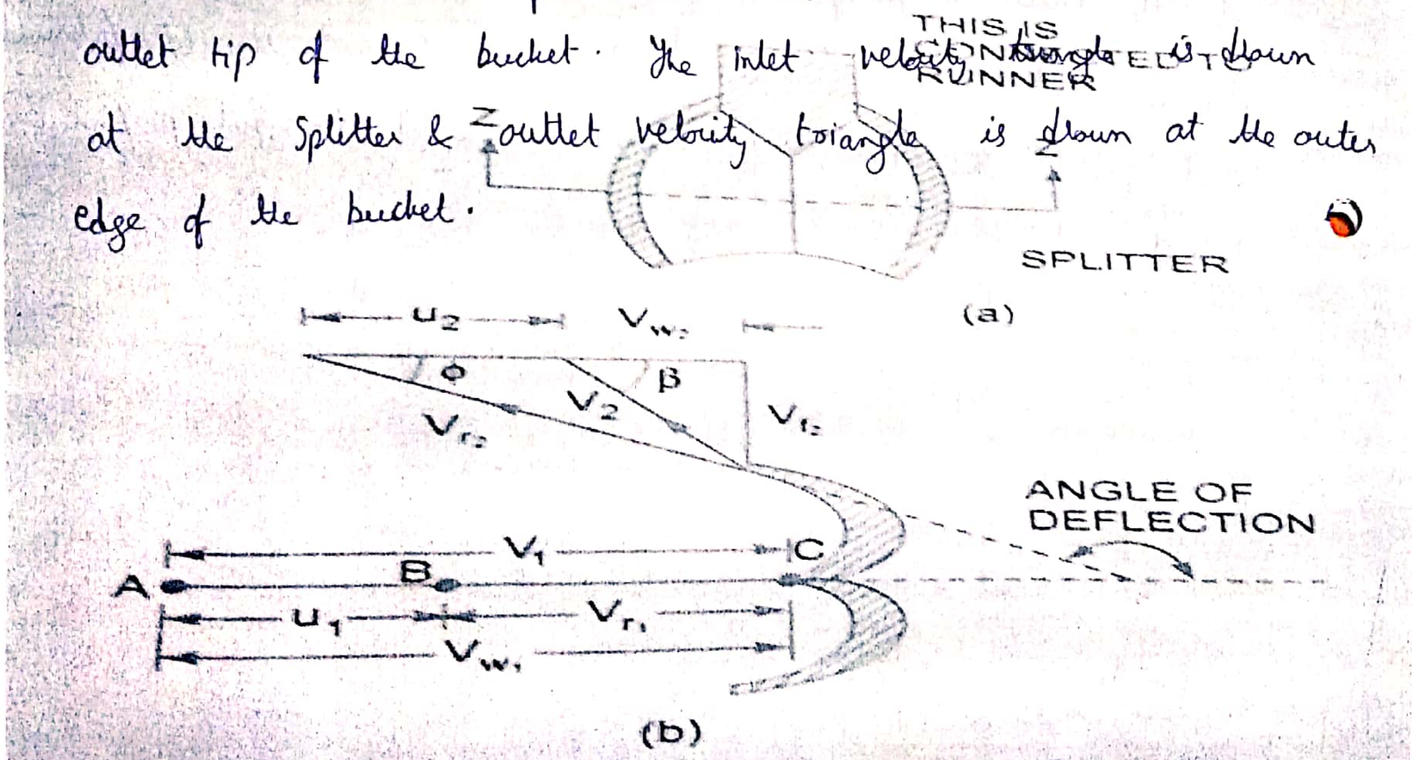

Fig. 18.5 *Shape of bucket.*

Let H = Net head acting on the Pelton wheel

$= H_g - h_f$

where H_g = Gross head & $h_f = \dfrac{4fLV^2}{D^* \times 2g}$

where D^* = Dia. of Penstock, N = Speed of the wheel in r.p.m

D = Dia. of the wheel, d = diameter of the jet

Then V_1 = Velocity of the jet at Inlet $= \sqrt{2gH}$ — (1)

$u = u_1 = u_2 = \dfrac{\pi D N}{60}$.

The velocity triangle at Inlet will be a straight line where

$V_{r_1} = V_1 - u_1 = V_1 - u$

$V_{w_1} = V_1$

$\alpha = 0°$ & $\theta = 0°$.

From the velocity triangle at Outlet, we have

$V_{r_2} = V_{r_1}$ & $V_{w_2} = V_{r_2}\cos\phi - u_2$

The force exerted by the jet of water in the direction of motion is given by eqn

$$F_x = \rho a V_1 [V_{w_1} + V_{w_2}] \quad \text{— (2)}$$

As the angle β is an acute angle, +ve sign should be taken. Also this is the case of series of vanes, the mass of water striking is $\rho a V_1$ & not $\rho a V_{r_1}$. In eqn (2), 'a' is the area of the jet which is given as

$$a = \text{Area of jet} = \frac{\pi}{4} d^2$$

Now, work done by the jet on the runner per second

$$= F_x \times u = \rho a V_1 [V_{w_1} + V_{w_2}] \times u \ \ Nm/s \quad \text{— (3)}$$

The

Power given to the runner by the jet

$$= \frac{\rho a V_1 [V_{w_1} + V_{w_2}] \times u}{1000} \text{ kW} \quad — (4)$$

Workdone/per Unit weight of water striking/s

$$= \frac{\rho a V_1 (V_{w_1} + V_{w_2}) \times u}{\text{Weight of water Striking/s}}$$

$$= \frac{\rho a V_1 [V_{w_1} + V_{w_2}] \times u}{\rho a V_1 \times g} = \frac{1}{g} [V_{w_1} + V_{w_2}] \times u \quad — (5)$$

The energy supplied to the jet ~~of~~ at Inlet is in the form of kinetic energy & is equal to $\frac{1}{2} m V^2$.

$\therefore$ K.E of jet per Second $= \frac{1}{2} (\rho a V_1) \times V_1^2$

$\therefore$ Hydraulic efficiency, $\eta_h = \frac{\text{Workdone per Second}}{\text{K.E of jet per Second}}$

$$= \frac{\rho a V_1 [V_{w_1} + V_{w_2}] \times u}{\frac{1}{2} (\rho a V_1) \times V_1^2}$$

$$\boxed{\therefore \eta_h = \frac{2 [V_{w_1} + V_{w_2}] \times u}{V_1^2}} \quad — (6)$$

Now $V_{w_1} = V_1$, $V_{r_1} = V_1 - u_1 = (V_1 - u)$

$\therefore V_{r_2} = (V_1 - u)$

& $V_{w_2} = V_{r_2} \cos\phi - u_2 = V_{r_2} \cos\phi - u = (V_1 - u) \cos\phi - u$

Substituting the Values of V_{w_1} & V_{w_2} in eqn (6)

$$\therefore \eta_h = \frac{2 [V_1 + (V_1 - u)\cos\phi - u] \times u}{V_1^2}$$

$$= \frac{2 [V_1 - u + (V_1 - u)\cos\phi] \times u}{V_1^2} = \frac{2 (V_1 - u)[1 + \cos\phi] \times u}{V_1^2} \quad — (7)$$

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The efficiency will be max. for a given value of V_1 when

$$\frac{d}{du}(\eta_h) = 0 \quad (or) \quad \frac{d}{du}\left[\frac{2u(V_1-u)(1+\cos\phi)}{V_1^2}\right] = 0$$

$$\Rightarrow \frac{(1+\cos\phi)}{V_1^2}\frac{d}{du}(2uV_1 - 2u^2) = 0$$

$$\Rightarrow \frac{d}{du}(2uV_1 - 2u^2) = 0 \qquad \left[\because \frac{1+\cos\phi}{V_1^2} \neq 0\right]$$

$$\Rightarrow 2V_1 - 4u = 0 \Rightarrow \boxed{u = \frac{V_1}{2}} \quad \text{— (8)}$$

Eqn (8), states that hydraulic efficiency of a Pelton wheel will be maximum when the velocity of the wheel is half of the velocity of the jet of water at Inlet. The expression for maximum efficiency will be obtained by substituting the value of $u = \frac{V_1}{2}$ in eqn (7)

$$\therefore \text{Max. } \eta_h = \frac{2\left(V_1 - \frac{V_1}{2}\right)(1+\cos\phi) \times \frac{V_1}{2}}{V_1^2}$$

$$= \frac{2 \times \frac{V_1}{2}(1+\cos\phi)\frac{V_1}{2}}{V_1^2}$$

$$\boxed{\therefore \eta_h = \frac{(1+\cos\phi)}{2}} \quad \text{— (9)}$$

Design foll

Points to be Remembered for Pelton wheel:

(i) The Velocity of the jet at Inlet is given by $V_1 = C_v\sqrt{2gH}$

where C_v = Coeff of velocity = 0.98 to 0.99

H = Net head on turbine.

(ii) The velocity of wheel (u) is given by $u = \phi\sqrt{2gH}$

where ϕ = Speed ratio. The Value of speed ratio varies from 0.43 to 0.48.

(iii) The angle of deflection of the jet through buckets is taken at 165° if no angle of deflection is given.

(iv) The mean dia. (or) the pitch diameter 'D' of the Pelton wheel is given by

$$u = \frac{\pi D N}{60} \quad (or) \quad D = \frac{60u}{\pi N}$$

(v) Jet Ratio: It is defined as the ratio of the pitch diameter (D) of the Pelton wheel to the diameter of the jet (d). It is denoted by 'm' &

$$m = \frac{D}{d} \quad (\approx 12 \text{ for most Cases})$$

(vi) No. of buckets on a runner is given by

$$Z = 15 + \frac{D}{2d} = 15 + 0.5m.$$

where m = Jet ratio.

(vii) No. of Jets: It is obtained by dividing the total rate of flow through the turbine by the rate of flow of water through a single jet.

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Design of Pelton Wheel: Design of Pelton wheel means the following data is to be determined.

(i) Diameter of the jet (d)

(ii) Diameter of wheel (D)

(iii) Width of the buckets which is $= 5 \times d$

(iv) Depth of the buckets which is $= 1.2 \times d$, &

(v) No. of buckets on the wheel.

Size of buckets means the width & depth of the buckets.

Radial Flow Reaction Turbines

Radial flow turbines are those turbines in which the water flows in the radial direction. The water may flow radially from outwards to inwards (i.e., towards the axis of rotation) or from inwards to outwards. If the water flows from outwards to inwards through the runner, the turbine is known as inward radial flow turbine. And if the water flows from inwards to outwards, the turbine is known as outward radial flow turbine.

Reaction turbine means that the water at the inlet of the turbine possesses kinetic energy as well as pressure energy. As the water flows through the runner, a part of pressure energy goes on changing into kinetic energy. Thus the water through the runner is under pressure. The runner is completely enclosed in an air-tight casing & casing & the runner is always full of water.

Main Parts of a Radial flow Reaction Turbine:

The main parts of a radial flow reaction turbine are:

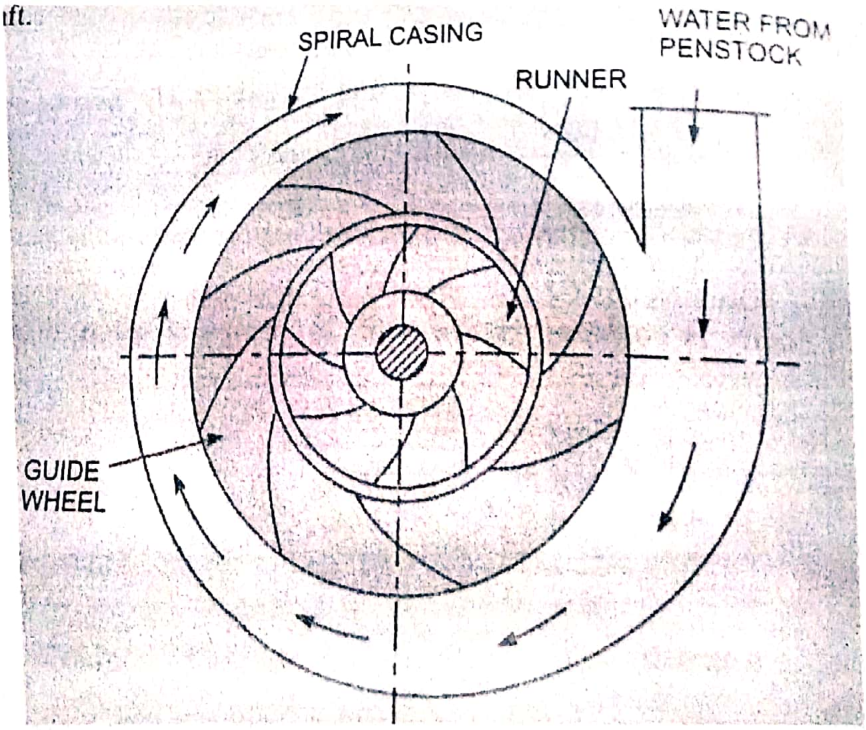
1.) Casing 2.) Guide mechanism 3.) Runner & (4.) Draft-tube.

1) Casing: The water from the penstocks enters the casing which is of spiral shape in which area of cross-section of the casing goes on decreasing gradually. The casing completely surrounds the runner of the turbine. The casing as shown in fig is made of spiral shape, so that the water may enter the runner at constant velocity throughout the circumference of the runner. The casing is made of concrete, cast steel (or) plate steel.

2) Guide Mechanism: It consists of a stationary circular wheel all round the runner of the turbine. The stationary guide vanes are fixed on the guide mechanism. The guide vanes allows the water to strike the vanes fixed on the runner without shock at inlet. Also by a suitable arrangement, the width between two adjacent vanes of guide mechanism can be altered so that the amount of water striking the runner can be varied.

3.) Runner: It is a circular wheel on which a series of radial curved vanes are fixed. The surface of the vanes are made very smooth. The radial curved vanes are so shaped that the water enters & ~~exits~~ leaves the runner without shock. The runners are made of cast steel, cast iron (or) stainless steel. They are keyed to the shaft.

4) <u>Draft tube</u>: The pressure at the exit of the runner of a reaction turbine is generally less than the atmospheric pressure. The water at exit cannot be directly discharged to the tail race. A tube (or) pipe of gradually increasing area is used for discharging water from the exit of the turbine to the tail race. This tube of increasing area is called draft tube.



Velocity

Inward Radial flow turbine:

Figure shows inward radial flow turbine, in which case the water from the casing enters the stationary guiding wheel. The guiding wheel consists of guide vanes which direct the water to enter the runner which consists of moving vanes. The water flows over the moving vanes in the inward radial direction & is discharged at the inner diameter of the runner. The outer diameter of the runner is the ~~first~~ inlet & inner diameter is the outlet.

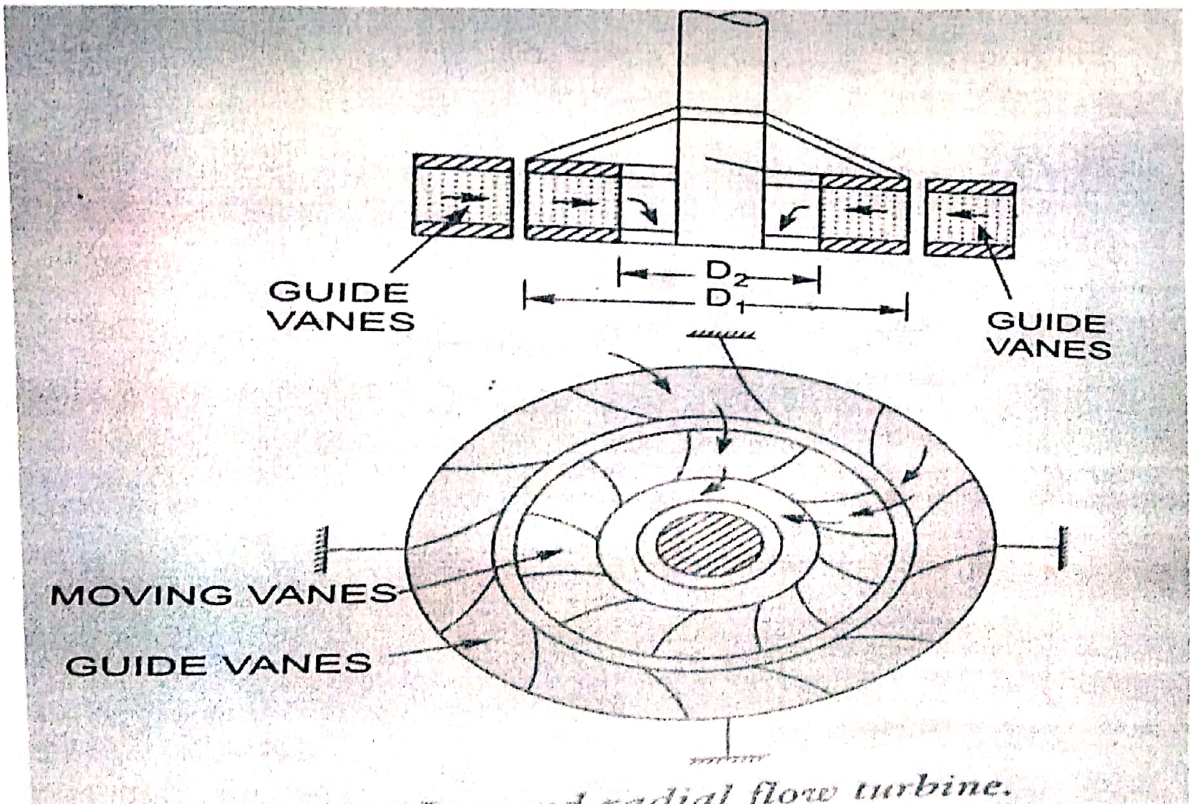


Fig. 18.11 *Inward radial flow turbine.*

Velocity Triangles & Work done by Water on Runner

The work done per second on the runner by water is given by eqn as $= \rho a V_1 [V_{w_1} u_1 \pm V_{w_2} u_2]$

$$= \rho Q [V_{w_1} u_1 \pm V_{w_2} u_2] \quad [\because a V_1 = Q] \quad -(1)$$

The eqn (1) also represents the energy transfer per second to the runner.

where, V_{w_1} = Velocity of whirl at Inlet,

V_{w_2} = " " at Outlet.

u_1 = Tangential Velocity of wheel at Inlet

$= \frac{\pi D_1 \times N}{60}$, where D_1 = Outer dia. of runner.

u_2 = Tangential velocity of wheel at Outlet.

$= \frac{\pi D_2 \times N}{60}$, where D_2 = Inner dia. of runner,

N = Speed of the turbine in r.p.m.

The work done per second per unit weight of water per second.

$$= \frac{\text{Workdone per Second}}{\text{Weight of water striking per Second}}$$

$$= \frac{\rho Q [V_{w_1} u_1 \pm V_{w_2} u_2]}{\rho Q \times g} = \frac{1}{g} [V_{w_1} u_1 \pm V_{w_2} u_2] - (2)$$

The eqn (2) represents the energy transfer per unit weight/s to the runner. This equation is known as Euler's eqn of hydrodynamics machines. This is also known as fundamental eqn of hydrodynamic machines & was given by Swiss Scientist L. Euler.

In Eqn (2), +ve sign is taken if angle β is an acute angle. If β is an obtuse angle then -ve sign is taken. If $\beta = 90°$, then $V_{w_2} = 0$ & work done per second per unit weight of water striking's become as $= \frac{1}{g} V_{w_1} u_1$ — (3)

$$\text{Hydraulic Efficiency}, \eta_h = \frac{R.P}{W.P} = \frac{\frac{W}{1000g}[V_{w_1}u_1 \pm V_{w_2}u_2]}{\left(\frac{W \times H}{1000}\right)}$$

$$= \frac{(V_{w_1}u_1 \pm V_{w_2}u_2)}{gH} \quad \text{— (4)}$$

where R.P = Runner Power i.e., power delivered by water to the runner.

W.P = Water Power.

If the discharge is radial at Outlet, then $V_{w_2} = 0$

$$\boxed{\eta_h = \frac{V_{w_1}u_1}{gH}} \quad \text{— (5)}$$

Definitions: The following terms are generally used in case of reaction radial flow turbines which are defined as:

(i) Speed ratio : The Speed ratio $= \frac{u_1}{\sqrt{2gH}}$ where u_1 = Tangential Velocity of wheel at Inlet

(ii) Flow Ratio : The ratio of the velocity of flow at Inlet (V_{f_1}) to the velocity given $\sqrt{2gH}$ is known as flow ratio (or) it is given as $= \frac{V_f}{\sqrt{2gH}}$ where H = Head on turbine.

(iii) Discharge of the Turbine : The discharge through a reaction radial flow turbine is given by

$$Q = \pi D_1 B_1 \times V_{f_1} = \pi D_2 \times B_2 \times V_{f_2}$$

where, D_1 = Dia. of runner at Inlet.

B_1 = Width of runner at Inlet 9

V_{f_1} = Velocity of flow at Inlet &

D_2, B_2 & V_{f_2} = Corresponding Values at Outlet.

If the thickness of vanes are taken into consideration, then the area through which flow takes place is given by $(\pi D_1 - n \times t)$ where n = No. of vanes on runner & t = thickness of each vane.

The discharge Q, then is given by $Q = (\pi D_1 - n \times t) B_1 \times V_{f_1}$

(iv) The head (H) on the turbine is given by $H = \frac{P_1}{\rho \times g} + \frac{V_1^2}{2g}$

where P_1 = Pressure at Inlet.

(v) Radial Discharge: This means the angle made by absolute velocity with the tangent on the wheel is 90° & the component of whirl velocity is zero. Radial discharge at Outlet means $\beta = 90°$ & $V_{w_2} = 0$, while radial discharge at Inlet means $\alpha = 90°$ & $V_{w_1} = 0$.

(vi) If there is no loss of energy when water flows through the Vanes then we have,

$$H - \frac{V_2^2}{2g} = \frac{1}{g}\left[V_{w_1} u_1 \pm V_{w_2} u_2\right]$$

Outward Radial Flow Reaction Turbine:

Figure Shows Outward radial flow reaction turbine in which the water ~~fl~~ from casing enters the stationary guide wheel. The guide wheel consists of guide vanes which direct water to enter the runner which is around the stationary guide wheel. The water flows through the vanes of the runner in the outward radial direction & is discharged at the outer diameter of the runner. The inner diameter of the runner is inlet & outer diameter is the Outlet.

The velocity triangles at inlet & outlet will be drawn by the same method that was adopted for inward flow turbine.

In this case as inlet of the runner is at the inner diameter of the runner, the tangential velocity at inlet will be less than that of at outlet, i.e., $u_1 < u_2$ as $D_1 < D_2$.

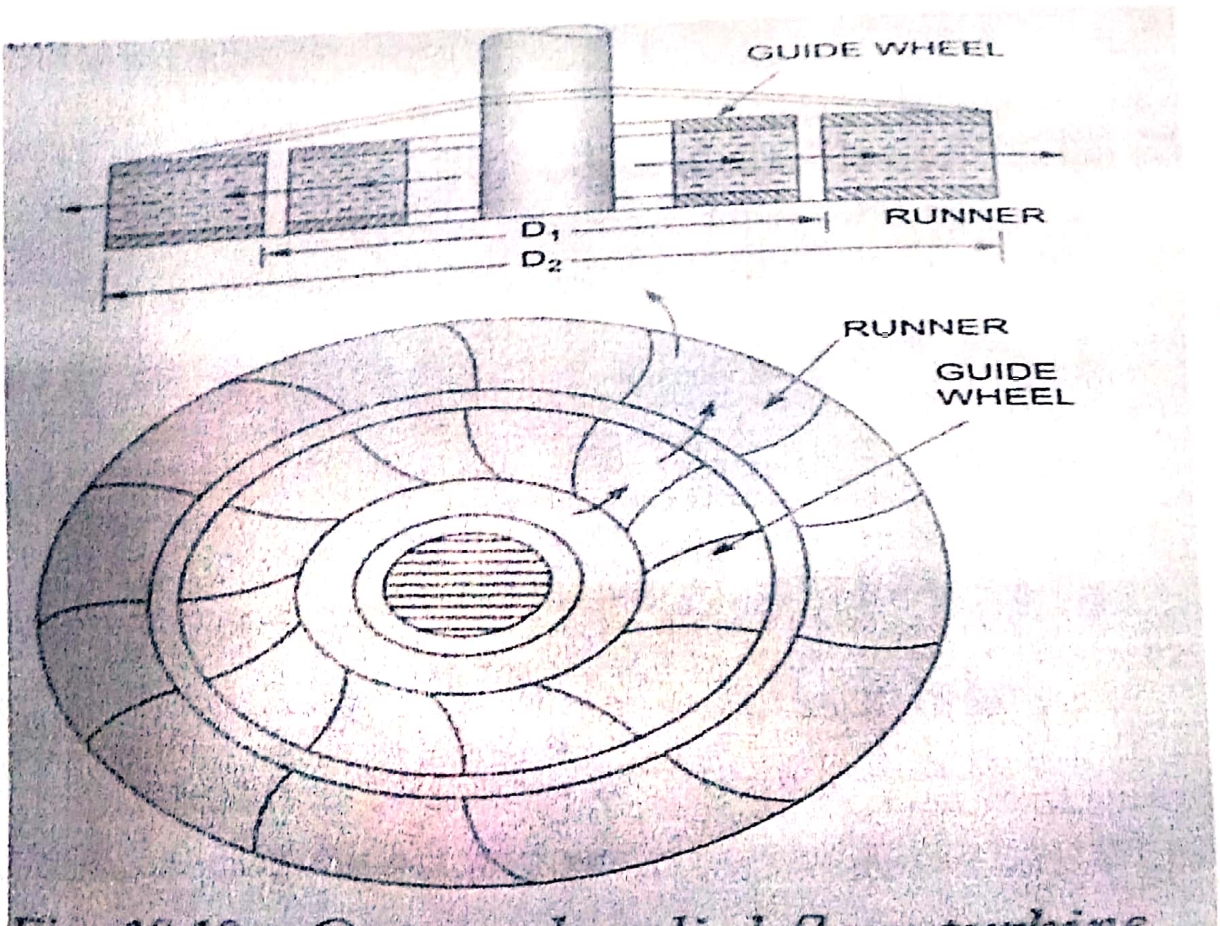


Fig. 18.18 Outward radial flow turbine.

Francis turbine:

The inward flow reaction turbine having radial discharge at outlet is known as Francis Turbine, after the name of J.B. Francis, an American engineer who in the begining designed inward radial flow reaction type of turbine. In the modern Francis turbine, the water enters the runner of the turbine in the radial direction at outlet & leaves in the axial direction at the inlet of the runner. Thus the modern Francis Turbine is a mixed flow type turbine.

The velocity triangle at Inlet & Outlet of the Francis turbine are drawn in the same way as in case of Inward flow reaction turbine. As in case of Francis turbine, the discharge is radial at Outlet, the velocity of whirl at Outlet (i.e., V_{w_2}) will be zero. Hence the work done by water on the runner per second will be $= \rho Q (V_{w_1} u_1)$

And work done per second per unit weight of water striking/s $= \frac{1}{g}[V_{w_1} u_1]$

Hydraulic efficiency will be given by, $\eta_h = \dfrac{V_{w_1} u_1}{gH}$

Important Relations for Francis Turbine:

1.) The ratio of width of the wheel to its diameter is given as $n = \frac{B_1}{D_1}$. The value of 'n' varies from 0.10 to 0.40.

2.) Flow ratio $= \dfrac{V_{f_1}}{\sqrt{2gH}}$ → Varies from 0.15 to 0.30.

3.) Speed Ratio $= \dfrac{u_1}{\sqrt{2gH}}$ → Varies from 0.6 to 0.9.

Axial Flow Reaction Turbine:

If the water flows parallel to the axis of the rotation of the shaft, the turbine is known as axial flow turbine. And if the head at the inlet of the turbine is the sum of pressure energy & kinetic energy & during the flow of water through runner a part of pressure energy is converted into kinetic energy, the turbine is known as reaction turbine.

For the axial flow reaction turbine, the shaft of the turbine is vertical. The lower end of the shaft is made larger which is known as 'hub' (or) 'boss'. The vanes are fixed on the hub & hence hub acts as a runner for axial flow reaction turbine. The following are the important type of axial flow reaction turbines:

1.) Propeller Turbine 2.) Kaplan Turbine.

When the vanes are fixed to the hub & they are not adjustable, the turbine is known as propeller turbine.

But if the vanes on the hub are adjustable, the turbine is known as a Kaplan Turbine, after the name of V. Kaplan, an Austrain Engineer. This turbine is suitable where a large quantity of water at low head is available. Fig (a) shows the runner of a kaplan turbine.

The main parts of a kaplan turbine are:

1.) Scroll casing
2.) Guide vanes mechanism
3.) Hub with vanes (or) runner of the turbine &.
4.) Draft tube.

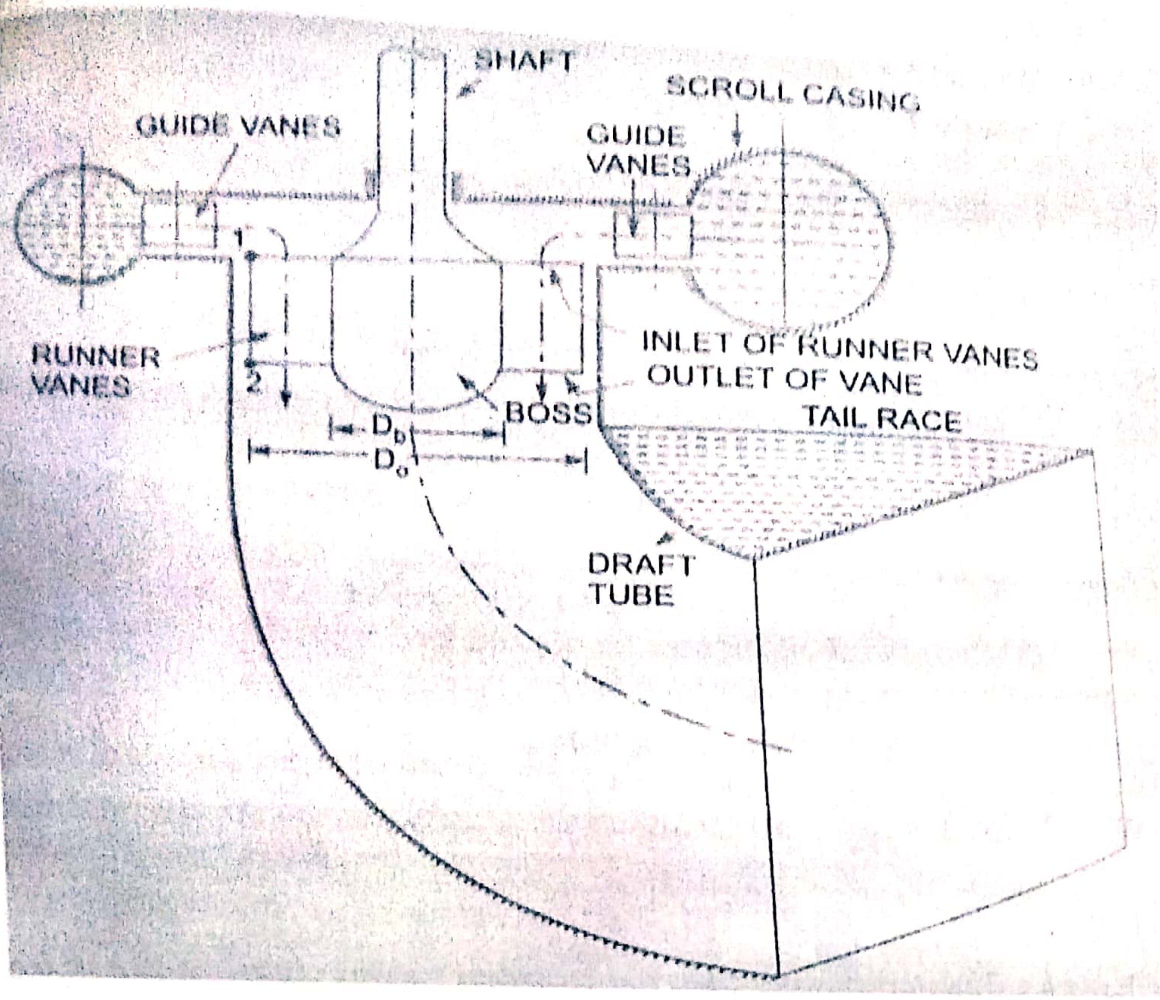


Fig (a): Main Parts of Kaplan Turbine.

Fig(a) shows all main parts of a kaplan turbine. The water from penstock enters the Scroll casing & then moves to the guide vanes. From the guide vanes, the water turns through 90° and flows axially through the runner as shown in above Figure. The discharge through the runner is obtained as

$$Q = \frac{\pi}{4}\left(D_o^2 - D_b^3\right) \times V_{f_1}$$

where D_o = Outer diameter of the runner,

D_b = Dia. of hub &

V_{f_1} = Velocity of flow at Inlet.

The Inlet & Outlet velocity triangles are drawn at the extreme edge of the runner vane corresponding to the points 1 & 2 as shown in Fig(a).

Some Important Points for Propeller (Kaplan Turbine):

1) The peripheral velocity at Inlet & Outlet are equal.

$$\therefore u_1 = u_2 = \frac{\pi D_o N}{60}$$ where D_o = Outer dia of Runner.

2) Velocity of flow at Inlet & Outlet are equal

$$\therefore V_{f_1} = V_{f_2}$$

3) Area of flow at Inlet = Area of flow at Outlet.

$$= \frac{\pi}{4}(D_o^2 - D_b^2)$$

Draft - Tube:

The draft-tube is a pipe of gradually increasing area which connects the outlet of the runner to the tail race. It is used for discharging water from the exit of the turbine to the tail race. This pipe of gradually increasing area is called a draft-tube. One end of the draft-tube is connected to the Outlet of the runner while the other end is sub-merged below the level of water in tail race. The draft-tube, in addition to serve a passage for water discharge, has the following two purposes also.

1) It permits a negative head to be established at the outlet of the runner & thereby increase the net head on the turbine. The turbine may be placed above the tail race without any loss of net head & hence turbine may be inspected properly.

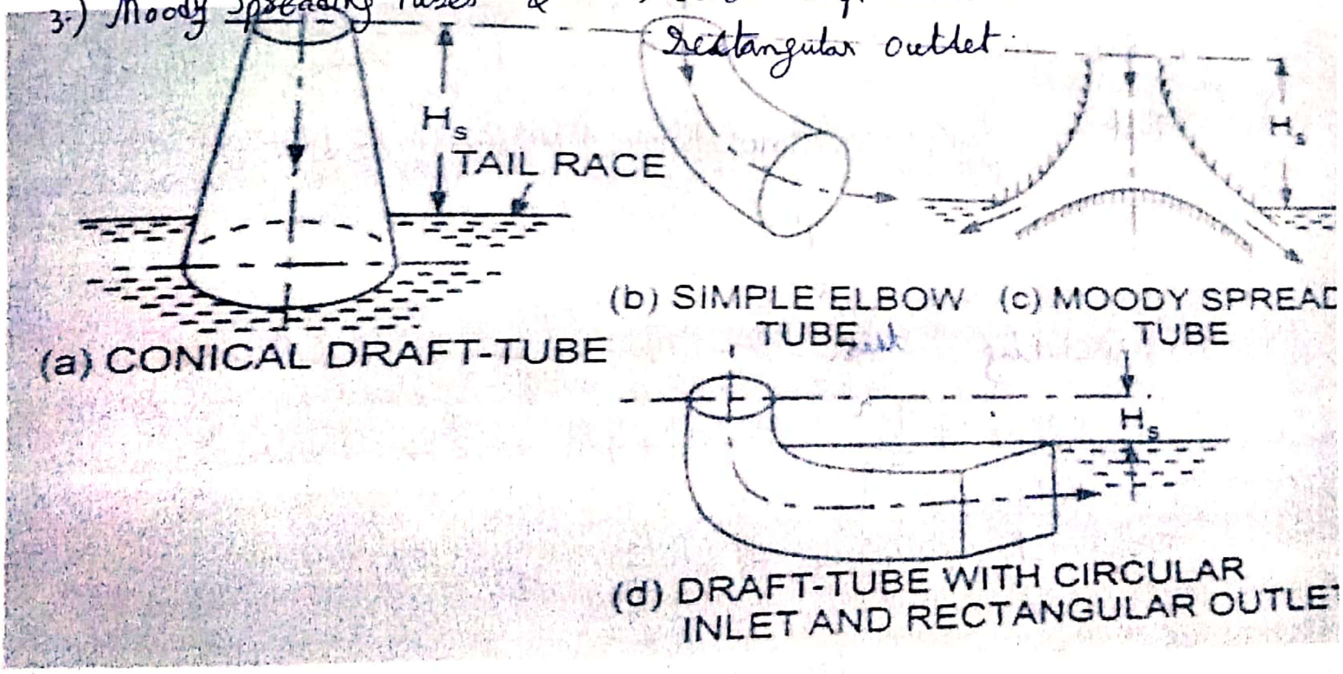
2) It converts a large proportion of the kinetic energy ($V_2^2/2g$) rejected at the outlet of the turbine into useful pressure energy. Without the draft tube, the kinetic energy rejected at the outlet of the turbine will go waste to the tail race.

Hence by using draft-tube, the net-head on the turbine increases. The turbine develops more power & also the efficiency of the turbine increases.

If a reaction turbine is not fitted with a draft-tube, the pressure at the outlet of the runner will be equal to atm. pr. The water from the outlet of the runner will discharge freely into the tail race. The net head on the turbine will be less than that of a reaction turbine fitted with a draft tube. Also without a draft-tube, kinetic energy ($V^2/2g$) rejected at the outlet of the runner will go waste to the tail race.

Types of Draft-Tubes:

1.) Conical draft-tubes
2.) Simple Elbow tubes
3.) Moody Spreading tubes &
4.) Elbow draft-tubes with Circular Inlet & Rectangular outlet.

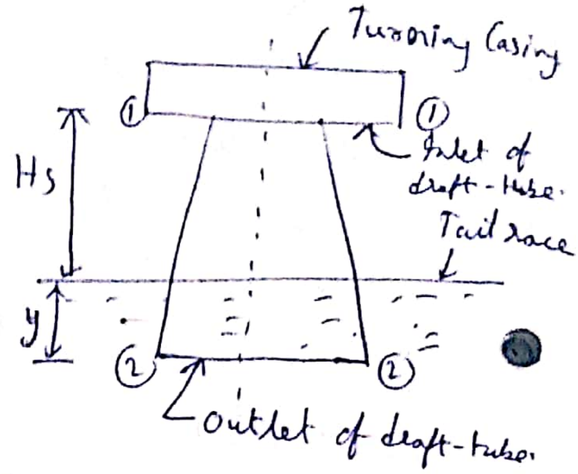


(a) CONICAL DRAFT-TUBE
(b) SIMPLE ELBOW TUBE
(c) MOODY SPREAD TUBE
(d) DRAFT-TUBE WITH CIRCULAR INLET AND RECTANGULAR OUTLET

The conical draft-tubes & Moody spreading draft-tubes are most efficient while simple elbow tubes & elbow draft-tubes with circular inlet & rectangular outlet require less space as compared to other draft-tubes.

<u>Draft-tube Theory</u>

Consider a capital draft-tube as shown in Fig.



Let H_s = Vertical height of draft-tube above the tail race.

y = Distance of bottom of draft-tube from tail race.

Applying Bernoulli's Eqn to Inlet (Section 1-1) & outlet (Section 2-2) of the draft-tube & taking section 2-2 as ~~shown~~ the datum line, we get

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + (H_s + y) = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + 0 + h_f \quad -①$$

where h_f = loss of energy b/w Sections ①-① & ②-②

But $\frac{P_2}{\rho g}$ = Atmospheric pressure head + y

$$= \frac{P_a}{\rho g} + y.$$

Substituting this value of $\frac{P_2}{\rho g}$ in eqn ①, we get.

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + (H_s + y) = \frac{P_a}{\rho g} + y + \frac{V_2^2}{2g} + h_f.$$

$$\Rightarrow \frac{P_1}{\rho g} + \frac{V_1^2}{2g} + H_s = \frac{P_a}{\rho g} + \frac{V_2^2}{2g} + h_f$$

$$\Rightarrow \frac{P_1}{\rho g} = \frac{P_a}{\rho g} + \frac{V_2^2}{2g} + h_f - \frac{V_1^2}{2g} - H_s.$$

$$= \frac{P_a}{\rho g} - H_s - \left(\frac{V_1^2}{2g} - \frac{V_2^2}{2g} - h_f\right)$$ where $\frac{P_1}{\rho g}$ is less than atmospheric pressure.

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Efficiency of Draft-tube: (η_d)

$$\eta_d = \frac{\text{Actual Conversion of kinetic head into Pressure head in the draft-tube}}{\text{kinetic head at the Inlet of draft-tube.}}$$

Let V_1 = Velocity of water at Inlet of draft-tube,
V_2 = " " " " Outlet " " ",
h_f = Loss of head in the draft-tube.

Theoritical conversion of kinetic ~~energy~~ head into pressure head in draft-tube $= \left(\frac{V_1^2}{2g} - \frac{V_2^2}{2g}\right)$

& Actual conversion of kinetic head into Pressure head $= \left(\frac{V_1^2}{2g} - \frac{V_2^2}{2g}\right) - h_f$.

$$\therefore \eta_d = \frac{\left(\frac{V_1^2}{2g} - \frac{V_2^2}{2g}\right) - h_f}{\left(\frac{V_1^2}{2g}\right)}$$

Specific Speed: (N_s)

It is defined as the Speed of a turbine which is ~~identical~~ identical in Shape, geometrical dimensions, blade angles, gate openings etc., with the actual turbine ~~but~~ but of Such a Size that it will develop unit power when working under unit head. The Specific Speed is used in comparing the different types of turbines, as every type of turbine has different specific Speed.

In M.K.S Units, unit power is taken as one horse power & unit head as one meter. But in S.I Units, Unit power is taken as one kilowatt & Unit head as one metre.

Derivation of the Specific Speed:

The Overall efficiency (η_o) of any turbine is given by,

$$\eta_o = \frac{\text{Shaft Power}}{\text{Water Power}} = \frac{\text{Power developed}}{\frac{\rho \times g \times Q \times H}{1000}} = \frac{P}{\frac{\rho \times g \times Q \times H}{1000}} \quad -(1)$$

where H = Head under which the turbine is working

Q = Discharge through turbine.

P = Power developed (or) Shaft Power.

From eqn (1), $P = \eta_o \times \frac{\rho \times g \times Q \times H}{1000}$

$\propto Q \times H$ (as η_o & ρ are Constant) $-(2)$

Now let D = Diameter of actual turbine,

N = Speed of actual turbine,

u = Tangential velocity of the turbine,

N_s = Specific Speed of the turbine

V = Absolute velocity of water.

The absolute Velocity, tangential Velocity & head on the turbine are related as, ~~$u \propto \sqrt{V}$~~ $u \propto V$, where $V \propto \sqrt{H}$

$\propto \sqrt{H} \quad -(3)$

But tangential Velocity 'u' is given by

$u = \frac{\pi D N}{60}$

$\propto DN \quad -(4)$

$\therefore$ From Eqns (3) & (4), we have

$\sqrt{H} \propto DN$ (or) $D \propto \frac{\sqrt{H}}{N} \quad -(5)$

The discharge through turbine is given by

Q = Area $\times$ Velocity

But Area $\propto B \times D$ (where B = Width)

$\propto D^2$ ($\because B \propto D$)

And Velocity $\propto \sqrt{H}$

$$\therefore Q \propto D^2 \times \sqrt{H}$$

$$\propto \left(\frac{\sqrt{H}}{N}\right)^2 \times \sqrt{H} \quad \left(\because \text{from Eqn } (5),\ D \propto \frac{\sqrt{H}}{N}\right)$$

$$\propto \frac{H^{3/2}}{N^2} \quad \text{—} (6)$$

Substituting the Value of Q in eqn (2), we get

$$P \propto \frac{H^{3/2}}{N^2} \times H \propto \frac{H^{5/2}}{N^2}$$

$$\boxed{\therefore P = K \frac{H^{5/2}}{N^2}}$$ where K = Constant of proportionality.

If P = 1, H = 1, the Speed N = Specific Speed, N_s. Substituting these Values in the above Eqn, we get

$$1 = \frac{K \times 1^{5/2}}{N_s^2}$$

$$\Rightarrow N_s^2 = K$$

$$\therefore P = N_s^2 \frac{H^{5/2}}{N^2} \Rightarrow N_s^2 = \frac{N^2 P}{H^{5/2}}$$

$$\therefore N_s = \sqrt{\frac{N^2 P}{H^{5/2}}} = \frac{N\sqrt{P}}{H^{5/4}} \quad \text{—} (8)$$

In Eqn (8), If P is taken in metric horse power the Specific Speed is obtained in M.K.S units. But if P is taken in kilowatts, the Specific Speed is obtained in S.I Units.

<u>Significance of Specific Speed</u>: Useful for selecting the type of the turbine and also the performance of a turbine can be predicated by knowing the Specific Speed of the turbine.

S.No	Specific Speed (M.K.S)	Specific Speed (S.I)	Type of turbine.
1.	10 to 35	8.5 to 30	Pelton wheel with Single jet
2.	35 to 60	30 to 51	Pelton wheel with 2 (or) more jets
3.	60 to 300	51 to 225	Francis ~~turbi~~ turbine
4.	300 to 1000	225 to 860	Kaplan (or) Propeller turbine

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Unit Quantities:

In order to predict the behaviour of a turbine working under varying conditions of head, speed output & gate opening, the results are expressed in terms of quantities which may be obtained when the head on the turbine is reduced to unity. The conditions of the turbine under unit head are such that the efficiency of the turbine remains unaffected. The following are the 3 important unit quantities: ~~which~~

(1) Unit Speed (2) Unit discharge (3) Unit power

Unit Speed (N_u) It is defined as the speed of a turbine working under a unit head (i.e., under a head of 1m).

Let N = Speed of a turbine under a head H.

H = Head under which a turbine is working.

u = Tangential Velocity.

The tangential velocity, absolute velocity of water & head on the turbine are related as

$$u \propto \sqrt{V} \quad \text{where } V \propto \sqrt{H}$$

$$\propto \sqrt{H} \quad \text{— (1)}$$

Also tangential velocity (u) is given by

$$u = \frac{\pi D N}{60} \quad \text{where } D = \text{Dia. of turbine.}$$

For a given turbine, the diameter (D) is constant.

$$\therefore u \propto N \text{ (or) } N \propto u \text{ (or) } N \propto \sqrt{H} \quad (\because \text{from eqn (1)}, u \propto \sqrt{H})$$

$$\therefore N = k_1 \sqrt{H} \quad \text{— (2)}$$

where k_1 is a constant of proportionality.

If head on the turbine becomes unity, the speed becomes unit speed (or) when $H = 1$, $N = N_u$

Substituting the value of k_1 in eqn (2), $N = N_u \sqrt{H}$ (or) $N_u = \frac{N}{\sqrt{H}}$ — (I)

Unit Discharge (Q_u): It is defined as the discharge passing a turbine, which is working under a unit head (i.e., 1 m).

Let H = Head of water on the turbine.

Q = Discharge passing through turbine when head is H on the turbine.

a = Area of flow of water

The discharge passing through a given turbine under a head 'H' is given by, Q = Area of flow × Velocity.

But for a ~~test~~ turbine, area of flow is constant & velocity is proportional to $\sqrt{H}$.

$$\therefore Q \propto \text{Velocity} \propto \sqrt{H}$$

(or) $$Q = k_2\sqrt{H} \quad - (3)$$

where k_2 is constant of proportionality

If $H = 1$, $Q = Q_u$

Substituting these values in eqn (3), we get

$$Q_u = k_2\sqrt{1.0} = k_2$$

Substituting the value of k_2 in eqn (3), we get

$$Q = Q_u\sqrt{H}$$

$$\boxed{\therefore Q_u = \frac{Q}{\sqrt{H}}}$$

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Unit Power: (P_u)

It is defined as the power developed by a turbine working under a unit head (i.e., under a head of 1m). It is denoted by the symbol 'P_u'. The expression for unit power is obtained as:

Let H = Head of water on the turbine,

P = Power developed by the turbine under a head of H,

Q = Discharge through turbine under a Head H.

The overall efficiency (η_o) is given as

$$\eta_o = \frac{\text{Power developed}}{\text{Water power}} = \frac{P}{\frac{\rho \times g \times Q \times H}{1000}}$$

$$\therefore P = \eta_o \times \frac{\rho \times g \times Q \times H}{1000}$$

$$\propto Q \times H$$

$$\propto \sqrt{H} \times H \qquad (\because Q \propto \sqrt{H})$$

$$\propto H^{3/2}$$

$$\therefore P = k_3 H^{3/2} \quad \text{—— (iv)}$$

where k_3 is a constant of proportionality.

When $H = 1m$, $P = P_u$

$$\therefore P_u = k_3 (1)^{3/2} = k_3$$

Substituting the value of k_3 in eqn (iv), we get

$$P = P_u H^{3/2}$$

$$\boxed{\therefore P_u = \frac{P}{H^{3/2}}}$$

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Use of Unit Quantities (N_u, P_u, Q_u):

If a turbine is working under different heads, the behaviour of the turbine can be easily known from the values of the unit quantities i.e, from the values of unit speed, unit discharge and unit power.

Let $H_1, H_2 \ldots$ are heads under which a turbine works,
$N_1, N_2 \ldots$ are the corresponding speeds,
$Q_1, Q_2 \ldots$ are the discharge and
P_1 & $P_2 \ldots$ are the power developed by the turbine

Using eqns $N_u = \frac{N}{\sqrt{H}}$, $Q_u = \frac{Q}{\sqrt{H}}$ & $P_u = \frac{P}{H^{3/2}}$ respectively,

$$\left.\begin{aligned} N_u &= \frac{N_1}{\sqrt{H_1}} = \frac{N_2}{\sqrt{H_2}} \\ Q_u &= \frac{Q_1}{\sqrt{H_1}} = \frac{Q_2}{\sqrt{H_2}} \\ P_u &= \frac{P_1}{\sqrt{H_1}} = \frac{P_2}{\sqrt{H_2}} \end{aligned}\right\} \quad \text{— (A)}$$

Hence if the speed, discharge & power developed by a turbine under a head are known, then by using eqn (A), the speed, discharge & power developed by the same turbine under a different head can be easily obtained.

Characteristic Curves of Hydraulic Turbines:

Characteristic Curves of a hydraulic turbine are the Curves, with the help of which the exact behaviour & performance of the turbine under different working Conditions, can be known. These curves are plotted from the results of the tests performed on the turbine under different working conditions.

The important parameters which are varied during a test on a turbine are:

1.) Speed (N) 2.) Head (H) 3.) Discharge (Q)

4.) Power (P) 5.) Overall efficiency (η_o) & 6.) Gate opening.

Out of the above six parameters, three parameters namely Speed (N), head (H) & discharge (Q) are independent parameters.

Out of the 3 independent parameters (N, H, Q) one of the parameter is kept Constant (Say H) & the variation of the other four parameters with respect to any one of the remaining two Independent variables (Say N & Q) are plotted & various curves are obtained. These curves are called characteristics curves.

The following are the important characteristic Curves of a turbine:

1.) Main Characteristic Curves (or) Constant Head Curves.

2.) Operating " " (or) " Speed ".

3.) Muschel Curves (or) " Efficiency "

Main Characteristic Curves or Constant Head Curves:

Main Characteristic Curves are Obtained by maintaining a constant head & a constant gate opening (G.O) on the turbine. The Speed of the turbine is varied by changing load on the turbine. For each value of the Speed, the corresponding values of power (P) & discharge (Q) are obtained. Then the overall efficiency (η_o) for each value of the [power ~~(P) & discharge (Q) are obtained.~~]* ~~the~~ speed is calculated. From these readings the values of unit Speed (N_u), unit power (P_u) & unit discharge (Q_u) are obtained. Taking N_u as abscissa, the values of Q_u, P_u, P & η_o are plotted as shown in fig (1) & (2).

By changing the gate opening, the values of Q_u, P_u & η_o and N_u are determined & taking N_u as abscissa, the values of Q_u, P_u, & η_o are plotted.

Fig (1) Shows the main Characteristic Curves for Pelton Wheel & Fig (2) Shows the main Characteristic Curves for Reaction (Francis & Kaplan) turbines.

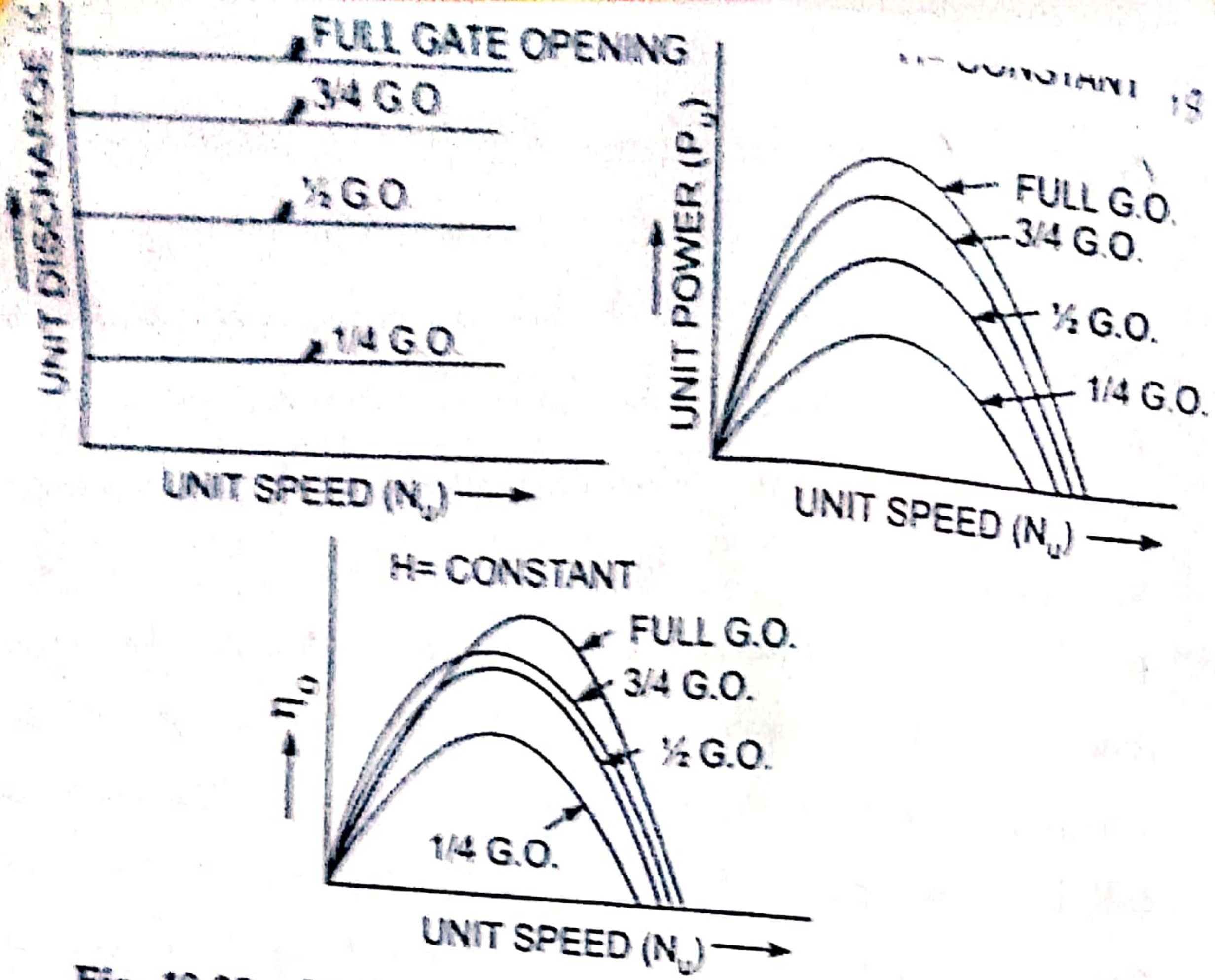
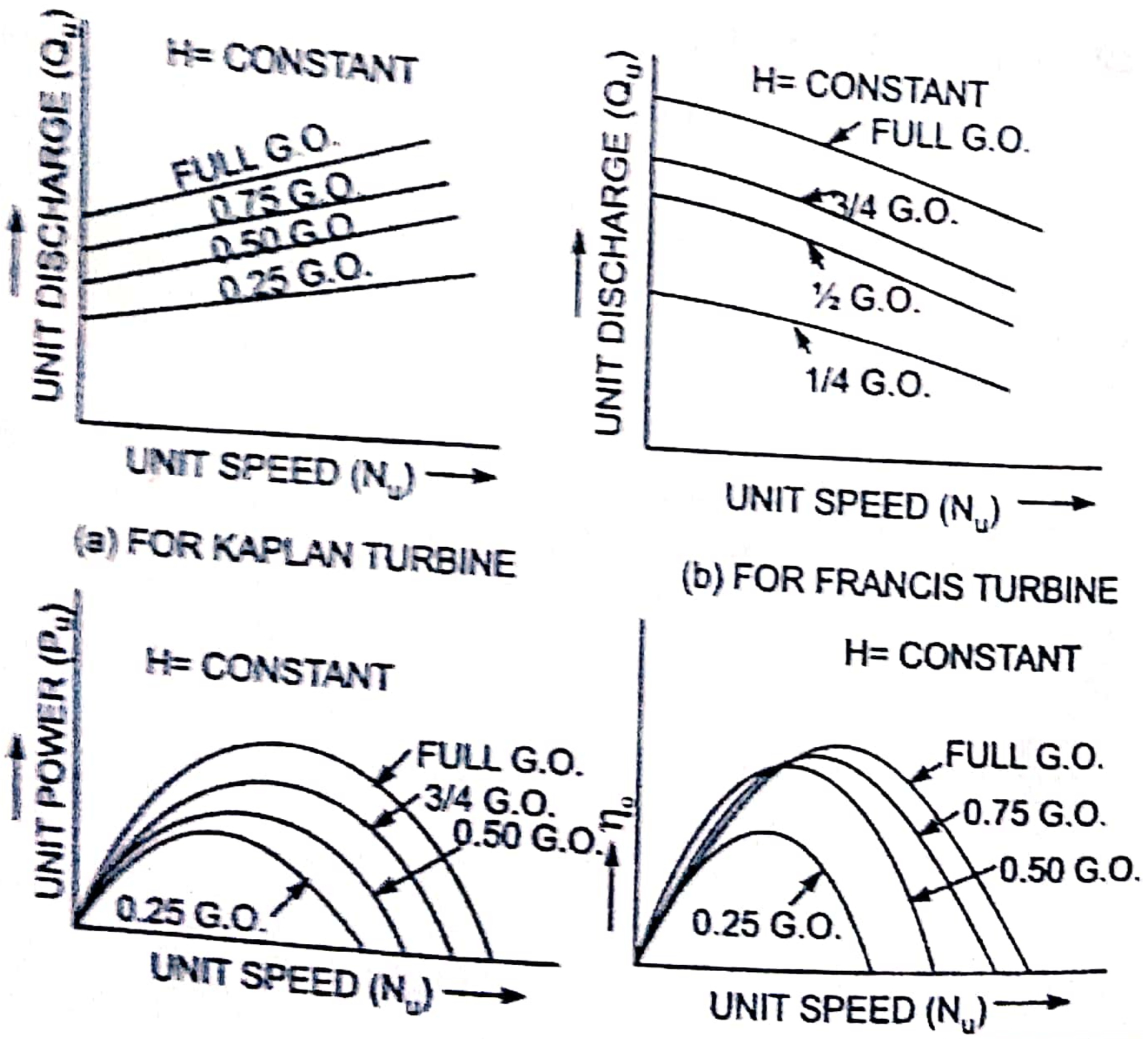
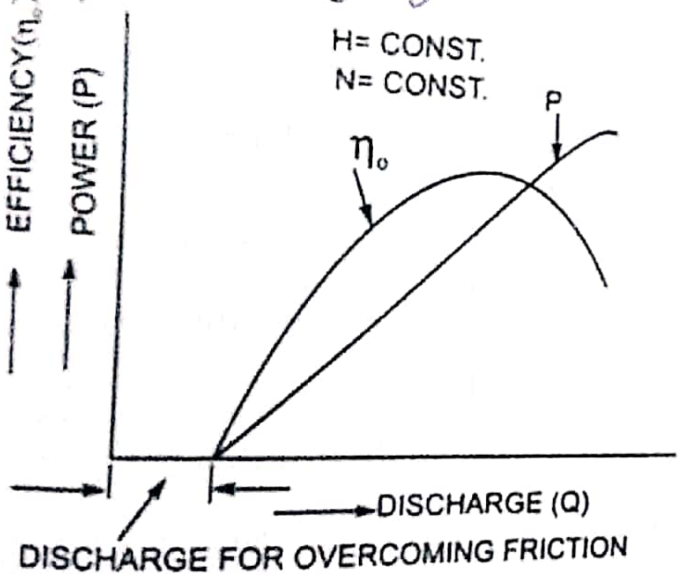


Fig. 18.35 *Main characteristic curves for a Pelton wheel.*



Operating Characteristic Curves (or) Constant Speed Curves:

Operating Characteristic Curves are plotted when the Speed of the turbine is constant. In case of turbines, the head is generally constant. As we know, there are 3 independent Parameters namely N, H & Q. For operating characteristics N & H are constant & hence the variation of power & efficiency w.r.t discharge 'Q' are plotted. The power curve for turbines shall not pass through the origin because certain amount of discharge is needed to produce power to overcome initial friction. Hence the power & efficiency curves will be slightly away from the origin on the x-axis, as to overcome initial friction certain amount of discharge will be required. Fig (3) shows the variation of power & efficiency with respect to discharge.



Constant Efficiency Curves (or) Muschel Curves (or) Iso-Efficiency Curves

These curves are obtained from the Speed vs efficiency & Speed Vs discharge curves for different gate openings. For a given efficiency from the N_u vs η_o curves, there are 2 speeds. From the N_u Vs Q_u curves, corresponding to two values of speeds there are 2 values of discharge. Hence for a given efficiency there are 2 values of discharge for a particular gate opening. This means for a given efficiency there are two values of speeds & two values of the discharge for a given gate ~~open~~ opening.

If the efficiency is max. there is only one value. These 2 values of speed & 2 values of discharge corresponding to a particular gate opening are opened are plotted as shown in Fig(4B) The procedure is repeated for different gate ~~opening~~ openings & the curves Q Vs. N are plotted. The points having same efficiencies are joined & these curves of same efficiency are called Iso-efficiency curves. These curves are helpful for determining the zone of constant efficiency & for predicating the performance of the turbine at various efficiencies

For plotting the Iso-efficiency curves, horizontal lines representing the same efficiency are drawn on the $\eta_o \sim$ speed curves. The points at which these lines cut the efficiency curves at various gate openings are transferred to the corresponding $Q \sim$ Speed curves. The points having the same efficiency are then joined by a smooth curves. These smooth curves represents the Iso-efficiency curve.

Governing of Turbines:

The governing of a turbine is defined as the operation by which the speed of the turbine is kept constant under all conditions of working. It is done automatically by means of a governor, which regulates the rate of flow through the turbines according to the changing load condition on the turbine.

Governing of a turbine is necessary as a turbine is directly coupled to an electric generator, which is required to run at constant speed under all fluctuating load conditions. The frequency of power generation by a generator of constant number of pair of poles under all varying condition should be ~~cond.~~ constant. This is only possible when the speed of the generator, under all changing load condition, is constant. The speed of the generator will be constant, when the speed of the turbine (which is coupled to the generator) is constant.

When the load on the generator decreases, the speed of the generator increases beyond the normal speed (constant speed) The the speed of the turbine also increases beyond the normal speed. If the turbine (or) generator is to run at constant (normal) speeds the rate of flow of water to the turbine should be decreased till the speed becomes ~~(normal)~~ normal. This process ~~of~~ by which the speed of the turbine (and hence of generator) is kept constant under varying condition of load is called governing.

Governing of Pelton Turbine (Impulse Turbine)

Governing of Pelton turbine is done by means of Oil pressure governor, which consists of the following parts:

1.) Oil Sump

2.) Gear pump also called oil pump, which is driven by the power obtained ~~by~~ from turbine Shaft.

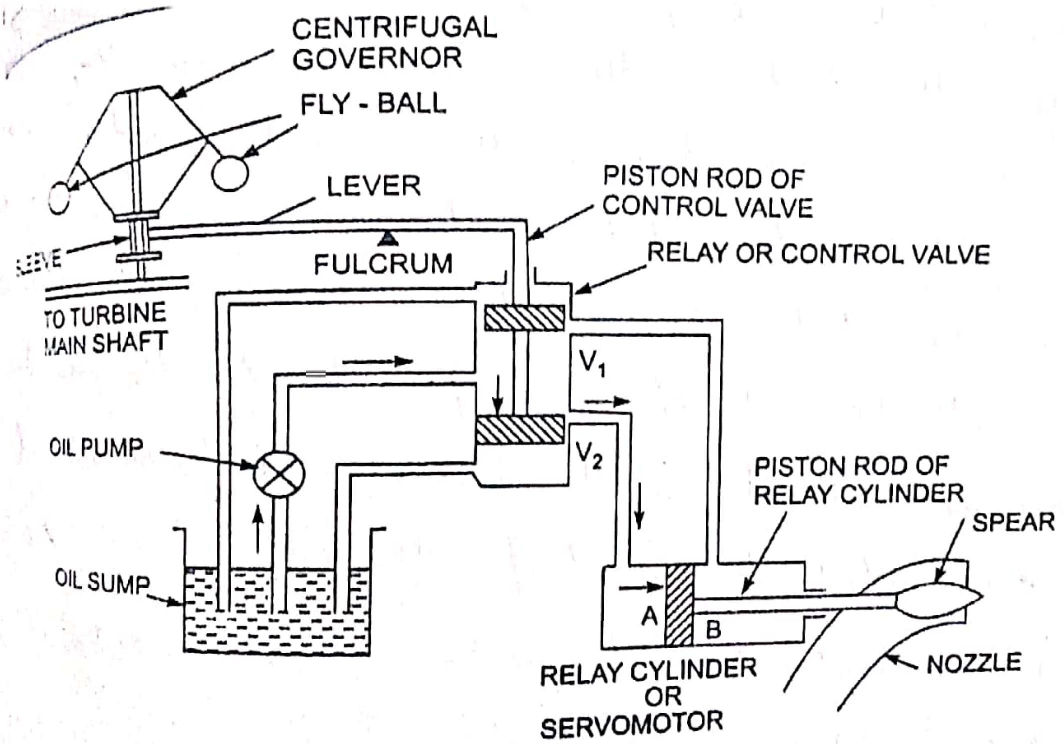
3.) The Servomotor also called relay cylinder.

4.) The control Valve or the distribution Valve or relay Valve.

5.) The Centrifugal governor or pendulum which is driven by belt or gear from the turbine shaft.

6.) Pipes connecting the oil Sump with the control Valve & control Valve with Servometer &

7.) The Spear rod & needle.



MODULE V

Introduction:

- The hydraulic Machine which converts the mechanical energy into hydraulic energy are called pumps. Here the hydraulic energy is in the form of pressure energy.

- If this conversion of this mechanical energy into pressure energy is by means of centrifugal force acting on the fluid, then that hydraulic machine is called as centrifugal pump.

- The flow in centrifugal pump is in radial outward direction which is reverse action of an inward radial flow reaction turbine.

- The Centrifugal pump works on the principle of forced vortex flow which means that when a certain mass of liquid is rotated by an external torque, the rise in pressure head of the rotating liquid takes place.

- The rise in pressure head at any point of the rotating liquid is proportional to the square of the tangential velocity of the liquid at that point (i.e., rise in pressure head $= \frac{V^2}{2g} = \frac{\omega^2 r^2}{2g}$)

- Thus at the Outlet of the Impeller, where radius is more, the rise in pressure head will be more & the liquid will be discharged at the outlet with a high pressure head. Due to this high pressure head, the liquid can be lifted to a high level.

Main Parts of a Centrifugal Pump:

1.) **Impeller:** The rotating part of a centrifugal pump is called 'Impeller'. It consists of a series of backward curved vanes. The impeller is mounted on a shaft which is connected to the shaft of an electric motor.

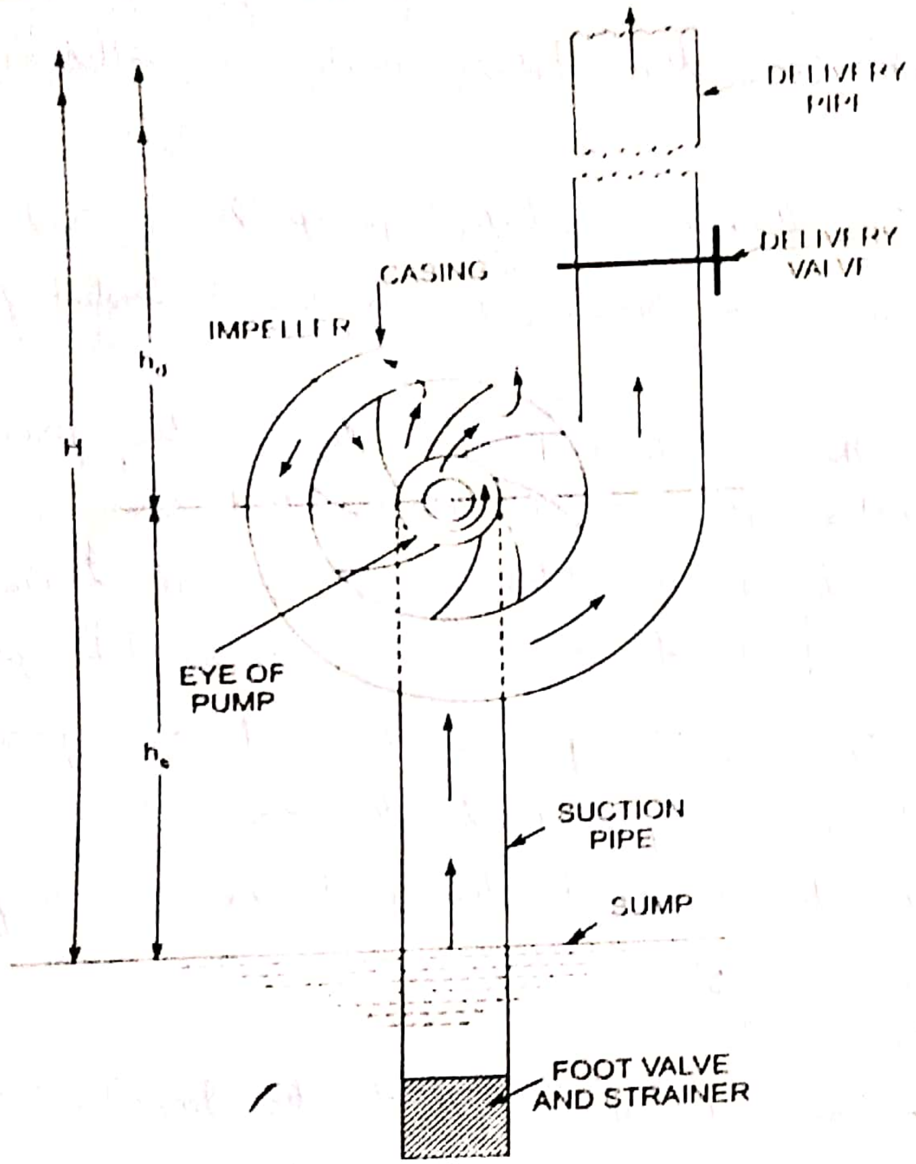


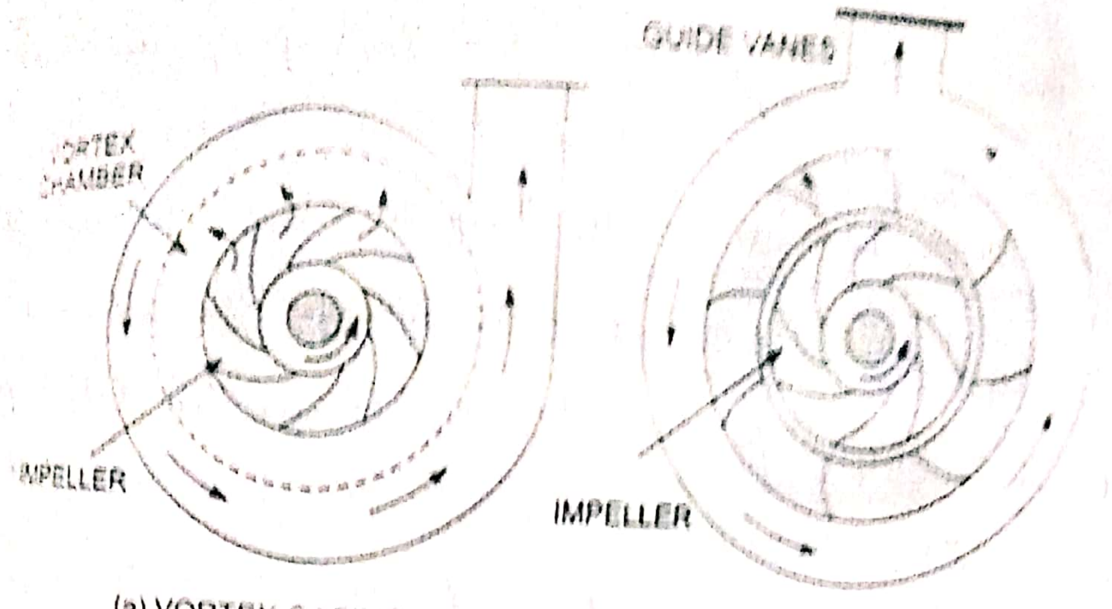
Fig-1 : Main Parts of a Centrifugal Pump

Casing:

The Casing of a Centrifugal pump is similar to the casing of a reaction turbine. It is an air-tight passage surrounding the impeller & is designed in such a way that the kinetic energy of the water discharged at the outlet of the Impeller is converted into pressure energy before the water leaves the Casing & enter the delivery pipe. The following 3 types of casings are commonly used:

(a) Volute Casing: Fig 1, shows the Volute Casing, which surrounds the Impeller; It is of Spiral type in which area of flow increases gradually. The increase in area of flow decreases the velocity of flow. The decrease in velocity increases the pressure of the water flowing through the casing. In case of this volute Casing, the efficiency of the pump increases slightly, as a large amount of energy is lost due to the formation of eddies in this type of Casing.

(b) Vortex Casing: Fig 2(a): In this Casing, a circular Chamber is introduced b/w the Casing & the Impeller as shown in Fig 2(a). This Circular Chamber helps in reduction of loss of energy due to formation of eddies to a considerable extent. Thus the Efficiency of the pump is more than the efficiency when only volute casing is provided.



(a) VORTEX CASING (b) CASING WITH GUIDE BLADES

(c) Casing With Guide Blades: (Fig 2 b)

Here the Impeller is surrounded by a Series of guide blades mounted on a ring which is known as diffuser. The guide Vanes are designed in Such a way that the water from the Impeller enters the guide van ~~with~~ without Stock.

Also the area of the guide Vanes increases, thus reducing the velocity of flow through guide Vanes and consequently increasing the pressure of water.

The water from the guide Vanes then passes through the surrounding casing which is in most of the cases concentric with the impeller as shown in Fig (2b).

Suction Pipe with a Foot Valve & a Strainer:

A pipe whose one end is connected to the inlet of the pump & the other end dips into water in a sump is known as suction pipe. A foot-Valve which is a non-return valve (or) one-way type of valve is fitted at the lower end of the Suction pipe. The foot valve opens only in the upward direction. A Strainer is also fitted at the lower end of the suction pipe.

4.) Delivery Pipe: A pipe whose one end is connected to the outlet of the pump & other end delivers the water at a required height is known as delivery pipe.

Work done by the Centrifugal Pump (or by Impeller) on Water

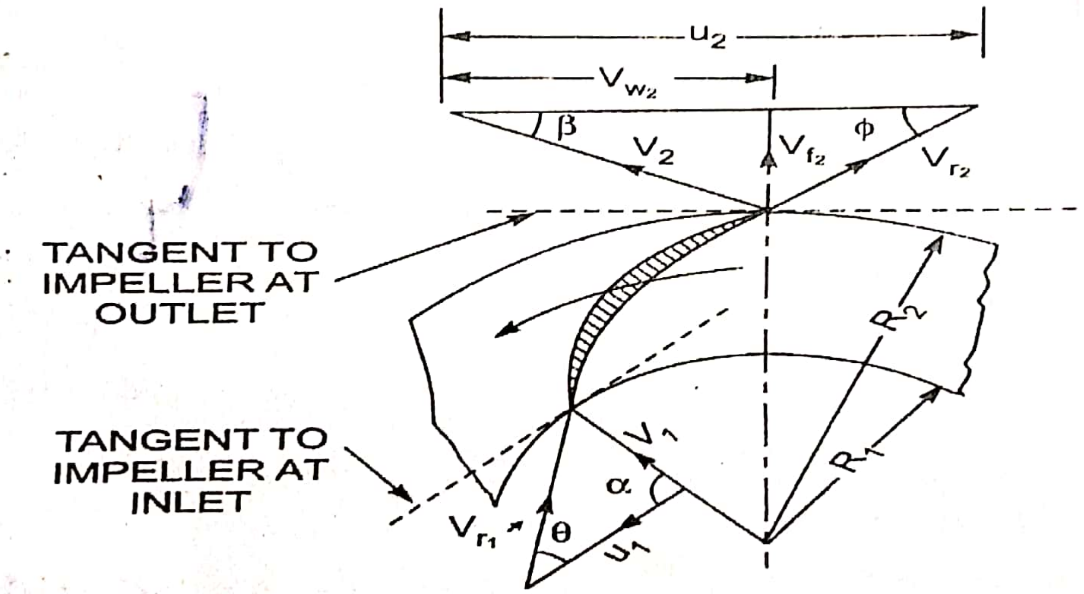
In case of the Centrifugal pump, work is done by the impeller on the water. The expression for workdone by the Impeller on the water is obtained by drawing velocity triangles at Inlet & Outlet of the Impeller.

The water enters the Impeller radially at Inlet for best efficiency of the pump, which means the absolute velocity of water at Inlet makes an angle of 90° with the direction of motion of the Impeller at Inlet. Hence angle $\alpha = 90°$ & $V_{w_1} = 0$.

Let N = Speed of the Impeller in r.p.m.,

D_1 = Diameter of Impeller at Inlet

u_1 = Tangential velocity of Impeller at Inlet $= \frac{\pi D_1 N}{60}$



D_2 = Diameter of impeller at Outlet

u_2 = Tangential velocity of Impeller at Outlet $= \frac{\pi D_2 N}{60}$

V_1 = Absolute velocity of water at Inlet

V_{r_1} = Relative velocity of water at Inlet

α, θ = Angle made by absolute Velocity (V_1) & relative velocity (V_{r_1}) at Inlet with the direction of motion of vane respectively.

V_2, V_{r_2}, β & ϕ are corresponding values at Outlet.

As the water enters the impeller radially which means the absolute velocity of water at Inlet is in the radial direction & hence angle $\alpha = 90°$ & $V_{w_1} = 0$

A ~~Centrifugal~~

Workdone by the Impeller on the water per sec per unit weight of water striking per second

$$= -[\text{Workdone in Case of turbine}]$$

$$= -\left[\frac{1}{g}(V_{w_1}u_1 - V_{w_2}u_2)\right] = \frac{1}{g}\left[V_{w_2}u_2 - V_{w_1}u_1\right]$$

$$= \frac{1}{g} V_{w_2} u_2 \quad (\because V_{w_1} = 0) \quad \text{—(1)}$$

Workdone by Impeller on water per Second $= \frac{W}{g} \cdot V_{w_2} \cdot u_2$

where W = Weight of water $= \rho \times g \times Q$

where Q = Volume of water

& Q = Area $\times$ Velocity of flow $= \pi D_1 B_1 \times V_{f_1}$

$= \pi D_2 B_2 \times V_{f_2}$

where B_1 & B_2 are width of Impeller at Inlet & Outlet & V_{f_1} & V_{f_2} are velocities of flow at Inlet & Outlet.

Eqn (1) gives the head Imparted to the water by the Impeller (or) energy given by impeller to water per unit weight per Second.

Definitions:

Types of Heads of a Centrifugal Pump:

1.) Suction Head (h_s): It is the vertical height of the Centre line of the centrifugal pump above the water surface in the tank (or) pump from which water is to be lifted as shown in Fig 1.

2.) Delivery Head (h_d): The vertical distance between the Centre line of the pump & the water surface in the tank to which water is delivered is known as delivery head.

3.) Static Head (H_s): The Sum of Suction head & delivery head is known as static head.

$$H_s = h_s + h_d.$$

4.) Manometric Head (H_m): It is the head against which a centrifugal pump has to work.

(a) H_m = Head Imparted by the Impeller to the water − Loss of head in the pump

$$= \frac{V_{w_2} \cdot u_2}{g} - \text{Loss of head in impeller \& Casing}$$

$$= \frac{V_{w_2} \cdot u_2}{g} \longrightarrow \text{If loss of pump is zero.}$$

(b) H_m = Total head at Outlet of the pump − Total head at the Inlet of the Pump.

$$= \left(\frac{P_o}{\rho g} + \frac{V_o^2}{2g} + Z_o\right) - \left(\frac{P_i}{\rho g} + \frac{V_i^2}{2g} + Z_i\right)$$

where $\frac{P_o}{\rho g}$ = Pr. head at Outlet of the pump $= h_d$

$\frac{V_o^2}{2g}$ = Velocity head at Outlet of the Pump $= \frac{V_d^2}{2g}$

Z_o = Vertical height of the Outlet of the pump from datum line &

$\frac{P_i}{\rho g}$, $\frac{V_i^2}{2g}$, Z_i = Corresponding values of pressure head, Velocity head & datum head at the inlet of pump.

i.e., h_s, $\frac{V_s^2}{2g}$, Z_s respectively.

(c) $H_m = h_s + h_d + h_{f_s} + h_{f_d} + \frac{V_d^2}{2g}$

where h_s = Suction head, h_d = Delivery head

h_{f_s} = Frictional head loss in Suction pipe

h_{f_d} = " " " " delivery "

V_d = Velocity of water in delivery pipe.

Efficiencies of a Centrifugal Pump:

In case of a centrifugal Pump, the power is transmitted from the shaft of the ~~electric~~ electric motor to the shaft of the pump & then to the impeller. From impeller, the power is given to the water. The power is decreasing from the Shaft of the pump to the impeller & then to the water.

The following are the important efficiencies of a centrifugal pump:

(a) Manometric Efficiency (η_{man}):

The ratio of the manometric head to the head imparted by the impeller to the water is known as manometric efficiency.

$$\eta_{man} = \frac{\text{Manometric head}}{\text{Head imparted by Impeller to water}}$$

$$= \frac{H_m}{\left(\frac{V_{w2} \cdot u_2}{g}\right)} = \frac{g H_m}{V_{w2} \cdot u_2}$$

The power at the Impeller of the pump is more than the power given to the water at Outlet of the pump. The ratio of the power given to water at Outlet of the pump to the power available at the impeller, is known as manometric efficiency.

The power given ~~by~~ to water at Outlet of the pump $= \frac{W H_m}{1000}$ kW

The power at the Impeller $= \frac{\text{Workdone by Impeller per Second}}{1000}$ kW

$$= \frac{W}{g} \times \frac{V_{w2} \times u_2}{1000} \text{ kW}$$

$$\therefore \eta_{man} = \frac{\frac{W \times H_m}{1000}}{\frac{W}{g} \times \frac{V_{w2} \times u_2}{1000}} = \frac{g \times H_m}{V_{w2} \times u_2}$$

Mechanical Efficiency (η_m)

The power at the Shaft of the Centrifugal pump is more than the power available at the Impeller of the pump. The ratio of the power available at the Impeller to the power of the Centrifugal pump is known as mechanical efficiency.

$$\eta_m = \frac{\text{Power at the Impeller}}{\text{Power at the Shaft}}$$

The power at the Impeller in kW = $\frac{\text{Workdone by Impeller/Sec}}{1000}$

$$= \frac{W}{g} \times \frac{V_{w_2} u_2}{1000}$$

$$\boxed{\eta_m = \frac{\frac{W}{g} \times \left(\frac{V_{w_2} u_2}{1000}\right)}{S.P}}$$ where S.P = Shaft Power.

(c) Overall efficiency (η_o): It is defined as ratio of power Output of the pump to the power input to the pump.

The power Output of the Pump in kW = $\frac{\text{Weight of water lifted} \times H_m}{1000}$

$$= \frac{W H_m}{1000}$$

Power Input to the pump = Power Supplied by the electric motor
= S.P of the Pump.

$$\boxed{\therefore \eta_o = \frac{\left(\frac{W H_m}{1000}\right)}{S.P}}$$

Also, $\boxed{\eta_o = \eta_{man} \times \eta_m}$

Minimum Speed for Starting a Centrifugal Pump:

If the pressure rise in the Impeller is more than equal to manometric head (H_m), the Centrifugal pump will delivering water. Otherwise, the pump will not discharge any water though the impeller is rotating. When impeller is rotating, the water in contact with the Impeller is also rotating. This is the case of forced vortex. In case of forced vortex, the Centrifugal head (or) head due to pressure rise in the Impeller

$$= \frac{\omega^2 r_2^2}{2g} - \frac{\omega^2 r_1^2}{2g} \quad \text{--- (1)}$$

where ωr_2 = Tangential velocity of Impeller at outlet $= u_2$ &
ωr_1 = " " " " Inlet $= u_1$

$\therefore$ Head due to pressure rise in Impeller $= \frac{u_2^2}{2g} - \frac{u_1^2}{2g}$

The flow of water will commence only if

Head due to pressure rise in impeller $\geq H_m$ (or) $\frac{u_2^2}{2g} - \frac{u_1^2}{2g} \geq H_m$

For minimum speed, we must have $\frac{u_2^2}{2g} - \frac{u_1^2}{2g} = H_m$ --- (2)

we have, $\eta_{man} = \frac{gH_m}{V_{w_2} u_2}$

$$\therefore H_m = \eta_{man} \times \frac{V_{w_2} \cdot u_2}{g}$$

Substituting this value of H_m in eqn (2)

we get, $\frac{u_2^2}{2g} - \frac{u_1^2}{2g} = \eta_{man} \times \frac{V_{w_2} \cdot u_2}{g}$ --- (3)

& we know that $u_2 = \frac{\pi D_2 N}{60}$ & $u_1 = \frac{\pi D_1 N}{60}$

Substituting the Values of u_2 & u_1 in eqn (3),

we get $$\frac{1}{2g}\left(\frac{\pi D_2 N}{60}\right)^2 - \frac{1}{2g}\left(\frac{\pi D_1 N}{60}\right)^2 = \eta_{man} \times \frac{V_{w_2} \times \pi D_2 N}{g \times 60}$$

$\Rightarrow$ Dividing the both Sides by $\frac{\pi N}{g \times 60}$,

we get $$\frac{\pi N D_2^2}{2 \times 60} - \frac{\pi N D_1^2}{2 \times 60} = \eta_{man} \times V_{w_2} \times D_2$$

$$\Rightarrow \frac{\pi N}{120}\left[D_2^2 - D_1^2\right] = \eta_{man} \times V_{w_2} \times D_2$$

$$\boxed{\therefore \quad N = \frac{120 \times \eta_{man} \times V_{w_2} \times D_2}{\pi \left[D_2^2 - D_1^2\right]}} \quad \text{—} \quad (4)$$

Eqn (4), gives the minimum starting speed of the Centrifugal pump.

Multi-Stage Centrifugal Pumps:

If a Centrifugal pump, consists of two (or) more impellers, the pump is called a multi-stage centrifugal pump. The Impellers may be mounted on the Same shaft (or) different shafts.

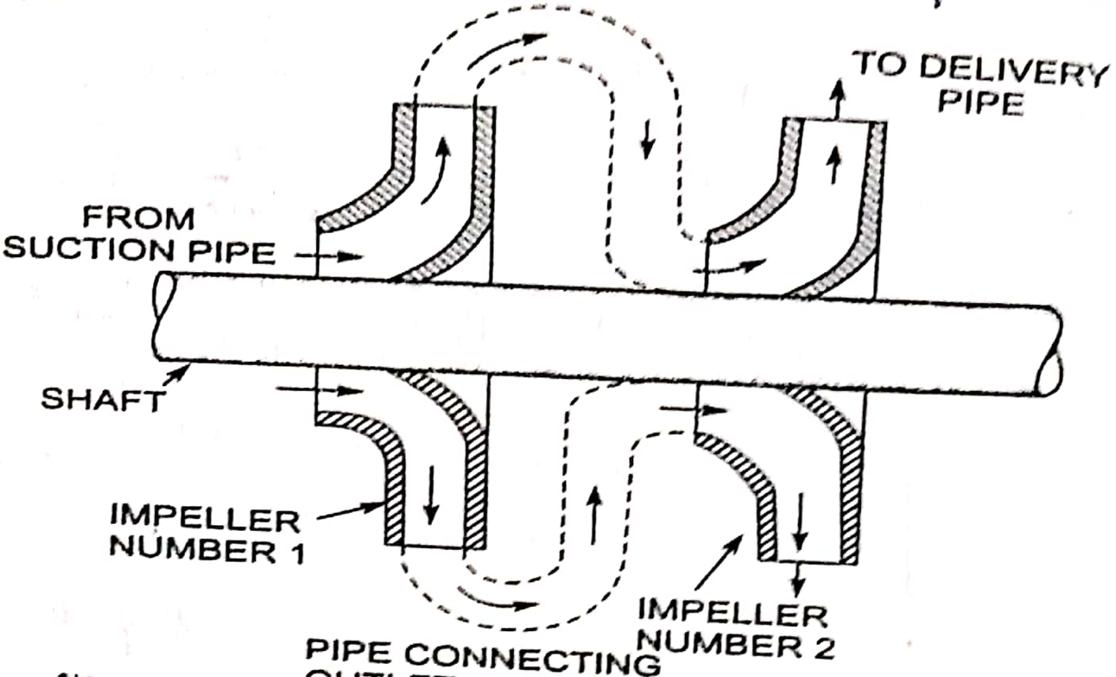
A multistage pump is having the following two important factors

1.) To produce a high head &
2.) To discharge a large quantity of liquid.

If a high head is to be developed, the Impellers are connected in series (or on the Same shaft) while for discharging large quantity of liquid, the Impellers (or pumps) are connected in parallel.

Multistage Centrifugal Pumps for High Heads:

For developing a high head, a number of Impellers are mounted in Series (or) on the Same shaft as shown in



The water from suction pipe enters the 1st Impeller at inlet & is discharged at increased

Fig. 19.12 *Two-stage pumps with impellers in series*

pressure. The water with increased pressure from the outlet of the 1st Impeller is taken to the Inlet of the 2nd Impeller with the help of a connecting pipe as shown in Fig (a). At the outlet of the 2nd impeller, the pressure of water will more than the pressure of water at the outlet of the 1st Impeller. Thus if more Impellers are mounted on the Same Shaft, the pressure at the outlet will be increased further.

Let n = Number of Identical Impellers mounted on the Same Shaft

H_m = Head developed by each Impeller.

Then total head developed $= n \times H_m$

The discharge passing through each Impeller is Same.

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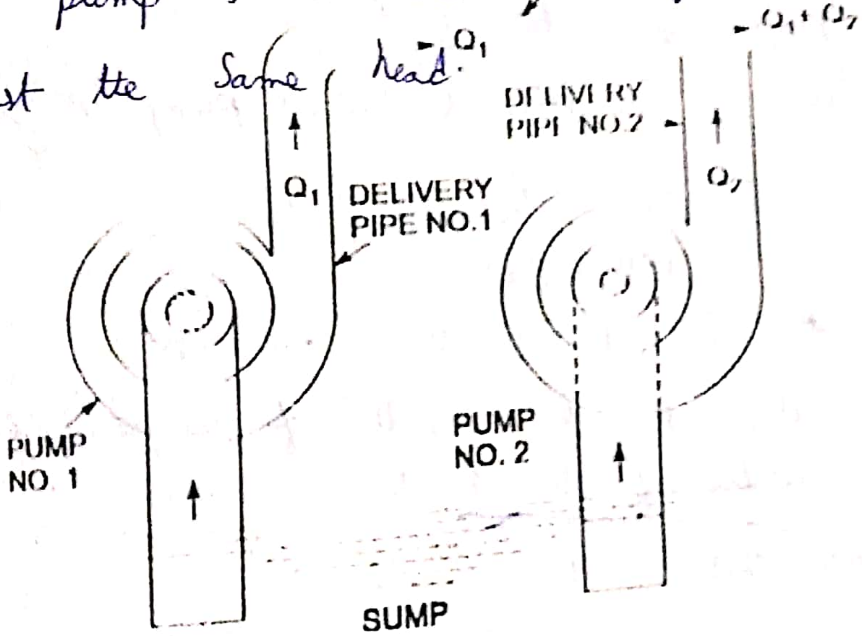
Multistage Centrifugal Pumps for High Discharge:

For obtaining high discharge, the pumps should be connected in parallel as shown in Fig(b). Each of the pumps lifts the water from a common pump & discharges water to a common pipe to which the delivery pipes of each pump is connected. Each pump is working against the same head.



Let n = Number of identical pumps arranged in parallel,

Q = Discharge from one pump.

$\therefore$ Total discharge $= n \times Q$

Specific Speed of a Centrifugal Pump (N_s):

The Specific Speed of a Centrifugal pump is defined as the Speed of a geometrically Similar pump which would deliver one cubic metre of liquid per Second against a head of one metre.

Expression for Specific Speed for a Pump:

The discharge Q, for a Centrifugal pump is given by the relation

$$Q = \text{Area} \times \text{Velocity of flow}$$

$$= \pi D \times B \times V_f$$

(or) $Q \propto D \times B \times V_f$ — ①

where D = Dia of the Impeller of the pump &

B = Width of the Impeller.

We know that $B \propto D$

$\therefore$ From eqn ①, we have, $Q \propto D^2 \times V_f$ — ②

We know, $u = \frac{\pi D N}{60} \propto DN$ — ③

Now the tangential velocity (u) & velocity of flow (V_f) are related to the manometric head (H_m) as

$u \propto V_f \propto \sqrt{H_m}$ — ④

Substituting the Value of u in eqn ③, we get

$\sqrt{H_m} \propto DN$ (or) $D \propto \frac{\sqrt{H_m}}{N}$

Substituting the Values of 'D' in eqn ②,

$$Q \propto \frac{H_m}{N^2} \times V_f$$

$$\propto \frac{H_m}{N^2} \times \sqrt{H_m} \quad [\because \text{from eqn ④}, V_f \propto \sqrt{H_m}]$$

$$\propto \frac{H_m^{3/2}}{N^2}$$

$$\therefore \boxed{Q = K\frac{H_m^{3/2}}{N^2}} \quad \text{--- } (5)$$

where K = Constant of proportionality

If $H_m = 1\,m$ & $Q = 1\,m^3/s$; N becomes $= N_s$

Substituting these values in eqn (5), we get

$$1 = K\frac{1^{3/2}}{N_s^2} = \frac{K}{N_s^2}$$

$$\therefore K = N_s^2$$

Substituting the value of K in eqn (5), we get

$$Q = N_s^2 \frac{H_m^{3/2}}{N^2}$$

$$\Rightarrow N_s^2 = \frac{N^2 Q}{H_m^{3/2}}$$

$$\therefore \boxed{N_s = \frac{N\sqrt{Q}}{H_m^{3/4}}}$$

Model Testing of Centrifugal Pumps:

Before ~~to~~ manufacturing the large sized pumps, their models which are in complete similarity with the actual pumps (also called prototypes) are made. Tests were conducted on the models and performance of the prototypes are predicted. The complete similarity b/w the ~~model~~ model & prototype (actual pump) will exist if the following conditions are satisfied.

1.) Specific Speed of Model = Specific Speed of prototype.

$$(N_s)_m = (N_s)_p$$

$$\Rightarrow \left(\frac{N\sqrt{Q}}{H_m^{3/4}}\right)_m = \left(\frac{N\sqrt{Q}}{H_m^{3/4}}\right)_p \quad \text{—— (1)}$$

2.) Tangential velocity (u) is given by $u = \frac{\pi D N}{60}$ also $u \propto \sqrt{H_m}$

$$\therefore \sqrt{H_m} \propto DN$$

$$\therefore \frac{\sqrt{H_m}}{DN} = \text{Constant}$$

$$\Rightarrow \left(\frac{\sqrt{H_m}}{DN}\right)_m = \left(\frac{\sqrt{H_m}}{DN}\right)_p \quad \text{—— (2)}$$

3.) We know that $Q \propto D^2 \times V_f$ where $V_f \propto u \propto DN$

$$\propto D^2 \times D \times N$$

$$\propto D^3 \times N$$

$$\therefore \frac{Q}{D^3 N} = \text{Constant} \Rightarrow \left(\frac{Q}{D^3 N}\right)_m = \left(\frac{Q}{D^3 N}\right)_p \quad \text{—— (3)}$$

4.) Power of the pump, $P = \frac{\rho \times g \times Q \times H_m}{75}$

$$\therefore P \propto Q \times H_m$$

$$\propto D^3 \times N \times H_m \quad (\because Q \propto D^3 N)$$

$$\propto D^3 N \times D^2 N^2 \quad (\because \sqrt{H_m} \propto DN)$$

$$\propto D^5 N^3$$

$$\therefore \frac{P}{D^5 N^3} = \text{Constant}$$

$$\Rightarrow \left(\frac{P}{D^5 N^3}\right)_m = \left(\frac{P}{D^5 N^3}\right)_p \quad \text{—— (4)}$$

vitation:

Cavitation is defined as the phenomenon of formation of vapour bubbles of a flowing liquid in a region where the pressure of the liquid falls below its vapour pressure & the sudden collapsing of these vapour bubbles in a region of high pressure. When the vapour bubbles collapse, a very high pressure is created. The metallic surfaces, above which these vapour bubbles collapse, is subjected to these high pressures, which cause pitting action on the surface. Thus cavities are formed on the metallic surface & also considerable noise & vibrations are produced.

Cavitation includes formation of vapour bubbles of the flowing liquid & collapsing of the vapour bubbles. Formation of vapour bubbles of the ~~following~~ flowing liquid take place only whenever the pressure in any region falls below vapour pressure. When the pressure of the flowing liquid is less than its vapour pressure, the liquid starts boiling & vapour bubbles are ~~vaps~~ formed. These vapour bubbles are carried along the flowing liquid to a higher pressure zones where these vapours condense & bubbles collapse. Due to sudden collapsing of the bubbles on the metallic surface, high pressure is produced & ~~metta~~ metallic surfaces are ~~ob~~ subjected to high local stresses. Thus the surfaces are damaged.

Precautions Against Cavitation:

(i) The pressure of the flowing ~~of~~ liquid in any part of the hydraulic system should not be allowed to fall below its vapour pressure. If the flowing liquid is water, then the absolute pressure head should not be below 2.5m of water.

(ii) The Special materials (or) coatings such as aluminium & Stainless Steel, which are cavitation resistant materials, be used.

Effects of Cavitation:

(i) The metallic surfaces are damaged & cavities are formed on the surface.

(ii) Due to Sudden collapse of Vapour bubble, considerable noise & vibrations are produced.

(iii) The efficiency of a turbine decreases due to cavitation. Due to pitting action, the surface of the turbine blades becomes rough & the force exerted by water on the turbine blades decreases. Hence, the workdone by water (or) output horse power becomes less & thus efficiency decreases.

Hydraulic Machines Subjected to Cavitation:

- Reaction turbines
- Centrifugal Pumps.

...vitation in Turbines :

In turbines, only reaction turbines are subjected to cavitation. In reaction turbine the cavitation may occur at the outlet of the runner (or) at the inlet of the draft tube where the pressure is considerably reduced (i.e., which may be below the vapour pressure of the liquid flowing through the turbine)

Due to cavitation, the metal of the runner vanes & draft-tube is gradually eaten away, which results in lowering the efficiency of the turbine. Hence, the cavitation in a reaction turbine can be noted by a sudden drop in efficiency. In order to determine whether ~~cavitation~~ cavitation will occur in any ~~point~~ portion of a reaction turbine, the critical value of Thoma's cavitation factor (σ, sigma) is calculated.

Thoma's Cavitation Factor for Reaction Turbines :

Prof. D. Thoma suggested a dimensionless number, called after his name Thoma's cavitation factor σ (sigma), which can be used for determining the region where cavitation takes place in reaction turbines. The mathematical expression for the Thoma's cavitation factor is given by

$$\sigma = \frac{H_b - H_s}{H} = \frac{(H_{atm} - H_v) - H_s}{H} \quad \text{--- (1)}$$

where H_b = Barometric pressure head in 'm' of water,

H_{atm} = Atmospheric " " " " "

H_v = Vapour " " " " "

H_s = Suction pressure at the outlet of reaction turbine in 'm' of water (or) height of turbine runner above the tail water surface.

H = Net head on the turbine in 'm'.

Cavitation in Centrifugal Pumps:

In Centrifugal Pumps, the cavitation may occur at inlet of the impeller of the pump, (or) at the suction side of the pumps, where the pressure is considerably reduced. Hence, if the pressure at the suction side of the pump drops below the vapour pressure of the liquid then the cavitation occur. The cavitation in a pump can be noted by a sudden drop in efficiency & head. In order to determine whether cavitation will occur in any portion of the suction side of the pump, the critical value of Thoma's cavitation factor (σ) is calculated.

Thoma's Cavitation Factor for Centrifugal pumps:

The mathematical expression for Thoma's cavitation factor for Centrifugal pump is given by

$$\sigma = \frac{(H_b) - H_s - h_{LS}}{H} = \frac{(H_{atm} - H_v) - H_s - h_{LS}}{H} \quad \text{II}$$

where H_{atm} = Atmospheric pressure head in 'm' of water (or) absolute pressure head at the liquid surface in pump.

H_v = Vapour pressure head in 'm' of water,

H_s = Suction " " " "

h_{LS} = Head lost due to friction in suction pipe &

H = Head developed by the pump.

The value of Thoma's cavitation factor (σ) for a particular type of turbine (or) pump is calculated from eqn I & II. This value of Thoma's cavitation factor (σ) is compared with critical cavitation factor (σ_c) for that type of turbine pump. If the value of σ is greater than σ_c, the cavitation will not occur in that turbine (or) Pump.

The following empirical relationships are used for obtaining the values of σ_c for different turbines:

For Francis turbines, $\sigma_c = 0.625\left(\frac{N_s}{380.78}\right)^2$

$= 431 \times 10^{-8} N_s^2$

For propeller turbines, $\sigma_c = 0.28\left\{ + \left[\frac{1}{7.5}\left(\frac{N_s}{380.78}\right)^3\right]\right.$

In the above expressions N_s is in (R.P.M, kW, m) units. The empirical relationships if N_s is in (r.p.m, h.p, m) units, would be as follows:

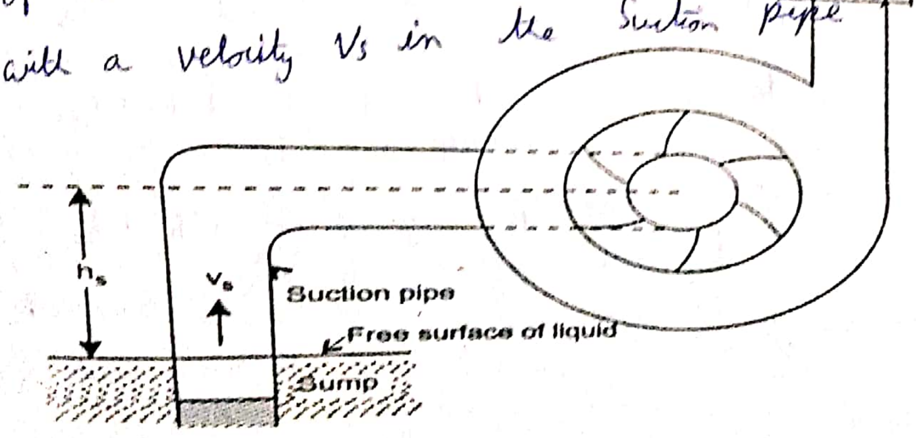
For Francis turbines, $\sigma_c = 0.625\left(\frac{N_s}{444}\right)^2$

$= 317 \times 10^{-8} \times N_s^2$

For propeller turbines, $\sigma_c = 0.28 + \left[\frac{1}{7.5}\left[\frac{N_s}{444}\right)^3\right]$

Maximum Suction Lift (or Suction Height):

Fig Shows a centrifugal pump that lifts a liquid from a sump, The free surface of liquid is at a depth of h_s below the pump axis. The liquid is flowing with a velocity V_s in the suction pipe



let h_s = Suction height (or lift)

Applying Bernoulli's Eqn at the free surface of liquid in the Sump & Section ① in the Suction pipe just at the inlet of the pump & taking the free Surface of liquid as datum line, we get

$$\frac{P_a}{\rho g} + \frac{V_a^2}{2g} + Z_a = \frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 + h_L \quad -①$$

where P_a = Atm. Pressure on the free Surface of liquid.

V_a = Velocity of liquid at the free Surface of liquid = 0

Z_a = Height of free Surface from datum line = 0

P_1 = Absolute pr. at the Inlet of Pump.

V_1 = Velocity of liquid through Suction Pipe = V_s

Z_1 = Height of inlet of pump from datum line = h_s

h_L = Loss of head in the foot Valve, Strainer & suction pipe = h_{fs}.

Hence eqn ①, after Substituting the above Values becomes as

$$\frac{P_a}{\rho g} + 0 + 0 = \frac{P_1}{\rho g} + \frac{V_s^2}{2g} + h_s + h_{fs}$$

$$\Rightarrow \frac{P_a}{\rho g} = \frac{P_1}{\rho g} + \frac{V_s^2}{2g} + h_s + h_{fs}$$

$$\boxed{\therefore \frac{P_1}{\rho g} = \frac{P_a}{\rho g} - \left(\frac{V_s^2}{2g} + h_s + h_{fs}\right)} \quad -②$$

For finding the max. suction lift, the pressure at the inlet of the pump should not be less than the vapour pressure of the liquid. Hence for the limiting case, taking the pressure at the inlet of pump equal to vapour pressure of the liquid, we get,

$P_1 = P_v$, where P_v = Vapour Pressure of the liquid.

Now the eqn (2), becomes

$$\frac{P_v}{\rho g} = \frac{P_a}{\rho g} - \left(\frac{V_s^2}{2g} + h_s + h_{f_s}\right)$$

$$\Rightarrow \frac{P_a}{\rho g} = \frac{P_v}{\rho g} + \frac{V_s^2}{2g} + h_s + h_{f_s} \quad (\because P_1 = P_v) \quad \text{— (3)}$$

Now $\frac{P_a}{\rho g}$ = Atm. pr. Head = H_a (meter of liquid)

$\frac{P_v}{\rho g}$ = Vapour pr. Head = H_v (meter of liquid)

Now eqn (3) becomes as

$$H_a = H_v + \frac{V_s^2}{2g} + h_s + h_{f_s}$$

$$\Rightarrow \boxed{h_s = H_a - H_v - \frac{V_s^2}{2g} - h_{f_s}} \quad \text{— (4)}$$

Above eqn gives the value of max. suction lift (or max. suction height) for a centrifugal pump. Hence, the suction height of any pump should not be more than that given by eqn (4). If the suction height of the pump is more, then vaporization of liquid at inlet of pump will take place & there will be a possibility of cavitation.

Net Positive Suction Head (NPSH)

It is defined as the absolute pressure head at the inlet to the pump, minus the vapour pressure head plus the velocity head.

$\therefore$ NPSH = Absolute pr. head at Inlet of the pump − Vapour pressure head (Absolute Units) + Velocity head

$$= \frac{P_1}{\rho g} - \frac{P_v}{\rho g} + \frac{V_s^2}{2g} \quad \text{——} (1)$$

($\because$ Absolute pressure at Inlet of pump $= P_1$)

But we know that, the Absolute pressure head at Inlet of the pump is given by

$$\frac{P_1}{\rho g} = \frac{P_a}{\rho g} - \left(\frac{V_s^2}{2g} + h_s + h_{f_s}\right)$$

Substituting this Value in Eqn (1), we get

$$NPSH = \left[\frac{P_a}{\rho g} - \left(\frac{V_s^2}{2g} + h_s + h_{f_s}\right)\right] - \frac{P_v}{\rho g} + \frac{V_s^2}{2g}$$

$$= \frac{P_a}{\rho g} - \frac{P_v}{\rho g} - h_s - h_{f_s}$$

$$= H_a - H_v - h_s - h_{f_s}$$

($\because \frac{P_a}{\rho g} = H_a$ = Atm. pr. head, $\frac{P_v}{\rho g} = H_v$ = Vapour Pr. Head)

$$\boxed{\therefore NPSH = \left[(H_a - h_s - h_{f_s}) - H_v\right]} \quad \text{——} (2)$$

The R.H.S of above eqn is the total suction head & hence NPSH is equal to total suction head.

So, NPSH may also be defined as the total head required to make the liquid flow through the suction pipe to the pump impeller.

...ng of a Centrifugal Pump:

Priming of a Centrifugal pump is defined as the ...ation in which the Suction pipe, casing of the pump & a ...ion of the delivery pipe upto the delivery valve is completely ...led up from outside source with the liquid to be raised ...y the pump before starting the pump. Thus the air from these parts of the pump is removed & these parts are filled with the liquid to be pumped.

Characteristics Curves of Centrifugal pumps:

Characteristic Curves of Centrifugal pumps are defined as those Curves which are plotted from the results of a no. of tests on the centrifugal pump. These curves are necessary to predict the behaviour & performance of the pump when the pump is working under different flow rate, head & Speed.

The following are the Important Characteristic Curves for pumps:

1.) Main Characteristic Curves
2.) Operating " " &
3.) Constant Efficiency (or) Muschel Curves.

Main Characteristic Curves:

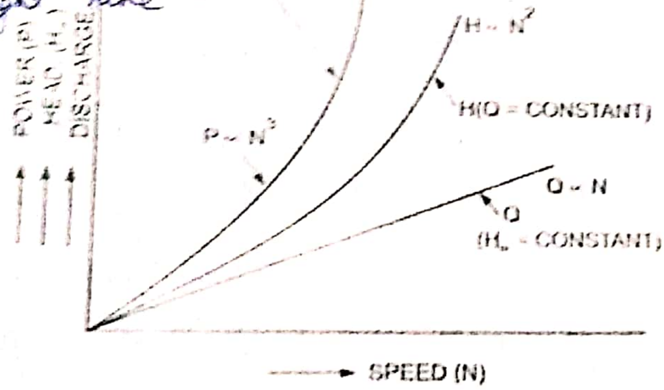
- ~~The~~ It consists of variation of head (Manometric Head, H_m) power & discharge w.r.t Speed.
- For plotting Curves of Manometric head Vs Speed, discharge is kept Constant.
- For plotting Curves of discharge Vs Speed, Manometric head (H_m) is kept constant.

→ For plotting curves of power Vs speed, the manometric head & discharge are kept constant. (Fig a) shows main characteristic curves of a pump.

→ From Eqn $\sqrt{H_m}/DN$ or $H_m \propto N^2 \Rightarrow$ Curve H_m Vs N is a parabolic curve

→ From Eqn P/D^5N^3 or $P \propto N^3 \Rightarrow$ Curve P Vs N is a cubic curve.

→ From Eqn $\frac{Q}{D^3N}$ or $Q \propto N \Rightarrow$ Curve Q Vs N is a straight line.



Operating Characteristic Curves:

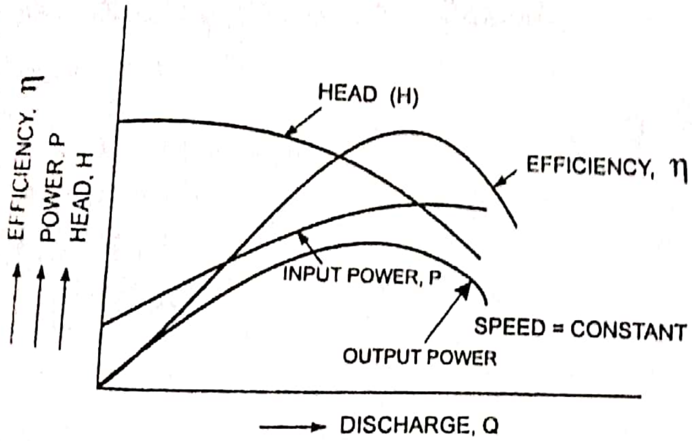
– If the speed is kept constant, the variation of manometric head, power & efficiency w.r.t discharge gives the operating characteristics of the pump. Fig (b) shows the operating characteristic curves of a pump.

– The input power curve for pumps shall not pass through the origin. It will be slightly away from the origin on the y-axis, as even at zero discharge some power is needed to overcome mechanical losses.

The head curve will have maximum value of head when discharge is zero.

→ The output power curve will start from origin as at $Q = 0$, output power $(\rho Q g H)$ will be zero.

→ The efficiency curve will start from origin as at $Q = 0$, $\eta = 0$ $\left(\because \eta = \frac{\text{Output}}{\text{Input}}\right)$



Constant Efficiency Curves:

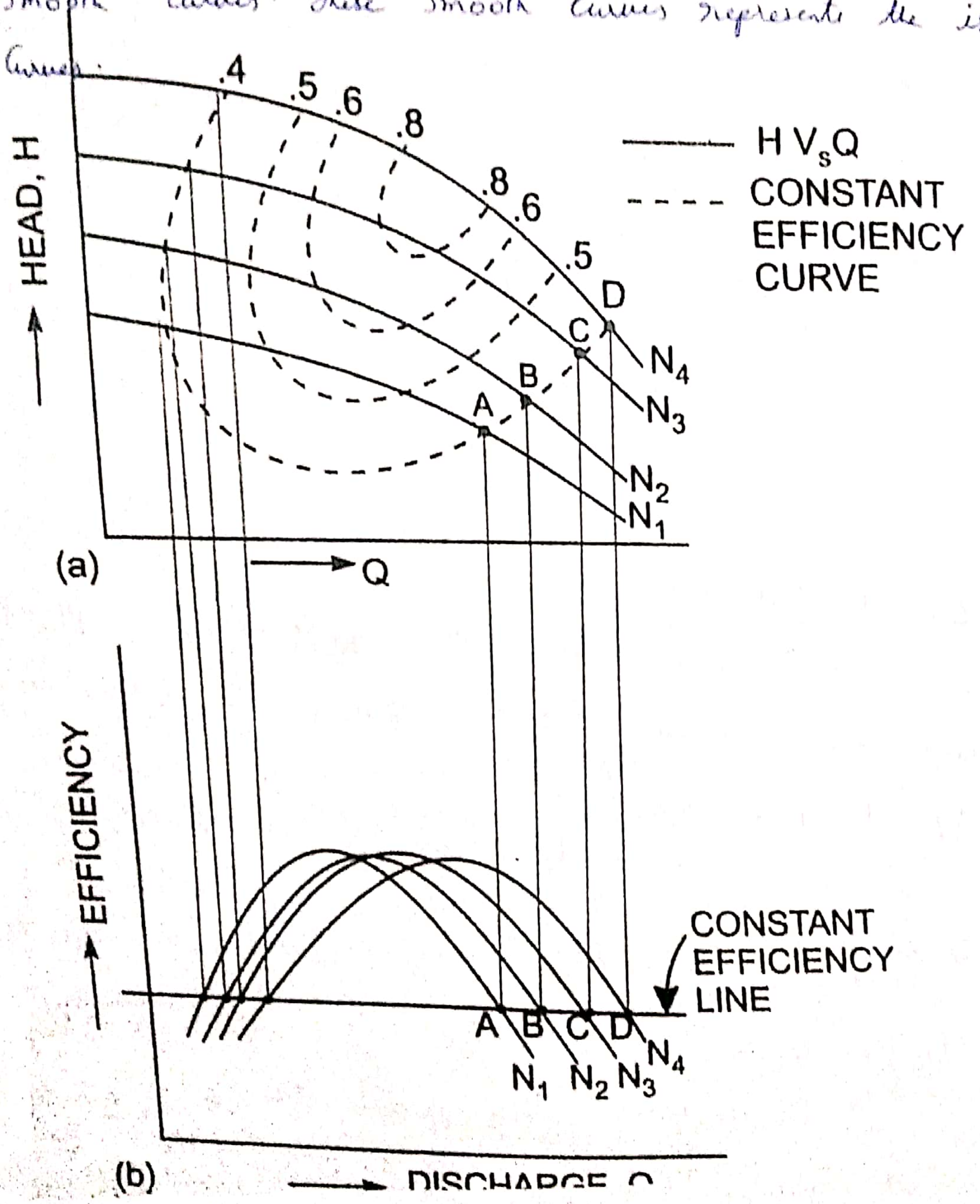
→ For obtaining constant efficiency curves for a pump, the head versus discharge curves & efficiency Vs discharge curves for different speed are used.

→ Fig (c) shows the head Vs discharge curves for different speeds.

→ Fig (d) shows the efficiency Vs discharge curves for the different speeds.

→ By combining these curves ($H \sim Q$ & $\eta \sim Q$ curves), Constant efficiency curves are obtained as shown in fig (c)

⇒ For plotting the constant efficiency curves (also kn... as iso-efficiency curves), horizontal lines representing con... efficiencies are drawn on the $\eta \sim Q$ curves. The points at which these lines cut the efficiency curves at various speeds, are transferred to the corresponding $H \sim Q$ curves. The points having the same efficiency are then jointed by smooth curves. These smooth curves represents the iso-efficiency curves.



Cavitation in Centrifugal Pumps:

Thoma's Cavitation factor is used to indicate whether Cavitation will occur in pumps.

Below eqn gives the value of Thoma's cavitation factor for pumps as

$$\sigma = \frac{(H_{atm} - H_v) - H_s - h_{f_s}}{H}$$

$$= \frac{(H_a - H_v - h_s - h_{f_s})}{H_m} \quad (\because H_s = h_s \ \& \ h_{f_s} = h_{f_s}) \ \& \ (H = H_m \text{ for Pumps})$$

But ~~from~~ we have, $H_a - H_v - h_s - h_{f_s} = NPSH$

$$\therefore \sigma = \frac{NPSH}{H_m} \quad \text{— (I)}$$

If the value of σ, calculated from above eqn I, is less than the Critical value, σ_c then cavitation will occur in the pumps.

The value of σ_c depends upon the Specific Speed of the pump $\left(N_s = \frac{N\sqrt{Q}}{H_m^{3/4}}\right)$.

The following empirical relation is used to determine the value of σ_c.

$$\sigma_c = 0.103\left(\frac{N_s}{1000}\right)^{4/3}$$

$$= 0.103\left(\frac{N_s^{4/3}}{(10^3)^{4/3}}\right) = \frac{0.103\,N_s^{4/3}}{10^4}$$

$$\boxed{\therefore \sigma_c = 1.03 \times 10^{-3}\,N_s^{4/3}}$$